Phys 531 Lecture 26 29 November 2005

Photons

Last time, finished polarization

Learned about retarders,
Jones matrices

Today: introduce quantum optics
Whole new way to look at light

Outline:

- Photon optics
- Photon detectors
- Quantum noise
- Quantum states

Photon optics not too hard

Helpful to know quantum mechanics
not totally necessary

Next time:
Quantum field theory of light

Photon Optics (Hecht 3.3.3)

Simple version of quantum theory

Say that light is really composed of particles
particles called *photons*

Energy of photon = $\hbar\omega$
$\hbar$ = Planck's constant = 1.054×10^{-34} J s
ω = oscillation frequency of light
(Don't worry about what is oscillating for now)

Polychromatic light:
many photons with different ω's

- Pulse of light with energy U (in J)

contains $N = \dfrac{U}{\hbar\omega}$ photons

N = *photon number*

- Beam with power P (in W = J/s)

delivers $\Phi = \dfrac{P}{\hbar\omega}$ photons/s

Φ = *photon flux*

- Beam with irradiance $I(\mathbf{r})$ (in W/m^2)

has $\phi = \dfrac{I}{\hbar\omega}\,\dfrac{\text{photons/s}}{\text{m}^2}$ at $\mathbf{r}$

ϕ = *photon flux density*

Also light with energy density $u(\mathbf{r})$ (in J/m^3)

has photon density $\frac{u}{\hbar\omega}$ $\left(\frac{\text{photons}}{\text{m}^3}\right)$

Example:

Sunlight has an irradiance of about 250 W/m^2
Assume average wavelength = 500 nm

Then average $\hbar\omega = 4 \times 10^{-19}$ J

Photon flux density $= 6 \times 10^{20}$ photons/(s m^2)

Looking at sun, pupil area $\approx 10^{-6}$ m^{-2}
collect about 6×10^{14} photons in 1 s

Photons are not "classical" particles!

→ propagate according to wave equation
not Newton's laws

Procedure: use wave techniques to calculate $I(\mathbf{r})$
Then I gives flux density

Imagine photons "follow" wave
like surfers in ocean

But photons and wave are inseparable

Best to interpret probabilistically:

Average number of photons in volume $d^3r =$

$$\langle N \rangle = \frac{u(\mathbf{r})\, d^3r}{\hbar\omega} = \frac{I(\mathbf{r})\, d^3r}{\hbar\omega c}$$

If $\langle N \rangle \ll 1$, interpret as probability to find one photon

Important:
No definite trajectory for individual photons

Picture surfers scrambling back and forth on wave

Avoids "two-slit" paradox:
In two slit interferometer, which slit does photon pass through?

Correct interpretation:
It doesn't matter

The *wave* passes through both

Interference pattern says where photons can go

Question: What if a wave passes through beam spitter, and the outputs are separated by a large distance? Does it make sense to ask where one of the photon is?

If wave determines photon distribution,
why use photons?

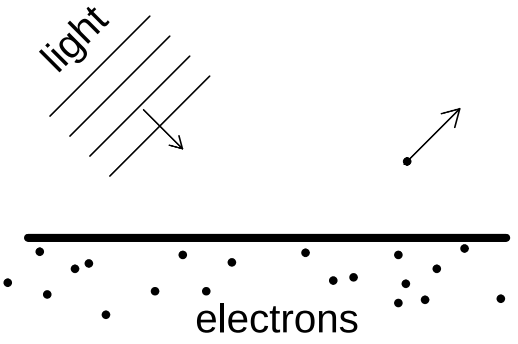
Because detectors see photons, not waves

Or: light can only transfer energy in units of $\hbar\omega$

So photons important whenever light is emitted or absorbed

Example: photoelectric effect

Shine light on metal:
Electrons absorb energy from light and escape

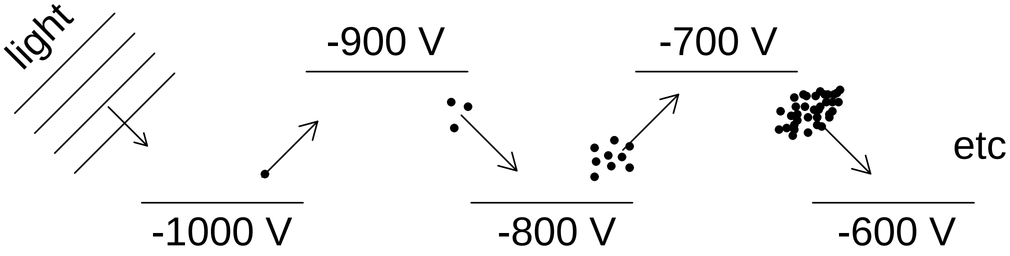


Find that maximum electron energy $= \hbar\omega$
$=$ max energy absorbed from photon

