Phys 531 Midterm Exam Due October 20

This is a take-home exam, due at 9:00 AM Monday October 20. You may spend as much time on it as you like over the weekend; I expect it should take 3–5 hours. It covers material from Saleh and Teich Chapters 1, 2, and 4 (through section 4.3). There are six problems, worth 10 points each.

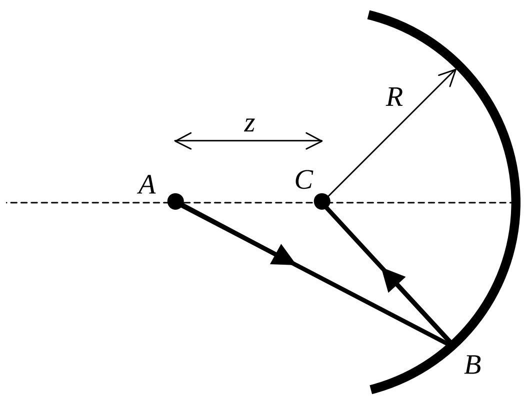
Instructions:
- You may use the textbook, your class notes, your homework assignments, and the homework solutions, but no other reference materials.
- You may not discuss the problems with other students.
- You may use a calculator, but not a computer.
- You must show all work for full credit.
- You may use any approximations justified by the conditions of a problem.

Use your own paper, but turn in all pages stapled together with this cover sheet.

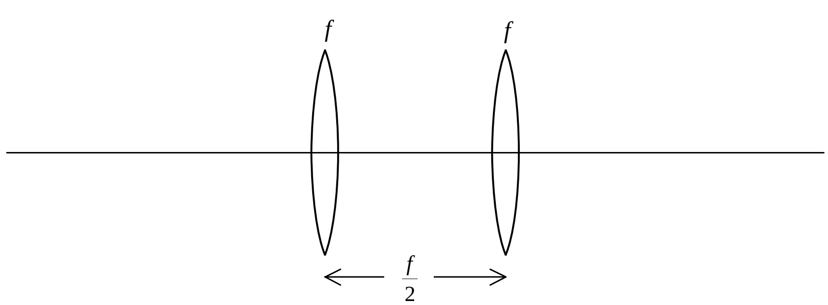
Name: ____

Signature: ____

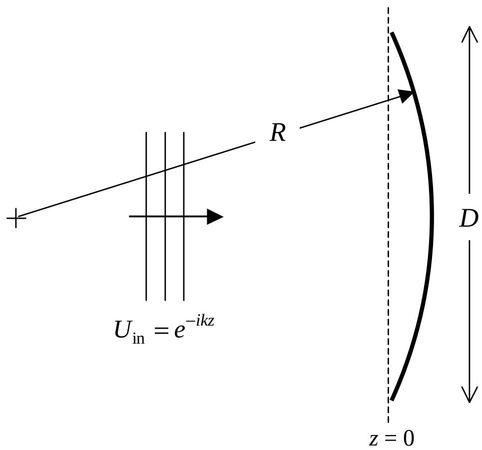
1. Consider light rays propagating from A to C near a concave spherical mirror of radius R. Point C is located at the center of curvature of the mirror. Point A is a distance z away, with positive z measured to the right. (The figure shows an example with $z < 0$.) Light can travel from A to C by reflecting off the mirror at some point B as shown.
(a) Use Fermat's principle to locate point B.
(b) If $z < 0$, is the optical path length $\overline{ABC}$ a minimum or a maximum with respect to nearby paths? (Specifically, paths $AB'C$ for nearby points B' on the mirror surface.)
(c) Answer the same question for $z > 0$.



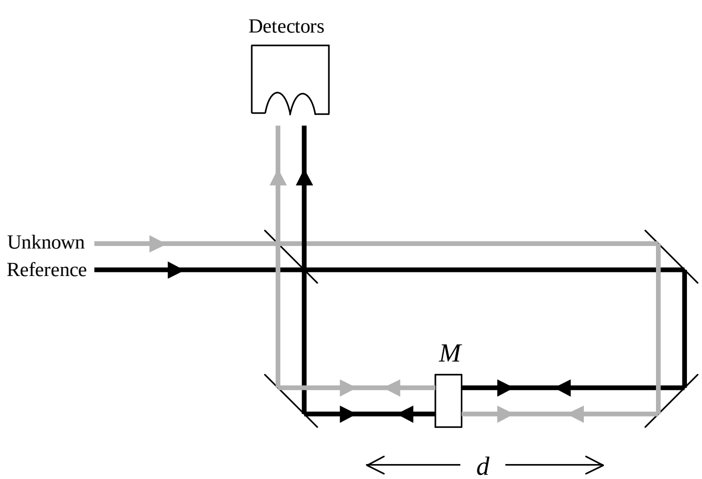
2. Consider an optical system consisting of two thin lenses with focal length f separated by a distance $f/2$, as shown.
(a) Find the ray matrix for this system.
(b) Find the system focal length and the locations of the two principle planes.
(c) If a point source is located on the optical axis a distance $3f/2$ in front of the system, find the location of the image and draw a sketch showing the incident and outgoing rays. Your sketch should illustrate the role played by the principle planes.



3. A plane wave with wavelength λ is normally incident on a concave spherical mirror of radius R, with $|R|$ large compared to both the diameter of the mirror D and D^2/λ. (Recall $R < 0$ for a convex mirror.) Locate the $z = 0$ plane at the edges of the mirror as shown.
(a) Find the complex transmitance function $t(x, y)$ of the mirror, measured from where the incoming wave crosses $z = 0$ to where the outgoing wave crosses $z = 0$.
(b) Show that the reflected light is a converging paraboloidal wave and find its center.



4. The interferometer shown below can be used to measure the wavelength of a laser beam. One of the two beams is from a reference laser with known wavelength λ_{ref}, and the other has wavelength λ to be determined. The double-sided mirror M can move, which produces an interference signal monitored by the detectors. Suppose that when the the mirror travels a distance d, the reference detector counts N_r interference maxima and the other detector counts N maxima.
(a) Use these values to relate λ to λ_{ref}.
(b) The uncertainty in N and N_r is $\pm 1/2$, since the counter can only register integer values. Estimate the uncertainty this causes in λ if $d = 0.5$ m, $\lambda_{ref} = 632.8$ nm, and $\lambda \approx 780$ nm.



5. Suppose a glass plate has thickness

$$d = d_0 + a \sin\left(\frac{2\pi x}{\Lambda}\right)$$

with $d_0 = 1$ mm, $a = 5$ nm, and $\Lambda = 1$ μm. The index of refraction of the glass is $n = 1.5$. The plate is illuminated by a plane wave at normal incidence, with amplitude A and a wavelength $\lambda = 500$ nm. Find the transmitted wave function $U(x, y, z)$ an arbitrary distance z from the plate.

6. A plane wave with intensity I_{in} and wavelength λ is normally incident on an opaque screen having a hole of area A. Calculate the intensity of light at the center of the Fraunhofer diffraction pattern (i.e., at $x = 0, y = 0$).