Phys 532 Assignment 8 Solutions ①

1. a) Bragg angle $\Theta = \frac{\lambda}{2\Lambda}$, full angle between beams is $2\Theta = \frac{\lambda}{\Lambda}$

λ: light wave length $= \frac{\lambda}{n} = 332\text{ nm}$ here

Λ = sound wavelength $= \frac{v_s}{\nu_s} = \frac{617\,\text{m/s}}{80\,\text{MHz}} = 7.7\,\mu\text{m}$

So $2\Theta = \frac{332\,\text{nm}}{7.7\,\mu\text{m}} = 0.043\text{ rad} = 2.47^\circ$

But this is angle in crystal. In air, larger by n since $\Theta_{air} = n\,\Theta_{glass}$

$$\boxed{2\Theta_{air} = 5.8^\circ}$$

b) Beam waist of $200\,\mu$m, estimate wave requires time

$T = \frac{200\,\mu\text{m}}{617\,\text{m/s}} = \boxed{0.32\,\mu\text{s}}$ to cross beam

For one full modulation cycle, need to turn light on <u>and</u> off, so max rate is

$f = \frac{1}{2T} = \boxed{1.5\text{ MHz}}$

2. Peak power $= \frac{1\text{ J}}{5\text{ ns}} = \boxed{2\times 10^8\text{ W}}$

Peak intensity $I = \frac{2P}{\pi w_0^2} = \boxed{5\times 10^{16}\ \frac{\text{W}}{\text{m}^2}}$

Peak E-field : $I = \frac{|E_0|^2}{2\mathfrak{z}_0}$ $\quad \mathfrak{z}_0 = 377\,\Omega$

so $E_0 = \sqrt{2\mathfrak{z}_0 I} = \boxed{6.2\times 10^{9}\ \frac{\text{V}}{\text{m}}}$

Over long times, $P_{avg} = \frac{1\text{ J}}{1\text{ s}} = \boxed{1\text{ W}}$

3. a) Flash energy $E_f = 20\,J$, $0.2\,J$ absorbed

ion in excited state has $h\nu = \frac{hc}{\lambda} = 1.9\times10^{-19}\,J$ energy

So, # of excited ions is $\frac{0.2\,J}{1.9\times10^{-19}\,J} = 1.1\times10^{18}$

Volume of laser medium is $10\,cm^3$, so

$$N_2 = \frac{1.1\times10^{19}}{10\,cm^3} = 1.1\times10^{17}\,cm^{-3} = \Delta N$$

So
$$\gamma_0 = \frac{\left(\frac{1.064\,\mu m}{1.5}\right)^2 (1.1\times10^{17}\,cm^{-3})}{8\pi\,(1\,ms)\,(2\times10^{11}\,Hz)} = 10.7\,m^{-1}$$

and
$$g_0 = e^{2\gamma_0 l} = e^{2.1} = \boxed{8.5 \gg \Gamma = 0.3}$$

b) Peak power $P = h\nu\,V\cdot\frac{1}{\tau_p}\,\frac{\Delta N_i}{2}$

V = mode volume $\approx l\times\pi w_0^2$

Take $\pi w_0^2 = A$, optimum case

Then $V\Delta N_i = 1.1\times10^{18}$

$$\tau_p \approx \frac{1}{\Gamma\nu_F} = \frac{1}{\Gamma}\cdot\frac{2d}{c} = \frac{1}{0.3}\cdot\frac{2\times0.3\,m}{3\times10^8\,\frac{m}{s}} = 6.7\,ns$$

$$P_{max} \approx (1.9\times10^{-19}\,J)(1.1\times10^{18})\left(\frac{1}{2\times6.7\,ns}\right) = \boxed{1.6\times10^{7}\,W}$$

Pulse duration $\approx \boxed{\tau_p = 6.7\,ns}$

4. For mode locked laser, have

(3)

$$E(t) = \sum_n A_n e^{i2\pi(\nu_0 + n\nu_F)t}$$

= sum of waves at frequencies $\nu + n\nu_F$

So expect $|A_n|^2 \propto P(\nu = \nu_0 + n\nu_F)$

$$A_n \propto e^{-\frac{n^2\nu_F^2}{2\sigma^2}}$$

If $\nu_F \ll \sigma$, replace sum by integral

$$E(t) \propto \int_{-\infty}^{\infty} d\nu \, A(\nu) \, e^{i2\pi\nu t}$$

$$A(\nu) = e^{-\frac{(\nu-\nu_0)^2}{2\sigma^2}}$$

Fourier transform of Gaussian:

$$E(t) \propto e^{i2\pi\nu_0 t} \, e^{-2\pi^2 t^2 \sigma^2} \qquad \text{(from Table A.1-1)}$$

Then power $P \propto |E|^2$, so

$$\boxed{P(t) \propto e^{-4\pi^2 t^2 \sigma^2}}$$

Gaussian, width $\Delta t \approx \frac{1}{2\pi\sigma}$