Phys 531 Final Exam December 10

Instructions:

This is an in-class exam that must be completed during the allotted 3 hour period. You may use your textbook and any handwritten notes for reference. You may also use a calculator and scratch paper as desired.

The exam has two sections. The first section consists of six multiple choice questions. For each question, clearly mark the appropriate answer(s) on the exam sheet. The questions are intended to be conceptual, so if you find yourself doing a detailed calculation, you are probably on the wrong track. No partial credit will be given for these questions, so you don't need to show your work. Each question is worth 5 points.

The second section consists of four problems. Work these problems on the scratch paper provided, which should be stapled to the exam before turning it in. For these problems, partial credit will be given and you must show all work for full credit. Each problem is worth 10 points.

Throughout the exam, you may assume light to be propagating in free space unless stated otherwise.

Name: ______________________________

Signature: ______________________________

Questions

1. A symmetric optical system in one which is unchanged if the order of its elements is reversed (where “elements” include both actual lens surfaces and distances of free propagation). Which of the following ray matrices represents a symmetric system?

(a) $\begin{bmatrix} 2 & 1\text{ m} \\ 3\text{ m}^{-1} & 2 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & 0\text{ m} \\ 4\text{ m}^{-1} & 2 \end{bmatrix}$

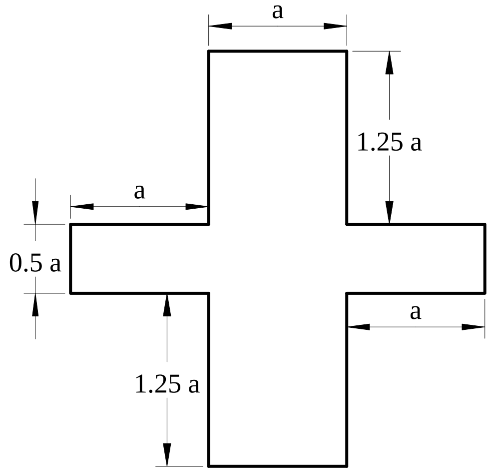
(c) $\begin{bmatrix} 0 & 1\text{ m} \\ -1\text{ m}^{-1} & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & 1\text{ m} \\ 1\text{ m}^{-1} & 1 \end{bmatrix}$

2. Which of the following wave functions represent monochromatic plane waves? Assume $c = \omega/k$ as appropriate, and $k = 2\pi/\lambda$. Mark all answers that apply.
(a) $u(\mathbf{r}, t) = f(z - ct)$
(b) $u(\mathbf{r}, t) = A\sin(\omega t - kz)$
(c) $U(\mathbf{r}) = i$
(d) $U(\mathbf{r}) = A\cos(\mathbf{k}\cdot\mathbf{r})$
(e) $U(\mathbf{r}) = A\exp\left[\frac{i\pi(x+y)}{\lambda}\right]\exp\left[-\frac{i\sqrt{2}\pi z}{\lambda}\right]$

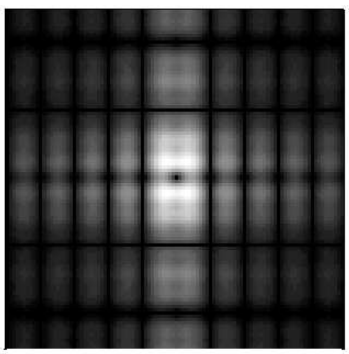
3. Consider two narrow slits in an opaque screen, separated by 0.2 mm. The slits are illuminated at normal incidence by stationary white light emitted from a distant point source. One of the slits is covered by a red filter, passing light only with wavelengths near 650 nm. The other slit is covered by a blue filter, passing only wavelengths near 450 nm. In the resulting Fraunhofer diffraction pattern:
(a) No interference effects from the two slits will be observed.
(b) Interference effects will be observed, at the center of the diffraction pattern.
(c) Interference effects will be observed, but only at a location offset from the center of the diffraction pattern.

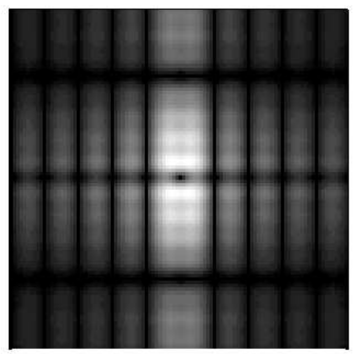
4. If a medium has complex index of refraction $\tilde{n} = 1.5 + 0.5i$, what is the phase velocity c within the medium?
(a) $c = 1.5c_0$ (b) $c = 0.667c_0$ (c) $c = 0.6c_0$ (d) $c = (0.6 - 0.2i)c_0$

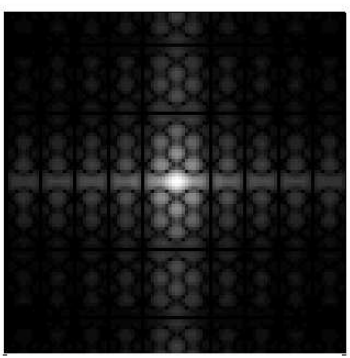
5. Consider the Fraunhofer diffraction pattern produced by the following aperture:

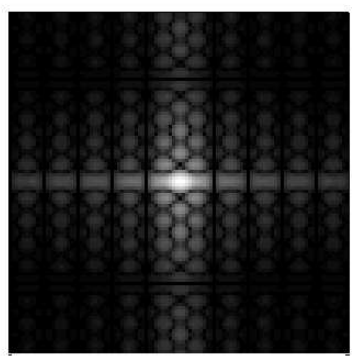


Which of the following images shows the correct pattern? The pictures are all shown to scale, with the width of each picture equal to $10\lambda d/\pi a$ for observation distance d and light wavelength λ. White areas indicate high intensity.

