04/06/05 Lecture 27. ①

Electro-optic effect : can be described in terms of impermeability tensor.

$$\eta_{ij}(\vec{E}) = \eta_{ij} + \underbrace{\sum_k r_{ijk} E_k}_{\text{Pockels}} + \underbrace{\sum_{\ell m} S_{ij\ell m} E_\ell E_m}_{\text{Kerr}}$$

Examples KDP (symmetry class $\bar{4}2m$) has

$$r = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ r_{41} & 0 & 0 \\ 0 & r_{41} & 0 \\ 0 & 0 & r_{63} \end{bmatrix}$$

Then index ellipsoid becomes

$$\frac{x^2}{n_o^2} + \frac{y^2}{n_o^2} + \frac{z^2}{n_e^2} + 2r_{41} yz E_x + 2r_{41} xz E_y + 2r_{63} xy E_z = 1$$

$(n_o = 1.51, \quad n_e = 1.47)$

How can we use this to make a modulater?
want to change effective n's. x & y are symmetric, so just consider applying E_x or E_z.

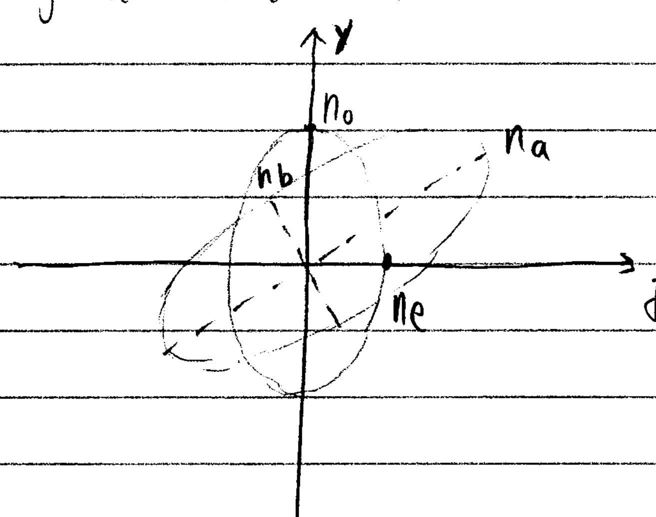
Apply E_x:

Only yz plane is affected: n_x remains n_o
So take yz = plane of polarization
→ light propagates along x

In yz plane, polarization ellipse is

$$\frac{y^2}{n_o^2} + \frac{z^2}{n_e^2} + 2 r_{41} yz E_x = 1$$

The electric field will rotate the ellipse from its original orientation.



Need to find new principal axes: easiest way is to diagonalize quadratic form

$$\begin{bmatrix} y & z \end{bmatrix} \begin{bmatrix} \frac{1}{n_o^2} & r_{41} E_x \\ r_{41} E_x & \frac{1}{n_e^2} \end{bmatrix} \begin{bmatrix} y \\ z \end{bmatrix}$$

Eigenvectors of matrix are principal axes, and eigenvalues are $\frac{1}{n_a^2}$, $\frac{1}{n_b^2}$.

Generally, eigenvalues of $\begin{bmatrix} A & C \\ C & B \end{bmatrix}$ is

$$\begin{vmatrix} A-\lambda & C \\ C & B-\lambda \end{vmatrix} = 0$$

$$\lambda^2 - (A+B)\lambda + AB - C^2 = 0$$

are $\lambda_\pm = \frac{1}{2}\left[A+B \pm \sqrt{(A-B)^2+4C^2}\right]$

Here $A-B = \frac{1}{1.51^2} - \frac{1}{1.47^2} = -0.0242$

suppose $E_x = 10\ \frac{kV}{mm} = 10^7\ \frac{V}{m}$ pretty large

Then $C = 8.6\ \frac{pm}{V} \cdot 10^7\ \frac{V}{m} = 8.6 \times 10^{-5}$

Then $4C^2 \ll (A-B)^2$ for any practical E_x

$$\lambda_\pm \simeq A - \frac{C^2}{(B-A)},\quad B + \frac{C^2}{B-A}$$

so $\frac{1}{n_a^2} = \frac{1}{n_o^2} - \frac{C^2}{B-A}$ $\qquad \frac{1}{n_b^2} = \frac{1}{n_e^2} + \frac{C^2}{B-A}$

$= \frac{1}{n_o^2} - \Delta\left(\frac{1}{n^2}\right)$ $\qquad = \frac{1}{n_e^2} + \Delta\left(\frac{1}{n^2}\right)$

$$\text{So} \quad \Delta\left(\frac{1}{n^2}\right) = 2\frac{\Delta n}{n^3} = \frac{c^2}{B-A}$$

$$\Delta n = \frac{1}{2} \cdot n^3 \frac{(r_{41} E_x)^2}{B-A}$$

$$\approx \frac{1}{2}(1.5)^3 \frac{(8.6\times10^{-5})^2}{0.0242}$$

$$\sim 5\times10^{-7}$$

Shift is not linear, and very small. Not good for modulation.

Now try applying E_z, light propagating along z.

Index ellipse:

$$\frac{x^2}{n_0^2} + \frac{y^2}{n_0^2} + 2xy\, r_{63} E_z = 1$$

Matrix is
$$\begin{bmatrix} \frac{1}{n_0^2} & r_{63}E_z \\ r_{63}E_z & \frac{1}{n_0^2} \end{bmatrix} \qquad A = B \quad \text{so}$$

$$\lambda_\pm = \frac{1}{n_0^2} \pm r_{63} E_z \quad \rightarrow \quad \text{linear effect}$$

$$\text{So} \quad \Delta n = \frac{1}{2} n^3 \Delta\lambda = \pm\frac{1}{2} n_0^3 r_{63} E_z$$

$$= \pm 1.8\times10^{-4} \text{ at } 10\,\mathrm{kV/mm}$$

much better.

* This is just like quantum mechanics:
off-diagonal perturbation only has
significant effect if original states are degenerate.

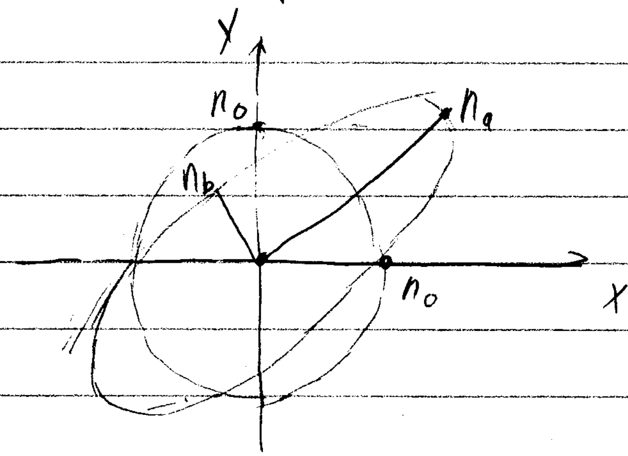
Also need eigenvectors:

easy to see new axes are:

$$x' = \frac{1}{\sqrt{2}}(x+y)$$

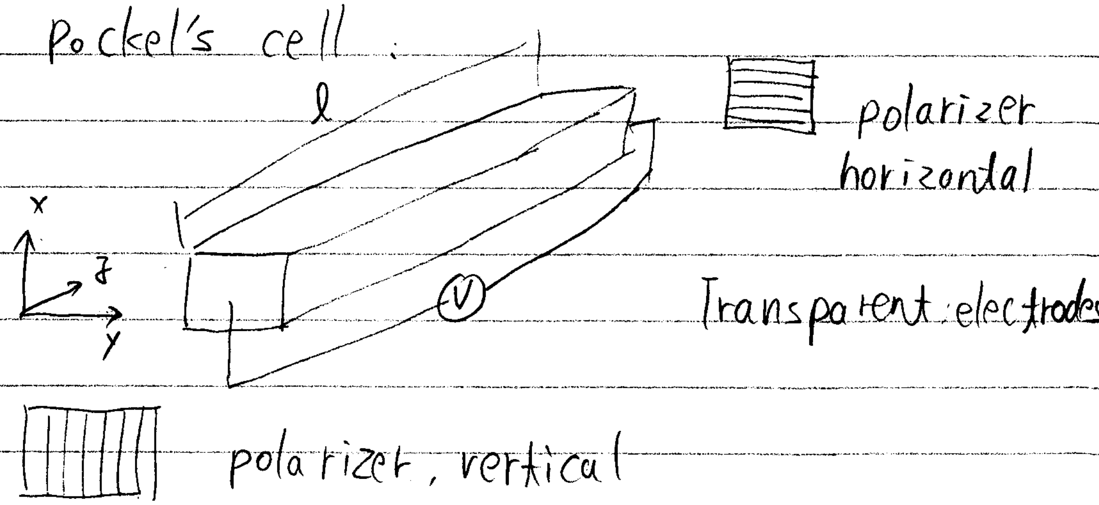
$$y' = \frac{1}{\sqrt{2}}(x-y)$$

Ellipse



$\hat{x}'$: $n_a = n_o + \Delta n$, $\hat{y}'$: $n_b = n_o - \Delta n$.

To make Pockel's cell:



Incident light polarized along $\hat{x} = \frac{1}{\sqrt{2}}(\hat{x}' + \hat{y}')$

So after crystal, have

$$\vec{E}_{out} \propto \frac{1}{\sqrt{2}}\left[\hat{x}' e^{+ik_0 l \Delta n_0} + \hat{y}' e^{-ik_0 l \Delta n_0}\right]$$

$$= \frac{1}{\sqrt{2}}\left[\hat{x}' + \hat{y}' e^{-2ik_0 l \Delta n_0}\right]$$

Phase shift $\quad \phi = (2k_0 l)\left(\frac{1}{2} n_0^3 r_{63} E_z\right)$

Here $E_z = \frac{V}{l}$, so $\phi = k_0 n_0^3 r_{63} V$

$$= \frac{2\pi}{\lambda_0} n_0^3 r_{63} V.$$

Recall:

$$I_{out} = I_{in} \sin^2\frac{\phi}{2}$$

So to get max output need $\phi = \pi$

$$\pi = \frac{2\pi}{\lambda} n_0^3 r_{63} V$$

Write $\quad V_\pi = \dfrac{\lambda}{2 n_0^3 r_{63}}$

at $\lambda = 633\,\text{nm}$, $\quad V_\pi = 8.7\ \text{kV}$

Actually, KDP is a hard example
Simpler if you can use a diagonal component of r.

For instance, r_{xxz} gives $\Delta n_x = \frac{1}{2} n_x^3 r_{13} E_z$.

If take light propagating along $\hat{z}$

ellipse is

$$\frac{x^2}{n_x^2} + \frac{y^2}{n_y^2} + r_{13} E_z x^2 = 1$$

axes don't change

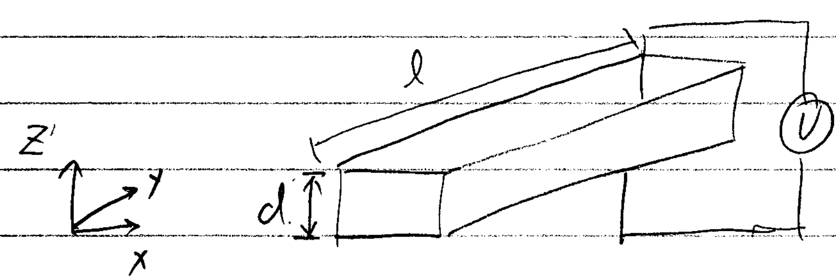
$$\hat{x} \to \hat{x}' \quad , \quad \hat{y} \to \hat{y}'$$

Also can take light propagating along $\hat{y}$, then

ellipse is

$$\frac{x^2}{n_x^2} + \frac{z^2}{n_z^2} + r_{13} E_z x^2 = 1$$

Then $\vec{E}$ is transverse to light



Don't need transparent electrodes
and can get better modulation using long skinny crystal

$$\phi = k l \frac{1}{2} n_x^3 Z_z r_{31}$$

$$= \frac{1}{2} k l n_x^3 r_{31} \frac{V}{d}$$

$$= \pi \frac{l}{\lambda_0 d} n_x^3 r_{31} V$$

$$V_\pi = \frac{\lambda_0 d}{l} \cdot \frac{1}{n_x^3 r_{31}}$$

muck smaller if $d \ll l$