Optical amplification.

Gain coefficient :

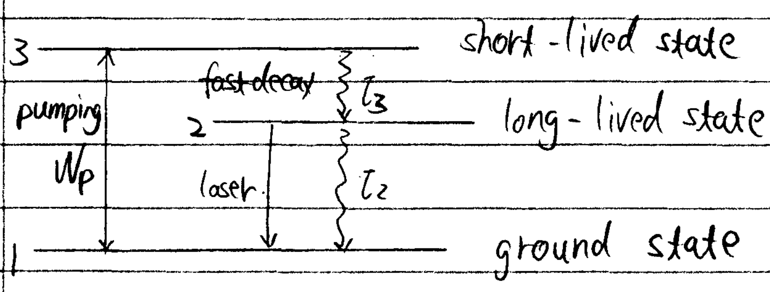
$$\gamma = \frac{\lambda^2}{8\pi t_{sp}} g(\nu)\, \Delta N$$

ΔN = population inversion $= N_2 - N_1 \dfrac{g_2}{g_1}$

Need pumping to achieve inversion

2 - level doesn't work.

3 - level



Analyze for equal degeneracies :

$$\frac{dN_1}{dt} = -W_p N_1 + W_p N_3 + \frac{1}{\tau_2} N_2$$

$$\frac{dN_2}{dt} = +\frac{1}{\tau_3} N_3 - \frac{1}{\tau_2} N_2$$

$$\frac{dN_3}{dt} = +W_p N_1 - W_p N_3 - \frac{1}{\tau_3} N_3$$

Solve in steady state

$$\frac{dN_1}{dt} = \frac{dN_2}{dt} = \frac{dN_3}{dt} = 0$$

$$N_3 = \frac{\tau_3}{\tau_2} N_2$$

So $$0 = -W_p N_1 + W_p \frac{\tau_3}{\tau_2} N_2 + \frac{1}{\tau_2} N_2$$

$$N_2 = N_1 \frac{\tau_2 W_p}{1+\tau_3 W_p}$$

Moreover $N_1 + N_2 + N_3 = N_a$: total atomic density

However, τ_3 is very short, level 3 a negligible stead-state population.

So : $$N_1 + N_2 \cong N_a$$

$$N_1 \left(1 + \frac{\tau_2 W_p}{1+\tau_3 W_p}\right) = N_a$$

$$N_1 = N_a \frac{1+\tau_3 W_p}{1+(\tau_2+\tau_3)W_p}$$

$$N_2 = N_a \frac{\tau_2 W_p}{1+(\tau_2+\tau_3)W_p}$$

$$\Delta N = N_2 - N_1 = N_a \frac{(\tau_2-\tau_3)W_p - 1}{1+(\tau_2+\tau_3)W_p}$$

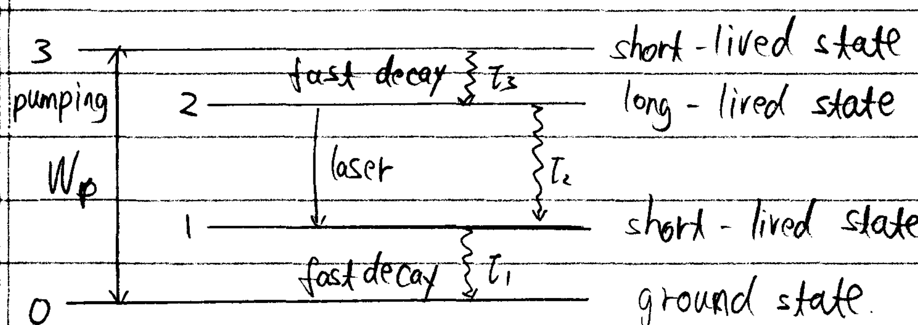
Usually $\tau_2 \gg \tau_3$

$$\Delta N = N_a \frac{\tau_2 W_p - 1}{1 + \tau_2 W_p}$$

Population inversion requires $W_p > \frac{1}{\tau_2}$, strong pumping

This works : "3-level" laser.

Even better : 4 level laser.



Again τ_3 is very short
If τ_1 is short, state 1 is naturally empty.
should get inversion for any pumping.

