Phys 531 Lecture 24 15 November 2005

Polarizers

Last time, discussed basics of polarization

Linear, circular, elliptical states

Describe by polarization vector $\widehat{\jmath}$

Today:
How to establish and manipulate polarization

Outline:

- Polarizers
  - Reflection, scattering, dichroism
  - Calculations with polarizers
- Birefringence

Next time: Retarders
- make circular and elliptical polarizations

Jones calculus
- matrix method for calculations

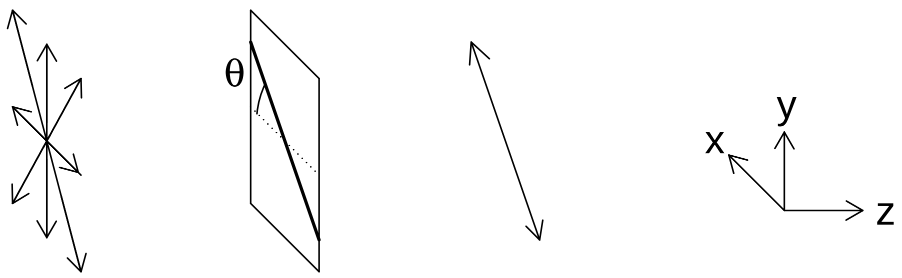
Polarizers (Hecht 8.2)

Most natural light sources are unpolarized

Obtain polarized light with *polarizer*
= "filter" passing only one polarization state

Usually transmit linear polarization

Plane of polarization given by *transmission axis*



Ideal polarizer:
Transmission for $\hat{\jmath} \parallel$ axis = 1
Transmission for $\hat{\jmath} \perp$ axis = 0

In general, say axis = $\mathbf{a}$
and $\mathbf{b} \perp \mathbf{a}$

Then in 1-2 basis, $\hat{\jmath} = j_{\parallel}\mathbf{a} + j_{\perp}\mathbf{b}$
transmit amplitude $j_{\parallel}$

To get: $\hat{\jmath} \cdot \mathbf{a}^* = j_{\perp}$
since $\mathbf{a} \cdot \mathbf{a}^* = 1$ and $\mathbf{b} \cdot \mathbf{a}^* = 0$

Usually $\mathbf{a}$ is real, write

$$\boxed{T = |j_{\parallel}|^2 = |\hat{\jmath}^* \cdot \mathbf{a}|^2}$$

If $\mathbf{a}$ real, have *linear polarizer*

Axis at angle θ, write $\mathbf{a} = \cos\theta\,\hat{\mathbf{x}} + \sin\theta\,\hat{\mathbf{y}}$

If $\hat{\jmath}$ linearly polarized $\hat{\jmath} = \cos\alpha\,\hat{\mathbf{x}} + \sin\alpha\,\hat{\mathbf{y}}$

Then $\hat{\jmath}^* \cdot \mathbf{a} = \hat{\jmath} \cdot \mathbf{a} = \cos\theta\cos\alpha + \sin\theta\sin\alpha$
$= \cos(\theta - \alpha)$

Gives *Malus's Law*:

For linear polarization incident on polarizer,

$$\boxed{I_{\text{out}} = I_{\text{in}}\cos^2(\theta - \alpha)}$$

$\theta - \alpha =$ angle between transmission axis
and incident plane of polarization

But $T = |\hat{\jmath}^* \cdot \mathbf{a}|^2$ is more general

Example:

If $\hat{\jmath}_{\text{inc}} = \hat{\mathbf{e}}_{\mathcal{R}} = \dfrac{\hat{\mathbf{x}} - i\hat{\mathbf{y}}}{\sqrt{2}}$, what is transmission
through linear polarizer at angle θ?

Have $\hat{\jmath}^* \cdot \mathbf{a} = \dfrac{\cos\theta + i\sin\theta}{\sqrt{2}} = \dfrac{1}{\sqrt{2}}e^{i\theta}$

So $T = \dfrac{1}{2}|e^{i\theta}|^2 = \dfrac{1}{2}$ independent of θ

Question: What would it mean if $\mathbf{a}$ were complex?

