Fourier Optics

Last time, studied diffraction examples

- diffraction from an array
- diffraction from circular aperture

Also gave important result:

- get Fraunhofer pattern at focal plane of lens

Today, pursue this idea: Fourier optics

- also discuss holograms

Fanciest demos of whole course!

Outline:

- Lens as Fourier transformer
- Optical filtering
- Phase constrast imaging
- Holography

End of Fourier transform unit

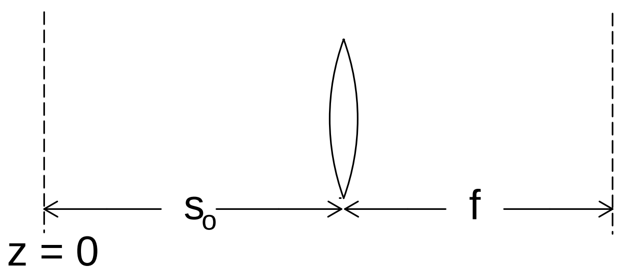
Next time, start interference applications

Optical Fourier Transform (Hecht 13.2)

Idea from end of previous class:

Suppose field $E(x, y, 0) = A(x, y)$ known

Introduce (thin) lens at $z = s_o$



Then at back focal plane of lens:

$$E(x, y, s_o + f) = \frac{C}{\lambda f} \mathcal{A}\left(\frac{kx}{f}, \frac{ky}{f}\right)$$

with

$$C = -ie^{ik(s_o+f)} \exp\left[-i\frac{(x^2 + y^2)(s_o - f)}{2k}\right]$$

Idea:

Lens maps $z = \infty$ to back focal plane

At $z = \infty$, have Fraunhofer pattern

(regardless of s_o)

Allows observation of Fraunhofer pattern

More important:

Gives access to Fourier transform $\mathcal{A}$

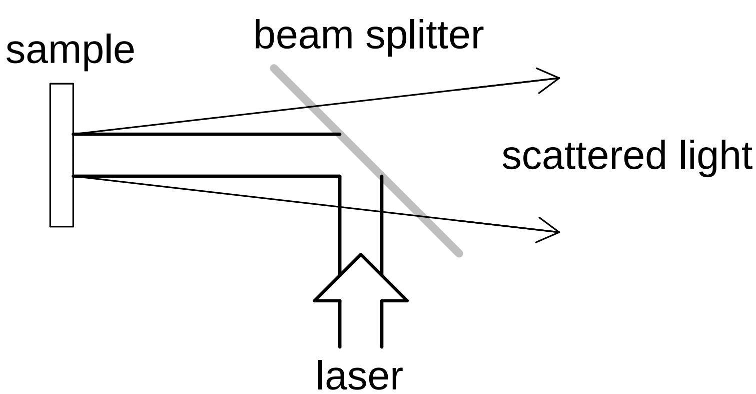
$\boxed{\text{Lens = Fourier transformer}}$

Transform is not just a mathematical tool!

Lens sometimes called "optical computer"

Useful if transform is what you really want

Example: Back scattering experiment

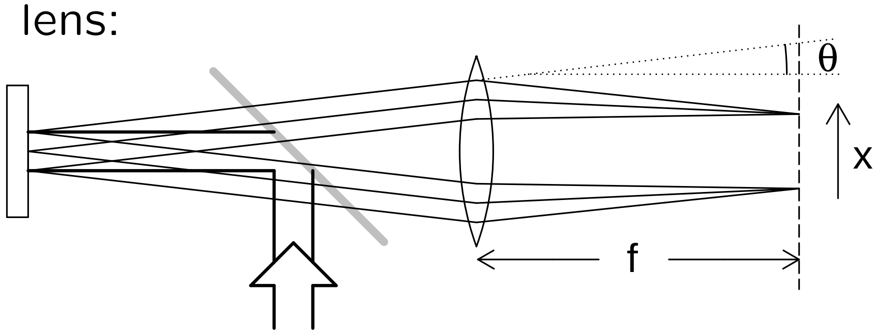


Shine laser on sample, light scattered in all directions

Angular distribution: information about surface

How to measure?

