02/18/05 Lecture 14

Lineshape function: $g(\nu)$

homogeneous broadening: Lorentzian

$$g(\nu) = \frac{\Delta\nu/2\pi}{(\nu-\nu_0)^2 + (\Delta\nu/2)^2}$$

$$\Delta\nu = \frac{1}{2\pi}\left(\frac{1}{\tau_1} + \frac{1}{\tau_2} + 2f_{col}\right)$$

$$g(\nu_0) = \frac{2}{\pi\Delta\nu}$$

inhomogeneous broadening: Gaussian

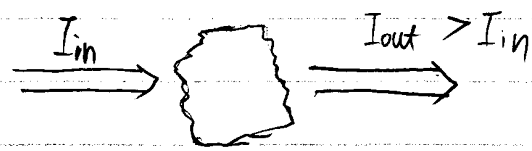
$$\overline{g(\nu)} = \sqrt{\frac{M\lambda^2}{2\pi kT}}\, e^{-\frac{M\lambda^2}{2kT}(\nu-\nu_0)^2}$$

$$\overline{\Delta\nu} = (8\ln 2)^{1/2}\sqrt{\frac{kT}{M\lambda^2}}$$

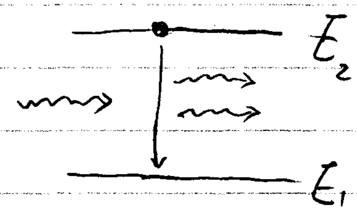
$$\overline{g(\nu_0)} = \sqrt{\frac{M\lambda^2}{2\pi kT}} = \frac{0.94}{\overline{\Delta\nu}}$$

Lasers: light amplification

Get amplification through stimulated emission.



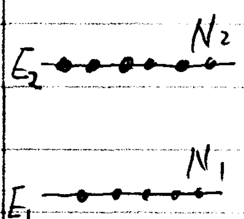
atom, in state 2.



In general, medium will contain atoms in both state 1 and state 2.

Define: N_1 = density of atoms in state 1
N_2 = density of atoms in state 2 (atoms/m^3)

ϕ: photon flux $= \frac{\text{photons}/s}{m^2}$



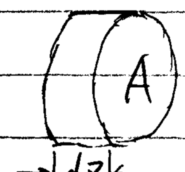
Each atom in state 1 absorbs photons at rate $W_{1\to 2} = \frac{\sigma}{h\nu} I g_2 = \sigma \phi g_2$

Each atom in state 2 emitts (stimulated) at rate

$$W_{2\to 1} = \sigma \phi g_1$$

Think about $\frac{d\phi}{dz}$

Say



n_γ = # of photons entering volume in time Δt

$= \phi \cdot A \cdot \Delta t$

n_1 = # of atoms in state 1 $= N_1 A\, dz$

n_2 = # of atoms in state 2 $= N_2 A\, dz$

Then # of photons leaving is n_γ'

$$n_\gamma' = n_\gamma - n_1 W_{1\to2} \Delta t + n_2 W_{2\to1} \Delta t$$

$$\phi' A \Delta t = \phi A \Delta t - N_1 A\, dz\, \sigma \phi g_2 \Delta t + N_2 A\, dz\, \sigma \phi g_1 \Delta t$$

flux out

$$\phi(z+dz) = \phi(z)\left[1 - N_1 g_2 \sigma dz + N_2 g_1 \sigma dz\right]$$

$$\frac{d\phi}{dz} = \sigma\left(N_2 g_1 - N_1 g_2\right)\phi$$

Recall $\quad g_1 \sigma = \frac{\lambda^2}{8\pi t_s} g(\nu)$

so

write $$\frac{d\phi}{dz} = \frac{\lambda^2 g(\nu)}{8\pi t_s}\left(N_2 - \frac{g_2}{g_1}N_1\right)\phi$$

Or in terms of intensity, $I = h\nu\phi$. so

$$\frac{dI}{dZ} = \frac{\lambda^2 g(\nu)}{8\pi t_s}\Delta N\, I$$

$$\Delta N \equiv N_2 - \frac{g_2}{g_1}N_1$$

If $\Delta N > 0$, I increases: gain

write $$\frac{dI}{dz} = \gamma I.$$

$$\gamma = \frac{\lambda^2 g(\nu)}{8\pi t_s}\Delta N,$$ "gain coefficient"

$$I(z) = e^{\gamma z} I(0)$$

If $\Delta N < 0$, I decreases: absorption.

