Discussed two general modulation techniques
EO : control phase using electric field
AO : deflect & shift frequency using acoustic wave

Next : pulsed lasers.
another type of modulation, often uses AO & EO
modulation (EO is more common)

In book (section 14.3)

Advantages of pulsed lasers :
Very high power :
Say 10 mJ pulse energy, 100 fs duration.
$P_{peak} = 10^{11}\ W = \frac{E}{\Delta t}$

Focus to $W_0 = 10\,\mu m$ spot $I \sim 6\times10^{20}\ \frac{W}{m^2} = \frac{2P}{\pi W_0^2}$

* Good for studying strong field effects

* Good timing resolution :
100 fs pulse lets you observe dynamics on 100 fs scale :
a. motion of molecules during chemical reactions
b. motion of electrons in high-lying states

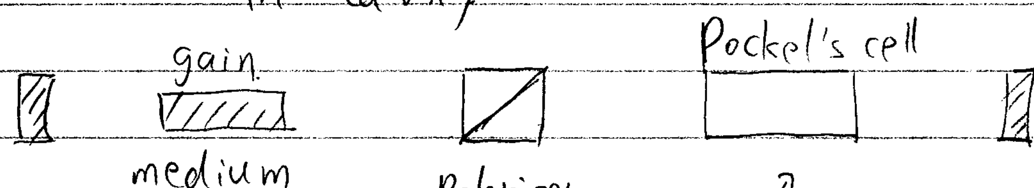
Two methods for making fast pulses
- Q-switching (switching the cavity Q-factor)
- mode locking

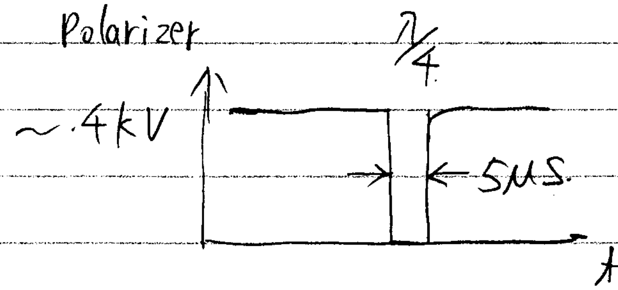
Start with Q-switching.

Idea: with strong pumping, can get very large small signal gain, $g_0 \gg g_t$ (g_t = threshold gain)

But saturation clamps gain $= g_t$

To avoid saturation by putting Pockel's cell in cavity

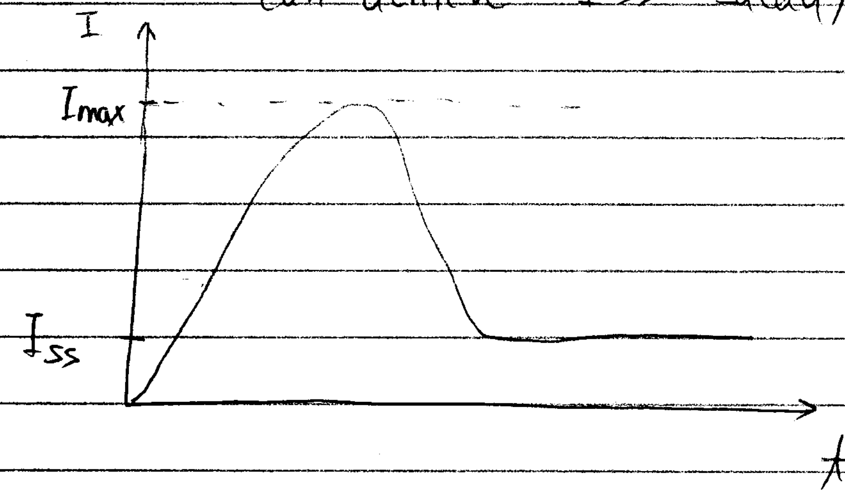




Normally, the high voltage applied on Pockel's cell makes it work as a quarter-wave plate.
The light after the polarizer will go through Pockel's cell twice before return to the polarizer. The twice passing the Pockel's cell will build up π phase shift, i.e. rotate the poliarization of light by $90°$ to be perpendicular to the polarizer, therefore reflected off the cavity.

So normally, cell is non-transmitting prevent lasing (losses very high)
So population inversion of medium will build up
no saturation : $g = g_0$.

When you want a pulse, switch off Pockel's cell high voltage, so there is no polarization rotation by PC
i.e. PC is transmitting.
High gain, so light builds up more quickly than light escapes cavity.
→ can achieve $I \gg$ steady-state I



"Atoms give up energy faster than energy leaves cavity → excess builds-up.

Can make a simple model for dynamics.

Two dynamical variables:
inversion ΔN
photon number density n_p $(I = h\nu c\, n_p)$

* Photon number satisfies:

$$\frac{dn_p}{dt} = -\frac{n_p}{\tau_p} + \Delta N\, W$$

τ_p: photon lifetime in cavity $= \frac{1}{\Gamma \Delta\nu_F}$ $\left(\Gamma = \frac{1}{\Delta\nu_F \tau_p}\right)$

W: stimulated transition rate:

$$W = \phi\, \sigma(\nu) = c\, n_p\, \sigma(\nu) \quad , \quad \text{and} \quad \Delta N_t = \frac{1}{c\, \tau_p\, \sigma(\nu)}$$

$$\text{so} \quad W = \frac{n_p}{\Delta N_t\, \tau_p}$$

$$\boxed{\frac{dn_p}{dt} = \frac{n_p}{\tau_p}\left(\frac{\Delta N}{\Delta N_t} - 1\right)}$$

* For atoms:

$$\frac{dN_2}{dt} = R - \frac{N_2}{t_{sp}} - W(N_2 - N_1)$$

N_2 & N_1 the upper and lower lasing levels

(assume R independen of N's
→ ok for four level system :)

During short pulse, atoms don't usually have time to decay out of state 1, so

$$N_1 + N_2 = \text{constant} = N_c$$

so
$$\frac{d}{dt}(\Delta N) = \frac{dN_2}{dt} - \frac{dN_1}{dt}$$
$$= \frac{dN_2}{dt} - \frac{d}{dt}(N_c - N_2)$$
$$= 2\frac{dN_2}{dt}$$

every decay $2 \to 1$, decrease ΔN by 2.