Use lens:



Focal plane: all light at angle θ focussed to x

$x = f\theta$

Observe focal plane with camera, gives $I(\theta)$

Note: idea easy in ray optics

Fourier: connect $\theta \leftrightarrow k_x$

So lens provides Fourier transform

Limitations:

- bandwidth limited by lens aperture

  max $k_x \approx kD/s_o$
- aberrations give errors

  worse for larger k_x
- Hard to measure E in focal plane

  I only has $|E|$, not phase

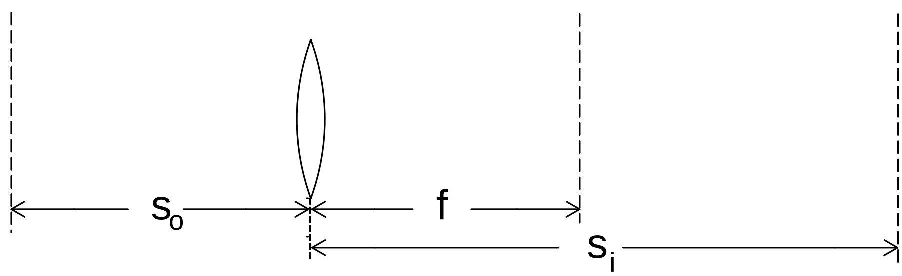
Still very useful

Discuss some applications

Optical Filtering (Hecht 13.2.3)

Don't just observe transform, manipulate it!

System:



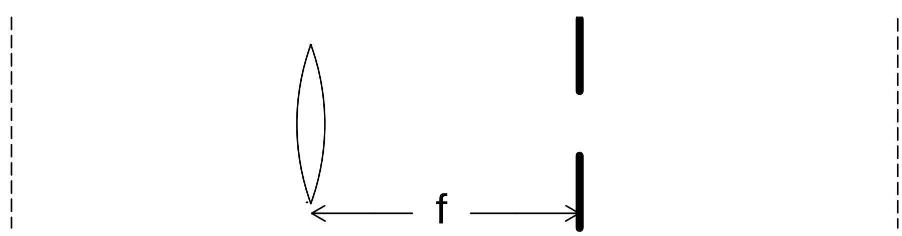
Get image of object at $\dfrac{1}{s_i} = \dfrac{1}{f} - \dfrac{1}{s_o}$

$\rightarrow$ inverts transform automatically

Introduce *filter* in focal plane

Characterize by transmission $t(x, y)$:

$$E_{\text{out}}(x, y) = t(x, y) E_{\text{in}}(x, y)$$



Example: filter = circular hole, radius a

Then $t(x, y) = \begin{cases} 1 & (x^2 + y^2 < a^2) \\ 0 & (\text{else}) \end{cases}$

What is effect?

Point (x, y) corresponds to (k_x, k_y) by

$$(k_x, k_y) = \left(\frac{kx}{f}, \frac{ky}{f}\right)$$

or

$$(x, y) = \left(\frac{k_x f}{k}, \frac{k_y f}{k}\right)$$

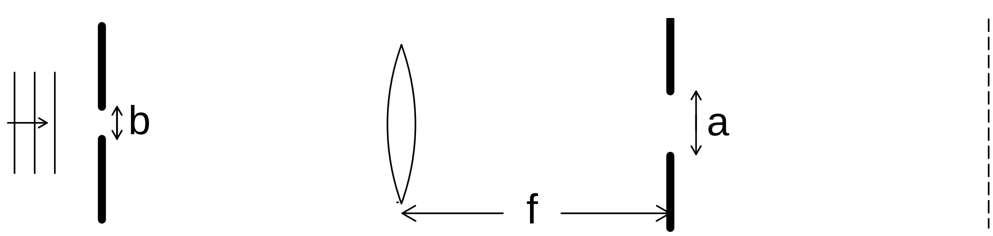
So aperture eliminates Fourier components with

$$\sqrt{k_x^2 + k_y^2} > \frac{ka}{f}$$

Called *low-pass filter*

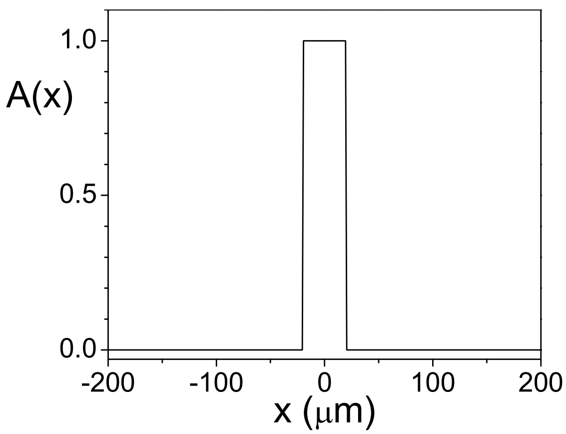
Generally, low-pass filter blurs image
Need high-k components to get sharp transitions

