Phys 531 Lecture 27 1 December 2005

Quantum Optics

Last time, discussed photon optics

Light energy comes in units of $\hbar\omega$
particles called photons
observable with good detector

One consequence: light is intrinsically noisy

Today: introduce proper quantum theory
Quantum field theory of light

Note, this work recognized with 2005 Nobel prize

Outline:

- Quantum fields
- Mode expansion
- Quantum states of light
- Photon tricks

Do need some QM for this
Material won't be on final

Warning: derivations today not rigorous,
some definitions simplified
Consult Scully and Zubairy for real deal

Next time: review . . . bring questions!

Quantum Field Theory

Want proper quantum theory for photon

From last time, expect
$E(\mathbf{r}) \sim$ wave function $\psi(\mathbf{r})$

Since:

- Probability $\propto |E|^2$
- Polarization analogous to spin
- E exhibits interference like ψ
- Wave equation $\sim$ Schrodinger equation

Intuition is right, but one problem

Number of photons is indefinite:
photons easy to create and destroy

"Normal" QM assumes fixed N
N = number of particles

Problem not just that N is unknown:

Can have quantum system in superposition

$$|\psi\rangle = \frac{1}{\sqrt{2}}\Big(\,|\text{atom + photon}\rangle + |\text{excited atom}\rangle\,\Big)$$

Photon in superposition of existing or not!

Not clear how to make Schr eqn for N variable...

Basically can't make quantum theory for photon
(as a particle)

Instead, make quantum theory for field $E(\mathbf{r})$
$\Rightarrow$ quantum field theory

Idea: make E itself a quantum variable
like $\mathbf{r}$ for electron

Say $\Psi(E)$ = state of EM field
like $\psi(\mathbf{r})$ = state of electron

Of course, really have $\Psi[E(\mathbf{r})]$:
E is itself a function

Example: polarization

Last time introduced "nonclassical" states like

$$\psi_{12} = \frac{1}{\sqrt{2}}\left(\widehat{X}_1\widehat{X}_2 + \widehat{Y}_1\widehat{Y}_2\right)$$

where $\widehat{X}$ = photon state polarized along x

(Here have definite $N = 2$)

No way to interpret ψ_{12} as any $\mathbf{E}$
Instead say field is superposition of different $\mathbf{E}$'s:

$$\Psi(E) = \frac{1}{\sqrt{2}}(E_X + E_Y)$$

where E_X = field polarized along x

Could measure this state with polarizer along x

Imagine sending repeated pulses:
each pulse in state Ψ

Each time get either 100% or 0% transmission
unpredictable per pulse

Works even if E_X and E_Y are classical states
with many photons

Simple version of Schrodinger's cat

Could also have superpositions of beam position, frequency, etc.

Mode Expansion

Still have problem that $E = E(\mathbf{r})$

How to handle $\Psi[E(\mathbf{r}))]$?

Solve by decomposing field into *modes*
Usually, mode = plane wave

Easier to work inside a box, volume V

Then only discrete values of $\mathbf{k}$ allowed
Get sums instead of integrals

Main advantage: makes normalization easier

Can write any $E(\mathbf{r},t)$ as

$$E(\mathbf{r},t) = \sum_{\mathbf{k}} A_{\mathbf{k}} e^{i(\mathbf{k}\cdot\mathbf{r}-\omega_{\mathbf{k}}t)}$$

so state of field uniquely specified by $\{A_{\mathbf{k}}\}$'s

Each mode $\mathbf{k}$ = independent degree of freedom
    Do QM on each independently

Treat each $A_{\mathbf{k}}$ as independent quantum variable
    Now a simple variable like X for a particle

But still one issue: $A_{\mathbf{k}}$ is complex

Really

$$E(\mathbf{r},t) = \sum_{\mathbf{k}} \text{Re}\,[A_{\mathbf{k}}] \cos(\mathbf{k}\cdot\mathbf{r}-\omega_{\mathbf{k}}t) \\ - \text{Im}[A_{\mathbf{k}}] \sin(\mathbf{k}\cdot\mathbf{r}-\omega_{\mathbf{k}}t)$$

