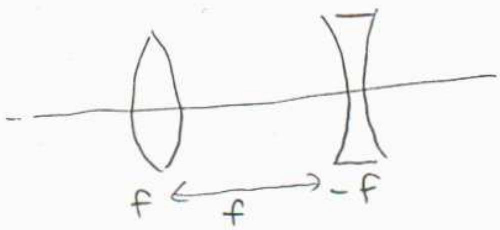
Solutions

532 Assignment 2

1. System is



$$M = \begin{bmatrix} 1 & 0 \\ \frac{1}{f} & 1 \end{bmatrix} \begin{bmatrix} 1 & f \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{bmatrix}$$

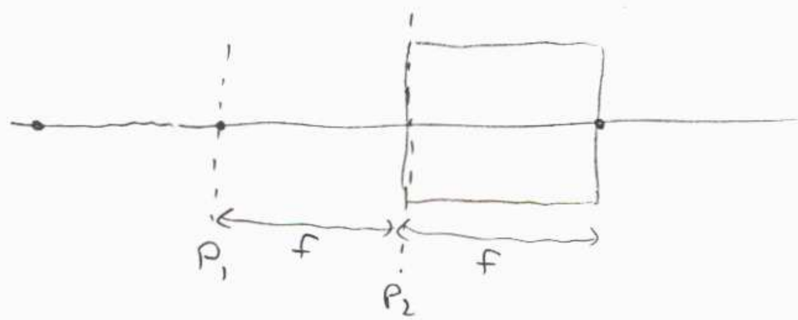
$$= \begin{bmatrix} 1 & f \\ \frac{1}{f} & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{bmatrix} = \boxed{\begin{bmatrix} 0 & f \\ -\frac{1}{f} & 2 \end{bmatrix}}$$

System focal length $= f$

$ffl = -\frac{D}{C} = 2f$

$bfl = -\frac{A}{C} = 0$

So principal planes are located:



(Since $f>0$, P_1 is located behind ~~in front~~ of front focal point, and P_2 is in front of back focal point.)

So, for instance, parallel input rays are focused to a point on the exit face. Rays emitted from a point $2f$ in front of system are collimated. Rays emitted from a point f in front of system emerge undeviated.

2. Eqs (1.3-9) and (1.3-10) give

$$y(z) = y_0 \cos\alpha z + \frac{\theta_0}{\alpha} \sin\alpha z$$

$$\theta(z) = -y_0\alpha \sin\alpha z + \theta_0 \cos\alpha z$$

For rays with y_0, θ_0 at $z = 0$.

This gives matrix for evolution in slab of length d

$$M = \begin{bmatrix} \cos\alpha d & \frac{1}{\alpha} \sin\alpha d \\ -\alpha \sin\alpha d & \cos\alpha d \end{bmatrix}$$

Accounting for surfaces, have

$$M_{TOT} = \begin{bmatrix} 1 & 0 \\ 0 & n_0 \end{bmatrix} \begin{bmatrix} \cos\alpha d & \frac{1}{\alpha} \sin\alpha d \\ -\alpha \sin\alpha d & \cos\alpha d \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{n_0} \end{bmatrix}$$

where we have taken $n_{slab} \simeq n_0$, which is valid for $\alpha^2 y^2 \ll 1$

So $$\boxed{M_{TOT} = \begin{bmatrix} \cos\alpha d & \frac{1}{n_0\alpha} \sin\alpha d \\ -n_0\alpha \sin\alpha d & \cos\alpha d \end{bmatrix}}$$

3. Plössl eye piece

a) $M_{TOT} = M_{S_6} M_{D_5} M_{S_5} M_{D_4} M_{S_4} M_{D_3} M_{S_3} M_{D_2} M_{S_2} M_{D_1} M_{S_1}$

where $M_{S_1} = \begin{bmatrix} 1 & 0 \\ \frac{1-n_1}{n_1 R_1} & \frac{1}{n_1} \end{bmatrix}$ $\quad M_{D_1} = \begin{bmatrix} 1 & 1.5 \\ 0 & 1 \end{bmatrix}$

and so forth. (Note M_{S_4}, M_{S_5} and M_{S_6} have negative R_3, R_2, R_1 respectively)

I used matlab,

get

$$M_{TOT} = \begin{bmatrix} 0.7993 & 6.6597 \text{ mm} \\ -0.0542 \text{ mm}^{-1} & 0.7993 \end{bmatrix}$$

