Interferometers

Last time, described laser beams

Explained how they propagate
in free space, optical systems

Today: Interferometers
Practical application of interference

Outline:

- Review interference
- Michelson interferometer
- Thin film interference
- Fabry-Perot interferometers

All from Hecht chapter 9

Next time: interference with incoherent sources

Interference

Looked at interference previously (lecture 13)

Then proceeded to transforms and diffraction

Today: go back and look at applications

For now consider monochromatic light
frequency ω

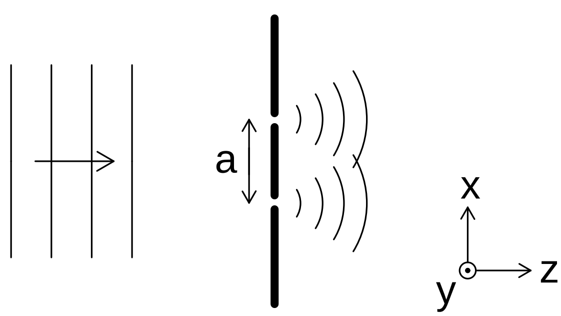
Polychromatic light next time

Basic interference formula:

$$E_{\text{tot}}(\mathbf{r},t) = E_1(\mathbf{r},t) + E_2(\mathbf{r},t)$$

$$|E_{\text{tot}}|^2 = |E_1|^2 + |E_2|^2 + E_1^* E_2 + E_1 E_2^*$$

Review simple example: two slit inteference



Setup:

- Slit width b, separation a along x
- Length along $y = L$: look at $y = 0$
- Incident amplitude E_0

From Fraunhofer formula:

$$E_{\text{tot}}(x, 0, d) = -\frac{ibL}{\lambda d}\, E_0\, e^{ikd}\, \text{sinc}\left(\frac{kxb}{2d}\right)\left(1 + e^{-ikxa/d}\right)$$

and

$$|E_{\text{tot}}|^2 = \left(\frac{bL}{\lambda d}\right)^2 \text{sinc}^2\left(\frac{kxb}{2d}\right)\left|1 + e^{-ikxa/d}\right|^2$$

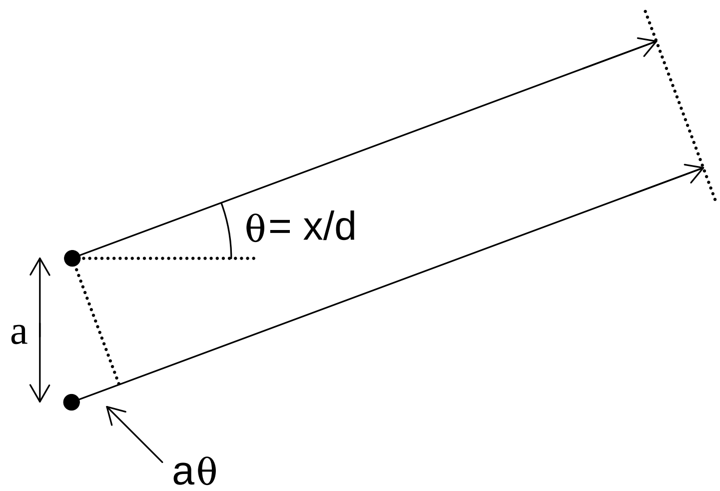
Interference described by $|1 + e^{-ikxa/d}|^2$ term

$$\begin{aligned}\left|1 + e^{-ikxa/d}\right|^2 &= \left(1 + e^{-ikxa/d}\right)\left(1 + e^{ikxa/d}\right) \\ &= 1 + 1 + e^{ikxa/d} + e^{-ikxa/d} \\ &= 2 + 2\cos\left(\frac{kxa}{d}\right)\end{aligned}$$