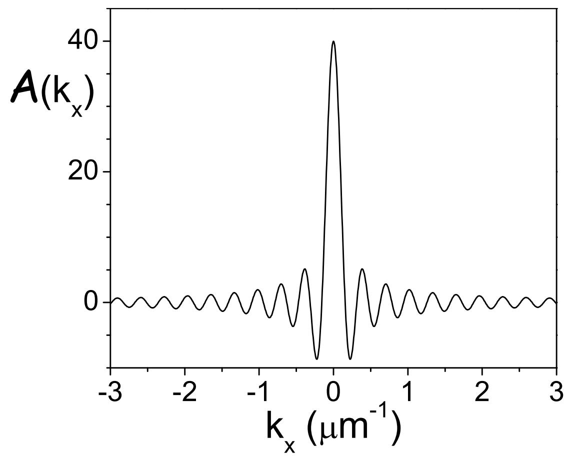
Example (1D):



Light with $\lambda = 500$ nm
Object = slit, width $b = 40\ \mu$m
Image with lens $f = 100$ mm
Filter with slit, width $a = 3.6$ mm

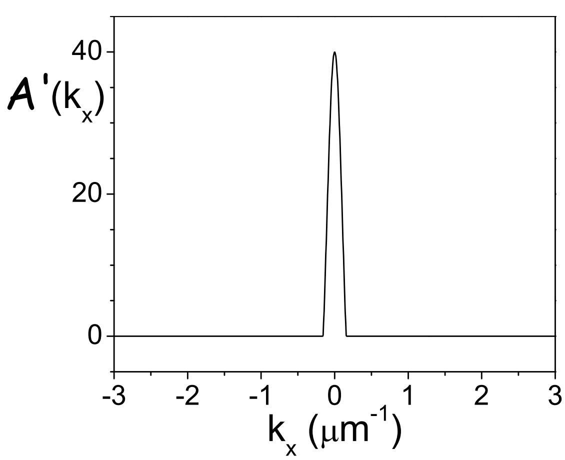
Incident field and transform:





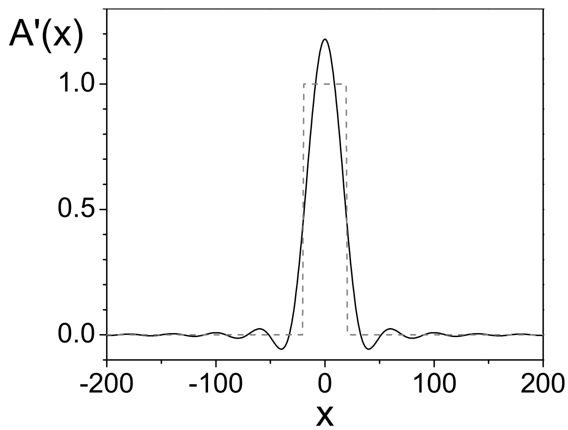
Filter cuts off $|k_x| > ka/2f = 0.23\ \mu\text{m}^{-1}$
approx at first zero of $\mathcal{A}$

So after filter, have



At image plane, get inverse transform of $\mathcal{A}'$

$= $ image of A, missing high k_x components



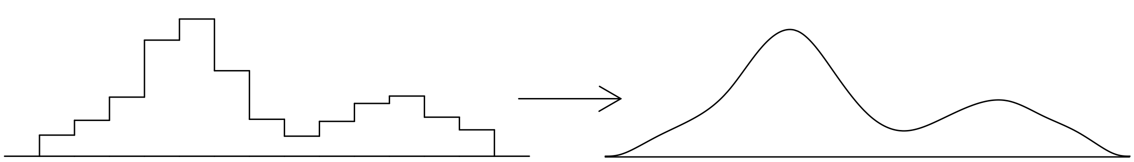
Sharp edges smoothed out

- avoid oscillations with better filter function

Works just like low pass filter in electronics

Applications:

- Remove high frequency noise
  example: dust on image
- Remove pixelation
  example: digital image projector



Question: What happens to the total power of the image when it is filtered?

