Gain reduced at high intensity : saturation

$$\gamma = \frac{\lambda^2}{8\pi t_{sp}} g(\nu) \frac{\Delta N_0}{1 + I/I_s}$$

$$= \frac{\gamma_0}{1 + I/I_s}$$

γ_0 = small-signal gain coefficient

Note: exact form of I_s depends on level structure.
Different for 2-level and 3-level schemes
Also changes in 4-level system if W_p is large

In general define I_s according to $\Delta N = \frac{\Delta N_0}{1 + I/I_s}$

Generally $\frac{1}{I_s} = \frac{\lambda^2}{8\pi t_s} \frac{g(\nu)}{h\nu} \tau_s$

(replace τ_2 by τ_s , τ_s : saturation lifetime.)

I_s: depends only on transition parameters

Another way to interpret saturation: power broadening

$$\gamma(\nu) = \frac{\lambda^2}{8\pi t_{sp}} g(\nu) \frac{\Delta N_0}{1 + I/I_s}$$

But $$\frac{g(\nu)}{1+I/I_s} = \frac{g(\nu)}{1+\frac{\lambda^2 I_s}{8\pi t_{sp}}\frac{I}{h\nu}g(\nu)}$$

For Lorentzian line,

$$g(\nu)=\frac{\Delta\nu/2\pi}{(\nu-\nu_0)^2+(\Delta\nu/2)^2}$$

so $$\frac{g(\nu)}{1+I/I_s} = \frac{\Delta\nu/2\pi}{(\nu-\nu_0)^2+\left(\frac{\Delta\nu}{2}\right)^2+\left(\frac{\lambda^2 I_s}{8\pi t_{sp}}\frac{I}{h\nu}\right)\frac{\Delta\nu}{2\pi}}$$

$$= \frac{\Delta\nu/2\pi}{(\nu-\nu_0)^2+\left(\frac{\Delta\nu}{2}\right)^2\left[1+\frac{\lambda^2 I_s}{8\pi t_{sp}}\frac{I}{h\nu}\underbrace{\frac{\Delta\nu/2\pi}{(\Delta\nu/2)^2}}_{g(\nu_0)}\right]}$$

$$= \frac{\Delta\nu/2\pi}{(\nu-\nu_0)^2+\left(\frac{\Delta\nu}{2}\right)^2\left[1+I/I_s(\nu_0)\right]}$$

$$\equiv \frac{\Delta\nu}{\Delta\nu_s}\,\frac{\Delta\nu_s/2\pi}{(\nu-\nu_0)^2+(\Delta\nu_s/2)^2}$$

$$= \frac{\Delta\nu}{\Delta\nu_s}\,g_s(\nu)$$

for $\Delta\nu_s = \Delta\nu\sqrt{1+I/I_s(\nu_0)}$

still looks Lorentzian, but linewidth increased by $\sqrt{1+I/I_s}$

& oscillator strength decreased by $\frac{1}{\sqrt{1+I/I_s}}$

so could also say

$$\gamma(\nu) = \frac{\lambda^2}{8\pi t_{sp}} \Delta N_0 \frac{\Delta\nu}{\Delta\nu_s} g_s(\nu)$$

Quick review: three important quantities

* (small-signal) gain coefficient:

$$\gamma_0 = \frac{\lambda^2}{8\pi t_{sp}} g(\nu) \Delta N_0$$

* line shape: $g(\nu) = \dfrac{\Delta\nu/2\pi}{(\nu-\nu_0)^2 + (\Delta\nu/2)^2}$ (homogeneous)

  or $\bar{g}(\nu)$ inhomogeneous

* saturation intensity: $I_s = \dfrac{8\pi}{\lambda^2} \dfrac{t_{sp}}{\tau_s} \dfrac{h\nu}{g(\nu)}$

In terms of cross section: $\sigma(\nu) = \dfrac{\lambda^2}{8\pi t_{sp}} g(\nu)$

$$\boxed{\gamma_0 = \sigma \Delta N_0}$$

So ΔN_0 gives "net" density of emitters.

$\gamma_0 dz$: prob. that an incident photon "hits" an atom in distance dz.

Saturation intensity :

$$\boxed{I_s = \frac{h\nu}{\sigma \tau_s}}$$

τ_s : saturation lifetime.

Then $I/I_s \approx$ probability that new photon "hits" atom before it's had a chance to relax .

$I = I_s$ means get $\frac{1 \text{ photon}}{(\text{cross section})(\text{sat. time})}$

Line shape :

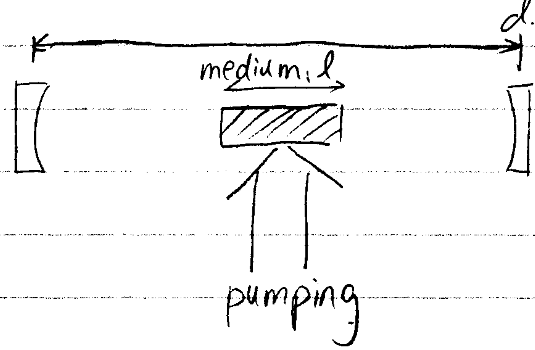
Can be either homogeneous : $g(\nu) = \frac{\Delta\nu/2\pi}{(\nu-\nu_0)^2 + (\Delta\nu/2)^2}$

or inhomogeneous : $\overline{g(\nu)} = \int g'(\nu)\, \rho(\nu_0')\, d\nu_0'$

Most important , width $\Delta\nu$: $g(\nu_0) = \frac{1}{\Delta\nu}$

Generally , $\sigma(\nu_0) \approx \frac{\lambda^2}{8\pi t_{sp} \Delta\nu}$

How lasers start lase.



0) Initially, no light

1) Turn on pumping:

establish inversion: ΔN_0

gain coefficient: γ_0

per round trip gain $\quad G = e^{2\gamma_0 l}$

But nothing to amplify yet.

2) Medium undergoes spontaneous emission:

some photons emitted into cavity modes

3.) Spontaneous light reflects, gets amplified by G

Also attenuated: cavity loss $= \Gamma$

Requires: $\quad G - 1 > \Gamma$

4) Intensity approaches I_s: gain decreases

5) Reach steady state when
amplification $(G-1) > $ losses Γ

$\rightarrow$ laser now "on"

This process typically takes a few ms, really limited by equilibration of atomic populations.

Key insight: in steady state:

$$\boxed{\text{round trip gain} = \text{round trip loss}}$$