Homework #8 Solutions
Due: Friday October 30, 1998

1 (Tipler 26-47). (*a*) The flux through the loop is $\Phi = B \cdot (\text{effective area}) = B \cdot Nab\cos\theta$, where $\theta = \omega t$ is the angle between the loop normal and the magnetic field B. From Faraday's law,

$$\mathcal{E} = -\frac{d\Phi}{dt} = -\frac{d}{dt}(NBab\,\cos\omega t) = NBab\omega\,\sin\omega t. \tag{1}$$

Note that the problem does not give you an explicit for for θ, which could just as easily have been defined as $\omega t - \delta$, with δ as any angle you wish to choose. Hence the "sin" in the derived equation could just as easily been "$\pm\sin$" or "$\pm\cos$".
(*b*) The amplitude of the emf out of this generator is $\mathcal{E}_0 = NBab\omega$. To have $\mathcal{E}_0 = 110$ V, with the given values for the other parameters, we must have an angular frequency $\omega = \mathcal{E}_0/NBab = (110)/[(1000)(2)(0.01)(0.02)] = 275$ rad/s.

2 (Tipler 26-58). There are a large number of ways to solve this problem. One (probably) very bad way is to look at the total magnetic flux $\Phi_1 + \Phi_2$. Unfortunately, since the book mentions flux explicitly, you may have been given the impression that this quantity is in some way a key to the solution you seek. The statement "…none of the flux from either passes through the other..." is meant to signify that the flux through inductor one (for example) is *not* given by

$$\Phi_1 = L_1 I_1 + M I_2, \tag{2}$$

which would mean that there is some *mutual* inductance between the two components. So, because we may write $\Phi_1 = L_1 I_1$ and $\Phi_2 = L_2 I_2$, we can also write that the voltage drop across inductor one is $\Delta V_1 = L_1(dI_1/dt)$, and similarly for the second inductor. As it so happens, when the inductors are in parallel, their voltage drops are the same (Kirchoff's first law):

$$\Delta V = L_1\frac{dI_1}{dt} = L_2\frac{dI_2}{dt}. \tag{3}$$

We would define an effective inductance L_{eff} using the relation

$$L_{\text{eff}} = \Delta V\left(\frac{dI}{dt}\right)^{-1}, \qquad \text{or} \qquad \frac{1}{L_{\text{eff}}} = \frac{1}{\Delta V}\frac{dI}{dt}, \tag{4}$$

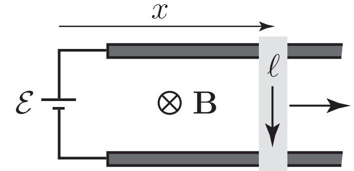
where $I = I_1 + I_2$ is the total current that goes into our two-inductor system. From Kirchoff's second law, $I = I_1 + I_2$, and so,

$$\frac{dI}{dt} = \frac{dI_1}{dt} + \frac{dI_2}{dt}. \tag{5}$$

We can use eq. (3) to substitute for the dI_k/dt terms:

$$\frac{1}{L_{\text{eff}}} = \frac{1}{\Delta V}\left(\frac{dI_1}{dt} + \frac{dI_2}{dt}\right) = \frac{1}{\Delta V}\left(\frac{\Delta V}{L_1} + \frac{\Delta V}{L_2}\right) = \frac{1}{L_1} + \frac{1}{L_2}. \tag{6}$$

3 (Tipler 26-67).



(*a*) The crossbar has a current that goes down the page, when the battery is attached as described in the problem. A current I flows through the bar, and therefore it will feel a force $F = I\ell B$ that is to the right, for a magnetic field B into the page. The problem now is, of course, to find the current that flows. Normally (no magnetic field), the current would simply be $\mathcal{E}/R$. The problem is that since the circuit is physically expanding, it's magnetic flux is changing and therefore, by Faraday's law, an emf is induced. The size of this "back"-emf, called "back" because it will always oppose the emf applied, is given by

$$|\mathcal{E}_{\text{back}}| = \frac{d\Phi}{dt} = \frac{d}{dt}(B\ell x) = B\ell\frac{dx}{dt} = B\ell v, \tag{7}$$

where x is the displacement of the crossbar bar (the placement of the origin turns out to not be very important), and $v = dx/dt$ is the velocity. The total emf is therefore $\mathcal{E} - |\mathcal{E}_{\text{back}}| = \mathcal{E} - B\ell v$, and the current that flows is $I = (\mathcal{E} - B\ell v)/R$. The force on the rod is therefore

$$F = I\ell B = \frac{(\mathcal{E} - B\ell v)\ell B}{R}, \tag{8}$$

and Newton's second law, $F = ma = m(dv/dt)$, is written

$$m\frac{dv}{dt} = \frac{(\mathcal{E} - B\ell v)\ell B}{R}. \tag{9}$$

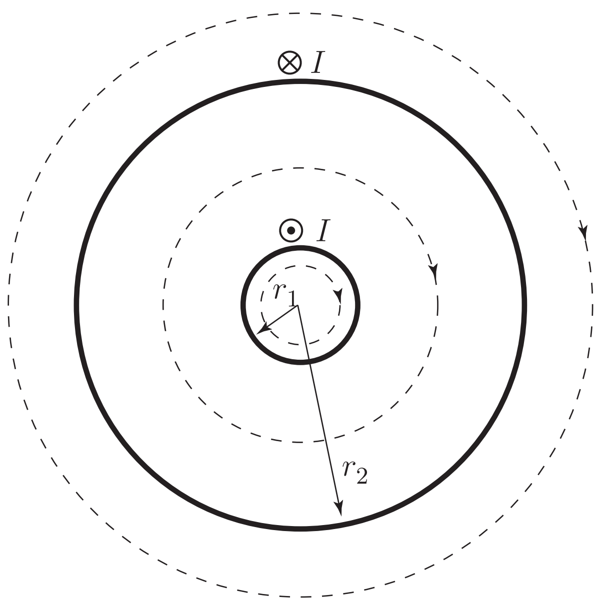
(b) The terminal velocity v_t is defined as the point at which $(dv/dt)_{v=v_t} = 0$. From above,