Other effect:

Light exitting ideal polarizer has $\hat{\jmath}_{\text{out}} = \mathbf{a}$

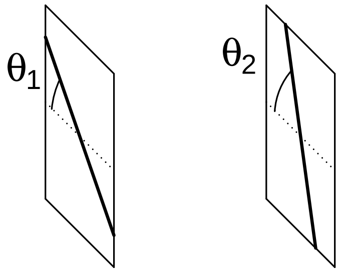
To see, write $\hat{\jmath}_{\text{in}} = j_{\|}\mathbf{a} + j_{\perp}\mathbf{b}$

$\mathbf{a}$ is transmitted
$\mathbf{b}$ is blocked

So $\hat{\jmath}_{\text{out}} = j_{\|}\mathbf{a} \to \mathbf{a}$
(amplitude $j_{\|}$ gives transmission)

Physically, $\perp$ component is absorbed or reflected,
only $\|$ component remains

If two polarizers: first at θ_1, second at θ_2



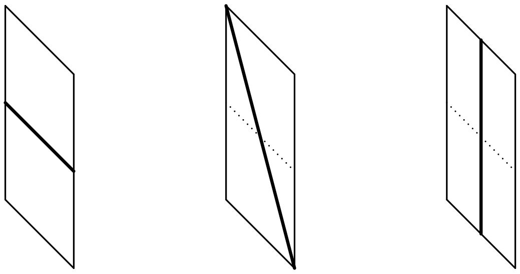
Output of first = polarized along θ_1

Transmission of second = $\cos^2(\theta_2 - \theta_1)$

Original version of Malus's Law

Or say three polarizers:

first at 0°, second at 45°, third at 90°



Without middle polarizer, transmission is zero

With all three, transmission is $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

Question: This seems counterintuitive. Where does the vertical component come from?

Real polarizers aren't perfect:

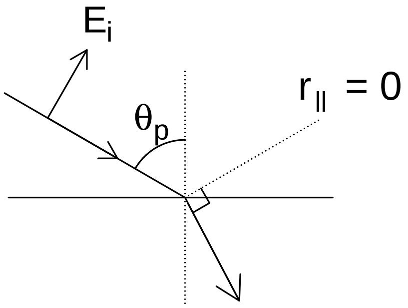
- Transmission for $\hat{\jmath} \parallel \mathbf{a} = T_0 < 1$
  (loss)
- Transmission for $\hat{\jmath} \perp \mathbf{a} = \epsilon > 0$
  (leakage)
- Output light not exactly polarized along $\mathbf{a}$
  (rarely specified)

Values depend on type of polarizer

Discuss types of polarizers

Constructing Polarizers (Hecht 8.3–8.6)

Already know one way to polarize light:
use Brewster's angle



When TM polarized light incident at angle
$\theta_p = \tan^{-1}(n_t/n_i)$

Get $r_\parallel = 0$

Two ways to make polarizer:

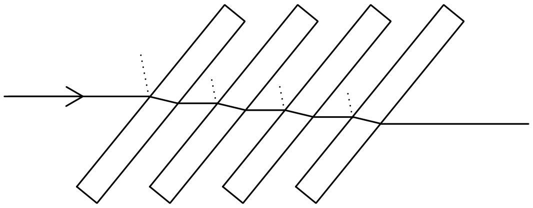
- Use reflected light: get $\perp$ component

Then loss is very high:
- glass, get $R_\perp \approx 0.2 \rightarrow$ lose 80%

Also, leakage is fairly high:
- hard to control angle accurately

- Better: use transmitted light
and many surfaces

Pile of plates polarizer:



Each surface transmits almost all of $I_{\|}$
and fraction $T_{\perp}$ of $I_{\perp}$
for glass, $T_{\perp} \approx 0.8$

For N plates, total $\perp$ transmission $= T_{\perp\text{tot}} = T_{\perp}^{2N}$

Say glass plates, $N = 10$: $T_{\perp\text{tot}} = 0.01$

Typically get total $T_{\|\text{tot}} = 0.5$

Pile of plates simple and robust
Good extinction for large N

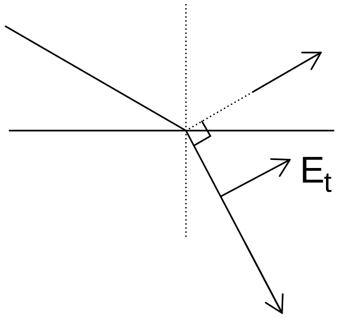
But often awkward to use:
thick, requires collimated light

Rarely used now in optics
Similar methods used for x-rays, other radiation

Polarization by Scattering

Brewster effect based on scattering properties:

Recall Brewster angle when $\mathbf{k}_{\text{ref}} \parallel \mathbf{E}_{\text{trans}}$