Doesn't depend on irradiance, just ω

Another example: photomultiplier tube

Light hits metal plate, detaches single electron



Accelerate electron to second plate:
detach more electrons

Cascade many plates, get big current pulse

See blip on output: detection of one photon

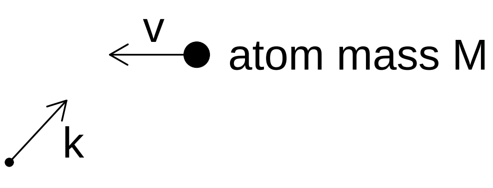
Other photon properties:

- Momentum $\mathbf{p} = \hbar\mathbf{k}$ (Hecht 3.3.4)

Interesting effect: suppose atom absorbs photon
makes internal transition $U_0 \to U_1$

Say atom velocity before absorption $= \mathbf{v}$

Velocity after absorption $= \mathbf{v} + \hbar\mathbf{k}/M$
gets kick from photon



Initial energy: $U_{\text{tot}} = U_0 + \frac{1}{2}Mv^2 + \hbar\omega$

Final energy: $U_{\text{tot}} = U_1 + \frac{1}{2}M\left(\mathbf{v} + \frac{\hbar\mathbf{k}}{M}\right)^2$

Energy conserved, so

$$\omega = \frac{U_1 - U_0}{\hbar} + \frac{\hbar k^2}{2M} + \mathbf{v}\cdot\mathbf{k}$$

First term: standard QM

Second term: “radiative correction”

Third term: Doppler shift

Derive Doppler effect from QM!

- Angular momentum (Hecht 8.1.5)

Circular polarization states have definite spin

Right circular: $\mathbf{L}_{\text{photon}} = -\hbar\hat{\mathbf{k}}$

Left circular: $\mathbf{L}_{\text{photon}} = +\hbar\hat{\mathbf{k}}$

Linear polarized states
= superposition of $\hat{\mathbf{e}}_{\mathcal{R}}$ and $\hat{\mathbf{e}}_{\mathcal{L}}$

Photon equally likelty to be “spin-up” and “spin-down”

Important for transition selection rules

Detecting Photons

Already mentioned one way to see single photons:
photo-mulitplier tube (PMT)

Are there other methods?

Can characterize light detectors by three parameters:

- quantum efficiency
- background rate
- saturation rate

Quantum efficiency $\varepsilon =$
probability that incident photon is detected

For PMT, typically about 5%

- limited by emission of first electron

Background rate $R_0 =$
count rate observed with no incident light

For good PMT, as low as ~ 1 count/s

- due to thermal emission of electrons

Saturation rate $R_{\text{sat}} \approx$
max measurable count rate

For PMT, about 10^6 counts/s
- takes about 1 μs to recover between ‘clicks’

To see single photons, need incident flux Φ with

$$R_0 < \varepsilon\Phi < R_{\text{sat}}$$

Impossible if $R_0 \geq R_{\text{sat}}$

Some detectors:

Device	ε (%)	R_0 (counts/s)	R_{sat} (counts/s)
PMT	1–20	< 1 to 10^5	$10^6 - 10^9$
Photodiode	20–90	10^4+	$< R_0$
Avalanche PD	30–70	10 to 5000	$10^6 - 10^7$

(Note PMTs have much larger area than APDs)

Quantum Noise

Important effect of photon theory:
light is noisy

Suppose laser beam with power P
measure with ideal photon counter

In time T, detect $N = PT/\hbar\omega$ photons

But photons are randomly distributed in wave

Don't expect to get exactly N every time

Guess that photons are distributed independently:

Probablity to detect a photon doesn't depend on when previous photon was detected

Why?
- Photons emitted by independent atoms
- Don't expect correlations in emission times

(Reconsider this assumption next time)

Then photons obey *Poisson statistics*

If on average you detect N photons in time T

then N fluctuates by $\boxed{\Delta N = N^{1/2}}$

Same as fluctuations in flipping coins,
most other statistical noise

Sometimes called “shot noise”:
- comes from photons being discrete “shots”

Better name is “quantum noise”

So no such thing as light with constant P

Always detect some fluctuations

Size of fluctuations depends on time scale

Measure for time T, get relative noise

$$\frac{\Delta P}{P} = \frac{\Delta N}{N} = \frac{1}{N^{1/2}} = \left(\frac{\hbar\omega}{PT}\right)^{1/2}$$

Noise goes up as T goes down
(just like normal noise goes down with averaging)

Question: If your detector has a quantum efficiency less than one, will the quantum noise scale as the square root of the number of **incident** photons, or the number of **measured** photons?

