Phys 531 Assignment 13 Solutions

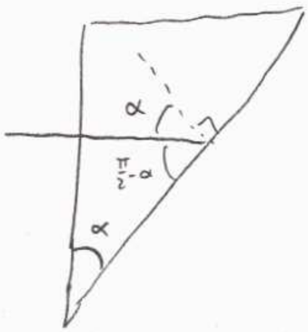
1. a) Get TIR if angle of incidence Θ is greater than Θ_c:

$$\sin\Theta_c = \frac{1}{n}$$

so here $\Theta_{ce} = \sin^{-1}\frac{1}{n_e} = 42.1^\circ$

$\Theta_{co} = \sin^{-1}\frac{1}{n_o} = 37.0^\circ$

Also, here $\Theta = \alpha$:



So if α is between $\boxed{37.0^\circ \text{ and } 42.1^\circ}$ ordinary ray is reflected while extraordinary ray is transmitted.

Since optic axis is $\perp$ to page, transmitted light is $\boxed{\text{TE polarized}}$

b) Need transmission through four surfaces for TE polarization.

Use $t = \dfrac{2n_1\cos\Theta_1}{n_1\cos\Theta_1 + n_2\cos\Theta_2}$ $\qquad T = \dfrac{n_2\cos\Theta_2}{n_1\cos\Theta_1}|t|^2$

First surface: $\Theta_1 = \Theta_2 = 0$ $\quad n_1 = 1$ $\quad n_2 = n_e = 1.49$

$t_1 = 0.80$ $\qquad T_1 = 0.96$

Second surface

$\theta_1 = \alpha$ take $\alpha = 39.5°$

$n_1 = 1.49$ $n_2 = 1$

$\sin\theta_2 = n_1 \sin\theta_1$ so $\theta_2 = 71.4°$

$t_2 = 1.56$ $T_2 = 0.68$

Third surface:

$\theta_1 = 71.4°$ $\theta_2 = \alpha = 39.5°$

$n_1 = 1$ $n_2 = 1.49$

$t_3 = 0.434$ $T_3 = 0.68$

Fourth surface:

$\theta_1 = \theta_2 = 0$

$n_1 = 1.49$ $n_2 = 1$

$t_4 = 1.19$ $T_4 = 0.96$

So total transmission is $T_1 T_2 T_3 T_4 = \boxed{0.43}$

or amplitude transmittance =

$0.8 \times 1.56 \times 0.434 \times 1.19 = 0.645$

2.

$$G(\tau) = \langle u^*(t)\, u(t+\tau) \rangle$$

$$= \left\langle \sum_{nm} a_n^* a_m\, e^{i 2\pi(\nu_m - \nu_n)t}\, e^{i 2\pi \nu_m \tau} \right\rangle$$

$$= \sum_{nm} \langle a_n^* a_m \rangle\, e^{i 2\pi(\nu_m - \nu_n)t}\, e^{i 2\pi \nu_m \tau}$$

Need $m = n$, so

$$\boxed{G(\tau) = \sum_{n=1}^{N} A_n\, e^{i 2\pi \nu_n \tau}}$$

$$S(\nu) = \int_{-\infty}^{\infty} G(\tau)\, e^{-i 2\pi \nu \tau}\, d\tau$$

$$= \sum_{n=1}^{N} A_n \int_{-\infty}^{\infty} e^{2\pi i(\nu_n - \nu)\tau}\, d\tau$$

$$\boxed{S(\nu) = \sum_n A_n\, \delta(\nu - \nu_n)}$$

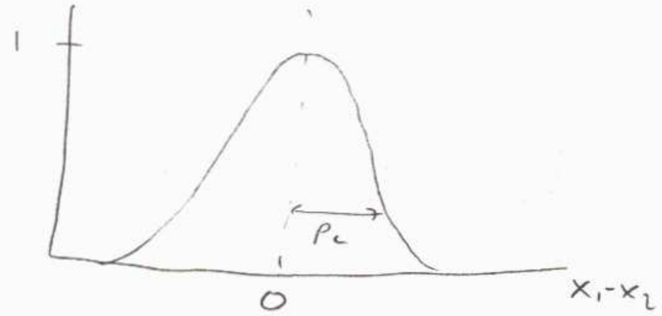
3. Given $G(x_1, x_2) = I_0\, e^{-\frac{x_1^2 + x_2^2}{w_0^2}}\, e^{-\frac{(x_1 - x_2)^2}{\rho_c^2}}$

Have $\boxed{I(x) = G(x,x) = I_0\, e^{-\frac{2x_1^2}{w_0^2}}}$

$$g(x_1, x_2) = \frac{G(x_1, x_2)}{\sqrt{I(x_1)\, I(x_2)}} = \frac{I_0\, e^{-\frac{x_1^2 + x_2^2}{w_0^2}}\, e^{-\frac{(x_1 - x_2)^2}{\rho_c^2}}}{I_0\, e^{-\frac{x_1^2}{w_0^2}}\, e^{-\frac{x_2^2}{w_0^2}}}$$

$$\boxed{g(x_1, x_2) = e^{-\frac{(x_1 - x_2)^2}{\rho_c^2}}}$$

So g is Gaussian:



So: G describes a beam with peak intensity I_0, beam width W_0. But the transverse coherence length is ρ_c, describes how well different portions of the beam are correlated.