Alternatively, make filter = small opaque spot

Disk radius a blocks components with

$$\sqrt{k_x^2 + k_y^2} < \frac{ka}{f}$$

Acts as *high-pass filter*

Not as directly useful as low-pass filter
(but related to phase contrast imaging...)

In slit example, say filter = stripe

Use width $a = 10$ mm

Cutoff $k_x = 0.63\ \mu\text{m}^{-1}$
Blocks five central maxima

Gives filtered fields:

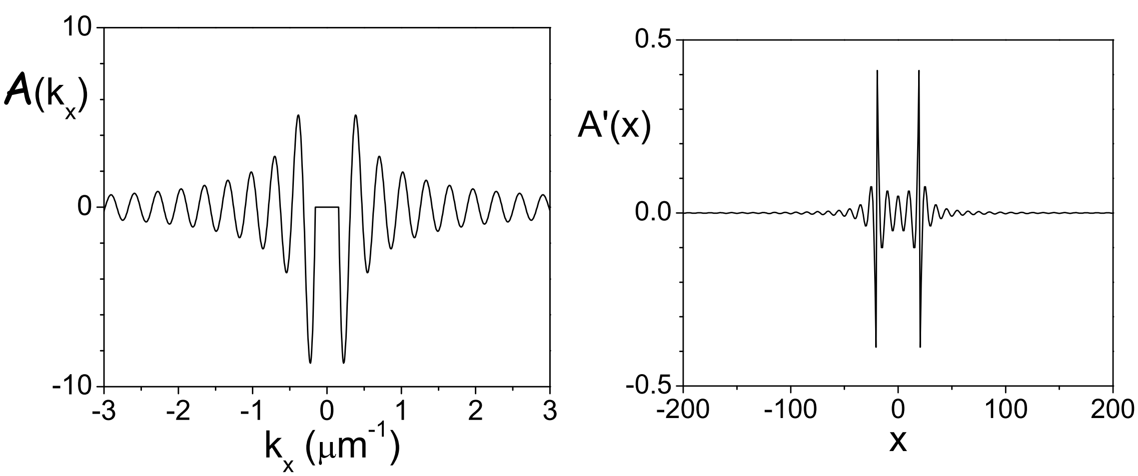


Image bright at edges of object

In general, many types of filter possible

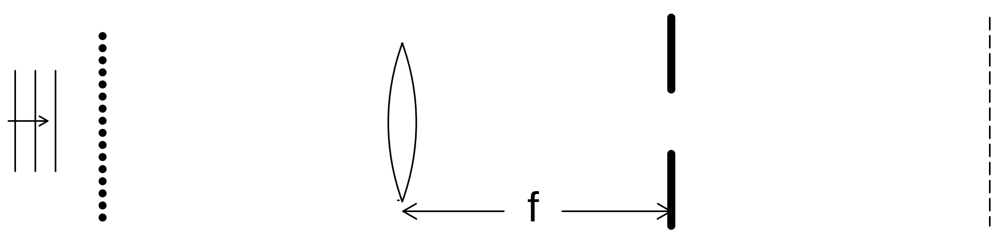
If known artifact or noise in image,
design filter to remove

Hecht figure 13.34, 13.35 has nice examples

Question: Suppose one used a "edge" filter that blocked all $k_x < 0$ and passed all $k_x > 0$. What would be the effect on the image?

