Phys 531 Assigment 12

Solutions

1. Fresnel equations give $R = |r|^2$

with $$r_x = \frac{n_1 \cos\theta_1 - n_2 \cos\theta_2}{n_1 \cos\theta_1 + n_2 \cos\theta_2}$$

$$r_y = \frac{n_2 \cos\theta_1 - n_1 \cos\theta_2}{n_2 \cos\theta_1 + n_2 \cos\theta_2}$$

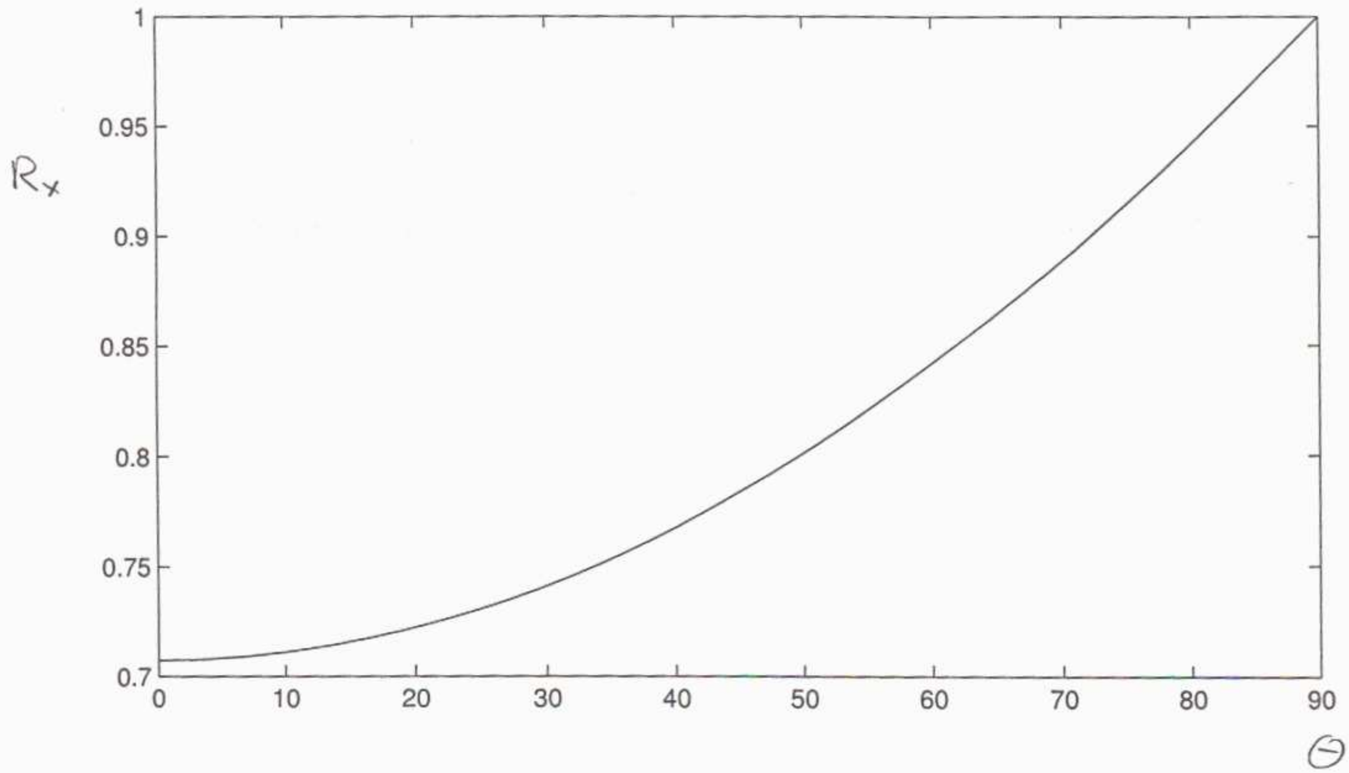
and $$\cos\theta_2 = \sqrt{1 - \left(\frac{n_1}{n_2}\right)^2 \sin^2\theta_1}$$