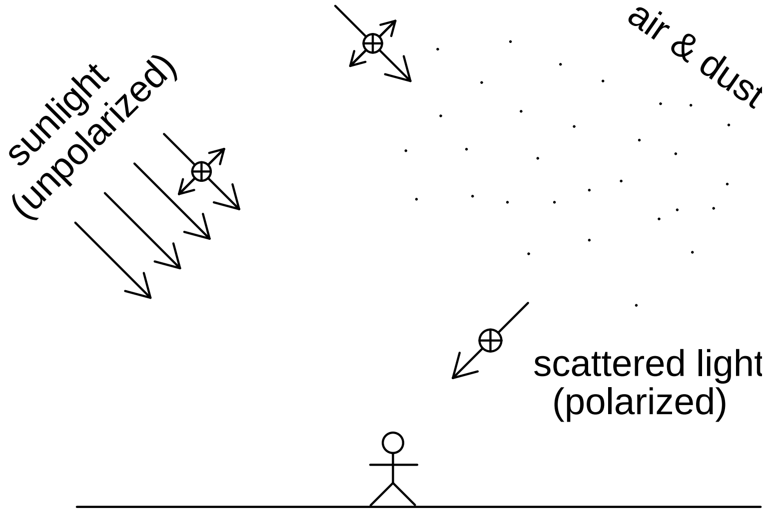


Atoms in glass can't radiate $\parallel \mathbf{E}$

(Charges radiate $\perp$ acceleration)

Scattered light is generally polarized

Example: light from sky



Not typically useful as polarizer

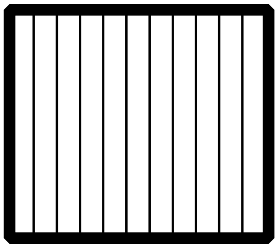
Dichroism (Hecht 8.3)

Dichroism = selective absorption of one (linear) polarization

Clearly useful for polarizers

Example: microwave polarizer

Array of parallel wires
spacing $\ll \lambda$



$\mathbf{E}$ aligned with wires: drive current
resistance → power dissipation
→ absorption

$\mathbf{E} \perp$ wires: little current, no absorption

Acts as a polarizer: transmits only $\mathbf{E} \perp$ wires

Watch out: graphically, want to picture vertical $\mathbf{E}$ "squeezing" through slots

Actual effect is just the opposite!

Optical version: wires → long polymer chains
Embed in clear plastic
Stretch plastic to align chains

Material called *polaroid*

Most common polarizer

Great for demos!

Characteristics of polaroid:

- Somewhat lossy: $T_0 \approx 0.7$
- Low leakage $\epsilon \approx 10^{-3}$
- Work best for visible light
- Cheap: \$1 for 5 cm square

Important restriction: limited to low power
(plastic can melt)

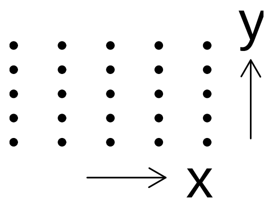
Don't use with high intensity beams
max $I \approx 1$ W/cm^2

Birefringence (Hecht 8.4)

Best polarizers based on *birefingence*
- Property of certain crystals

Generally, different directions not equivalent

Possible crystal lattice:



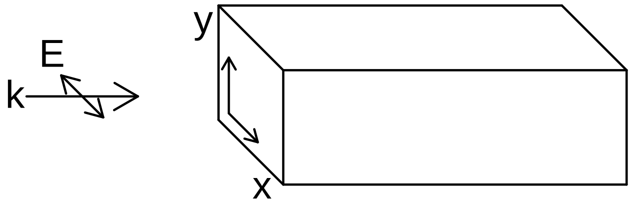
(dots = atoms)

x and y axes different

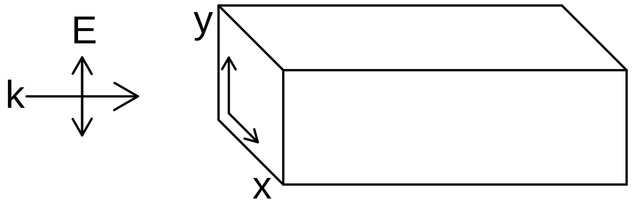
Note x and y determined by crystal

In asymmetric crystal, index of refraction n
depends on direction of $\mathbf{E}$

If $\mathbf{E}$ along x then have $n = n_x$:



If $\mathbf{E}$ along y then $n = n_y$:



All crystals have three basic symmetry axes
for now, label x, y and z

Call n_x, n_y, n_z = *principle indices of refraction*

Three different kinds of crystals:

- isotropic: $n_x = n_y = n_z$
  - not birefringent
- uniaxial: $n_x = n_y \neq n_z$
  - z axis special: called optic axis
- biaxial: $n_x \neq n_y \neq n_z$
  - optical properties complicated

Question: Can a liquid be birefringent?