Demo:

Set up as in examples



Object 1 = wire grid = array of square holes
Object 2 = circular hole

Filter = circular aperture (low pass)
or sphere mounted on glass slide (high pass)

Cameras view transform plane and image plane

Phase Contrast Imaging (Hecht 13.2.4)

Filters can let you see things otherwise invisible

Suppose object is *phase object*

Example (1D):

$$A(x) = \begin{cases} iE_0 & (|x| < b/2) \\ E_0 & (|x| > b/2) \end{cases}$$

Then $I(x) = \frac{1}{2\eta_0}|E_0|^2 =$ constant I_0

Object not visible with conventional detector

Phase objects = variations in index of refraction
w/ little absorption or reflection

Simple example:
glass plate with 250 nm-deep slot
(since 250 nm of glass $\rightarrow \pi/2$ phase shift)

Real life examples:
- bacteria in water
- density variations in water or air
- stresses in glass or plastic

All hard to see normally

Phase objects have nontrivial transform

Easy way to calculate:

Write $A(x) = E_0 + B(x)$

Then

$$B(x) = A(x) - E_0 = \begin{cases} E_0(i-1) & (|x| < b/2) \\ 0 & (|x| > b/2) \end{cases}$$

Like normal slit function, amplitude $E_0(i-1)$

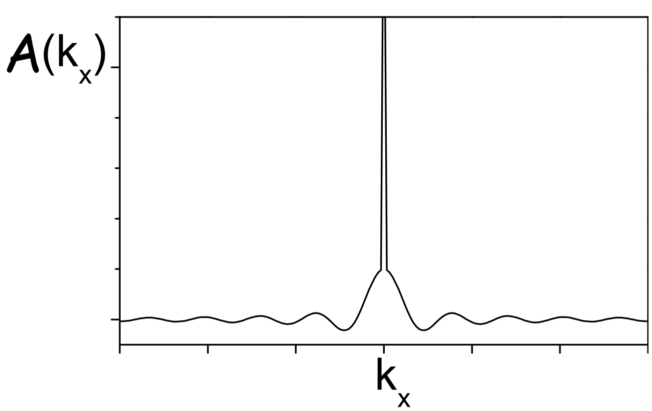
Transform

$$\mathcal{B}(k_x) = E_0(i-1)b\,\mathrm{sinc}\left(\frac{k_x b}{2}\right)$$

Also, transform of constant E_0 is $2\pi E_0 \delta(k_x)$

So $A(x) = E_0 + B(x)$ has transform

$$\mathcal{A}(k_x) = E_0\left[2\pi\delta(k_x) + (i-1)b\,\mathrm{sinc}\left(\frac{k_x b}{2}\right)\right]$$



In focal plane of lens, δ-function $\rightarrow$ Airy pattern
"spike", width 1.22 $\lambda f/D$

Width of sinc function $= 2\lambda f/b$

If lens diameter $D \gg$ slit width b,
central spike small compared to sinc pattern

Suppose high pass filter, size $a \approx \lambda f/D$

Blocks most of spike, negligible effect on sinc

Then $\mathcal{A}'(k_x) \approx E_0(i-1)b\,\mathrm{sinc}\left(\frac{k_x b}{2}\right)$
$=$ transform of ordinary slit function

So image looks like ordinary slit:

$$A'(x) = \begin{cases} E_0(i-1) & (|x| < b/2) \\ 0 & (|x| > b/2) \end{cases}$$

Irradiance:

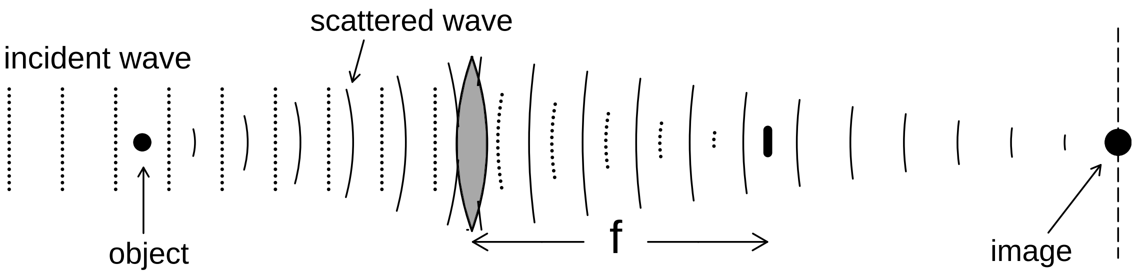
$$I'(x) = \begin{cases} 2I_0 & (|x| < b/2) \\ 0 & (|x| > b/2) \end{cases}$$

Observe bright slit on dark background!