Cosine term from interference of E_1 and E_2

Interference phase $= kxa/d$

Get same result from geometrical picture



Phase difference between E_1 and $E_2 = ka\theta$
$= kxa/d$

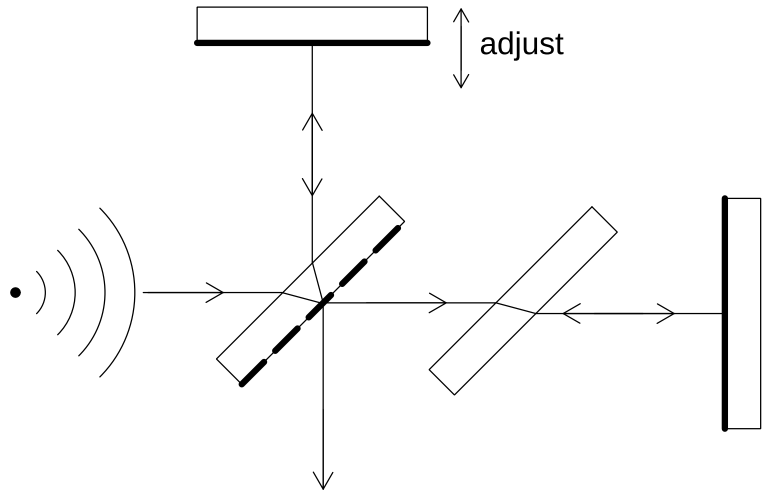
= argument of interference term

Two-slit system is simple *interferometer*
= device that measures interference between two (or more) fields

Allows measurement of phase differences:
often useful

Two-slit interference hard to apply
Look at some better techniques

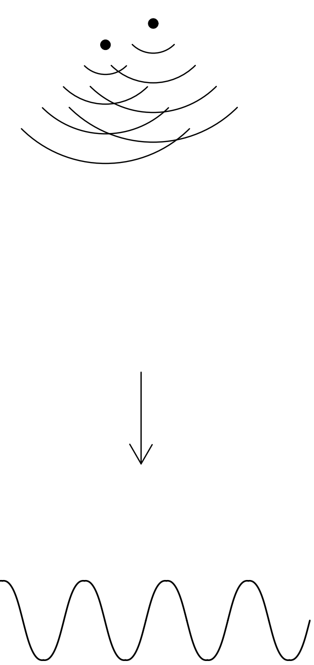
Michelson Interferometer (Hecht 9.4.2)



Heavy black lines = mirror surface
Dashed black line = beamsplitter surface

"Compensation plate" makes arms equivalent

At output, see two sources
reflection from each mirror



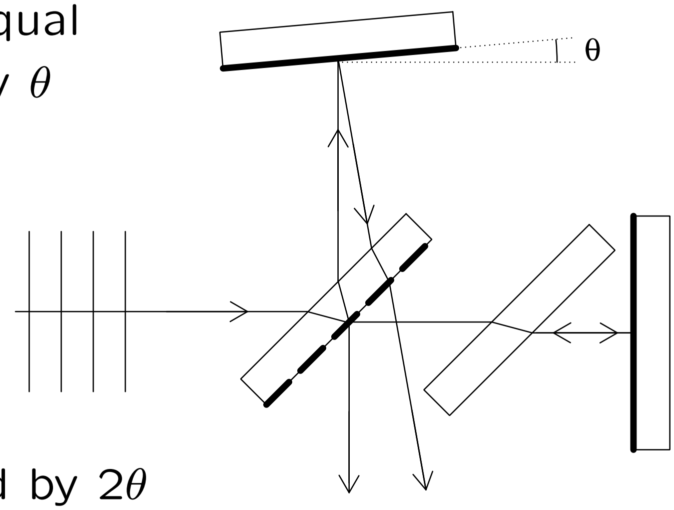
Interference pattern depends on
- mirror positions
- real source location

Mirror tilted:
- sources displaced horizontally

Arm lengths different:
- sources displaced vertically

Example:

