Phys 531 Assignment 6 Due October 26

1. For a simple lens, it's not too hard to do an exact ray tracing calcuation. Here you can estimate the geometrical spot size produced by a collimated beam normally incident on a plano-convex lens. Let the lens have an index of refraction $n = 1.5$, radius of curvature $R = 50$ mm, diameter $D = 2$ cm, and center thickness $t = 3$ mm. Note: when calculating intermediate results, be sure to carry enough decimal places to avoid round-off error.

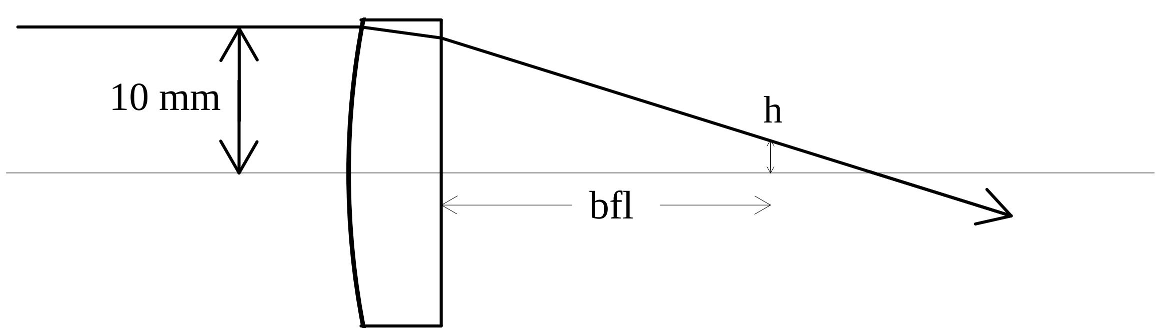
To proceed, start with a ray entering at the edge of the lens and solve for the point where it intersects the glass. The best way to do this is to set up a coordinate system and find an equation for the surface. Use this equation to get the intersection point.

Next, find the angle of the ray in the glass. You'll need to find the normal to the surface at the intersection point and use Snell's Law.

Then find the point where the ray intersects the second surface. Again, solve this exactly, without making any paraxial approximation.

Finally, calculate the height h of the ray at the paraxial focal plane. I'll give you the paraxial front focal length = 100 mm and back focal length = 98 mm for the lens oriented as shown. You can use h as an estimate of the geometrical spot size. (A smaller spot can be obtained by adjusting the focal plane, but don't worry about that here.)

Now reverse the orientation of the lens, and perform the calculation again. How do your spot sizes compare?



2. Consider the interference pattern produced when two plane waves cross at an angle of 2θ. The two waves have equal amplitudes and frequencies. Field E_1 has a wave vector $\mathbf{k}_1 = k_0(\cos\theta\hat{\mathbf{z}}+\sin\theta\hat{\mathbf{x}})$ while E_2 has wave vector $\mathbf{k}_2 = k_0(\cos\theta\hat{\mathbf{z}}-\sin\theta\hat{\mathbf{x}})$. Calculate the squared magnitude of the total field as a function of x and z. In the $z = 0$ plane, what is the distance between adjacent nodes?

3. Suppose that two plane wave fields,

$$E_{\text{tot}} = Ae^{i(k_1 z-\omega_1 t)} + Ae^{i(k_2 z-\omega_2 t)}$$

are propagating through a medium with a complex index of refraction,

$$\tilde{n}(\omega) = n(\omega) + i\frac{\alpha c}{\omega}.$$

You can assume the α is independent of ω. As usual, $k = \tilde{n}\omega/c$. Calculate $|E_{\text{tot}}|^2$ as a function of z and t. Show that the propagation velocity of the beat note depends only on the real part of the index.

4. (10 points) For the function $f(t) = e^{-\gamma|t|}$:
(a) Calculate the Fourier transform $F(\omega)$
(b) Calculate the product $\Delta t\Delta\omega$, here interpreting Δt and $\Delta\omega$ as the full-width at half-max of f and F respectively.
(c) Calculate explicitly

$$\int_{-\infty}^{\infty} |f(t)|^2 dt$$

and

$$\int_{-\infty}^{\infty} |F(\omega)|^2 d\omega.$$

and show that Parseval's theorem is satisfied.
(d) Calculate explicitly the Fourier transform of $g(t) = e^{-i\omega_0 t} f(t)$. (That is, do the integral, don't just use the translations properties.)
Hint: the easiest way to deal with $|t|$ is to separate your integrals into two parts, one over negative t and one over positive t.