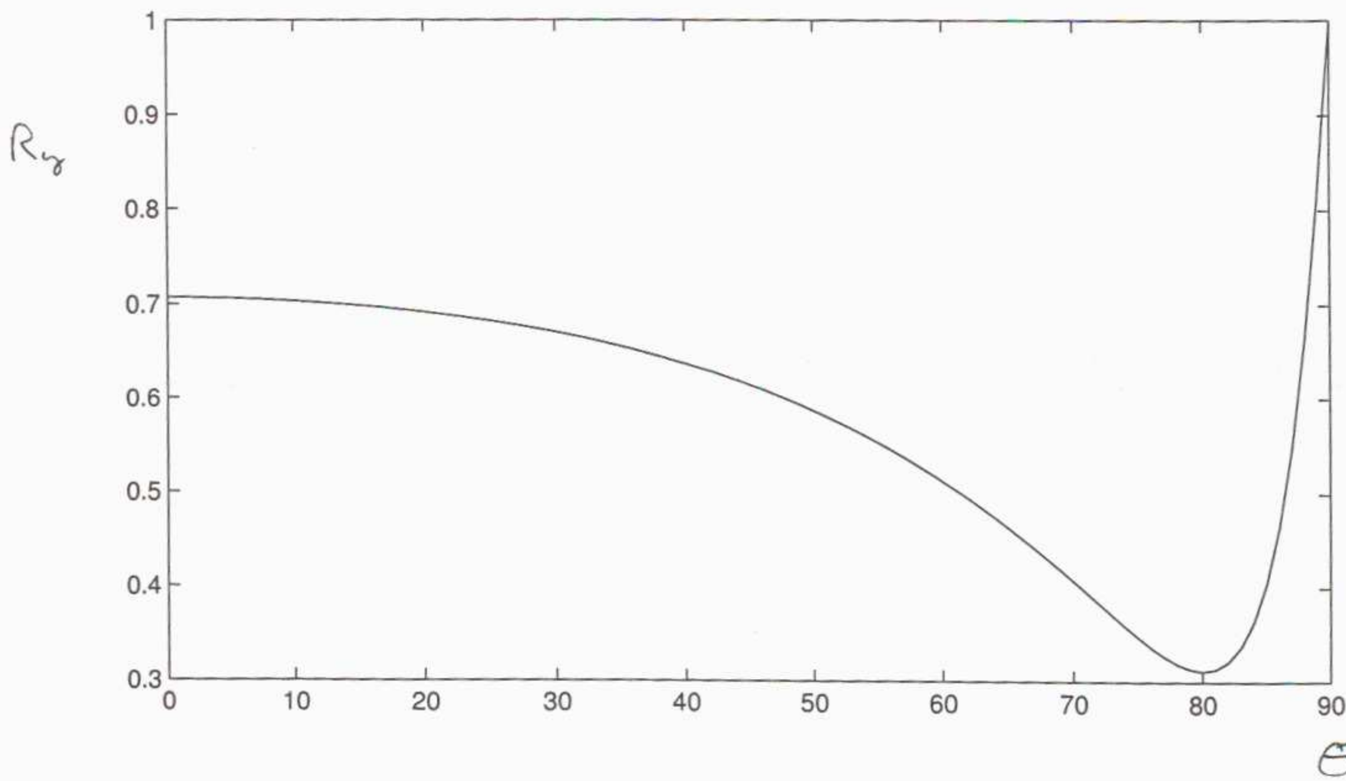
Here we have $n_1 = 1$ $\quad n_2 = 3+5i$

Calculate with MATLAB:

```
% HW 12 Problem 1

n = 3+5*i;
q1 = [0:90]*pi/180;
c1 = cos(q1);
s1 = sin(q1);

c2 = (1- n^(-2)*s1.*s1).^0.5;

rx = (c1-n*c2)./(c1+n*c2);
ry = (n*c1 - c2)./(n*c1+c2);

Rx = abs(rx).^2;
Ry = abs(ry).^2;

subplot(2,1,1)
plot(q1*180/pi,Rx)
subplot(2,1,2)
plot(q1*180/pi,Ry)
```

Get plots shown.