- Distant source
- Arm lengths equal
- Mirror tilted by θ



Output beams tilted by 2θ

Get plane wave interference pattern

For small θ:

$$E_1 = E_0 e^{i(kz-\omega t)}$$

$$E_2 = E_0 e^{i(kz+2k\theta x-\omega t)}$$

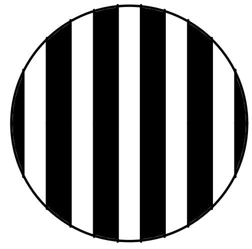
Interference pattern

$$|E_1 + E_2|^2 = |E_0|^2 |1 + e^{i2k\theta x}|^2$$

$$= 2|E_0|^2 \Big[1 + \cos(2k\theta x)\Big]$$

Observe vertical stripes

Periodicity $\Delta x = \lambda/2\theta$



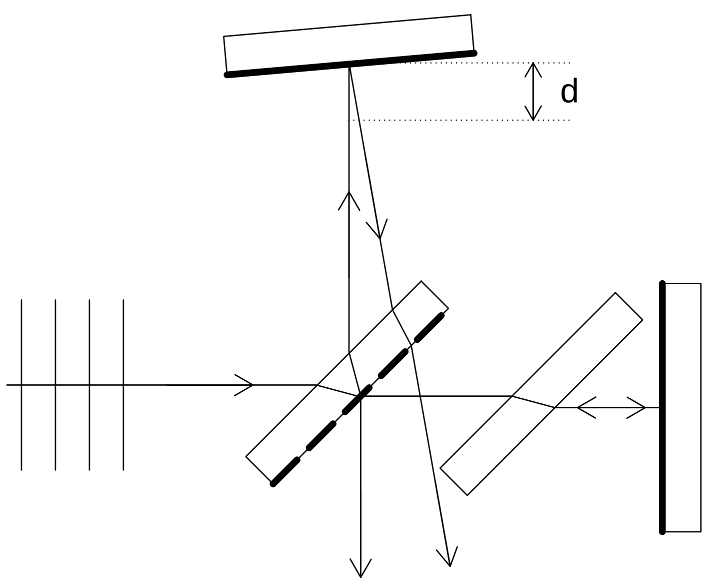
Call stripes "fringes"

As $\theta \to 0$ central fringe expands to fill output

If mirrors not perfectly flat,
get wavy pattern from mirror distortion

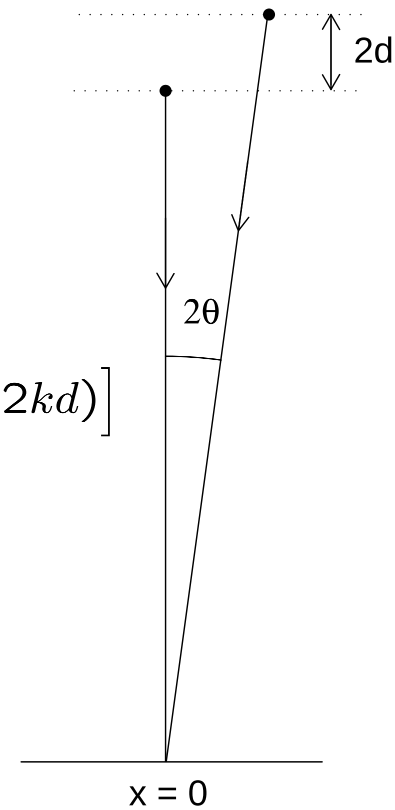
Useful for testing mirrors

What if we also adjust position of mirror?
Offset position by d



For small θ, upper arm length increases by $2d$

Effect on pattern:



$E_1 = E_0 e^{i(kz-\omega t)}$

$E_2 = E_0 e^{i(kz+kx\theta+2kd-\omega t)}$

Get

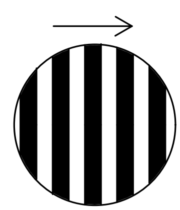
$|E_{\text{tot}}|^2 = 2|E_0|^2\Big[1+\cos(2k\theta x + 2kd)\Big]$

Peaks at $2kx\theta + 2kd = 2\pi m$

integer m

$$x_m = \frac{1}{\theta}\left(\frac{m\lambda}{2} - d\right)$$

Peaks slide across field as d changes



As $\theta \to 0$, pattern is uniform

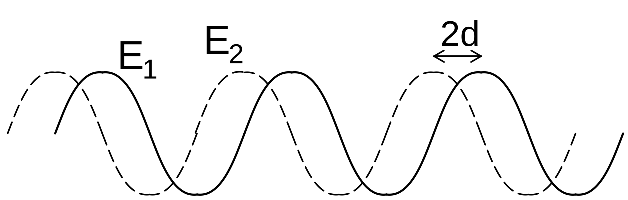
- oscillates between bright and dark with d

Periodicity in d: $2k\Delta d = 2\pi$

$$\Delta d = \frac{\lambda}{2}$$

Change d by $\lambda/4$, output changes bright $\to$ dark

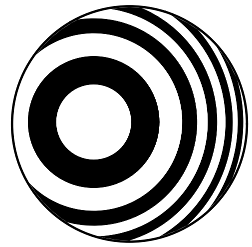
Easy to visualize how waves interfere:



What if source not at infinity?

Get interference of spherical waves, not plane waves

Observe rings, not stripes



Tilting θ adjusts center of rings

Changing d makes rings expand or contract

Obtain uniform output when $d \approx 0$

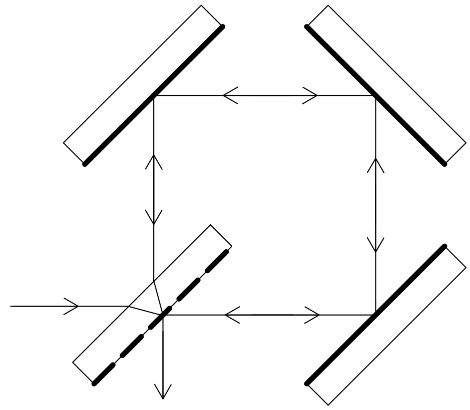
Question: If interferometer is adjusted to give uniform dark output, where is the energy going?

Applications:

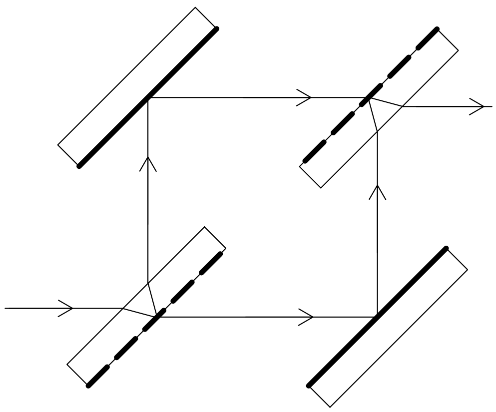
- Originially for testing aether theory
- Test surface accuracy of optics
  Variant: Twyman-Green interferometer
  (Hecht 9.8.2)
- Measure index of refraction of gases
  - Put gas cell in one arm, vary pressure
  - Count fringes
- FTIR spectroscopy
  - More complicated, polychromatic source

Other Interferometers

Sagnac →



Beams travel same path
Senstive to rotations



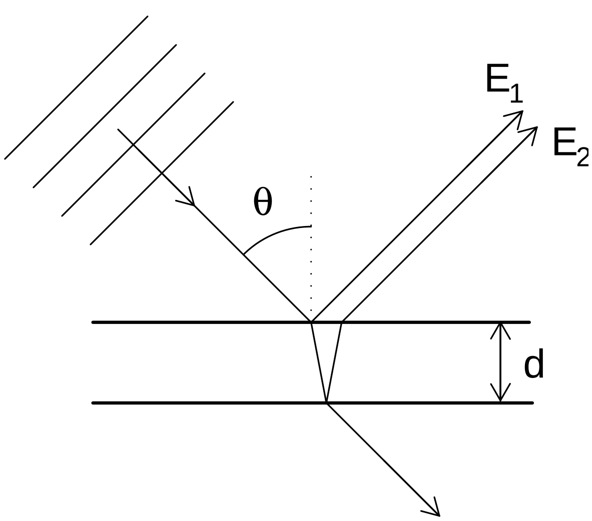
← Mach-Zehnder

Completely independent paths
Used as fiber optics switch

Parallel Plate Interferometer (Hecht 9.4.1)