$$0 = \frac{(\mathcal{E} - B\ell v_t)\ell B}{R} \quad \Rightarrow \quad v_t = \frac{\mathcal{E}}{B\ell}. \tag{10}$$

(c) The current when $v = v_t$ is given by $I = (\mathcal{E} - B\ell v_t)/R = 0$. This can be done without resort to algebra if you remember that $F = I\ell B$ and that the force $F = 0$ because it reached terminal velocity, so we must have $I = 0$.

Now is a good time to look back at a similar problem you had in chapter 24 and ask yourself how this problem is different.

4 (Tipler 26-80).



(a) The setup of this problem is very similar to that of Example 25-10 in Tipler. You should know that the symmetry of the problem is such that the magnitude of the **B** field is constant along circles that are concentric with the cable and have surface normals along the axis of the cable. Three examples of such curves are drawn (dotted) in the accompanying figure. In addition, the direction of the **B** field at all points along that circle is tangent to it, and has a sense given by the right hand rule. We therefore have that $\mathbf{B} \cdot d\boldsymbol{\ell} = B\, d\ell$ at all points along the curve. It is also for that reason that we choose our curves to travel clockwise (as drawn), since the current inside is into the page. With all of these preliminaries out of the way, we can make the seemingly simple statement that one side of Ampère's law is given by

$$\oint_C \mathbf{B} \cdot d\boldsymbol{\ell} = B\, 2\pi r. \tag{11}$$

Now the right hand side of Ampère's law is multivalued:

$$\mu_0 I_C = \mu_0 \begin{cases} 0 & r < r_1 \\ I & r_1 < r < r_2 \\ I + (-I) = 0 & r > r_2 \end{cases} . \tag{12}$$

Note that in the last line above, I_C has two parts because for $r > r_2$, C encloses both currents. The inner current goes into the page, which is the same direction as the surface normal of C (which is given by the right hand rule), so it counts as positive. The outer current is coming out of the page, is anti-parallel to the surface normal of C, and therefore counts as negative. Putting all of Ampère's law together, we get

$$B = \begin{cases} 0 & r < r_1 \\ \dfrac{\mu_0 I}{2\pi r} & r_1 < r < r_2 \\ 0 & r > r_2 \end{cases} . \tag{13}$$

(b) The magnetic energy density u_m (written η_m in Tipler) is given by

$$u_m = \frac{B^2}{2\mu_0} = \frac{1}{2\mu_0}\left(\frac{\mu_0 I}{2\pi r}\right)^2 = \frac{\mu_0 I^2}{8\pi^2 r^2}, \tag{14}$$

for the region between the conductors ($r_1 < r < r_2$).
(c) The total energy in the magnetic field between the conductors is

$$U_m = \int_V u_m dV = \int_{r_1}^{r_2} u_m(r)\,(\ell 2\pi r\, dr) = \frac{\mu_0 I^2 \ell}{4\pi}\int_{r_1}^{r_2} \frac{dr}{r} = \frac{\mu_0 I^2 \ell}{4\pi} \ln\frac{r_2}{r_1}. \tag{15}$$

(d) We can use the fact that for an inductor, $U_m = LI^2/2$ to obtain the inductance of this coaxial cable:

$$U_m = \frac{1}{2}LI^2 = \frac{\mu_0 I^2 \ell}{4\pi}\ln\frac{r_2}{r_1} \quad \Rightarrow \quad \frac{L}{\ell} = \frac{\mu_0}{2\pi}\ln\frac{r_2}{r_1}. \tag{16}$$

This example isn't really that artificial. Inductance, as it is formally defined, can be difficult to evaluate, and so seemingly roundabout techniques such as this are often employed to find the desired expressions.

5 (Tipler 27-20). (a) The field inside a ferromagnetic material such as iron is K_m times that of the applied field B_{app}. The solenoid is the source for the applied field here, and is $\mu_0 nI$. The field in the material is therefore $B = K_m\mu_0 nI$, and the flux Φ = (field)(effective area) = $(K_m\mu_0 nI)(n\ell A)$. Since $\Phi = LI$, we have $L = K_m\mu_0 n^2 A\ell$.
(b) The energy in the solenoid is

$$U_m = \frac{1}{2}LI^2 = \frac{1}{2}(K_m\mu_0 n^2 A\ell)\left(\frac{B}{K_m\mu_0 n}\right)^2 = \frac{B^2 A\ell}{2K_m\mu_0}, \tag{17}$$

where we have written the current in terms of the magnetic field it induces.
(c) The energy density $u_m \equiv U_m/(\text{volume}) = U_m/(A\ell) = B^2/2K_m\mu_0 = B^2/2\mu$, where $\mu \equiv K_m\mu_0$.