Approximation : assume W_p is weak enough that
N_0 is constant. $N_0 \cong N_a$
Valid if $W_p \ll \min\left\{\frac{1}{\tau_i}\right\}$

Then have $$\frac{dN_3}{dt} = +W_p N_0 - W_p N_3 - \frac{1}{\tau_3} N_3$$

$$\approx \underbrace{W_p N_a}_{\equiv R \text{ constant pumping rate.}} - \frac{1}{\tau_3} N_3$$

So $\frac{dN_3}{dt} = R - \frac{1}{\tau_3} N_3 = 0$

$$\frac{dN_2}{dt} = +\frac{1}{\tau_3} N_3 - \frac{1}{\tau_2} N_2 = 0$$

$$\frac{dN_1}{dt} = +\frac{1}{\tau_2} N_2 - \frac{1}{\tau_1} N_1 = 0$$

$$N_3 = R\tau_3, \quad N_2 = R\tau_2, \quad N_1 = R\tau_1$$

$$\Delta N = N_2 - N_1 = R(\tau_2 - \tau_1)$$

get inversion as long as $\tau_2 > \tau_1$

More general, consider degeneracy

$$\Delta N = R\left(\tau_2 - \frac{g_2}{g_1}\tau_1\right)$$
$$= W_p N_a \left(\tau_2 - \frac{g_2}{g_1}\tau_1\right)$$

get inversion for any rate, as long as $\frac{\tau_2}{g_2} > \frac{\tau_1}{g_1}$

Most lasers use four level scheme.

So if we achieve $\Delta N > 0$, get gain.

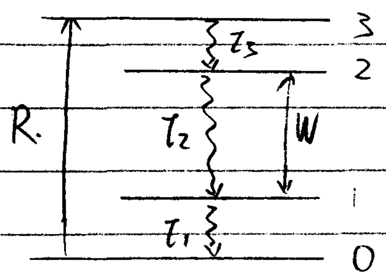
Total gain $G = e^{\gamma d}$ for distance d.

With this gain, get laser lasing. however, gain must stop eventually, otherwise we'd get infinite power out of a laser.

Effect called saturation.

gain reduced at high intensity.

comes from including laser light in rate equations.



Drive $2 \leftrightarrow 1$ at rate W

write $W_{2\to 1} = W g_1$

$W_{1\to 2} = W g_2$

$$W = \sigma \frac{I}{h\nu}$$

Then $$\frac{dN_3}{dt} = +R - \frac{1}{\tau_3} N_3 = 0 \quad \Rightarrow \quad N_3 = R\tau_3$$

$$\frac{dN_2}{dt} = +\frac{1}{\tau_3} N_3 - \frac{1}{\tau_2} N_2 - W g_1 N_2 + W g_2 N_1 = 0$$

$$\frac{dN_1}{dt} = +\frac{1}{\tau_2} N_2 - \frac{1}{\tau_1} N_1 + W g_1 N_2 - W g_2 N_2 = 0$$

add eqn's for 1 & 2

$$R - \frac{1}{\tau_1} N_1 = 0 \qquad \text{so } N_1 = \tau_1 R$$

Then $$R - \left(\frac{1}{\tau_2} + W g_1\right) N_2 + W g_2 \tau_1 R = 0$$

$$N_2 = R \frac{1 + W g_2 \tau_1}{\frac{1}{\tau_2} + W g_1} = \tau_2 R \frac{1 + W g_2 \tau_1}{1 + W g_1 \tau_2}$$

$$\Delta N = \tau_2 R \frac{1 + W g_2 \tau_1}{1 + W g_1 \tau_2} - \frac{g_2}{g_1} \tau_1 R$$

$$= R \frac{\tau_2 (1 + W g_2 \tau_1) - \frac{g_2}{g_1} (1 + W g_1 \tau_2) \tau_1}{1 + W g_1 \tau_2}$$

$$\Delta N = R \frac{\left(\tau_2 - \frac{g_2}{g_1}\tau_1\right)}{1 + W g_1 \tau_2}$$

write $\Delta N = \dfrac{\Delta N_0}{1 + W g_1 \tau_2}$

ΔN_0 = population inversion achieved with no laser light present
$\equiv$ small signal inversion

Use $W g_1 = W_{2\to1} = \dfrac{\lambda^2}{8\pi t_{sp}} g(\nu) \dfrac{I}{h\nu}$

so $\Delta N = \dfrac{\Delta N_0}{1 + \frac{\lambda^2 \tau_2}{8\pi t_{sp}} \frac{g(\nu)}{h\nu} I} \to 0$ as $I \to \infty$

Define $I_s(\nu) = \dfrac{8\pi t_s h\nu}{\lambda^2 \tau_2 g(\nu)}$

So $\Delta N = \dfrac{\Delta N_0}{1 + I/I_s}$

and gain $\gamma = \dfrac{\lambda^2}{8\pi t_s} g(\nu) \dfrac{\Delta N_0}{1 + I/I_s} \equiv \dfrac{\gamma_0}{1 + I/I_s}$

γ_0 = small-signal gain coefficient

when $I = I_s$, gain reduced by factor of 2