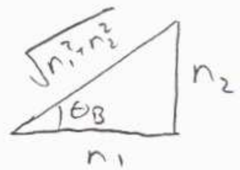

R_x
1
0.95
0.9
0.85
0.8
0.75
0.7
0 10 20 30 40 50 60 70 80 90
Θ


R_y
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0 10 20 30 40 50 60 70 80 90
Θ

2. Brewster's angle given by $\tan\theta_B = \frac{n_2}{n_1}$

Refraction angle is then θ_2 with

$$n_2 \sin\theta_2 = n_1 \sin\theta_B$$



Have $\sin\theta_B = \frac{n_2}{\sqrt{n_1^2+n_2^2}}$

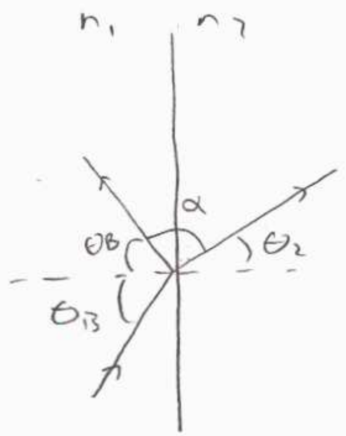
so $\sin\theta_2 = \frac{n_1}{\sqrt{n_1^2+n_2^2}}$

But this is just $\cos\theta_B = \frac{n_1}{\sqrt{n_1^2+n_2^2}}$

So $\sin\theta_2 = \cos\theta_B$

and $\theta_2 = \frac{\pi}{2} - \theta_B$

This means refracted and reflected waves are orthogonal:



If $\theta_B + \theta_2 = \frac{\pi}{2}$

Then $\boxed{\alpha = \frac{\pi}{2}}$

Makes sense: dipoles in medium 2 don't radiate in direction of $\vec{p}$, so no radiation at right angle to $\vec{k}_2$

3. Get TIR for $\theta > \theta_c$ with

$$\sin\theta_c = \frac{n_2}{n_1} = \frac{1}{n_1} \quad \text{if } n_2 = 1$$

If $\theta > \theta_c$

$$\cos\theta_2 = \sqrt{1-\left(\frac{n_1}{n_2}\right)^2 \sin^2\theta_1} = i\sqrt{\left(\frac{n_1}{n_2}\right)^2 \sin^2\theta_1 - 1} \equiv ib$$

So $r_x = \dfrac{n_1\cos\theta_1 - \cos\theta_2}{n_1\cos\theta_1 + \cos\theta_2} = \dfrac{n_1\cos\theta_1 - ib}{n_1\cos\theta_1 + ib}$

(= TE)

$$= \frac{(n_1^2\cos^2\theta_1 - b^2) - 2ibn_1\cos\theta_1}{n_1^2\cos^2\theta_1 + b^2}$$

Gives phase shift

$$\tan\phi_x = -\frac{2bn_1\cos\theta_1}{n_1^2\cos^2\theta_1 - b^2}$$

Also, $r_y = \dfrac{\cos\theta_1 - n_1\cos\theta_2}{\cos\theta_1 + n_1\cos\theta_2} = \dfrac{\cos\theta_1 - in_1b}{\cos\theta_1 + in_1b}$

(= TM)

$$= \frac{(\cos^2\theta_1 - n_1^2b^2) - 2ibn_1\cos\theta_1}{\cos^2\theta_1 + n_1^2b^2}$$

So $\tan\phi_y = -\dfrac{2bn_1\cos\theta_1}{\cos^2\theta_1 - n_1^2b^2}$

If $n_1 = 1.5$, then $\theta_c = 41.81°$

So if $\theta_1 = 1.2\,\theta_c = 50.17°$

$$\cos\theta_1 = 0.641$$

Then $b = \sqrt{(1.5)^2 \sin^2 50.17° - 1} = 0.572$

$$\tan \phi_x = -\frac{2 \times 0.572 \times 1.5 \times 0.641}{(1.5 \times 0.641)^2 - (0.572)^2}$$

$$= -1.841$$

$$\phi_x = -61.5° \quad \text{or} \quad +118.5°$$

and

$$\tan \theta_y = -\frac{2 \times 0.572 \times 1.5 \times 0.641}{(0.641)^2 - (1.5 \times 0.572)^2}$$

$$= 73.5° \quad \text{or} \quad -106.5°$$

To resolve 180° uncertainty:

