Phys 532 Assignment 11 — Due April 16

1. **Pulsed vs. CW Harmonic Generation:** Suppose a particular second-harmonic generation arrangment is run with CW input power $P(\omega)$ and output power $P(2\omega)$ such that the conversion efficiency is $P(2\omega)/P(\omega) = \epsilon \ll 1$. Now drive it instead with pulsed laser light having a pulse duration of Δt and repetition period T. If the *average* input power is still the same $P(\omega)$, find the new value of the conversion efficiency. (Note that once $\epsilon \sim 1$, the approximation that the field at ω is undepleted breaks down. Assume ϵ is small enough that you don't have to account for this.)

2. **Non-critical Phase Matching:** As mentioned in class, $LiNbO_3$ can be used with noncritical phase matching to frequency-double light with a wavelength of 1064 nm. Suppose instead you wish to double 1100 nm light to a wavelength of 550 nm. Dmitriev, Gurzadyan and Nikogosyan's *Handbook of Nonlinear Optical Crystals* lists the indices of refraction for $LiNbO_3$ as

$$n_o^2 = 4.9130 + \frac{0.1173 + 1.65 \times 10^{-8} T^2}{\lambda^2 - (0.212 + 2.7 \times 10^{-8} T^2)^2} - 0.0278\lambda^2$$

$$n_e^2 = 4.5567 + 2.605 \times 10^{-7} T^2 + \frac{0.0970 + 2.70 \times 10^{-8} T^2}{\lambda^2 - (0.201 + 5.4 \times 10^{-8} T^2)^2} - 0.0224\lambda^2$$

at temperature T in K and wavelength λ in μm. Given that $LiNbO_3$ has a nonlinear coefficient $d_{31} = 4 \times 10^{-23}$ C/V^2, find:
(a) The temperature required for noncritical phase matching
(b) The optimum output power obtained for CW input power of 100 mW and a 3-cm long crystal.

3. **Difference Frequency Generation:** A tunable laser source in the infrared can be obtained by difference-frequency generation between two diode lasers. Suppose diodes at $\lambda_1 = 680$ nm and $\lambda_2 = 808$ nm are used to generate light at $\lambda_3 = 4.3$ μm, using proustite, which has indicies of refraction

$\lambda(\mu\text{m})$	n_o	n_e
0.680	2.9685	2.6996
0.808	2.8865	2.6366
4.3	2.7347	2.5203

(a) If each diode can be tuned by ± 3 nm, what range of wavelengths in the infrared can be obtained?
(b) Suppose a phase matching scheme uses the d_{16} nonlinear coefficient, along with an extraordinary ray at λ_1, an ordinary ray at λ_2, and an extraordinary ray at λ_3. Draw a sketch showing the light polarizations and crystal axes for this case. (Don't worry about varying the azimuthal angle ϕ, just decide whether $\phi = 0°$ or $\phi = 90°$ is more appropriate.)
(c) Determine the phase matching angle θ between the crystal z axis and the light propagation direction. What is the effective nonlinear coupling d'?

4. **Phase Matching for Spontaneous Parametric Fluorescence:** Consider spontaneous parametric fluorescence in a crystal of $LiNbO_3$, as sketched in Fig 11.4 below. Here many different frequencies are produced, with light of wavelength λ propagating at an angle θ determined by the phase matching conditions. Suppose the crystal is oriented as shown, with a pump at $\lambda_3 = 532$ nm polarized along the crystal z axis. Determine the output angle and polarization for light of wavelength 2 μm. The required indicies of refraction can be determined from the formulas of problem 1, with $T = 300$ K.

Phys 822 students only:

5. **Walk-Off:** An important limitation of second-harmonic generation is beam walk-off. This occurs because when a wave propagates through an anisotropic crystal in a direction which is not a symmetry axis, the direction of the phase velocity $\mathbf{k}$ is not the same as the direction of the energy flow $\mathbf{S} = \frac{1}{2}\mathbf{E}\times\mathbf{H}$. Saleh and Teich explain this phenomenon in Section 6.3D. Phase matching occurs when $\mathbf{k}^{2\omega} = 2\mathbf{k}^{\omega}$, so $\mathbf{S}(2\omega)$ points in a different direction than $\mathbf{S}(\omega)$. The fundamental and harmonic beams will therefore diverge as they propagate, which limits the length of crystal that can be effectively used.

Figure 11.5 below reproduces Saleh and Teich Figure 6.3-12(b). Here k_x and k_z are the components of $\mathbf{k}$ along the crystal x and z axes, and θ is the angle between $\mathbf{k}$ and the z axis, as normal. The ellipse represents the constraint $|\mathbf{k}| = n(\theta)k_0$, where $n(\theta)$ is the index of refraction for e-polarized light and k_0 is vacuum wavenumber of the light. The ellipse can also be expressed as

$$\frac{k_x^2}{n_o^2} + \frac{k_z^2}{n_e^2} = k_0^2$$

for $k_x = n(\theta)k_0 \sin\theta$ and $k_z = n(\theta)k_0\cos\theta$. The Poynting vector $\mathbf{S}$ is perpedicular to the ellipse, and makes an angle α to the z axis.

(a) Calculate α in terms of θ, n_o, and n_e.

(b) In class, we considered the example of frequency-doubling a ruby laser ($\lambda = 694$ nm) in KDP. The relevant indices of refraction are $n_o(\omega) = 1.5055$, $n_o(2\omega) = 1.5357$, $n_e(\omega) = 1.5357$, and $n_e(2\omega) = 1.4897$. The phase matching angle is 53.5°, with the beam at ω o-polarized and the beam at 2ω e-polarized. Calculate the angle between the Poynting vectors $\mathbf{S}(\omega)$ and $\mathbf{S}(2\omega)$ in this case.

Fig. 11.4

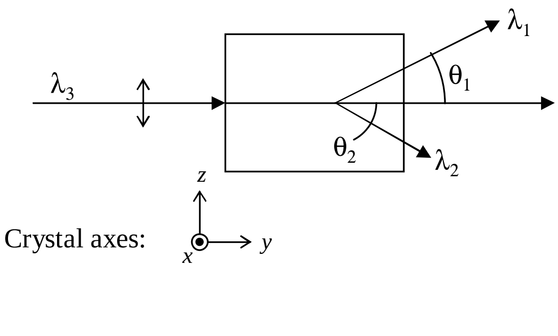

Fig. 11.5