Phys 531 Lecture 27 6 December 2005

Final Review

Last time: introduction to quantum field theory

Like QM, but field is quantum variable
rather than x, p for particle

Understand photons, noise, weird quantum states

Today:
Summarize everything

Outline:

- Final exam format
- Summary, key points
- Questions

Final Exam

In class: Friday December 9, 2:00-5:00 PM

Eight problems, 10 points each

Partial credit on all

Can use:

- Lecture notes
- Homework assignments and solutions
- Textbook

Bring a calculator

Scratch paper provided or bring your own

Problems have varying difficulty:

do easy ones first!

No long calculations
(if you know what you're doing)

Don't waste time on algebra:
Explain what you want to do and move on

Content:

Comprehensive, over full course

Roughly 1/4 from before midterm
3/4 from after midterm

All drawn from lectures,
but book useful for a few questions

Review

I. Maxwell's Equations

II. Ray Optics

III. Fourier Optics

IV. Interference

V. Polarization and photons

Review key points from each

I. Maxwell's Equations

A. Maxwell $\rightarrow$ wave equation

Light is a wave

- Electric and magnetic fields oscillate
- Energy related to field amplitudes

$$I = \frac{|E_0|^2}{2\eta_0}$$

- Simplest solution = plane wave

Complex representation $\mathbf{E} = \mathbf{E}_0 e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}$

B. Light in matter: Two pictures

Index of refraction and scattered waves

Index: $v = c/n$

$\lambda = \lambda_0/n$

$\mathbf{k} = n\mathbf{k}_0$

Easy way to include wave scattered by medium

Gives correct results

Scattering: $E_{\text{tot}} = E_{\text{inc}} + E_{\text{scat}}$

Scattered field produced at $\mathbf{r} \propto E(\mathbf{r})$

Sometimes easier to understand

C. Interfaces

Snell's Law, Law of Reflection:

Direction of transmitted, reflected waves

Understand both with Fermat's Principle

Fresnel equations:

Amplitude of transmitted, reflected waves

Generally complex:

- Total internal reflection
- Metallic media

II. Ray Optics

A. Thin lenses (paraxial limit)

$$\frac{1}{s_i} + \frac{1}{s_o} = \frac{1}{f}$$

Sign conventions important

Curved mirror $\approx$ lens with $f = -R/2$

B. Stops

Set field of view and brightness of image

When cascading systems, want exit pupil of first and entrance pupil of second to coincide

C. Optical Systems

- Cascade multiple elements with ray matrices

$$\mathcal{M}_{\text{tot}} = \mathcal{M}_N \mathcal{M}_{N-1} \ldots \mathcal{M}_2 \mathcal{M}_1$$

$$\mathbf{v}_{\text{out}} = \mathcal{M}_{\text{tot}} \mathbf{v}_{\text{in}}$$

- System ray matrix also gives thick lens picture:

Arb. system like thin lens

"Input" at front principle plane

"Output" at back principle plane

- Aberrations limit performance

Calculate with exact ray tracing

III. Fourier Optics

A. Fourier transform

$$\mathcal{E}(\mathbf{k}) = \iiint E(\mathbf{r}) e^{-i\mathbf{k}\cdot\mathbf{r}}\, d^3r$$

$$E(\mathbf{r}) = \frac{1}{(2\pi)^3} \iiint \mathcal{E}(\mathbf{k}) e^{i\mathbf{k}\cdot\mathbf{r}}\, d^3k$$

Express any field as sum of plane waves

Evaluate using tables and shifting properties

B. Propagation

Given $E(x, y, 0) = A(x, y)$, get $\mathcal{A}(k_x, k_y)$

Each component $e^{i(k_x x + k_y y)} \to e^{i(k_x x + k_y y + \kappa z)}$

with $\kappa = \sqrt{k^2 - k_x^2 - k_y^2}$

Convolution form:

$$E(x, y, z) = \iint A(X, Y) h(X - x, Y - y) dX dY$$

with h = inverse transform of $e^{i\kappa z}$

Either way, $E(x, y, z)$ determined by $A(x, y)$

C. Diffraction

- Fresnel: small angle (paraxial)

$$\kappa \approx k - \frac{k_x^2 + k_y^2}{2k}$$

- Fraunhofer: large distance

Get

$$E(x, y, d) \approx -\frac{i}{\lambda d} e^{i\phi} \mathcal{A}\left(\frac{kx}{d}, \frac{ky}{d}\right)$$

So $k_x \to kx/d = k\theta_x$ and $k_y \to ky/d = k\theta_y$

D. Applications

- Grating: sharp lines good for spectroscopy

- Lens: Fraunhofer pattern in focal plane

- Fourier filter: use aperture to modify transform

- Holography: make aperture function to create
  arbitrary diffracted field

IV. Interference

A. Gaussian Beams

Good approximation to laser beam

Characteristic $I(\rho) = I_0 e^{-2\rho^2/w^2}$

Remains focussed over length $z_0 = \pi w_0^2/\lambda$

Propagate using $q = z - iz_0$

Optical system: q_{in} and q_{out} related by
ray matrix

B. Interferometers

Device that exhibits interference

Basic equation: $E_{\text{tot}} = E_1 + E_2$

$$\Rightarrow I_{\text{tot}} = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$$

ϕ = phase difference between E_1 and E_2

Examples:

- Two slits: simplest
- Michelson: most common
- Fabry-Perot: multiple reflections
  Useful for spectroscopy

C. Coherence Theory

Theory of random waves

Temporal: when $E(t)$ fluctuates
Polychromatic light

Spatial: when $E(\mathbf{r})$ fluctuates
Extended light source

Generally have both fluctuations

Handle with $\Gamma_{12}(\tau) = \langle E(\mathbf{r}_1, t+\tau) E^*(\mathbf{r}_2, t)\rangle$

Gives contrast of interference pattern

Also $\Gamma_{11}(\tau) \to S(\omega)$ power spectral density

V. Polarization and photons

A. Polarization

Vector nature of electric field

Polarization states:

- Linear: $\mathbf{E}$ oscillates in plane

- Circular: $\mathbf{E}$ rotates in circle

- Elliptical: $\mathbf{E}$ traces out ellipse (most general)

Polarization components:

- Polarizer: transmits one state
  blocks orthogonal state

- Quarter-wave plate:
  converts linear $\leftrightarrow$ circular

- Half-wave plate:
  rotates plane of linear polarization

Calculate effects with Jones matrices

B. Quantum Optics

Light comes in packets called photons

Energy $= \hbar\omega$

But photons still propagate as wave

Typically have noise $\propto N^{1/2}$

Quantum field theory:

Not on final

Questions?