Filter converts phase variation of object to irradiance variation in image

Called *phase contrast imaging*

Understand idea from scattered waves:



Think of object as small, transparent lens

Several types of phase contrast imaging

High pass filter = *dark field* method
(also called dark ground: background is dark)

Demo: same setup as before
Object = scotch tape pattern

Similar idea: use edge filter
block δ function + half of scattered field

Called *schlieren* method
Easier to align, don't need small dot

Demo: movie!

Third method: bright field

Like dark field, but different filter

Instead of opaque spot, use *phase plate*
= glass plate with small phase spot

$$t(x,y) = \begin{cases} -i & (x^2 + y^2 < a^2) \\ 1 & (x^2 + y^2 > a^2) \end{cases}$$

Suppose object field is

$$A(x,y) = E_0 e^{i\phi(x,y)}$$

where ϕ is phase shift due to object

Typically $\phi \ll 1$, so

$$A(x,y) \approx E_0[1 + i\phi(x,y)]$$

Like in dark field example, get transform

$$\mathcal{A}(k_x,k_y) = 2\pi E_0 \delta(k_x)\delta(k_y) + iE_0\Phi(k_x,k_y)$$

where Φ is transform of ϕ

If phase plate radius $a \ll$ width of Φ,
then filtered transform is:

$$\mathcal{A}'(k_x, k_y) \approx -2\pi i E_0 \delta(k_x)\delta(k_y) + iE_0\Phi(k_x, k_y)$$

Image gives inverse transform:

$$A'(x, y) = -iE_0 + iE_0\phi(x, y)$$

with irradiance

$$\begin{aligned} I'(x, y) &= I_0[1 - \phi(x, y)]^2 \\ &\approx I_0[1 - 2\phi(x, y)] \end{aligned}$$

Looks like regular absorptive object

Signal depends linearly on ϕ
- Main advantage of bright field

Dark field, schlieren give signal $\propto \phi^2$
$\Rightarrow$ Bright field signal stronger for small ϕ

Disadvantage: bright background
- Source of noise
- Not as pretty

Bright field common in biology

Holograms

Another neat diffraction trick

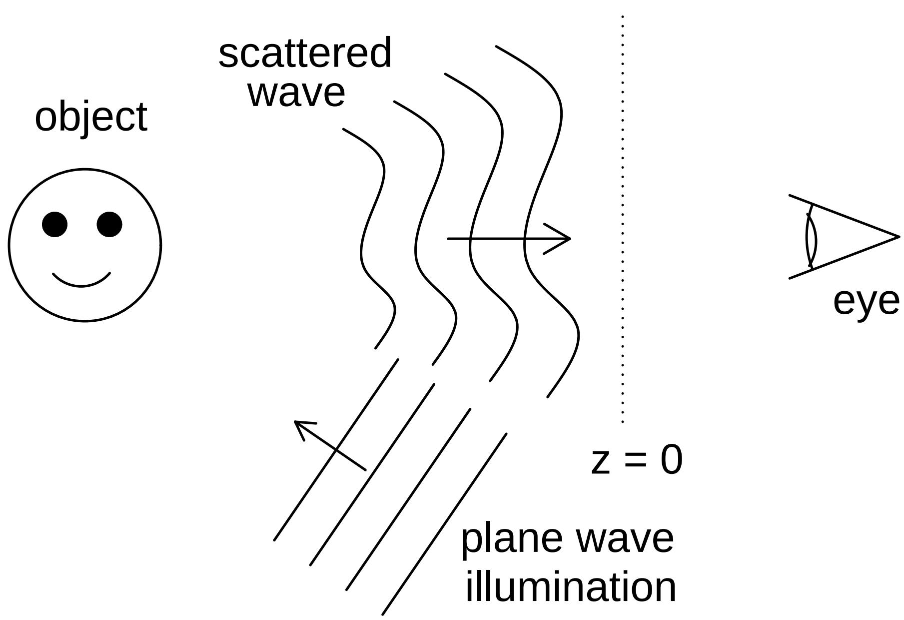
Question: Why is it so easy to tell the difference between an object and a photograph of the object?

