Instructions: This is an in-class exam that must be completed during the allotted 3 hour period. You may use your class notes and the textbook for reference. You may also use a calculator and scratch paper as desired.

The exam consists of nine problems, each worth 10 points. For all problems, partial credit will be given, so be sure to show your work and explain your reasoning when appropriate.

1. Suppose you wish to couple light between two indentical optical fibers by simply placing their ends together as shown. The single-mode fibers have a core diameter of $2 \mu \mathrm{~m}$, while the electric field of their mode field falls off to $1 / e$ of its peak value at a radius of $5 \mu \mathrm{~m}$ for light of wavelength $1.5 \mu \mathrm{~m}$. Roughly how large can the spacing $d$ between fiber ends be while still maintaining good coupling efficiency? (You need not specify exactly how "good" the efficiency is, because I only want an estimate for the maximum gap size.)

2. Suppose you measure the transmission of light through a 1-cm long crystal, and obtain the results plotted below. Here the transmission $T$ is defined to be the ratio of the output intensity to the input intensity. The observed absorption is due to a single non-degenerate atomic transition of a dopant ion. The doping concentration is $N=10^{15} \mathrm{~cm}^{-3}$ and the index of the host medium is $n=2$. Determine the spontaneous emission rate $t_{s}$ for the transition.

3. Suppose an argon ion laser is oscillating at a wavelength of 488 nm . It has a 1 -m long cavity filled with gas that supports a Gaussian mode with an approximately uniform beam area of $5 \times 10^{-6} \mathrm{~m}^{2}$. The output coupler transimission is $5 \%$, and additional losses amount to $3 \%$ per round trip. The laser transition has a spontaneous emission time of $7 \times 10^{-9} \mathrm{~s}$ and a homogeneously broadened linewidth $\Delta \nu=3 \mathrm{GHz}$. If the laser's output power is 200 mW , what is the saturated gain coefficient $\gamma$ for the medium?
4. If an optical parametric oscillator is pumped by laser pulses having a duration of 1 ps , but the OPO cavity has a photon lifetime of 1 ns , what will be the duration of the OPO output pulse? Explain your reasoning.
5. Suppose a uniaxial electro-optic material has nonzero EO coefficients $r_{13}=$ $r_{41}=r_{43}$. Design a phase modulator using this material: draw a clear picture showing a possible orientation of the $x, y$, and $z$ crystal axes, the orientation of the light polarization, and the direction of the applied electric field. (Again, I'm looking for a phase modulator, not anything else!)
6. Is it possible to use the $d_{11}, d_{22}$, or $d_{33}$ nonlinear optical coefficients for second harmonic generation in a critically phase matched uniaxial crystal? For each element, if you think the answer is no, explain the problem. If you think the answer is yes, draw a sketch showing an example setup. (The sketch should show the crystal axes and the polarizations of all the beams involved.)
7. Consider a uniaxial medium having anomalous dispersion, so that $d n / d \omega<0$. If it has indices of refraction $n_{o}(\omega)=2.0, n_{e}(\omega)=2.5$ and $n_{o}(2 \omega)=1.9, n_{e}(2 \omega)=2.3$, what is the phase matching angle for second harmonic generation?
8. If an optical parametric amplifier has a transparency range from 300 nm to $3 \mu \mathrm{~m}$ and is pumped at 532 nm , what wavelengths of light can be amplified?
9. Ordinarily, second harmonic generation is rather inefficient for CW beams, due to the high intensities required. However, the efficiency can be increased by placing the nonlinear crystal inside the cavity of the CW laser, as the intensity within the cavity is very high. Analyze this situation:

Suppose the laser oscillates at $\omega$, and it has a small signal gain of $g_{0}$ per round trip, a saturation intensity $I_{s}$, and a total cavity loss of $\Gamma_{i} \ll 1$ per round trip. The nonlinear crystal has a conversion parameter of $\beta$, such that it produces an intensity $I(2 \omega)$ equal to $\beta I(\omega)^{2}$. The cavity mirrors allow light of frequency $2 \omega$ to escape.
(a) How will the cavity loss (at $\omega$ ) be modified by the presence of the nonlinear crystal?
(b) In terms of the parameters given, calculate the intensity of light at $2 \omega$ that will be produced.
(c) Evaluate your answer to (b) in the limit $\Gamma_{i} \rightarrow 0$.