Real and imaginary part of $A_{\mathbf{k}}$ both needed
    really *two* quantum variables per mode

Define

$$q_{\mathbf{k}} = \sqrt{\frac{\epsilon_0 V}{2\hbar\omega_{\mathbf{k}}}}\ \text{Re}\,A_{\mathbf{k}} \qquad p_{\mathbf{k}} = \sqrt{\frac{\epsilon_0 V}{2\hbar\omega_{\mathbf{k}}}}\ \text{Im}A_{\mathbf{k}}$$

Then

$$E(\mathbf{r},t) = \sqrt{\frac{2\hbar\omega}{\epsilon_0 V}} \sum_{\mathbf{k}} (q_{\mathbf{k}} + ip_{\mathbf{k}}) e^{i(\mathbf{k}\cdot\mathbf{r} - \omega_{\mathbf{k}} t)}$$

To do QM, need Hamiltonian

Know classical energy for mode $\mathbf{k}$ is $U = uV$

So

$$\begin{aligned} H_{\mathbf{k}} &= \frac{\epsilon_0 V}{2} |A_{\mathbf{k}}|^2 \\ &= \left(\frac{\epsilon_0 V}{2}\right) \left(\frac{2\hbar\omega_{\mathbf{k}}}{\epsilon_0 V}\right) \left(q_{\mathbf{k}}^2 + p_{\mathbf{k}}^2\right) \\ &= \hbar\omega \Big(q_{\mathbf{k}}^2 + p_{\mathbf{k}}^2\Big) \end{aligned}$$

This Hamiltonian is familiar

Simple harmonic oscillator has

$$\begin{aligned} H &= \frac{p^2}{2m} + \frac{1}{2} m\omega^2 x^2 \\ &= \frac{1}{2m} \Big[p^2 + (m\omega x)^2\Big] \end{aligned}$$

Mode H looks the same, up to scale factor in q

Scale factors not important, so conclude:

Each mode of field acts like a quantum harmonic oscillator

Two main results:

(1) Energy eigenstates are $E_n = \hbar\omega(n + \frac{1}{2})$
for integer n

$\Rightarrow$ photons!

(2) q and p don't commute: $[q, p] = \frac{i}{2}$
for our scale factors

Then can't know q and p simultaneously:

$$\Delta q \Delta p \geq \frac{1}{4}$$

What do q and p correspond to physically?

Have $A \propto q + ip$ = amplitude of mode

Field $\propto q \cos(\mathbf{k} \cdot \mathbf{r} - \omega t) - p \sin(\mathbf{k} \cdot \mathbf{r} - \omega t)$

So q is amplitude of wave $\sim$ cos()
p is amplitude of wave $\sim$ sin()

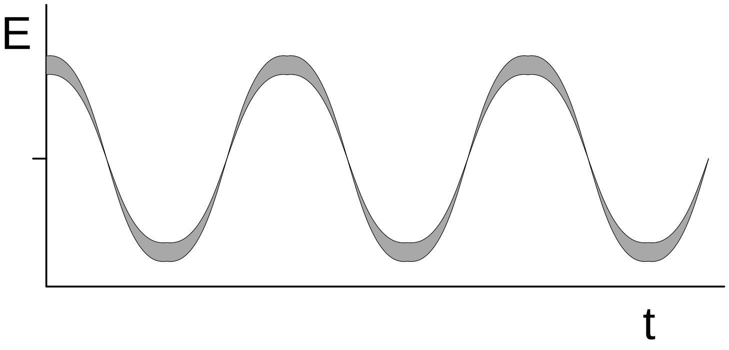
Call components "quadratures" of wave

Of course, arbitrary which is which
usually imagine a "reference oscillator" $\sim$ cos
Define modes relative to it