All real detectors also have "technical noise"
= noise sources besides quantum mechanics

For instance:
- noise in background signal
- electrical pick-up
- vibration-induced noise
- noise from temperature drifts

etc.

Most often, technical noise dominates

Quantum noise typically important for:

- Photon counting applications
  (N is low, so $\Delta N/N$ is large)

- High speed applications (T is small)
  → need high power to measure fast signals

Can distinguish quantum and technical noise:
Quantum: noise $\propto P^{1/2}$
Technical: noise $\propto P$ or indep of P

Quantum States

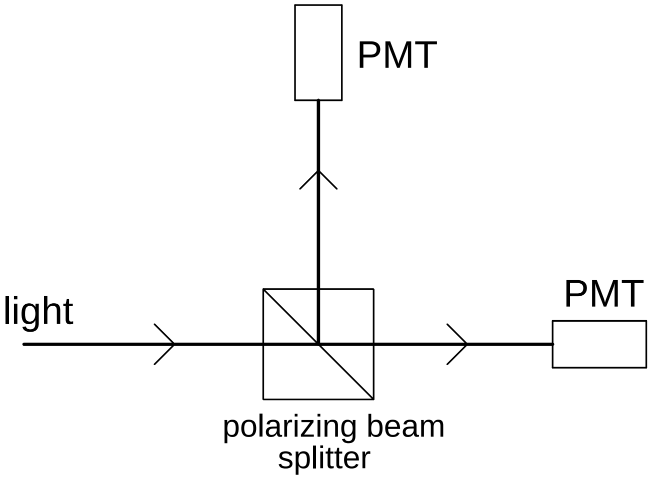
So far, haven't really needed quantum mechanics
just idea that light energy is discrete

But quantum mechanics of light is interesting too

Easiest to see using polarization:
Suppose beam with $\hat{\jmath} = \cos\alpha\hat{\mathbf{x}} + \sin\alpha\hat{\mathbf{y}}$

Count number of photons polarized along x, y

For instance:



If average photon number = N, expect

$N_x = \cos^2\alpha\ N$

$N_y = \sin^2\alpha\ N$

on average

If $N << 1$, often see no photons

but when one is detected, have prob

$\cos^2\alpha$ to be along x

$\sin^2\alpha$ along y

Similar to QM for particle in superposition state

Natural to describe photon with wave function

$\psi = \cos\alpha\widehat{X} + \sin\alpha\widehat{Y}$

where

$\widehat{X}$ = state polarized along x

$\widehat{Y}$ = state polarized along y

Note $\psi \sim \hat{\jmath}$, for right interpretation of $\hat{\mathbf{x}}$, $\hat{\mathbf{y}}$

More generally, have

$\psi(\mathbf{r}) \sim \mathbf{E}(\mathbf{r})$

Electric field of wave ≈ wave function of photons

Optics and quantum mechanics very similar

But quantum mechanics has more possibilities

Imagine light pulse with two photons

QM allows polarization state

$$\psi_{12} = \chi \equiv \frac{1}{\sqrt{2}}\left(\hat{X}_1\hat{Y}_2 + \hat{Y}_1\hat{X}_2\right)$$

Always measure one photon polarized along x
and one along y

But light not polarized in any usual way

Compare: light polarized at 45° has

$$\begin{aligned}\psi_{12} &= \left[\frac{1}{\sqrt{2}}\left(\hat{X}_1 + \hat{Y}_1\right)\right]\left[\frac{1}{\sqrt{2}}\left(\hat{X}_2 + \hat{Y}_2\right)\right]\\ &= \frac{1}{2}\left(\hat{X}_1\hat{X}_2 + \hat{X}_1\hat{Y}_2 + \hat{Y}_1\hat{X}_2 + \hat{Y}_1\hat{Y}_2\right)\end{aligned}$$

Have 50% chance to measure both photons in same state

Not the same as state χ

States like χ called *nonclassical*:
Wavefunction can't be expressed as an ordinary wave

Generally, don't worry about it:
Almost all light sources produce classical light

(Hard work to make state like χ)

But non-classical states are useful

Tricks like quantum noise suppression, quantum teleportation, quantum computing

Summary:

- Photons $\approx$ "particles" of light
  - but photons follow wave
- Photon energy $\hbar\omega$, momentum $\hbar k$,
  ang. momentum $\hbar$
- Detectors measure photons, not wave
  good detectors count individual photons
- Photon statistics → quantum noise
- Photon wave function $\approx$ electric field
  - but nonclassical states possible