So
$$\frac{d}{dt}\Delta N = 2R - \frac{2N_2}{t_{sp}} - 2W\Delta N$$

$$N_2 = \frac{1}{2}(N_2 + N_1 + N_2 - N_1)$$
$$= \frac{1}{2}(N_c + \Delta N)$$

$$\frac{d}{dt}\Delta N = \underbrace{2R - \frac{N_c}{t_{sp}}}_{\text{constant}, \; = \frac{\Delta N_0}{t_s}} - \frac{\Delta N}{t_{sp}} - 2W\Delta N$$

$$\Delta N_0 = 2Rt_s - N_c$$

$$\frac{d}{dt}\Delta N = \frac{1}{t_s}[\Delta N_0 - \Delta N] - 2W\Delta N$$

$$\frac{d}{dt}\Delta N = \frac{1}{t_s}[\Delta N_0 - \Delta N] - 2\frac{\Delta N}{\Delta N_t}\frac{n_p}{\tau_p}$$

But t_{sp} is long compared to pulse duration neglect.

$$\boxed{\frac{d}{dt}\Delta N = -2\,\frac{\Delta N}{\Delta N_t}\,\frac{n_p}{\tau_p}}$$

$$\boxed{\frac{dn_p}{dt} = \frac{n_p}{\tau_p}\left(\frac{\Delta N}{\Delta N_t} - 1\right)}$$

Nonlinear coupled equations, can't solve.

But can find some things

Peak photon density achieved when

$$\frac{dn_p}{dt} = 0 \quad \Rightarrow \quad \Delta N = \Delta N_t$$

when gain is exhausted.

Can find $n_p(\Delta N)$:

$$\frac{dn_p}{d\Delta N} = \frac{dn_p/dt}{d\Delta N/dt} = -\frac{1}{2}\,\frac{\left(\frac{\Delta N}{\Delta N_t} - 1\right)}{\Delta N/\Delta N_t}$$

$$= -\frac{1}{2} + \frac{\Delta N_t}{2\Delta N}$$

so

$$n_p(\Delta N) = \int_{\Delta N_i}^{\Delta N_f} \frac{1}{2}\left(\frac{\Delta N_t}{\Delta N} - 1\right) d\Delta N$$

$$= \frac{1}{2}\left[\Delta N_t \ln\frac{\Delta N_f}{\Delta N_i} - \Delta N_f - \Delta N_i\right]$$

So peak photon density at $\Delta N_f = \Delta N_t$

$$n_p(max) = \frac{1}{2}\left[\Delta N_t \ln \frac{\Delta N_t}{\Delta N_i} - \Delta N_t + \Delta N_i\right]$$

$$= \frac{\Delta N_i}{2}\left[1 + \frac{\Delta N_t}{\Delta N_i} \ln \frac{\Delta N_t}{\Delta N_i} - \frac{\Delta N_t}{\Delta N_i}\right]$$

If $\Delta N_i \gg \Delta N_t$ (normal case)

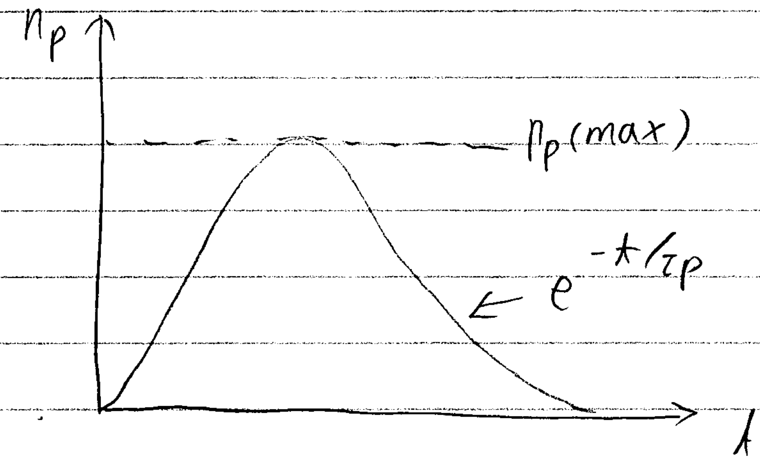
$$\boxed{n_p(max) = \frac{\Delta N_i}{2}}$$

Peak output power

$$I_{out} = h\nu c \cdot \frac{\partial n_p}{\partial t}\Big|_{out} = \frac{h\nu c}{\tau_p} n_p$$

$$I_{out} = \frac{h\nu c}{\tau_p} \cdot \frac{\Delta N_i}{2}$$

Can't solve time evolution exactly, but expect



Duration ~ τ_p: time required for photons to escape cavity

$$\tau_p \sim \frac{1}{\Gamma \Delta \nu_F} \qquad \Delta \nu_F = \frac{c}{L} \quad \text{up to } 1\,GHz$$

$\Gamma \sim$ loss $\sim 10\%$
(need to keep $\Delta N_i \gg \Delta N_t$!)

$$\tau_p \sim \frac{1}{0.1 \times 10^9\,Hz} = 10^{-8}\,s = 10\,ns$$

Try hard, get pulse duration ~ 1ns, or a bit lower
can't do much better with Q-switches.