Quantum States of Light

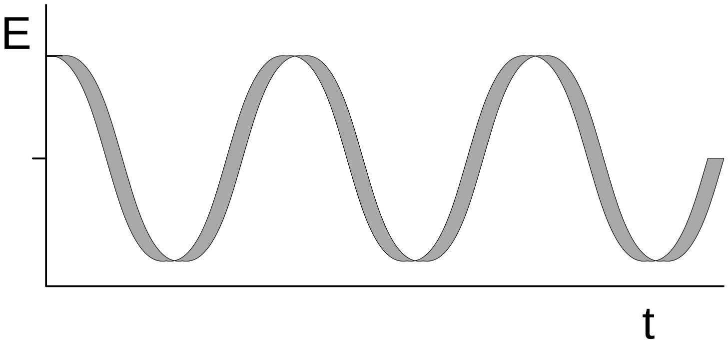
Since $\Delta q \Delta p \geq 1/4$,
amplitudes can't have definite values

Draw picture:
Say $\langle q \rangle$ large, $\langle p \rangle = 0$ and $\Delta q \gg \Delta p$



Amplitude of wave is uncertain

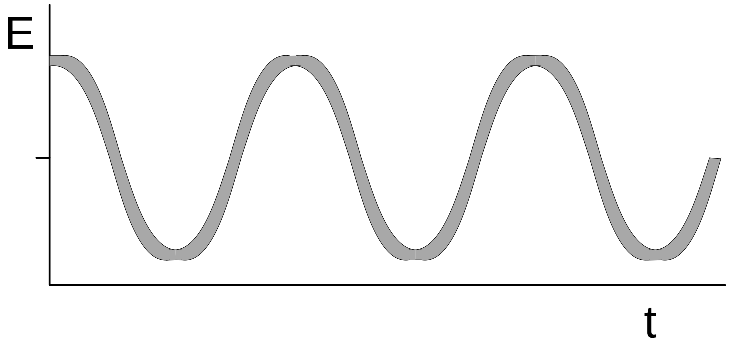
Or, if $\langle q \rangle$ large but $\Delta q \ll \Delta p$:



Amplitude well-defined, but phase uncertain

Question: What if $\langle p \rangle$ and Δq were large, and $\langle q \rangle$ and Δp were small? Would the phase or amplitude be more certain?

Normally, have $\Delta q = \Delta p = 1/2$



Uncertainty in both amplitude and phase

This is typical state produced by laser:
called "coherent state"

(Another poorly chosen name)

Label coherent state by $\alpha = \langle q \rangle + i \langle p \rangle$
= expectation value of field amplitude

(Note α is just a complex number)

Have $|\alpha|^2 = \langle q \rangle^2 + \langle p \rangle^2$

Ignoring uncertainties,

$$|\alpha|^2 \simeq q^2 + p^2 = H/\hbar\omega \simeq N$$

Can't really ignore uncertainties, but true that:
$|\alpha|^2 = \langle N \rangle$,
average number of photons in mode

What is uncertainty in N?

Have $N \simeq q^2 + p^2$

$$\begin{aligned} \text{So } \Delta N &\simeq \left[\left(\frac{\partial N}{\partial q}\right)^2 \Delta q^2 + \left(\frac{\partial N}{\partial p}\right)^2 \Delta p^2\right]^{1/2} \\ &\simeq \left[(2q)^2 \Delta q^2 + (2p)^2 \Delta p^2\right]^{1/2} \\ &\simeq \left(q^2 + p^2\right)^{1/2} \\ &\simeq N^{1/2} \end{aligned}$$

since $\Delta q = \Delta p = 1/2$

Same quantum noise from last time!

If $\Delta q \neq \Delta p$, called “squeezed state”
Generally don’t have $\Delta N = N^{1/2}$

Useful for precision interferometry:
- reduce noise in component measured
- increase noise in component not measured

Generate squeezed states in nonlinear crystals
Rather difficult to achieve

Local expert: Olivier Pfister

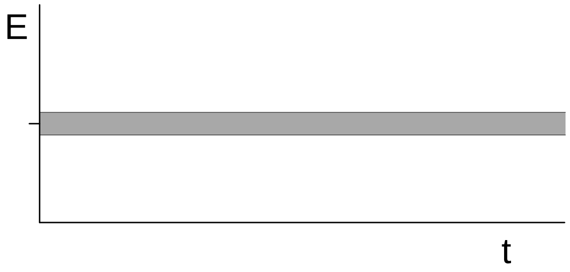
Can have coherent state with $\alpha = 0$