write $$\frac{dI}{dz} = -\alpha I$$

$$\alpha = -\frac{\lambda^2 g(\nu)}{8\pi t_s}\Delta N = -\gamma$$ "absorption coefficient"

$$I(z) = I(0)e^{-\alpha z}$$

In lasers, we want gain.

however, in normal situations, have $\Delta N < 0$:

stat. mech. says that, in thermal equilibrium,

$$\frac{\text{occupation of state at } E_2}{\text{occupation of state at } E_1} = e^{-(E_2-E_1)/k_BT} < 1$$

with degeneracy: $$\frac{N_2/g_2}{N_1/g_1} = e^{-(E_2-E_1)/kT}$$

So $$\frac{g_1}{g_2}\frac{N_2}{N_1} < 1$$

$$N_2 < \frac{g_2}{g_1} N_1 \Rightarrow \Delta N < 0$$

So to make a laser, medium cannot be in thermal equilibrium.

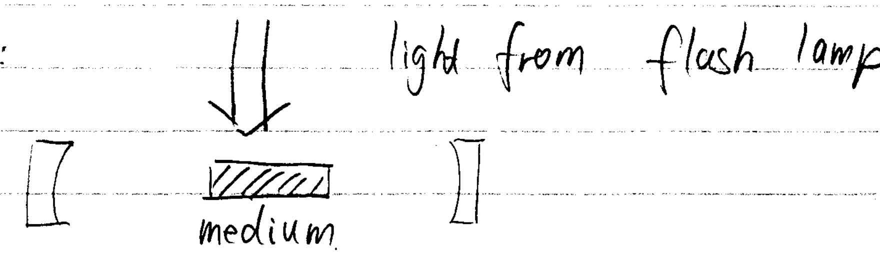
⇒ Requires power sources to set up & maintain inversion.

Termed "Pumping"

First idea: (warning doesn't work)

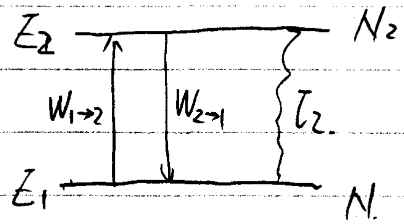
Drive $1 \to 2$ by external source.

Example:



(Hope for laser to convert incoherent lamp light into coherent laser beam)

Can be described by simple model: Rate equations.



pumping drives transitions at rate W_p

$$W_{p(1\to2)} = \frac{\sigma}{h\nu} I_p g_2$$

But if pump from $1 \to 2$, also pump from $2 \to 1$

$$W_{p(2\to1)} = \frac{\sigma}{h\nu} I_p g_1$$

Decay from $2 \to 1$ $\qquad \tau_2$

So population obey:

$$\frac{dN_1}{dt} = -W_{P_{12}} N_1 + W_{P_{21}} N_2 + \frac{1}{\tau_2} N_2$$

$$\frac{dN_2}{dt} = -\frac{dN_1}{dt}$$

Solve in steady state. $\frac{dN_1}{dt} = \frac{dN_2}{dt} = 0$

$$N_2 = N_1 \frac{W_{P_{12}}}{W_{P_{21}} + \frac{1}{\tau_2}}$$

$$= N_1 \frac{\frac{\sigma I_P}{h\nu} g_2}{\frac{\sigma I_P}{h\nu} g_1 + \frac{1}{\tau_2}}$$

$$= \frac{g_2}{g_1} N_1 \frac{1}{1 + \frac{h\nu}{\sigma I_p \tau_2 g_1}}$$

$$= \frac{g_2}{g_1} N_1 \left[1 - \frac{x}{1+x} \right]$$

$$x = \frac{h\nu}{\sigma I_p \tau_2 g_1}$$

so:

$$\Delta N = N_2 - \frac{g_2}{g_1} N_1 = -\frac{g_2}{g_1} N_1 \frac{x}{1+x} \leq 0$$

no inversion

Best case $\Delta N = 0$, when either $I_p \to \infty$, or $\tau_2 \to \infty$

need a more complex system