Focus on uniaxial:
Symmetry like a cylinder: x and y interchangable

Terminology: call $n_x, n_y = n_o$
ordinary index

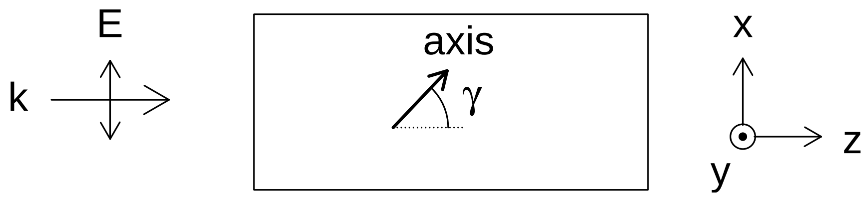
Call $n_z = n_e$: extraordinary index

Common optical materials:
Calcite: $n_o = 1.658$, $n_e = 1.486$
Quartz: $n_o = 1.544$, $n_e = 1.553$

Other examples: ice, mica, sapphire, LiNbO_3

What happens if $\mathbf{k}$ is not along a crystal axis?

Example:



Light propagates along z

$\mathbf{E}$ along x

optic axis at angle γ in xz-plane

Question: What is index if $\mathbf{E}$ along y?

For $\mathbf{E}$ along x, get effective index n_{eff}:

$$\boxed{\frac{1}{n_{\text{eff}}^2} = \frac{\cos^2\gamma}{n_o^2} + \frac{\sin^2\gamma}{n_e^2}}$$

If $\gamma = 0^\circ$, $n_{\text{eff}} = n_o$

if $\gamma = 90^\circ$, $n_{\text{eff}} = n_e$

Otherwise n_{eff} between n_o and n_e

Derivation a bit hard, won't go through

See Klein and Furtak §9.4

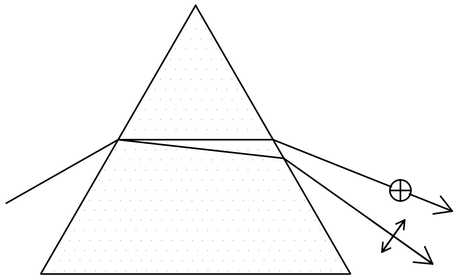
Probably cover in Phys 532

Upshot:

In birefringent materials, n depends on polarization

Simple polarizer:

Calcite prism, axis $\perp$ to page



$\perp$ and $\|$ polarizations have different n's

Deflected by different amounts

Separate outputs with lens or free propagation

Example of a *polarizing beam splitter*

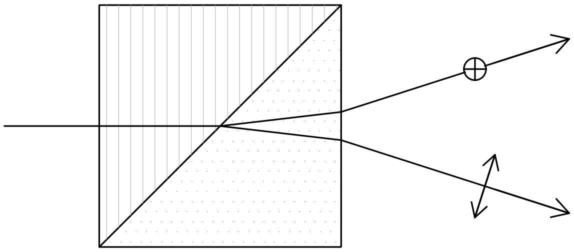
= polarizer with two outputs
one for each state

But not a good design:

- Deflection depends on λ
- Significant reflection from surfaces
- Large common deflection inconvenient

Improve by putting two prisms together

Wollaston prism:



Typical angular separation = 15-20°

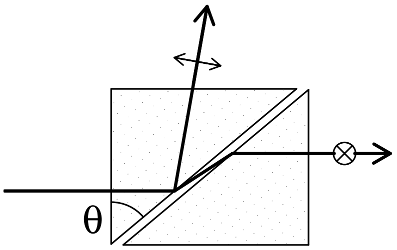
Good performance:

- Loss $\approx$ 10%, or 1% if AR coated
- Leakage $\sim 10^{-5}$
- Works at high power

Various other designs, see optics catalogs

Another method: Glan-Thompson

Uses total internal reflection



Again calcite, with optical axis $\perp$ page

Choose prism angle so that $n_e \sin\theta < 1 < n_o \sin\theta$

$\theta = 40°$ works

Then o-light is TIR, e-light is transmitted

(Gap is too big for frustrated TIR)

Performance similar to Wollaston
- low loss, low leakage
- high power capacity

Advantage: larger beam separation
no deviation of e beam
(deviation exaggerated in picture)

Wollaston and Glan-Thompson expensive
\$300-\$500 or more

Summary:

- Polarizers transmit one polarization
  $T = T_0 |\hat{\jmath}^* \cdot \mathbf{a}|^2$
- Most polarizers dichroic or birefringent
  Birefringent better, more \$
- Birefringence: n depends on $\hat{\jmath}$
  - Uniaxial crystal: one special direction
- Retarders use birefringence
  - Quarter-wave plate: make circ polarization
  - Half-wave plate: rotate linear polarization