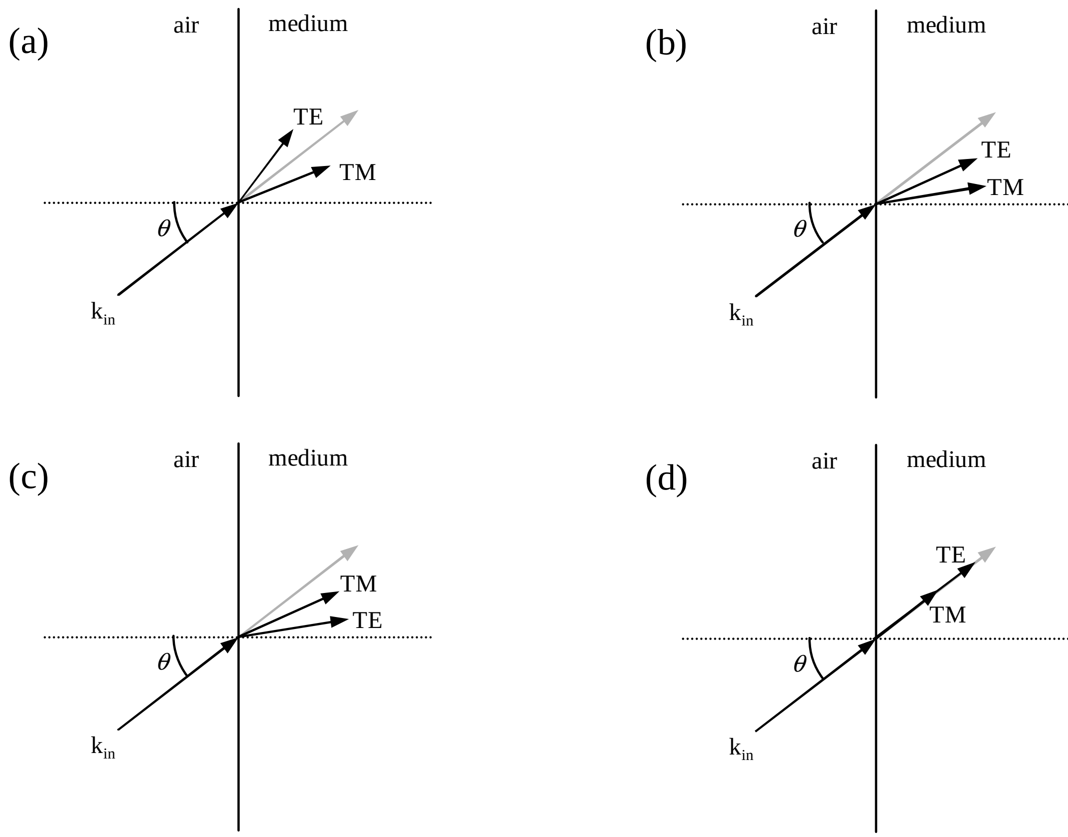








6. Suppose a plane wave is incident on a uniaxial crystal with $n_e > n_o > 1$. The angle of incidence θ is the same as the angle between the optic axis and the surface normal; i.e., the incident light is parallel to the optic axis. Which of the following pictures correctly shows the refraction angles for the TM and TE polarizations in this case? The gray arrow indicates the direction of the optic axis.



Problems

7. Suppose that in the $z=0$ plane, a wave travelling towards posistive z has the values

$$U(x,y,z=0) = A\left[\cos\left(\frac{\pi x}{\lambda}\right)\cos\left(\frac{\pi y}{\lambda}\right) - 1\right],$$

where λ is the wavelength of the wave. Then the intensity at the origin $I(0,0,0)$ is zero. Determine all other axial locations with $I(0,0,z)=0$.

8. Consider a linearly polarized plane wave, with

$$\mathbf{E}_{in}(\mathbf{r}) = A\hat{\mathbf{x}}\exp(-ikz)$$

normally incident on a plane mirror at $z=0$. To obtain the reflected wave, model the mirror as a perfect conductor, having index of refraction $\tilde{n} = n + i\alpha/2k$ with $\alpha \to \infty$. Calculate the total electric field $\mathbf{E}$ and magnetic field $\mathbf{H}$ resulting at each point z.

9. Some materials exhibit a phenomenon called circular dichroism, in which right- and left-handed circular polarized light are absorbed by different amounts. Suppose a sample of such a material has complex transmission coefficients t_R for RCP and t_L for LCP. Calculate the Jones matrix for this object, in the usual xy basis.

10. Suppose a random wave $U_0(t)$ is superposed with a time-delayed version of itself, to produce a resultant wave

$$U(t) = \frac{1}{\sqrt{2}}\left[U_0(t) + U_0(t-a)\right]$$

where a is the time delay. One means of accomplishing this is shown below. If the original wave has a spectral density $S_0(\nu)$, what is the spectral density of the resultant wave?