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r2 = 22.22;
r3 = -30.62;
d1 = 1.5;
d2 = 2.8;
d3 = 1.5;
n0 = 1;
n1 = 1.5189;
n2 = 1.6771;

ms1 = [1 0; (n0 - n2)/n2/r1  n0/n2];
ms2 = [1 0; (n2 - n1)/n1/r2  n2/n1];
ms3 = [1 0; (n1 - n0)/n0/r3  n1/n0];
ms4 = [1 0; -(n0 - n1)/n1/r3  n0/n1];
ms5 = [1 0; -(n1 - n2)/n2/r2  n1/n2];
ms6 = [1 0; -(n2 - n0)/n0/r1  n2/n0];

md1 = [1 d1; 0 1];
md2 = [1 d2; 0 1];
md3 = [1 d3; 0 1];
md4 = md2;
md5 = md1;

mtot = ms6*md5*ms5*md4*ms4*md3*ms3*md2*ms2*md1*ms1
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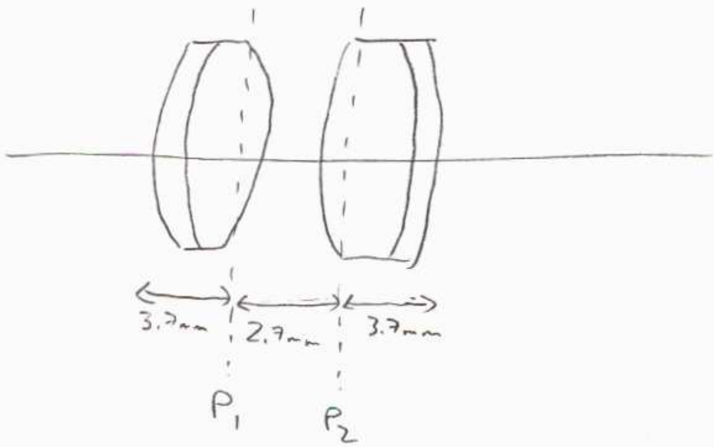
Check, $\det M_{TOT} = 1$ ✓

b) $f = -\frac{1}{C} = 18.45\,mm$

$ffl = -\frac{D}{C} = 14.75\,mm$

$bfl = -\frac{A}{C} = ffl$

So, principal planes are $18.45 - 14.75 = 3.7\,mm$ from vertices



4. Wave equation is $\nabla^2 u = \left(\frac{d^2}{dx^2} + \frac{d^2}{dy^2} + \frac{d^2}{dz^2}\right)u = \frac{1}{c^2}\frac{d^2u}{dt^2}$

a) $\frac{du}{dz} = \frac{1}{t}$ $\quad \frac{d^2u}{dz^2} = 0$

$\frac{du}{dt} = -\frac{z}{t^2}$ $\quad \frac{d^2u}{dt^2} = \frac{2z}{t^3}$ $\quad \boxed{\text{not a solution}}$

b) $\frac{du}{dz} = 2z$ $\quad \frac{d^2u}{dz^2} = 2$

$\frac{du}{dt} = 2v^2 t$ $\quad \frac{d^2u}{dt^2} = 2v^2$ $\quad \boxed{\text{solution with } c = v}$

c) $$\frac{du}{dz} = -2z\, e^{-(z^2+v^2t^2)}$$

$$\frac{d^2u}{dz^2} = -2e^{-(z^2+v^2t^2)} + 4z^2 e^{-(z^2+v^2t^2)} = (4z^2-2)\, e^{-(z^2+v^2t^2)}$$

$$\frac{du}{dt} = -2v^2t\, e^{-(z^2+v^2t^2)}$$

$$\frac{d^2u}{dt^2} = -2v^2 e^{-(z^2+v^2t^2)} + 4v^4t^2 e^{-(z^2+v^2t^2)} = v^2(4v^2t^2-2)e^{-(z^2+v^2t^2)}$$

$$\neq c^2 \frac{d^2u}{dt^2}\text{, so } \boxed{\text{not a solution}}$$

d) $$\frac{du}{dz} = k \cos kz \cos\omega t$$

$$\frac{d^2u}{dz^2} = -k^2 \sin kz \cos\omega t$$

$$\frac{du}{dt} = -\omega \sin kz \sin\omega t$$

$$\frac{d^2u}{dt^2} = -\omega^2 \sin kz \cos\omega t$$

$$= \frac{\omega^2}{k^2}\frac{d^2u}{dz^2}$$

so, solution with $\boxed{c = \frac{\omega}{k}}$

e) $$\frac{du}{dx} = \frac{-1}{(x+y+2z-vt)^2} \qquad \frac{d^2u}{dx^2} = \frac{2}{(x+y+2z-vt)^3}$$

$\frac{d^2u}{dy^2}$ is the same, and $\frac{d^2u}{dz^2} = \frac{8}{(x+y+2z-vt)^3}$

$$\nabla^2 u = \frac{12}{(x+y+2z-vt)^3}$$

$$\frac{du}{dt} = \frac{v}{(x+y+2z-vt)^2}, \quad \frac{d^2u}{dt^2} = \frac{2v^2}{(x+y+2z-vt)^3} = \frac{v^2}{6}\nabla^2 u.$$

$\boxed{\text{Solution with } c = \frac{v}{\sqrt{6}}}$