Very simple setup:

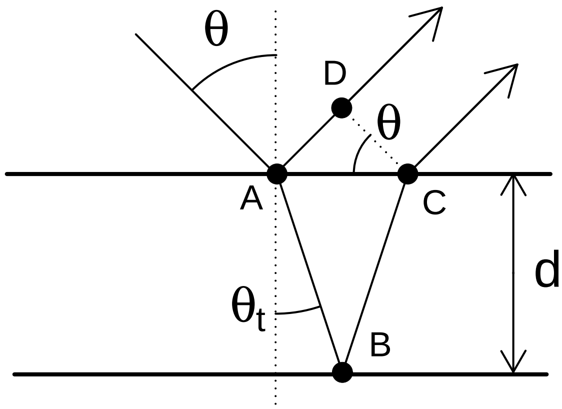
Plane wave incident on glass plate



Look at interference
of reflected beams

What is phase difference?

Get from optical path difference:



OPL for $E_1 = \overline{AD}$

OPL for $E_2 = n(\overline{AB} + \overline{BC})$

From geometry $\overline{AB} = \overline{BC} = \dfrac{d}{\cos\theta_t}$

Also have $\overline{AC} = 2d\tan\theta_t$
and $\overline{AD} = \overline{AC}\sin\theta = 2d\tan\theta_t\sin\theta$

Then

$$\Delta\mathcal{S} = 2n\overline{AB} - \overline{AD} = \frac{2nd}{\cos\theta_t} - \frac{2d\sin\theta_t\sin\theta}{\cos\theta_t}$$

Use $\sin\theta = n\sin\theta_t$

$$\Delta\mathcal{S} = \frac{2nd}{\cos\theta_t} - \frac{2nd\sin^2\theta_t}{\cos\theta_t} = \frac{2nd(1-\sin^2\theta_t)}{\cos\theta_t}$$

$$= \frac{2nd\cos^2\theta_t}{\cos\theta_t}$$

So $\Delta\mathcal{S} = 2nd\cos\theta_t = 2d\sqrt{n^2 - \sin^2\theta}$

However, get additional phase shift from reflection

Fresnel relations: if no TIR,
π phase shift for internal vs. external reflection

$$\text{Then } |E_{\text{tot}}|^2 = |E_0|^2 \left|1 - e^{ik\Delta\mathcal{S}}\right|^2$$
$$= 2|E_0|^2 \Big[1 - \cos(2nkd\cos\theta_t)\Big]$$

Note reflected power $\propto |E_{\text{tot}}|^2$ oscillates with θ

Zero when
$$\cos\theta_t = \frac{2\pi m}{2nkd} = \frac{m\lambda}{2nd} \quad \text{for integer } m$$

Note interference depends on λ

Reason why oil films, soap bubbles look colored:
For some θ, blue light has a maximum
and red light has a minimum

Question: Why don't we see colors in light reflected from ordinary glass windows?

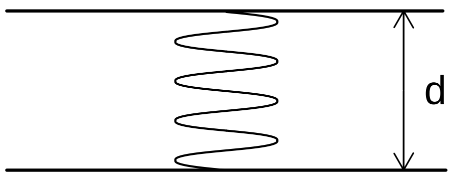
Note if $\theta = 0$, then $\theta_t = 0$

No reflection when $2nd = m\lambda$

or $d = m\dfrac{\lambda'}{2}$

λ' = wavelength in medium

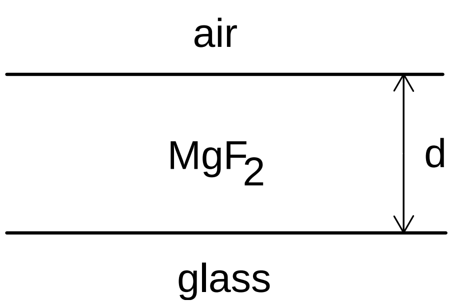
Simple picture:



Perfect transmission when wavelengths "fit" medium

- standard resonance condition

Use this idea for anti-reflection coating



Put layer of MgF_2 on glass air interface

$n = 1.38$

Get reflection from both surfaces,

set thickness so that waves cancel

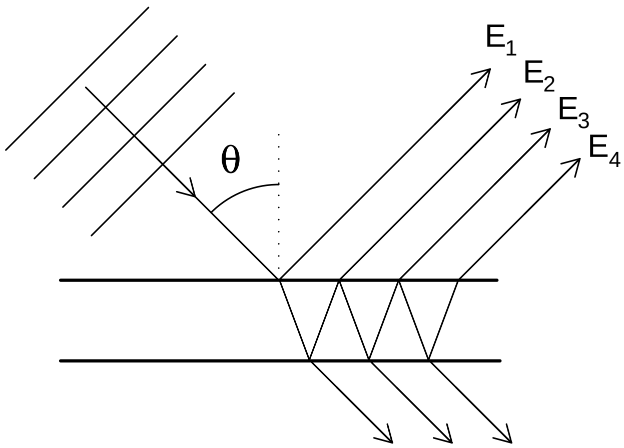
Amplitudes E_1 and E_2 not equal: get $R \approx 1\%$

- do better with multiple layers (Hecht 9.7)

Fabry-Perot Interferometer (Hecht 9.6)

We considered only one reflection
from each surface

Really multiple reflections



When R is not small, need to sum all reflections
Mirrored plate = *Fabry-Perot interferometer*

Have

$$E_{\text{ref}} = \sum_{N=1}^{\infty} E_N$$

We can evaluate this

Use:

t = amplitude transmittance air → glass
t' = amplitude transmittance glass → air
r = amplitude reflectance air → air
r' = amplitude reflectance glass → glass

Get from Fresnel equations

Look at each term

Suppose incident field E_0

First reflection just reflects air $\rightarrow$ air: $E_1 = rE_0$

Second reflection:

transmit air $\rightarrow$ glass: t

reflect glass $\rightarrow$ glass: r'

transmit glass $\rightarrow$ air: t'

Also acquires phase $e^{i\delta}$ with $\delta = 2nkd\cos\theta_t$

So $E_2 = tr't'e^{i\delta}E_0$

Third reflection:

Like E_2 but two additional reflections r'

and additional phase shift $e^{i\delta}$

So $E_3 = tr'^3t'e^{2i\delta}E_0$

Get additional factor of $(r')^2e^{i\delta}$ for each order

Generally

$$E_N = tt'\,(r')^{2N-3}\,e^{(N-1)i\delta}\,E_0$$

(but $N = 1$ is special)

So total reflected field is

$$E_{\text{ref}} = \left[r + tt'r'e^{i\delta} \left(1 + r'^2 e^{i\delta} + r'^4 e^{2i\delta} + \ldots \right) \right] E_0$$

Terms in parentheses are geometric sum:

$$1 + r'^2 e^{i\delta} + r'^4 e^{2i\delta} + \cdots = \sum_{N=0}^{\infty} x^N$$

for $x = r'^2 e^{i\delta}$

Then

$$\sum_{N=0}^{\infty} x^N = \frac{1}{1-x} = \frac{1}{1 - r'^2 e^{i\delta}}$$

So

$$E_{\text{ref}} = \left(r + \frac{tt'r'e^{i\delta}}{1 - r'^2 e^{i\delta}} \right) E_0$$

Can simplify further:

Still have $r' = -r$

Also, for nonabsorbing medium have $tt' = 1 - r^2$

Can prove from Fresnel, or see Hecht 4.10

Substitute, get

$$E_{\text{ref}} = \left[r - \frac{(1 - r^2) r e^{i\delta}}{1 - r^2 e^{i\delta}} \right] E_0$$

