

1. It is possible to obtain group velocities  $v$  larger than  $c_0$  for light passing through a dilute atomic vapor. Recall (S&T, equation 5.5-18) that near a resonance, the electric susceptibility of a medium is given by

$$\chi(\nu) = \chi_0 \frac{\nu_0/2}{(\nu_0 - \nu) + i\Delta\nu/2}$$

where  $\nu_0$  is the resonant frequency and  $\Delta\nu$  is the width of the resonance. Assume  $\chi_0$  is small enough that  $|\chi(\nu_0)| \ll 1$ .

(a) Calculate the group velocity at frequency  $\nu_0$ . Show that it can have arbitrarily large or negative values, even under the restriction  $|\chi(\nu_0)| \ll 1$ .

(b) Suppose a pulse of duration  $\Delta t$  propagates a distance  $L$  through such a medium where  $v > c_0$ . The pulse will exit the medium a time

$$\tau = L \left( \frac{1}{c_0} - \frac{1}{v} \right)$$

earlier than a pulse with speed  $c_0$  would have. However, this requires that the absorption  $\alpha L$  be no larger than 1, so that the pulse is not strongly attenuated. Also, the bandwidth of the pulse (i.e., the range of frequencies present) must be much smaller than  $\Delta\nu$ , to avoid strong dispersion effects. By imposing these requirements, show that  $\tau \ll \Delta t$ .

2. Saleh and Teich, problem 6.1-1, page 235.

3. Saleh and Teich, problem 6.1-3, page 235.

4. Saleh and Teich, problem 6.1-4, page 235. Note, I don't get the  $\pi/2$  phase shift referred to in the problem.

5. Consider light linearly polarized along the  $x$  axis, which is incident on a polarizer with transmission axis at angle  $\theta$  to the  $x$  axis, followed by a polarizer with transmission axis along the  $y$  axis. Calculate the fraction of the incident power transmitted through the system. What is the maximum value of the transmission, and what angle  $\theta$  achieves this?