Phys 532 Final Exam Solutions

1. Expect beam emitted from fiber to be approximately Gaussian,

$W_0 \approx 5\mu m$

So, can propagate a distance $z_0 \approx \frac{\pi W_0^2}{\lambda}$ before diverging significantly.

$$z_0 = 50\mu m$$

So, expect fiber ends must be closer than this: $\boxed{d < 50\mu m}$

2. Know $T = e^{-\alpha \ell}$ α: absorption coefficient

Since $\ell = 1cm$, $T = \frac{1}{2}$, get $\alpha = \frac{\ln 2}{1\,cm} = 0.7\,cm^{-1}$

Also, $\alpha = \frac{\lambda^2}{8\pi t_s} g(\nu) N$

Here $g(\nu)$ is flat, so

$$g(\nu) = \frac{1}{\Delta\nu} \quad (3\times10^{14}\,Hz < \nu < 3.03\times10^{14}\,Hz)$$
$$= 0 \quad \text{otherwise}$$

$$\Delta\nu = 3\times10^{12}\,Hz$$

So $t_s = \frac{\lambda^2}{8\pi\alpha}\frac{1}{\Delta\nu} N$

Use $\lambda_0 = \frac{c}{\nu} = \frac{3\times10^{8}\,m/s}{3\times10^{14}\,Hz} = 1\,\mu m$

$\lambda = \lambda_0/n = 500\,nm$

So $t_s = \frac{(500nm)^2}{8\pi \times 0.7cm^{-1}} \frac{1}{3\times10^{12}\,Hz} \times 10^{15}\,cm^{-3} = \boxed{4.7\times10^{-8}\,s}$

3. For any laser in steady state, gain = loss

So $g = 2\gamma l = \Gamma = 0.08$

$$\gamma = \frac{0.08}{2l} = \boxed{0.04\ m^{-1}}$$

4. Regardless of pump duration, output light requires time τ_p to exit cavity.

So, output pulse duration will be $\sim \boxed{1\,ns}$

5. Have $r_{13} = r_{xxz}$

$r_{41} = r_{yzx}$

$r_{43} = r_{yzz}$

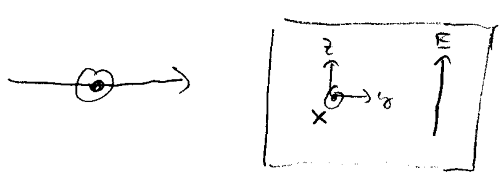
Index ellipsoid:

$$\frac{x^2}{n_o^2} + \frac{y^2}{n_o^2} + \frac{z^2}{n_e^2} + 2r_{13}x^2E_z + 2r_{41}yzE_x + 2r_{43}yzE_z = 1$$

If $n_o \neq n_e$, r_{41} & r_{43} terms will not give significant effect.

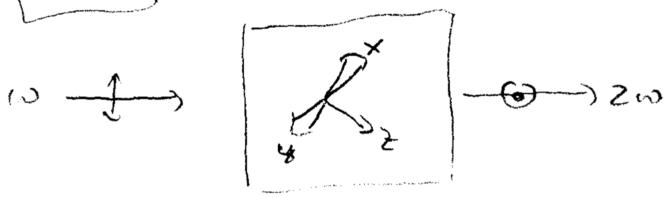
So use r_{13} term:

Apply E_z, polarize light along x



6. $d_{11} = d_{xxx}$ and $d_{22} = d_{yyy}$

$\boxed{\text{Yes:}}$

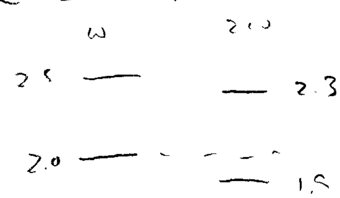


Same picture works in either case

$d_{33} = d_{zzz}$

$\boxed{\text{No:}}$ Only way for a beam to have an E_z component is if it is an extraordinary beam.

But, can't phase match two e-beams.



7. Need $n_e(2\omega) = n_o(\omega)$

$$\frac{1}{n_o(\omega)^2} = \frac{\cos^2\theta}{n_o(2\omega)^2} + \frac{\sin^2\theta}{n_e(2\omega)^2}$$

$$= \frac{1}{n_o(2\omega)^2} + \sin^2\theta\left(\frac{1}{n_e(2\omega)^2} - \frac{1}{n_o(2\omega)^2}\right)$$

$$\sin^2\theta = \frac{\frac{1}{n_o(\omega)^2} - \frac{1}{n_o(2\omega)^2}}{\frac{1}{n_e(2\omega)^2} - \frac{1}{n_o(2\omega)^2}} = \frac{\frac{1}{2.0^2} - \frac{1}{1.9^2}}{\frac{1}{2.3^2} - \frac{1}{1.9^2}} = 0.307$$