Since $n_1^2 \cos^2 \theta_1 - b^2 > 0$

r_x has real part > 0

imag part < 0

So $-90 < \phi_x < 0$

and $\phi_x = -61.5°$

Since $\cos^2 \theta_1 - n_1^2 b^2 < 0$,

r_y has real part < 0

and imag part < 0

So $-180 < \phi_y < -90$

$\phi_y = -106.5°$

Then retardance $\boxed{\Gamma = \phi_x - \phi_y = +45°}$

4. Have $n_e = 1.553$

$n_o = 1.544$

a) Retardance $\Gamma = k_o(n_e - n_o)d$

$$\frac{\Gamma}{d} = \frac{2\pi}{\lambda}(n_e - n_o)$$

$$= \frac{2\pi}{633\,nm}(1.553 - 1.544) = \boxed{89.3\ \frac{rad}{mm}}$$

b) Acts as quarter wave retarder when

$$\Gamma = \frac{\pi}{2}(2n+1) \quad \text{integer } n$$

$$= \pi\left(n + \tfrac{1}{2}\right)$$

So $$d = \frac{\pi\left(n+\frac{1}{2}\right)}{89.3\ \frac{rad}{mm}} = \boxed{3.52 \times 10^{-2}\,mm\ \left(n + \tfrac{1}{2}\right)}$$

5. a) For TE polarization, $n = n_o$ because $E \perp$ axis

So, $$\sin\theta_1 = n_o \sin\theta_2$$

$$\boxed{\sin\theta_{2o} = \frac{1}{n_o}\sin\theta_1}$$

For TM polarization, have

$$\sin\theta_1 = n(\theta_2)\sin\theta_2$$

with $$\frac{1}{n(\theta_2)^2} = \frac{\cos^2\theta}{n_o^2} + \frac{\sin^2\theta}{n_e^2}$$

Where θ = angle between $\vec{k}_2$ and optic axis

Then $\Theta = \frac{\pi}{2} - \Theta_2$ here, so

$$\frac{1}{n(\theta_2)^2} = \frac{\sin^2\theta_2}{n_o^2} + \frac{\cos^2\theta_2}{n_e^2}$$

This gives

$$\frac{\sin^2\theta_2}{\sin^2\Theta_1} = \frac{1}{n(\theta_2)^2} = \frac{\sin^2\theta_2}{n_o^2} + \frac{\cos^2\theta_2}{n_e^2}$$

$$= \frac{\sin^2\theta_2}{n_o^2} + \frac{(1-\sin^2\theta_2)}{n_e^2}$$

or

$$\sin^2\theta_2\left[\frac{1}{\sin^2\theta_1} - \frac{1}{n_o^2} + \frac{1}{n_e^2}\right] = \frac{1}{n_e^2}$$

$$\sin^2\Theta_2 = \frac{1}{n_e^2}\,\frac{1}{\frac{1}{\sin^2\theta_1} - \frac{1}{n_o^2} + \frac{1}{n_e^2}} = \frac{\sin^2\Theta_1}{n_e^2\left(1 - \frac{\sin^2\theta_1}{n_o^2} + \frac{\sin^2\theta_1}{n_e^2}\right)}$$

$$= \frac{\sin^2\Theta_1}{\left[n_e^2 + \sin^2\Theta_1\left(1 - \frac{n_e^2}{n_o^2}\right)\right]}$$

Or

$$\boxed{\sin\Theta_{2e} = \frac{\sin\Theta_1}{\sqrt{n_e^2 + \sin^2\theta_1\left(1 - \frac{n_e^2}{n_o^2}\right)}}}$$

or, $$\tan\Theta_{2e} = \frac{1}{\sqrt{\frac{n_e^2}{\sin^2\theta_1} - \frac{n_e^2}{n_o^2}}}$$