Phys 532 Assignment 8 Due March 26

1. **Kerr Modulator:** Suppose an intensity modulator is constructed by placing an isotropic Kerr medium between two polarizers, as in Saleh and Teich Fig 18.1-6. Calculate the output intensity as a function of applied voltage, in terms of the medium length ℓ, medium thickness d, index of refraction n, Kerr coeffcient s, and light wavelength λ_0. (If you want to worry about polarization details, assume that the light propagates along z and is polarized along x, the electric field is applied along x, and you are using the s_{xxxx} Kerr coefficient. The polarizers are at $\pm 45°$ to x.)

2. **Biaxial Crystal:** Consider a simple example of light propagating through a biaxial crystal, where $n_x \neq n_y \neq n_z$. Suppose the propagation direction of the light is $\hat{\mathbf{k}} = \sin\theta\hat{\mathbf{x}} + \cos\theta\hat{\mathbf{z}}$. Determine the directions and effective indices for the two principal polarizations of the light.

3. **Designing a Modulator:** In section 18.1-B (see figure 18.1-5), Saleh and Teich describe how the electro-optic effect can be used in a Mach-Zehnder interferometer to construct a intensity modulator integrated with a fiber-optic system. Design a modulator of this type using the material $LiNbO_3$. Select the orientation of the crystal and the polarization of the guided wave so as to obtain the smallest possible half-wave voltage V_π. Note that for an integrated modulator, it is not easy to apply an electric field along the direction of light propagation, so you can assume a transverse field configuration.

Example 18.2-1 has some discussion of the properties of $LiNbO_3$. However, to be complete, you should consider all possible system configurations, and not just the one discussed in the example.

If the active region has length $\ell = 1$ mm and width $d = 5$ μm, and the wavelength $\lambda_0 = 850$ nm, calculate V_π. The refractive indices for $LiNbO_3$ are $n_o = 2.29$ and $n_e = 2.17$, and the non-zero electro-optic coefficients are $r_{33} = 30.9$ pm/V, $r_{13} = 8.6$ pm/V, $r_{22} = 2.4$ pm/V, and $r_{51} = 28$ pm/V.

4. **Electro-optic Phase Modulation:** The electro-optic effect can also be used for frequency modulation. For example, suppose a $LiNbO_3$ crystal of length ℓ is oriented with a laser beam propagating along x and polarized along z. An oscillating electric field $E_1 \cos \Omega t$ is applied along the z direction. The electric field of the laser itself oscillates as $E_L = E_0 \exp i(\omega_0 t - kx)$.
(a) Show that after exiting the crystal, the laser field has a time dependence of $\exp i(\omega_0 t + \delta \cos \Omega t)$, and determine δ.
(b) If the "instantaneous" frequency of the laser $\omega(t)$ is defined by

$$\frac{dE_L}{dt} = i\omega(t)E_L,$$

find the range of instantaneous frequencies sampled by the laser output.
(c) In the limit $\delta \ll 1$, show that the laser electric field can be expressed as a sum of three components oscillating at ω_0, $\omega_0 + \Omega$, and $\omega_0 - \Omega$, and find their relative amplitudes.

Frequency modulation can thus be thought of either as a variation of the instantaneous frequency, or as the generation of additional frequency components at $\omega_0 \pm \Omega$. (This is a little counter-intuitive when δ is small, since the instantaneous frequency never actually equals $\omega_0 \pm \Omega$!)

822 students only:

5. **Strong Phase Modulation:** In problem 4, if δ is not small then the modulated optical field can still be expressed as

$$E_L = E_0 \sum_n A_n(\delta) \exp i(\omega_0 t + n\Omega t).$$

Determine the coefficients A_n. How large must δ be to have equal amplitudes for the "carrier" at $n = 0$ and the first "sidebands" at $n = \pm 1$? (Hint: the answer involves Bessel functions.)