02/09/05 Lecture 10

Next big topic: Laser theory.

Starting with how light interacts with matter.

Matter (atoms): quantum mechanics, (always)

Light: three possible approaches

* Classical E&M theory: treat light as waves obeying Maxwell's equations.

Almost sufficient to describe laser, need little input from quantum mechanics.

* photon theory: semi-classical approach
  simplified quantum approach.

treat light as quantum particles, that are absorbed and emitted in discrete events.

* Quantum Optics: treat light as quantum wave coupled to quantum atoms.

This approach is the most accurate, but also hardest.

We'll take photon theory approach. This approach use the concept of photon from quantum mechanical treatment of light, but simplifies things by glossing over details of emission & absorption events.

Recall basic laser idea:
put fluorescent medium in a cavity,
every round trip light passes through medium
→ gets amplified.
study amplification / absorption process.

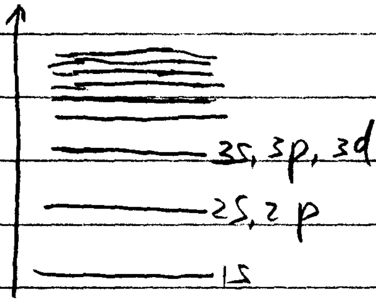
First briefly review quantum levels of matter, and light quanta : photon, then focus on their interactions.

Start with medium.

QM says that a given type of matter has certain allowed values for internal energy called energy levels

Examples

* Hydrogen atom



levels can be degenerate
= multiple distinct states
with same energy

# of states = degeneracy = g

Note: only relative degeneracy usually matters

In H, ground state is 1s : $g_1 = 1$
But electron has spin : $g_1 = 1 \times 2 = 2$
But nucleus has spin : $g_1 = 1 \times 2 \times 2 = 4$

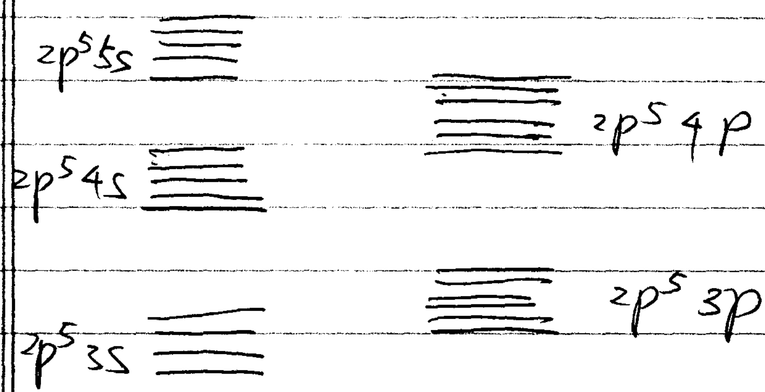
Other levels have same contributions :

for $n=2$ $\quad g_2 = 2s + 2p_{-1} + 2p_0 + 2p_1 = 4$

consider electron spin $\quad g_2 = 4 \times 2 = 8$
further more consider nucleus spin $\quad g_2 = 4 \times 2 \times 2 = 16$

only $\frac{g_2}{g_1}$ usually matters, so ignore spins
since they make same contribution for both $n=1$
and $n=2$ states, and take $g_1 = 1$
$g_2 = 4$

* Ne : more complicate atom. degeneracy is broken

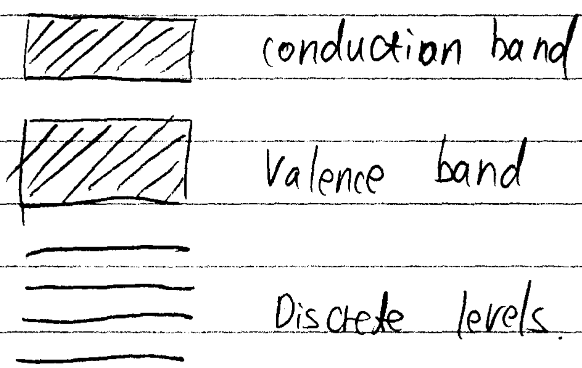


Each level has many nearby "sublevels"

Lump sublevels together or treat separately, depending on energy resolution of process considered.

Narrow linewidth laser : treat separately.
Broad linewidth : lump together.

* Si : solid, have so many nearby states, they can't be distinguished, → energy band



Typically need to integrate over energies in band ...
We'll more stick to discrete-level systems.

Light:

Consists of photons: each photon frequency ν has energy $E = h\nu$

Photons are associated with modes
Given mode contains integer # of photons

Mode: $E_{\vec{n}} \sim \sin\frac{n_x \pi x}{d} \sin\frac{n_y \pi y}{d} \sin\frac{n_z \pi z}{d}$

for cube cavity

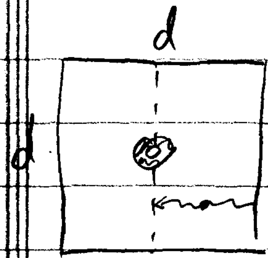
$E_{n,l,m} \sim$ Hermite-Gaussian mode
for laser cavity

Although we have been talking about Gaussian mode for laser cavity, but to start the discussion of light & matter think about cube cavity mode for now, which is easier since field is uniform. The results should be easy to generalize to Gaussian mode.

Note: free space — limit of cube as $d \to \infty$

Interactions :

Say a cavity contains both a photon and an atom, what happens?



photon bounces between cavity walls passes through plane of atom at rate $\frac{c}{d}$ $\left(\sim \frac{1}{t} = \frac{1}{d/c}\right)$

2D picture of cube cavity

Every time photon passes plane of atom, has probability p' to be absorbed.

p' depends on size of cavity, smaller cavity p' lager.

write. $p' = \frac{\sigma}{A}$ A: cross sectional area of cavity $= d^2$

σ: "cross section" of atom (having units of area.

Treat atom as little ball with cross area σ. photon is absorbed if it hits ball.

So $p' = \frac{\sigma}{d^2}$

Absorb at rate $\frac{c}{d} \cdot \frac{\sigma}{d^2}$ (prob. per unit time)

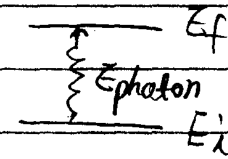
$= \frac{c}{V} \cdot \sigma$ $\qquad V$: cavity volume

If N photons, absorption at rate

$$P_{abs} = N \frac{c}{V} \sigma$$

However $\sigma = \sigma(\nu)$ is frequency-dependent.

Atom can absorb light if $E_{photon} \approx E_f - E_i$ for two atomic levels i & f



So $\sigma(\nu)$ has peaks at atomic resonances.

To absorb a photon at $\sim E_f - E_i$, atom need to initially be in state i : the lower energy state.

Depends on state of atom, there are another two possible, but less obvious process.

Stimulated emission:

QM is invariant under time reversal so if atom is initially in higher state f, can emit photons at rate.

$$P_{stim} = N \frac{c}{V} \sigma(\nu)$$

$\sigma(\nu)$ same as for absorption, since its a characteristics of atom.

Note, stimulated emission depends on N:

presence of photons in a mode "encourages" atom to emit photon into that mode.

Spontaneous emission:

even if cavity is empty, atom is higher state f can emit photons at rate

$$P_{spont} = \frac{c}{V} \sigma(\nu)$$

again, same $\sigma(\nu)$

can only be clearly explain by QED: zero-point energy.