Some points:

- We percieve scattered light in either case
- Get same irradiance from each

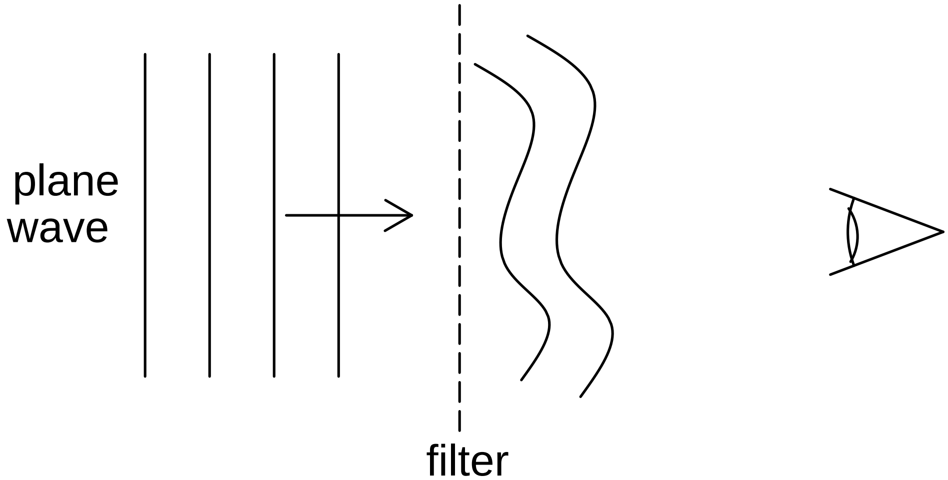
To get 3D effect, recreate complete scattered field

Real object:



In some plane $z = 0$, have $E(x, y, 0) = A(x, y)$

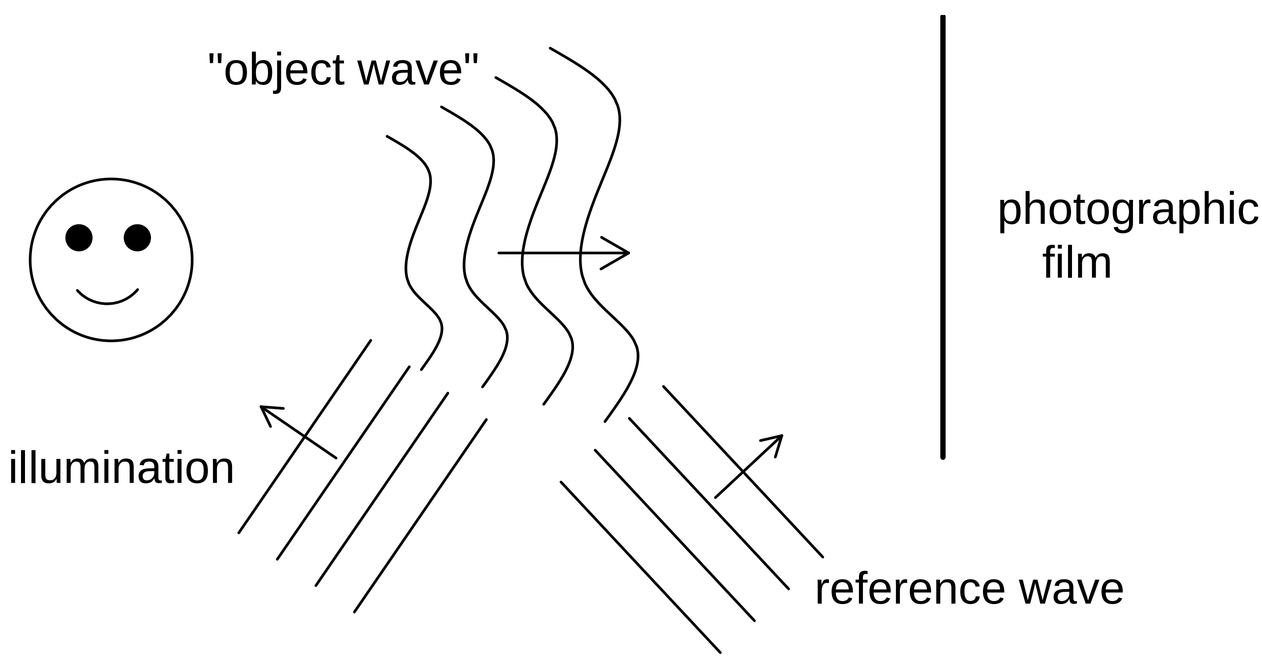
Could recreate field with filter $t(x, y) \propto A(x, y)$



Since $E_{\text{out}}(x, y) = tE_0 \propto A(x, y)$
propagates as original E

How to make filter?