4.

Point at $(x,0,d)$ is at distance $\sqrt{x^2+d^2}$ from source

Relative to $(0,0,d)$, get time delay

$$\tau = \frac{1}{c_0}\left(\sqrt{x^2+d^2} - d\right)$$

So $g(\vec{r}_1, \vec{r}_2) = g(\tau)$ for this τ

Given spectrum is Lorentizien with coherence time τ_c

If $S(\nu) = \frac{\Delta\nu}{2\pi} \frac{1}{(\nu-\nu_0)^2 + (\Delta\nu/2)^2}$

Then $g(\tau) = e^{-|\tau|/\tau_c} e^{i 2\pi \nu_0 \tau}$ according to Example 10.1-1 on page 353

Note if $\tau_c = 10\,ps$, then $g(\tau)$ is non-zero only if $\frac{1}{c_0}\left(\sqrt{x^2+d^2} - d\right)$ is not much larger than 10^{-11} s

Or, $\sqrt{x^2+d^2} - d \approx 3\times10^{-3}\,m = 3\,mm$

But $d = 10$ cm, so must have $x \ll d$

Can expand square root:

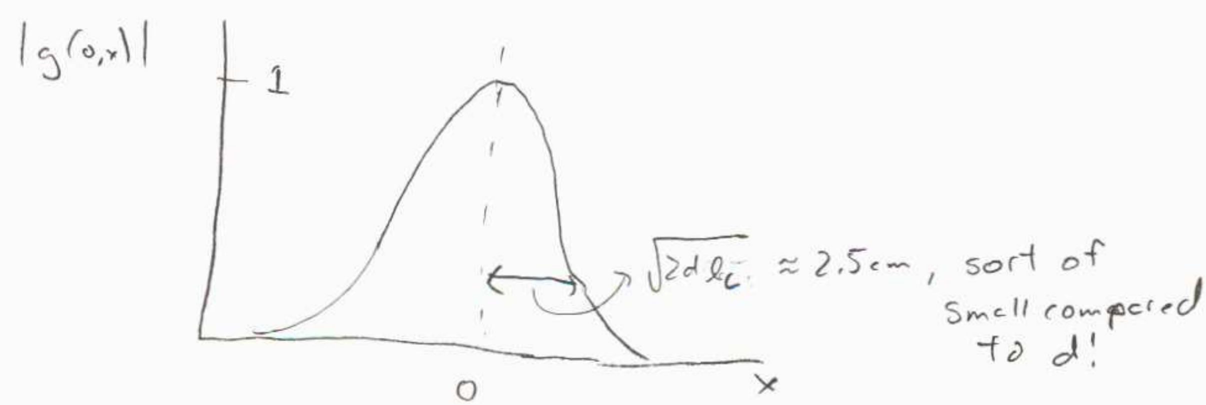
$$\sqrt{x^2+d^2} \approx d + \frac{x^2}{2d}$$

and $\tau = \frac{1}{c_0}\frac{x^2}{2d}$

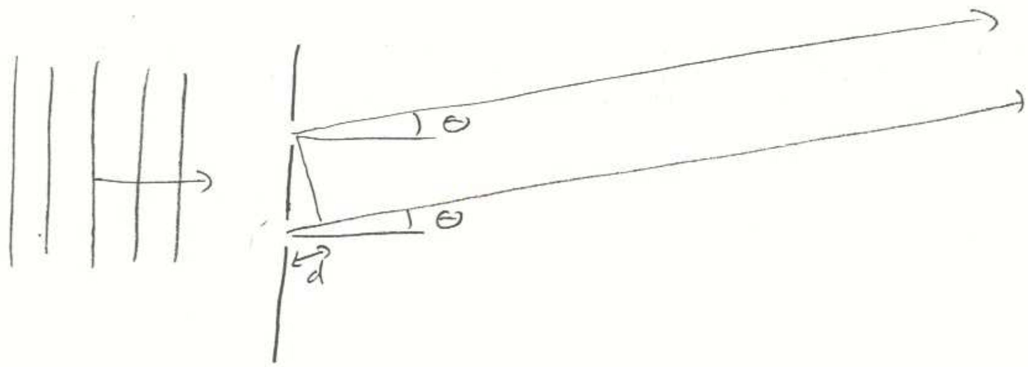
Finally

$$\boxed{g(0,x) = e^{-\frac{1}{\tau_c c_0}\frac{x^2}{2d}}\, e^{i\frac{2\pi\nu_0}{c_0}\frac{x^2}{2d}}}$$

$\tau_c c_0 = \ell_c = 3\,mm$



5. If source has coherence time τ_c, number of fringes is limited:



Path lengths differ by d

$$\text{So, need } \frac{d}{c_0} \lesssim \tau_c$$

$$d \leq c_0 \tau_c$$

Number of fringes is $N = \frac{d}{\lambda} \lesssim \frac{1}{\lambda} c_0 \tau_c$

$$N \approx \frac{l_c}{\lambda} \quad \text{for } l_c = c_0 \tau_c$$

So sources have:

	λ	l_c	N
Sun	0.6 μm	0.8 nm	1.3
LED	1 μm	20 μm	20
Lamp	0.6 μm	600 μm	1000
HeNe	0.6 μm	20 cm	3×10^5
HeNe*	0.6 μm	300 m	5×10^8