$$\boxed{\theta = 33.6^\circ}$$

8. Have $\frac{1}{\lambda_1} + \frac{1}{\lambda_2} = \frac{1}{\lambda_3}$ $\quad \lambda_3 = 532\text{nm}$

Need λ_1 & λ_2 between 300nm & 3μm

If $\lambda_2 = 3\mu m$, $\lambda_1 = 647\text{nm}$

So, output ranges from $\boxed{647\text{nm to } 3\mu m}$

9. a) Power produced at 2ω is lost from light at ω

So, when light passes through crystal, have

$$I(\omega) \rightarrow I(\omega) - I(2\omega)$$
$$= I(\omega) - \beta I(\omega)^2$$

In one round trip, get

$$I(\omega) \rightarrow (1-\Gamma_i) I(\omega) - \beta I(\omega)^2$$
$$= I(\omega) - \Gamma_i I(\omega) - \beta I(\omega)^2$$

So acts like $\boxed{\Gamma = \Gamma_i + \beta I(\omega)}$

b) As always, have gain = loss

$$g = \Gamma$$

$$\frac{g_0}{1+\frac{I}{I_s}} = \Gamma_i + \beta I$$

Solve for I

$$g_0 = (\Gamma_i + \beta I)\left(1 + \frac{I}{I_s}\right)$$
$$= \Gamma_i + \beta I + \frac{\Gamma_i}{I_s} I + \frac{\beta}{I_s} I^2$$

$$I^2\left(\frac{\beta}{I_s}\right) + I\left(\beta + \frac{\Gamma_i}{I_s}\right) + (\Gamma_i - g_0) = 0$$

$$I = \frac{I_s}{2\beta}\left[-\left(\beta + \frac{\Gamma_i}{I_s}\right) + \sqrt{\left(\beta + \frac{\Gamma_i}{I_s}\right)^2 - 4\frac{\beta}{I_s}(\Gamma_i - g_0)}\right]$$

Need $g_0 > \Gamma_i$, so $I = \frac{I_s}{2\beta}\left[\sqrt{\left(\beta - \frac{\Gamma_i}{I_s}\right)^2 + 4 g_0 \frac{\beta}{I_s}} - \beta - \frac{\Gamma_i}{I_s}\right]$

But then

$$I(2\omega) = \beta\, I(\omega)^2$$

$$= \beta \frac{I_s^2}{4\beta^2}\left[\left(\beta - \frac{\Gamma_i}{I_s}\right)^2 + 4 g_0 \frac{\beta}{I_s} + \left(\beta + \frac{\Gamma_i}{I_s}\right)^2 - 2\left(\beta + \frac{\Gamma_i}{I_s}\right)\sqrt{\left(\beta - \frac{\Gamma_i}{I_s}\right)^2 + 4 g_0 \frac{\beta}{I_s}}\,\right]$$

$$= \frac{I_s^2}{4\beta}\left[2\beta^2 + 2\frac{\Gamma_i^2}{I_s^2} + 4 g_0 \frac{\beta}{I_s} - 2\left(\beta + \frac{\Gamma_i}{I_s}\right)\sqrt{\left(\beta - \frac{\Gamma_i}{I_s}\right)^2 + 4 g_0 \frac{\beta}{I_s}}\,\right]$$

$$= \frac{1}{2}\beta I_s^2 + \frac{1}{2}\frac{\Gamma_i^2}{\beta} + g_0 I_s - \frac{1}{2}\left(I_s^2 + \frac{\Gamma_i I_s}{\beta}\right)\sqrt{\left(\beta - \frac{\Gamma_i}{I_s}\right)^2 + 4 g_0 \frac{\beta}{I_s}}\,\Big]$$

$$\boxed{I(2\omega) = \frac{1}{2}\beta I_s^2 + \frac{1}{2}\frac{\Gamma_i^2}{\beta} + g_0 I_s - \frac{1}{2}\left(\beta I_s^2 + \Gamma_i I_s\right)\sqrt{\left(1 - \frac{\Gamma_i}{\beta I_s}\right)^2 + \frac{4 g_0}{\beta I_s}}}$$

c) If $\Gamma_i \to 0$

$$I(2\omega) \to \frac{1}{2}\beta I_s^2 + g_0 I_s - \frac{1}{2}\beta I_s^2\sqrt{1 + \frac{4 g_0}{\beta I_s}}$$

$$\boxed{I(2\omega) = g_0 I_s - \frac{1}{2}\beta I_s^2\left(\sqrt{1 + \frac{4 g_0}{\beta I_s}} - 1\right)}$$