Phys 531 Final Exam December 17 2:00-5:00 PM

Instructions:

This is an in-class three-hour exam. You may refer to your textbook, lecture notes, and homework solutions. A calculator will be useful for some problems.

The exam consists of eight problems of varying difficulty. Each problem is worth 10 points and partial credit will be awarded. For maximum credit, be sure to explain your methods and write your solutions out clearly.

Some students are taking the exam early. These students should refrain from discussing the exam with anyone until after the regularly scheduled time.

1. A linearly polarized Gaussian beam has a wavelength of 633 nm, a power of 100 mW and is collimated with a beam waist of 1 mm. Find:
(a) the maximum irradiance
(b) the maximum electric field
(c) the maximum magnetic field
(d) the maximum photon density (photons/m^3)
of the beam.

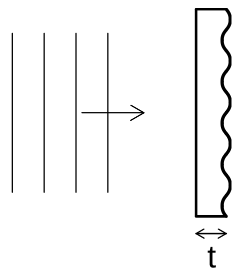
2. Suppose you wish to image a printed circuit board onto a CCD camera. The board is a square with side length 5 cm. The CCD sensor is a square with side length 7 mm. You require the image size to match the CCD size. The board is in a machine that prevents you from putting optics closer than 15 cm, but you would otherwise like to have the camera close to the board to minimize the space required. You have lenses available with focal lengths of 12 mm, 26 mm, 42 mm and -17 mm. Based on these constraints, design a single-lens imaging system. Choose the best lens for the job and specify the locations of the lens and the camera sensor.

3. Evaluate the Fourier transform $\mathcal{F}(k_x, k_y)$ of the function

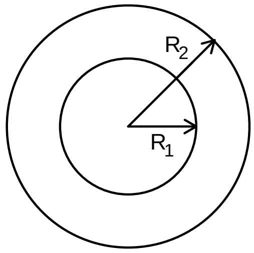
$$f(x,y) = \begin{cases} A\delta(x - x_0) + Be^{i\kappa x} & \text{if } |y| < b \\ 0 & \text{else} \end{cases}$$

Here A, B, κ, x_0 and b are real constants.

4. Suppose a piece of glass ($n = 1.5$) has a thickness that varies as $t = t_0 + a \sin \beta x$. If a plane wave $E = E_0 e^{i(kz-\omega t)}$ is normally incident on the plate, what is the transmitted field a distance d from the plate? You can take $ka \ll 1$ and $\beta < k$. Assume the plate is anti-reflection coated so that reflection losses are negligible.

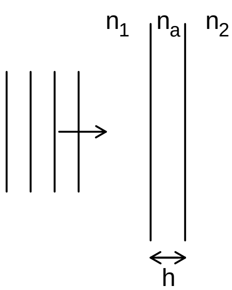


5. Calculate the Fraunhofer diffraction pattern for a ring-shaped aperture as shown. The aperture transmits light for $R_1 < \rho < R_2$, and is otherwise opaque. Assume a plane wave at normal incidence with wave number k.



6. Suppose left-circular polarized light passes through a half-wave plate with axis at angle θ to the x-axis. What is the polarization state of the output?

7. Consider an anti-reflection coating on an interface between materials with index n_1 and n_2. A thin layer of thickness h and index n_a is applied at the boundary, such that at normal incidence the reflections from the $n_1 \rightarrow n_a$ and $n_a \rightarrow n_2$ interfaces interfere destructively.
(a) What should the index n_a be so that reflection coefficients for the two surfaces have equal magnitudes?
(b) What should the thickness h be so that the reflected waves cancel? The light has vacuum wavelength λ.



8. Suppose a distant extended source subtends an angle η and produces monochromatic light with wavelength λ. If the light is collected with a lens of focal length f and diameter $D \ll f$, estimate the lateral coherence length of the light in the the focal plane.