Then $\langle N \rangle = 0$: no photons present

Called *vacuum state*

Electric field also zero on average

But still have fluctuations $\Delta q = \Delta p = 1/2$:



Fluctuations called vacuum noise

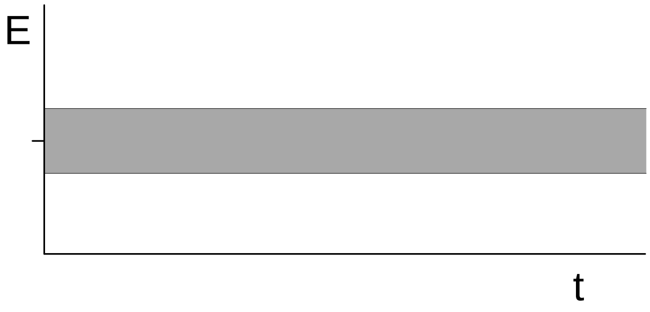
Help explain spontaneous emission

Another possible state: Fock state

Has definite energy $(N + 1/2)\hbar\omega$

$\rightarrow$ mode contains N photons

Both Δq and Δp large, $= \frac{1}{2}\sqrt{2N+1}$



Note vacuum state is also a Fock state

Fock state is ideal "state with one photon" or
"state with two photons", etc.

Has perfectly well defined amplitude,
indefinite phase
(gives no interference when combined with another beam)

Coherent state = superposition of Fock states

$$\Psi_\alpha = \sum_n c_n \Psi_n$$

with $c_n = e^{-|\alpha|^2/2} \frac{\alpha^n}{\sqrt{n!}}$

(Note $|c_n|^2$ are same as in Poisson distribution)

Fock states difficult in infinite-volume beams
need $N \to \infty$ or else $I \to 0$

Usually implement as pulses with definite N

Best experimental sources:
Semiconductor quantum dots

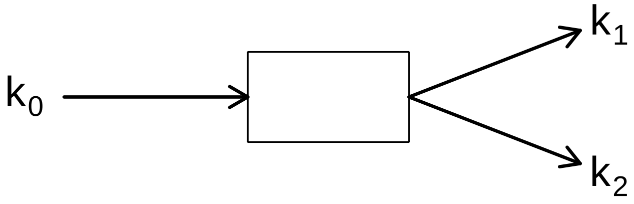
Excite with regular light pulse,
get single photon out

But only emit photon with ~10% efficiency

Most common nonclassical source:
"Parametric down conversion"

Use special crystal with nonlinear interactions

Can "split" one photon into two:
$\omega_0 \to \omega_1 + \omega_2$
with $\omega_0 = \omega_1 + \omega_2$ (conserve energy)
$\mathbf{k}_0 = \mathbf{k}_1 + \mathbf{k}_2$ (conserve momentum)



Input pulse (ω_0) in coherent state

Produces output pulse of form

$$\Psi_{\text{out}} = \sum_n c_n \Psi_n(k_1)\Psi_n(k_2)$$

where $\Psi_n(k)$ is Fock state for mode $\mathbf{k}$

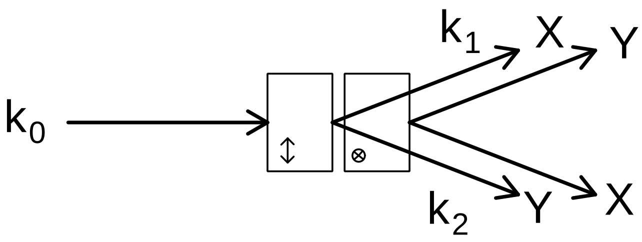
Typically operate with $c_0 \approx 1$, c_1 small,
higher c's negligible

Then usually get nothing, but sometimes get pair of single photons

Polarization depends on crystal setup

possible to produce state $\frac{1}{\sqrt{2}}(X_1Y_2 + Y_1X_2)$

One way:



Crystals thin, can't tell where photons emitted

Then say modes are *entangled*
    quantum states not separable

Gives interesting effects:

- Detect one photon, know that other is present
Acts ≈ like perfect 1-photon source
(don't need entanglement)

- Violate Bell's inequality

- Send messages immune to eavesdropping
(quantum cryptography)

- Transfer arbitrary quantum state
(quantum teleportation)

Can see how teleportation works:

Start with two photons, state

$$\frac{1}{\sqrt{2}}(X_1Y_2 + Y_1X_2)$$

Send one to Alice, one to Bob

Bob has third photon in unknown state

$$\Upsilon = aX_3 + bY_3$$

Wants to send state to Alice w/o actually transmitting photon

Bob makes special measurement of his photons in basis:

$$\Psi_A = \frac{1}{\sqrt{2}}(X_2X_3 + Y_2Y_3)$$
$$\Psi_B = \frac{1}{\sqrt{2}}(X_2X_3 - Y_2Y_3)$$
$$\Psi_C = \frac{1}{\sqrt{2}}(X_2Y_3 + X_2Y_3)$$
$$\Psi_D = \frac{1}{\sqrt{2}}(X_2Y_3 - X_2Y_3)$$

(Need special setup to make this measurement)

Basis states called "Bell states"

Before measurement, total field state is

$$\frac{1}{\sqrt{2}}(aX_1Y_2X_3 + aY_1X_2X_3 + bX_1Y_2X_3 + bY_1X_2Y_3)$$

Rewrite by adding and subtracting many terms:

$$\begin{aligned}\frac{1}{2^{3/2}}\Big(&aY_1X_2X_3 + aY_1Y_2Y_3 + bX_1X_2X_3 + bX_1Y_2Y_3\\ &+aY_1X_2X_3 - aY_1Y_2Y_3 - bX_1X_2X_3 + bX_1Y_2Y_3\\ &+aX_1X_2Y_3 + aX_1Y_2X_3 + bX_1X_2Y_3 + bY_1Y_2X_3\\ &-aX_1X_2Y_3 + aX_1Y_2X_3 + bX_1X_2Y_3 - bY_1Y_2X_3\Big)\end{aligned}$$

This factors into

$$\begin{aligned}\frac{1}{2^{3/2}}\Big[&(aY_1 + bX_1)(X_2X_3 + Y_2Y_3)\\ &+(aY_1 - bX_1)(X_2X_3 - Y_2Y_3)\\ &+(aX_1 + bY_1)(X_2Y_3 + Y_2X_3)\\ &-(aX_1 - bY_1)(X_2Y_3 - Y_2X_3)\Big]\end{aligned}$$

or

$$\begin{aligned}\frac{1}{2}\Big[&(aY_1 + bX_1)\Psi_A + (aY_1 - bX_1)\Psi_B\\ &+ (aX_1 + bY_1)\Psi_C - (aX_1 - bY_1)\Psi_D\Big]\end{aligned}$$

When Bob makes measurement, wavefunction
collapses

Then Alice's photon in state related to Υ

Bob knows result of his measurement

Calls Alice on phone and tells her how to change her state to Υ
(easy using $\lambda/2$ and $\lambda/4$ plates)

Now Alice has state Υ
indistinguishable from original photon 3

If we could do this with 10^{23} atoms instead of one photon, could teleport a mouse

Teleportation theory:

C. H. Bennett et al. Phys. Rev. Lett. **70** 1895 (1993)

Some experiments:

D. Bouwmeester et al. Nature **390** 575 (1997)

T. C. Zhang et al. Phys. Rev. A **67** 033802 (2003)

M. D. Barrett et al. Nature **429** 737 (2004)

Summary

- Quantum optics = quantum field theory
  Electric field is quantum variable
- Each field mode = harmonic oscillator
  Photons = excited states
- Different states have different noise
  Coherent state = typical laser
  Fock state = definite photon number
- Can have entangled field modes
  Tricks like teleportation