Simplify to

$$E_{\text{ref}} = \frac{r(1 - e^{i\delta})}{1 - r^2 e^{i\delta}} \, E_0$$

Then irradiance

$$I_{\text{ref}} = r^2 \frac{|1 - e^{i\delta}|^2}{|1 - r^2 e^{i\delta}|^2} \, I_0$$

$$\boxed{= \frac{2R(1 - \cos\delta)}{1 + R^2 - 2R\cos\delta} \, I_0}$$

for $R = r^2$ = reflectance of single surface

I_0 = incident irradiance

Work out transmission in similar way

Find $E_{\text{trans}} = \left(\dfrac{1 - r^2}{1 - r^2 e^{i\delta}} \right) E_0$

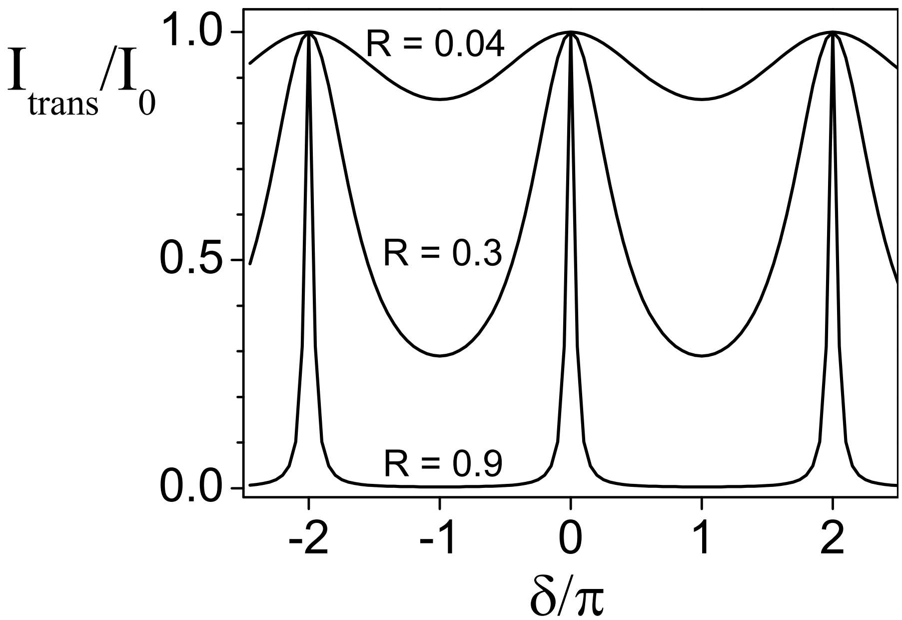
and

$$\boxed{I_{\text{trans}} = \frac{1 - 2R + R^2}{1 + R^2 - 2R\cos\delta} \, I_0}$$

Find that $I_{\text{trans}} = I_0 - I_{\text{ref}}$ as expected

Recall $\delta = 2nkd\cos\theta_m$
depends on d, λ, θ

Plot I_{trans}/I_0 as function of δ



Transmission = 1 when $\delta = 2\pi m$

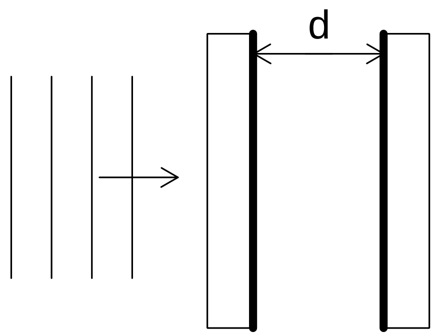
Same condition for reflection = 0 in original calc

Peaks narrower for higher R:
For $R \approx 1$, full width at half-max = $2(1 - R)$

Can get R up to 0.99999

Very narrow transmission peaks:
useful for spectroscopy

Fabry-Perot spectrometer:

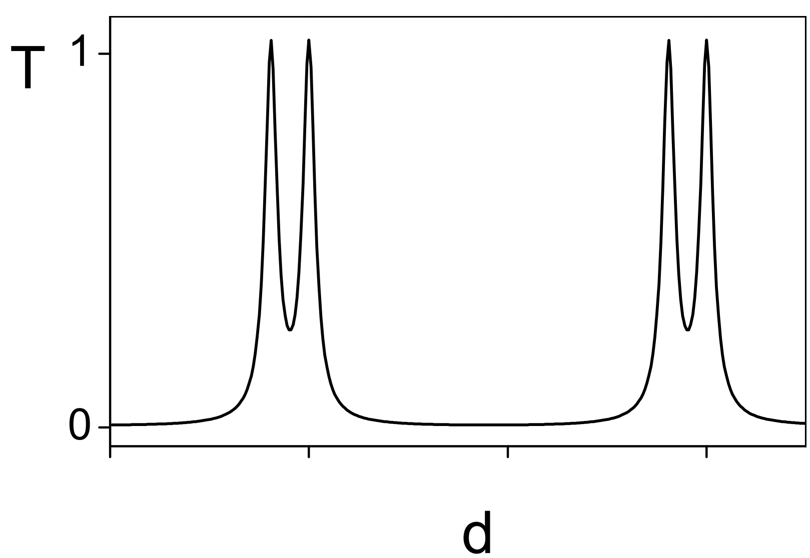


Scan mirror separation d:

Large transmission when $d = m\lambda/2$

Suppose source has two frequencies ω_1 and ω_2

Get two transmission peaks



Peaks at $d = m\lambda_1/4$ and $d = m\lambda_2/4$

(large integer m)

Peaks resolved if $\delta_1 - \delta_2 > \Delta \approx 2(1-R)$
where $\delta_1 = 2k_1 d$ and $\delta_2 = 2k_2 d$

Need $k_1 - k_2 > \dfrac{1-R}{d}$

or $\omega_1 - \omega_2 > (1-R)\dfrac{c}{d}$

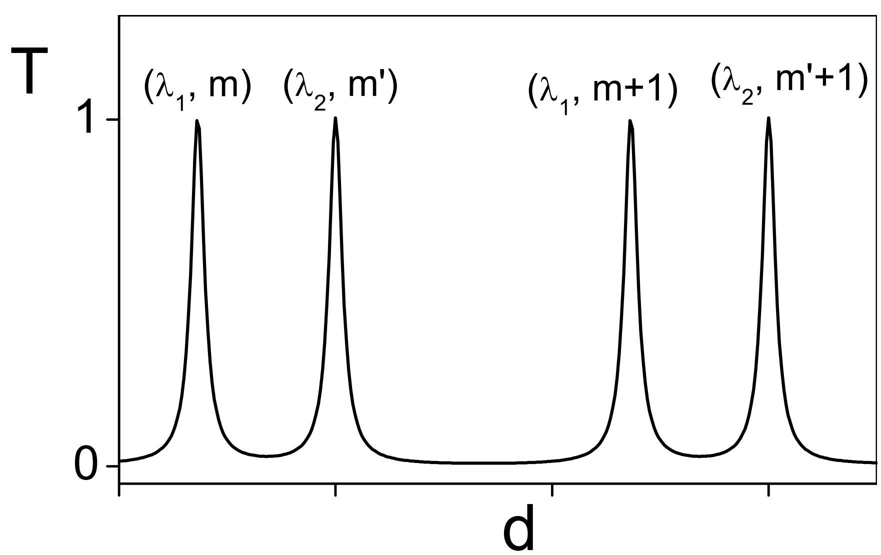
If $R = 0.999$ and $d = 3$ cm, get $\Delta\omega = 10^7$ rad/s
or $\Delta\nu = 1.6$ MHz

This is incredible resolution:

Optical frequency $= 6 \times 10^{14}$ Hz
so $\Delta\nu/\nu \approx 10^{-9}$

If $\Delta\omega > c/2d$, peaks with different m's overlap

Can't tell which m is which, so $\Delta\omega$ is ambiguous



Typically use grating spectrometer to measure m

Summary:

- Interferometer = device that measures phase
- Michelson: beamsplitter, mirrors control light
- Thin plate: two-beam interference
  Can eliminate reflection
- Fabry-Perot: multiple-beam interference
  Useful for spectroscopy