Easiest way is with interference



At film, get interference pattern

$$E_{\text{tot}} = E_{\text{obj}} + E_{\text{ref}}$$

Record irradiance

$$|E_{\text{tot}}|^2 = |E_{\text{obj}}|^2 + |E_{\text{ref}}|^2 + E_{\text{obj}}E^*_{\text{ref}} + E^*_{\text{obj}}E_{\text{ref}}$$

Make $t \propto |E_{\text{tot}}|^2$

Film contains information about both waves

To observe, illuminate film with same E_{ref}

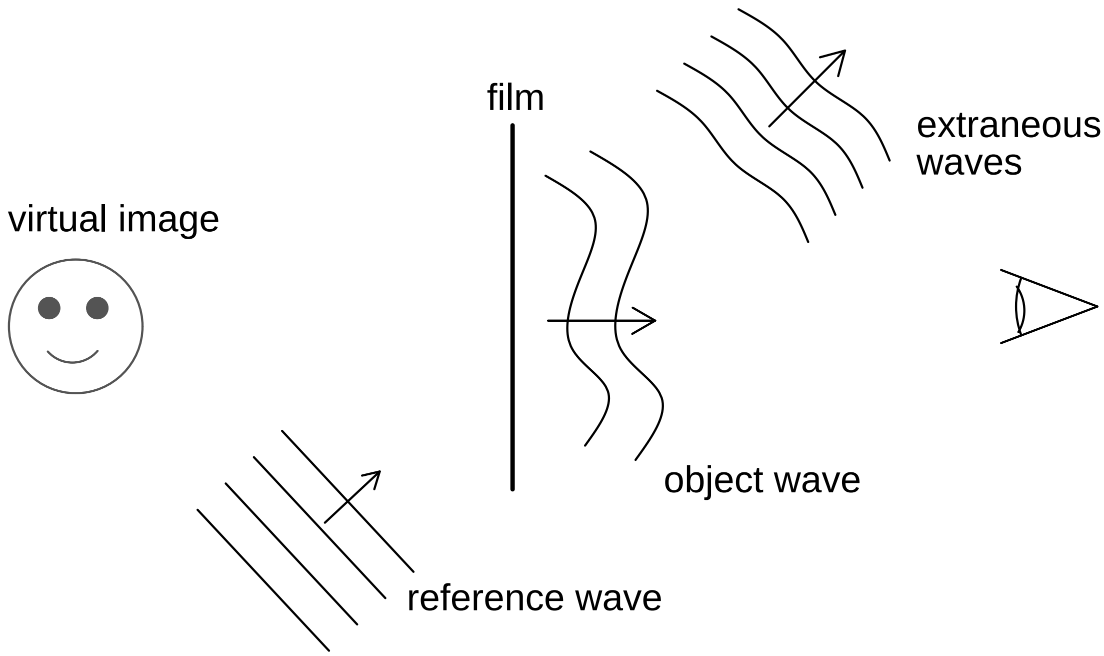
Then $A(x, y) = t(x, y)E_{\text{ref}}$

$$A(x, y) \propto |E_{\text{obj}}|^2 E_{\text{ref}} + |E_{\text{ref}}|^2 E_{\text{ref}} + E^*_{\text{obj}}(E_{\text{ref}})^2 + \boxed{E_{\text{obj}}|E_{\text{ref}}|^2}$$

Since E_{ref} is plane wave, $|E_{\text{ref}}|^2$ is constant

So boxed term $\propto E_{\text{obj}}$, = desired field

Other terms:
waves propagating in other directions



Percieve 3D representation of object

Of course, have demo!

Simple holograph method requires laser

Possible to avoid:
    holograph on credit card uses ambient light

Laser method easiest to understand
    - also works best

Summary:

- Lens computes Fourier transform
- Put filter in transform plane
  - low pass = aperture: smooths image
  - high pass = spot: accentuates edges
- Use filters for phase contrast imaging
  - dark field = opaque spot
  - schlieren = straight edge
  - bright field = phase plate
- Holograph = filter recreates object field
  - make filter using interference