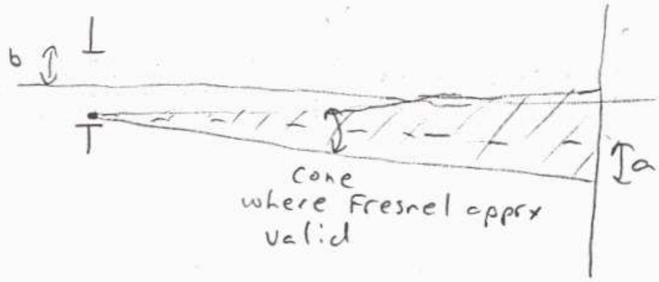
Phys 531 Assignment 7 Solutions

1. Fresnel: Each point in aperture produces diffracted wave, that can be approximated with Fresnel within cone radius $a^4 \ll 4d^3\lambda$



So, if Fresnel approx is to be valid for all source points for field on axis at $z=d$, need $b^4 \ll 4d^3\lambda$

or, $d \gg \left(\frac{b^4}{4\lambda}\right)^{1/3} = \left(\frac{1\,cm^4}{4 \times .5\,\mu m}\right)^{1/3} = 17\,cm$

Fraunhofer: need $N_F' \ll 1$

$$\frac{b^2}{\lambda d} \ll 1$$

$$d \gg \frac{b^2}{\lambda} = \boxed{200\,m}$$

Furthermore, for the Fraunhofer approximation, d must really be larger than 200m, perhaps $d \sim 1\,km$.
While for Fresnel, need $d^3 \gg (17\,cm)^3$, satisfied by, say $d \sim 30\,cm$

2. If laser has diameter 2mm, expect range of spatial frequencies $\Delta\nu \sim \frac{1}{2\,mm}$

Then $\Delta\theta \approx \lambda \Delta\nu = \frac{633\,nm}{2\,mm} = 3 \times 10^{-4}$ rad

So, spot size on moon is about $\Delta x = d\Delta\theta$

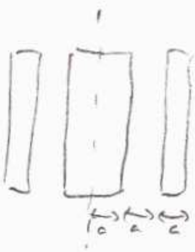
$$= 3.76 \times 10^5\,km \cdot 3 \times 10^{-4}\,rad$$

$$\boxed{\Delta x \approx 120\,km}$$

3. Use $U(x,y,d) = h_0\, F\left(\frac{x}{\lambda d}, \frac{y}{\lambda d}\right)$

Evaluate $F(\nu_x, \nu_y) = \int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dy\; t(x,y)\, e^{2\pi i(\nu_x x + \nu_y y)}$

For three slits get



$$= \int_{-b/2}^{b/2} dy\, e^{2\pi i \nu_y y} \left[\int_{-3a}^{-2a} e^{2\pi i \nu_x x}\, dx + \int_{-a}^{a} e^{2\pi i \nu_x x}\, dx + \int_{2a}^{3a} e^{2\pi i \nu_x x}\, dx \right]$$

$$= b\,\mathrm{sinc}(\pi b \nu_y) \left[a e^{-i 2\pi \nu_x \cdot \frac{5a}{2}} \mathrm{sinc}(\pi a \nu_x) + 2a\,\mathrm{sinc}(2\pi a \nu_x) + a e^{+i 2\pi \nu_x \frac{5a}{2}} \mathrm{sinc}(\pi a \nu_x) \right]$$

Used result from Assignment 3, problem 5,

$$\int_{y_0-c}^{y_0+c} e^{-2\pi i \nu_y y}\, dy = 2c\, e^{-2\pi i \nu_y y_0}\, \mathrm{sinc}\, 2\pi \nu_y c$$

So $F(\nu_x, \nu_y) = ab\,\mathrm{sinc}(\pi b \nu_y)\left[2\,\mathrm{sinc}(2\pi a \nu_x) + 2\cos(5\pi \nu_x a)\,\mathrm{sinc}(\pi \nu_x a) \right]$

So $$\boxed{U(x,y,d) = \frac{i}{\lambda d}\, 2ab\, e^{-ikd}\, \mathrm{sinc}\left(\frac{\pi b y}{\lambda d}\right)\left[\mathrm{sinc}\left(\frac{2\pi a x}{\lambda d}\right) + \cos\left(\frac{5\pi a x}{\lambda d}\right) \mathrm{sinc}\left(\frac{\pi a x}{\lambda d}\right) \right]}$$

Square to get $I(x,y)$

4. Here $U_{in} = A e^{-ik(z+\theta x)}$

So after slit, have $U(x,y,z=0) = A e^{-i2\pi \frac{\theta}{\lambda} x}$ with slit

$= 0$ outside

So, $$F(\nu_x, \nu_y) = A \int_{-D_y/2}^{D_y/2} e^{2\pi i \nu_y y} dy \int_{-D_x/2}^{D_x/2} e^{2\pi i \nu_x x} e^{-2\pi i \frac{\theta}{\lambda} x} dx$$

$$= A D_y \operatorname{sinc}(\pi \nu_y D_y)\, D_x \operatorname{sinc}\left[\pi D_x\left(\nu_x - \frac{\theta}{\lambda}\right)\right]$$

and $$U(x,y,d) = A D_x D_y \operatorname{sinc}\left(\frac{\pi y D_y}{\lambda d}\right) \operatorname{sinc}\left[\pi D_x\left(\frac{x}{\lambda d} - \frac{\theta}{\lambda}\right)\right]$$

$$= \boxed{A D_x D_y \operatorname{sinc}\left(\frac{\pi y D_y}{\lambda d}\right) \operatorname{sinc}\left[\frac{\pi D_x}{\lambda d}(x - \theta d)\right]}$$

So diffraction pattern is just translated by distance θd

Means pattern propagates at angle θ: