Polarization of Light

Last time, finished interference discussion

Considered interference with extended sources

Next three lectures: polarization
explore vector nature of light

Today: basic ideas

No Fourier transforms required!

Outline:

- Notation and conventions
- Polarization states
- Basis states
- Unpolarized light
- Polarization and quantum mechanics

Follow book more closely again: Chapter 8

Note, book neglects complex notation until §8.13
- we'll use from beginning

Next time:
Generating and manipulating polarized light

Conventions

Have been ignoring vector nature of $\mathbf{E}$

- Not very important for diffraction
- Simplifies calculations

But it is important for many things

Already saw in Fresnel relations

When is polarization important?

When can we ignore polarization?

- Imaging problems
- Interference/diffraction for beams at small angles

When is it important?

- Transmittance/reflectance calcs
- Superposing beams at large angles
- Detailed interactions with matter:

  Birefringent materials, surface effects, atomic/molecular transitions, nonlinear optics, magneto-optical effects, electro-optical effects, . . .

Beyond this course, but common applications

Review what we know:

Plane wave solution is

$$\mathbf{E}(\mathbf{r},t) = \mathbf{E}_0 e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}$$

complex vector amplitude $\mathbf{E}_0$, know $\mathbf{k}\cdot\mathbf{E}_0 = 0$

Standard configuration: take $\mathbf{k} = k\hat{\mathbf{z}}$
Then $\mathbf{E}_0 = E_{0x}\hat{\mathbf{x}} + E_{0y}\hat{\mathbf{y}}$

Real fields are

$$E_x(z,t) = |E_{0x}|\cos(kz - \omega t + \phi_x)$$
$$E_y(z,t) = |E_{0y}|\cos(kz - \omega t + \phi_y)$$

Phases ϕ_x, ϕ_y are independent

Relation between phases sets polarization
- along with amplitudes $|E_{0x}|, |E_{0y}$

Define phase difference $\varepsilon = \phi_y - \phi_x$

Write

$$E_x(z,t) = |E_{0x}|\cos(kz - \omega t + \phi)$$
$$E_y(z,t) = |E_{0y}|\cos(kz - \omega t + \phi + \varepsilon)$$

don't worry about overall phase ϕ

In complex form:

$$E_x(z,t) = |E_{0x}|e^{i\phi}e^{i(kz-\omega t)}$$

$$E_y(z,t) = |E_{0y}|e^{i(\phi+\varepsilon)}e^{i(kz-\omega t)}$$

Define complex amplitude

$$E_0 = \sqrt{|E_{0x}|^2 + |E_{0y}|^2}\, e^{i\phi}$$

and polarization vector = Jones vector =

$$\hat{\jmath} = \frac{|E_{0x}|}{|E_0|}\,\hat{\mathbf{x}} + \frac{|E_{0y}|}{|E_0|}e^{i\varepsilon}\,\hat{\mathbf{y}}$$

Then $\mathbf{E}(z,t) = E_0\hat{\jmath}e^{i(kz-\omega t)}$

Polarization States (Hecht 8.1)

Look at $\mathbf{E}$ for different ε

Take $\phi = 0$ for simplicity

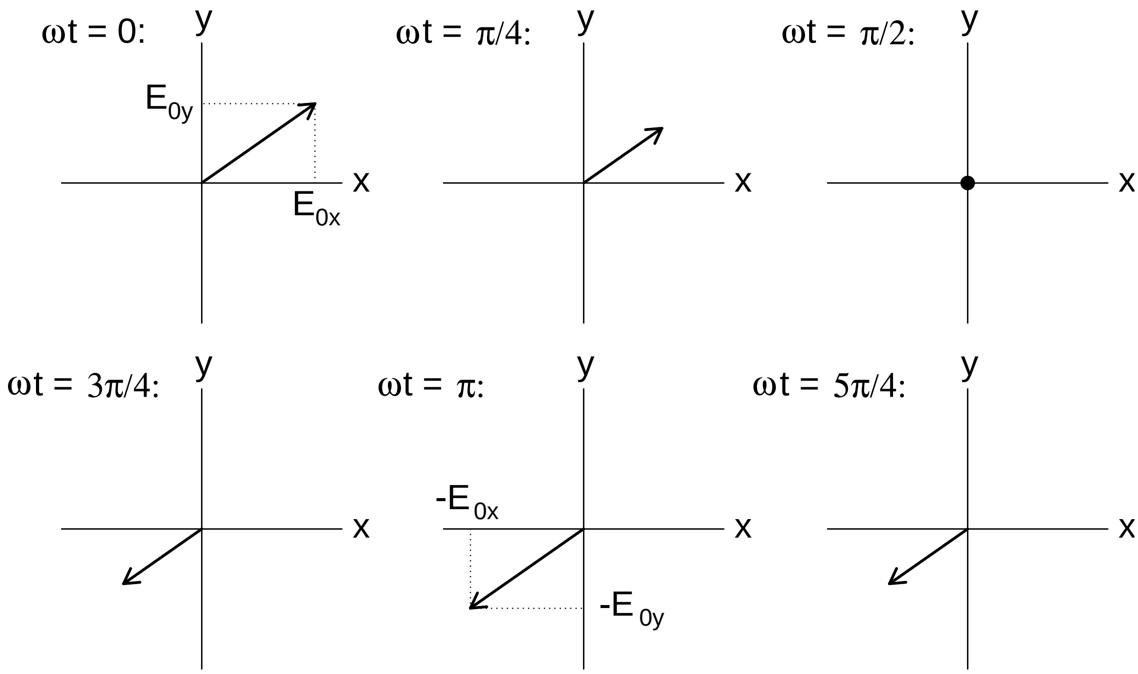
Suppose $\varepsilon = 0$

Then $E_x(z,t) = |E_{0x}|\cos(kz-\omega t)$

$E_y(z,t) = |E_{0y}|\cos(kz-\omega t)$

When E_x is maximum, so is E_y
When E_x is zero, so is E_y

Trace $\mathbf{E}(t)$ in $z = 0$ plane:



$\mathbf{E}$ oscillates along line:

state called *linear polarization*

In 3D, $\mathbf{E}$ oscillates in plane

plane called *plane of polarization*

Snapshot of $\mathbf{E}(z, t)$ looks like cosine function

lying in plane of polarization

Used linearly polarized light in original derivations

only $\hat{\mathbf{x}}$ or $\hat{\mathbf{y}}$

More generally, allow any plane

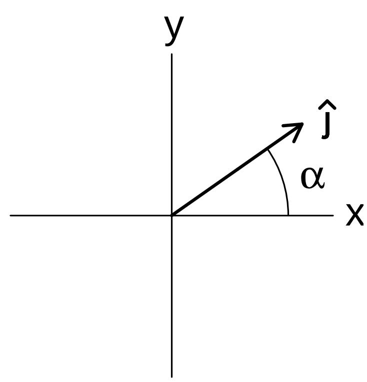
Complex notation:

$$\mathbf{E}(z,t) = E_0 \hat{\jmath} e^{i(kz-\omega t)}$$

with

$$\hat{\jmath} = \cos\alpha\, \hat{\mathbf{x}} + \sin\alpha\, \hat{\mathbf{y}}$$

for $\alpha = \tan^{-1}(E_{0y}/E_{0x})$



Plane of polarization spanned by $\mathbf{k}$ and $\hat{\jmath}$

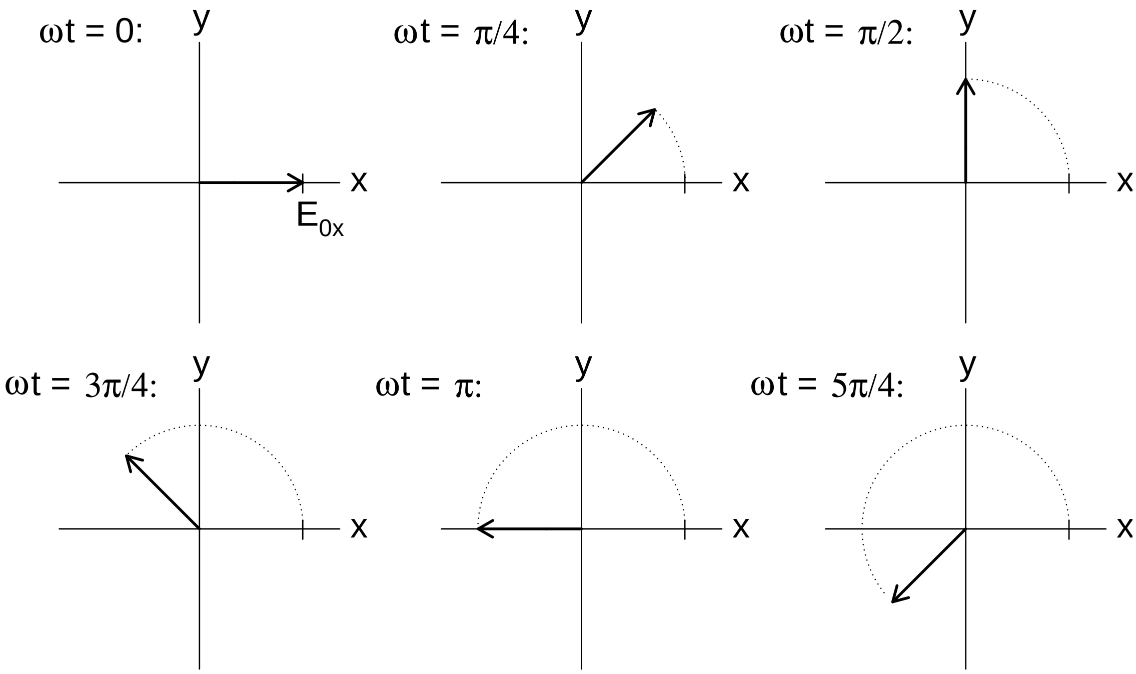
Question: How are polarization states with $\hat{\jmath}$ and $-\hat{\jmath}$ different?

Another special case: $\varepsilon = \pm\pi/2$
and $|E_{0x}| = |E_{0y}|$

Then $E_x(z,t) = |E_{0x}| \cos(kz - \omega t)$

and $E_y(z,t) = |E_{0x}| \cos(kz - \omega t \pm \pi/2)$
$= \mp |E_{0x}| \sin(kz - \omega t)$

Plot in $z = 0$ plane for $\varepsilon = +\pi/2$
So $E_x(z,t) = |E_{0x}| \cos(\omega t)$
$E_y(z,t) = |E_{0x}| \sin(\omega t)$



$\mathbf{E}$ rotates in circle

Called *circular polarization*

Note if $\varepsilon = -\pi/2$, $\mathbf{E}$ rotates in opposite direction

Call $\varepsilon = -\pi/2$ *right-circular* polarization (RCP)
$\varepsilon = +\pi/2$ *left-circular* polarization (LCP)

At fixed t, $\mathbf{E}(z)$ traces out helix = corkscrew
RCP: right-handed screw along $\widehat{\mathbf{k}}$
LCP: left-handed screw along $\widehat{\mathbf{k}}$

RCP vs. LCP very easy to mix up

LCP:

- For fixed z, $\mathbf{E}$ rotates in counter-clockwise sense
  - when light propagating toward observer

- For fixed t, $\mathbf{E}$ rotates in clockwise sense as z increases

Because of sign difference in $kz - \omega t$ factor

Also, if complex convention is $e^{i(\omega t - kz)}$
then sign of phases reversed

Fortunately, rarely need to know which is which

Complex notation:

$$\mathbf{E}(z,t) = E_0 \hat{\jmath} e^{i(kz - \omega t)}$$

with

$$\hat{\jmath} = \frac{1}{\sqrt{2}}(\hat{\mathbf{x}} - i\hat{\mathbf{y}}) \qquad \text{(RCP)}$$

$$\hat{\jmath} = \frac{1}{\sqrt{2}}(\hat{\mathbf{x}} + i\hat{\mathbf{y}}) \qquad \text{(LCP)}$$

and $E_0 = \sqrt{2} E_{0x}$

Question: As $\mathbf{E}$ rotates, amplitude is always E_{0x}. So why do we have $E_0 = \sqrt{2} E_{0x}$?

Linear and circular are only special polarizations

General case: *elliptical polarization*

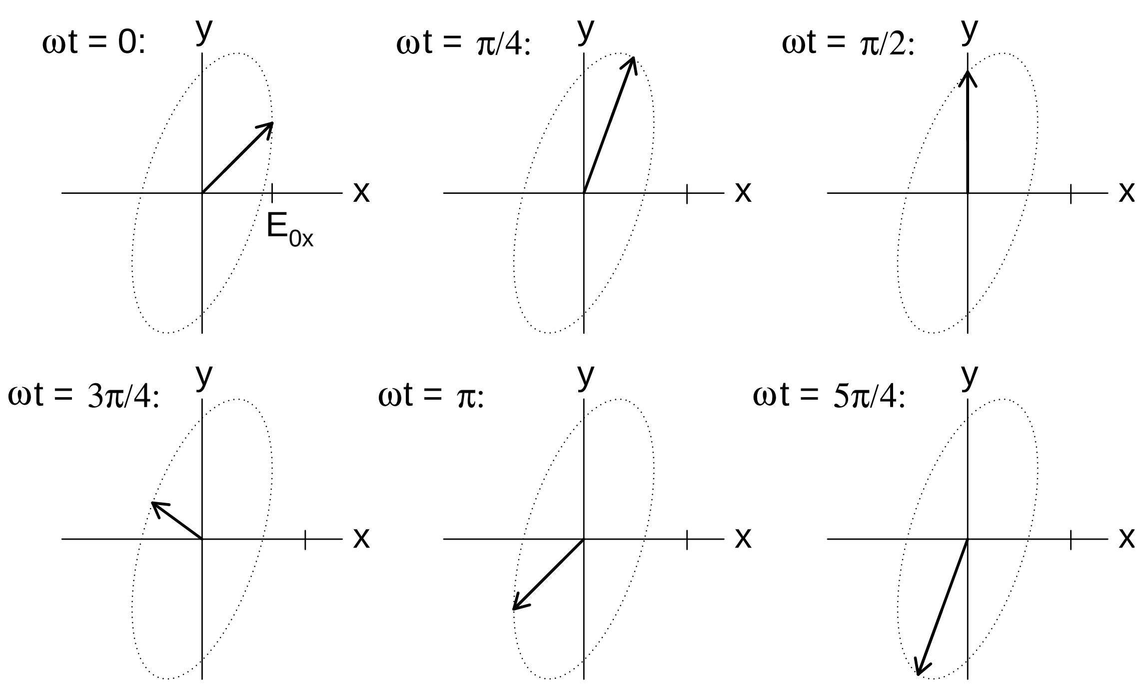
Example:

$$|E_{0y}| = 2|E_{0x}| \qquad \varepsilon = \pi/3$$

Then for $z = 0$:

$$E_x(t) = |E_{0x}| \cos(\omega t)$$
$$E_y(t) = 2|E_{0x}| \cos(\omega t - \pi/3)$$

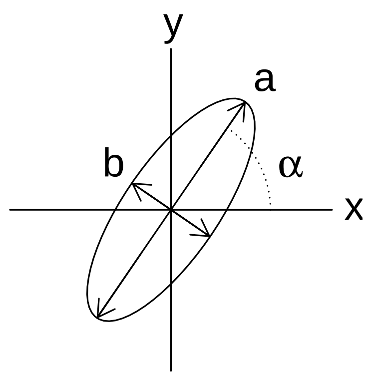
Traces out ellipse:



Equation of ellipse from $|E_{0x}|$, $|E_{0y}|$ and ε:

$$\frac{E_x^2}{|E_{0x}|^2} + \frac{E_y^2}{|E_{0y}|^2} - \frac{2E_xE_y\cos\varepsilon}{|E_{0x}||E_{0y}|} = \sin^2\varepsilon$$

Characterize by angle α and eccentricity $e = a/b$



Define $p = |E_{0y}|/|E_{0x}|$

Then angle of axes α:

$$\tan 2\alpha = \frac{2p\cos\varepsilon}{1-p^2} \qquad \text{(Note: axis ambiguous)}$$

and eccentricity e:

$$e^2 = \frac{1+p^2+\sqrt{1+2p^2\cos 2\varepsilon + p^4}}{1+p^2-\sqrt{1+2p^2\cos 2\varepsilon + p^4}}$$

For example shown, $\alpha = 73.1^\circ$ and $e = 2.48$

These formulas hard to find!

General properties:

- eccentricity = 0 for $\varepsilon = 0$
  (linear polarization)
- eccentricity max for $\varepsilon = \pm\pi/2$
  (circ. if $|E_{0x}| = |E_{0y}|$)
- right-handed rotation for $\varepsilon < 0$
- left-handed for $\varepsilon > 0$

Complex notation:

$$\mathbf{E}(z,t) = E_0 \hat{\jmath} e^{i(kz-\omega t)}$$

with

$$E_0 = \sqrt{|E_{0x}|^2 + |E_{y0}|^2}$$

$$\hat{\jmath} = \cos\beta\,\hat{\mathbf{x}} + e^{i\varepsilon}\sin\beta\,\hat{\mathbf{y}}$$

$$\tan\beta = p = \frac{|E_{0y}|}{|E_{0x}|}$$

Note β not the same as ellipse angle α

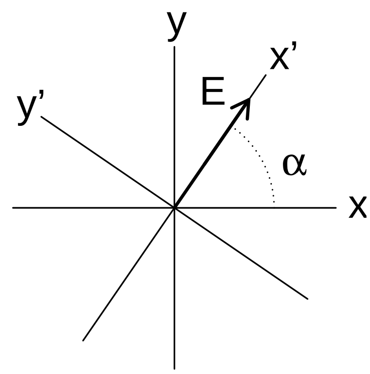
Find $\tan(2\alpha) = \tan(2\beta)\cos(\varepsilon)$

Choice of Basis

So far have used x, y coordinates

Problems often easier in different coords

Example:
linearly polarized light at angle α



Define $x' = x \cos\alpha + y \sin\alpha$

$$y' = -x \sin\alpha + y \cos\alpha$$

Then light polarized along x'

$$\mathbf{E}(z,t) = E_0 \hat{\mathbf{x}}' e^{i(kz-\omega t)}$$

Set of unit vectors = *basis*

Can also use complex $\hat{\jmath}$'s as basis

Most often use circular states

$$\hat{\mathbf{e}}_{\mathcal{R}} = \frac{1}{\sqrt{2}}(\hat{\mathbf{x}} - i\hat{\mathbf{y}}) \qquad \hat{\mathbf{e}}_{\mathcal{L}} = \frac{1}{\sqrt{2}}(\hat{\mathbf{x}} + i\hat{\mathbf{y}})$$

Then $\hat{\mathbf{x}} = \frac{1}{\sqrt{2}}(\hat{\mathbf{e}}_{\mathcal{R}} + \hat{\mathbf{e}}_{\mathcal{L}})$

$$\hat{\mathbf{y}} = \frac{i}{\sqrt{2}}(\hat{\mathbf{e}}_{\mathcal{R}} - \hat{\mathbf{e}}_{\mathcal{L}})$$

Useful if circ. polarizations are important for you

Optical elements have simple effect in some bases,
complicated in others

Convenient to go back and forth

For complex bases, look at orthogonality condition

Need two basis vectors $\hat{\mathbf{e}}_1$ and $\hat{\mathbf{e}}_2$, with $\hat{\mathbf{e}}_1 \perp \hat{\mathbf{e}}_2$

For complex vectors, $\perp$ means $\hat{\mathbf{e}}_1^* \cdot \hat{\mathbf{e}}_2 = 0$

Example: if $\hat{\mathbf{e}}_1 = \frac{\sqrt{3}}{2}\hat{\mathbf{x}} + i\frac{1}{2}\hat{\mathbf{y}}$

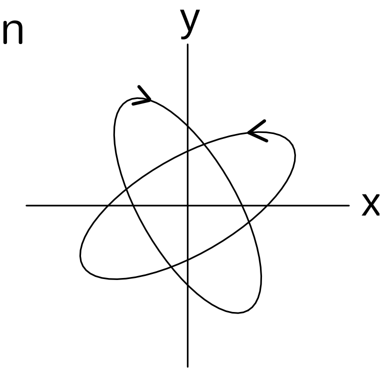
then $\hat{\mathbf{e}}_2 = \frac{1}{2}\hat{\mathbf{x}} - i\frac{\sqrt{3}}{2}\hat{\mathbf{y}}$

since $\hat{\mathbf{e}}_1^* \cdot \hat{\mathbf{e}}_2 = \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} = 0$

In general, $\sin\beta\hat{\mathbf{x}} + e^{i\varepsilon}\cos\beta\hat{\mathbf{y}}$ is orthogonal to
$\cos\beta\hat{\mathbf{x}} - e^{i\varepsilon}\sin\beta\hat{\mathbf{y}}$

Graphically:

Orthogonal polarizations rotated 90°
and opposite sense of rotation



Question: What state is orthogonal to LHC polarization, and does it satisfy $\hat{\mathbf{e}}^* \cdot \hat{\mathbf{e}}_{\mathcal{L}} = 0$?

Unpolarized Light (Hecht 8.1.4)

Extend ideas of coherence

Know $E(t)$ is coherent (over time τ) if

$$\Gamma(\tau) = \langle E(t+\tau)E^*(t)\rangle \approx \left\langle |E|^2 \right\rangle$$

For quasi-monochromatic light $E(t) = E_0 e^{-i(\omega t - \phi)}$
where ϕ fluctuates

Then $\Gamma(\tau) = |E_0|^2 e^{-i\omega t}\left\langle e^{i[\phi(t+\tau)-\phi(t)]}\right\rangle$

Coherent: $\phi \sim$ constant over time τ

Coherence affects polarization

For incoherent light,

$$E_x(z,t) = |E_{0x}| \cos(kz - \omega t + \phi_x)$$
$$E_y(z,t) = |E_{0y}| \cos(kz - \omega t + \phi_y)$$

ϕ_x and ϕ_y vary randomly

All polarization effects average out:
Say light is *unpolarized*

Light can be incoherent but polarized

Suppose $E_x(z,t) = |E_{0x}| \cos(kz - \omega t + \phi)$
$E_y(z,t) = |E_{0y}| \cos(kz - \omega t + \phi + \varepsilon)$

with ϕ fluctuating but ε constant

Then E_x and E_y components fluctuate together

- Alternatively, could just have $E_{0y} = 0$

Either way, see polarization effects

Describe unpolarized light =
equal "mixture" of any two orthogonal states

Add irradiances of each, not fields

$I_{\text{tot}} = I_1 + I_2$

If system transmits $\hat{\mathbf{e}}_1$ with transmittance T_1,
$\hat{\mathbf{e}}_2$ with transmittance T_2

Get $I_{\text{out}} = T_1 I_1 + T_2 I_2$

No interference effects

Example:
sunlight = 50% linear $\|$ + 50% linear $\perp$

Transmission through surface $\langle T \rangle = \dfrac{1}{2}(T_\| + T_\perp)$

Doesn't matter what is $\hat{\mathbf{x}}$, what is $\hat{\mathbf{y}}$

Or: sunlight = 50% RHC + 50% LHC

Suppose some material absorbs all RHC:
Get 50% transmittance

As before, work in whatever basis is easiest
- Here, don't need to recalculate state

Connection to Quantum Mechanics

Mathematics of polarization

= math of quantum two-level system

Examples:

- Electron in magnetic field $\Leftarrow$
- Two atomic levels coupled by laser
- Single proton in NMR

Doesn't mean that light is quantum mechanical!

- means that two-level systems are classical

Develop analogy to electron in field

Apply QM understanding to light

- $\mathbf{k} \leftrightarrow \mathbf{B}$ (magnetic field) - along z
- $\hat{\jmath} \leftrightarrow |\psi\rangle$
- basis states $\leftrightarrow$ basis states
- LHC $\rightarrow$ spin up along z
- RHC $\rightarrow$ spin down along z
- Linear polarized along x = spin along x
- Unpolarized light = mixture states
  (w/ density matrix)

Optical devices

$\Rightarrow$ operators (measurement or unitary)

Connect to photon optics in last two lectures

Summary:

- Linear polarization: $\mathbf{E}$ oscillates in plane
  $\hat{\jmath} = \cos\alpha\,\hat{\mathbf{x}} + \sin\alpha\,\hat{\mathbf{y}}$
- Circular polarization: $\mathbf{E}$ winds in helix
  $\hat{\jmath} = (\hat{\mathbf{x}} \pm i\hat{\mathbf{y}})/\sqrt{2}$
- More generally, $\mathbf{E}$ traces out ellipse
  $\hat{\jmath} = \cos\beta\,\hat{\mathbf{x}} + e^{i\varepsilon}\sin\beta\,\hat{\mathbf{y}}$
- Work in whatever basis is convenient
  - Just like QM
- Unpolarized light:
  mixture of orthogonal states