02/02/05 Lecture 7

A cavity has matrix of single round trip

$$M_1 = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

A beam has parameter q_1, after one round trip. the parameter of beam becomes q_2, according to ABCD Law

$$q_2 = \frac{A q_1 + B}{C q_1 + D}$$

Want beam to return to same state.

so demand $q_2 = q_1 = q$.

solve $$q = \frac{Aq + B}{Cq + D}$$

$$Cq^2 + Dq = Aq + B$$

$$Cq^2 + (D-A)q - B = 0$$

$$q = \frac{1}{2C}\left[A - D \pm \sqrt{(D-A)^2 + 4BC}\right]$$

use $BC = AD - 1$

So $(D-A)^2 + 4BC = D^2 - 2AD + A^2 + 4AD - 4$
$= D^2 + 2AD + A^2 - 4$
$= (D+A)^2 - 4$

So $q = \frac{1}{C}\left[\frac{A-D}{2} + \sqrt{\left(\frac{A+D}{2}\right)^2 - 1}\right]$

since $q = z + iz_0$, must be complex number for a confined Gaussian beam,

so $\boxed{\text{stability need } \left|\frac{A+D}{2}\right| < 1}$

Finding modes of a cavity.

Mode = solution of Helmholtz equation that satisfies boundary conditions of a system.

= stationary solution of Maxwell's equations

Different from general techniques to find modes in Z&K,

What's used to find a laser mode in a cavity is called "self-consistent" approach. (only valid in paraxial limit)

Again:

stability condition $\left|\frac{A+D}{2}\right| < 1$

then in stable cavity

$$q = \frac{1}{C}\left[\frac{A-D}{2} \pm i\sqrt{1-\left(\frac{A+D}{2}\right)^2}\right]$$

$$= z + i z_0$$

since $z_0 = \frac{\pi W_0^2}{\lambda}$, take "+" sign, if $C>0$

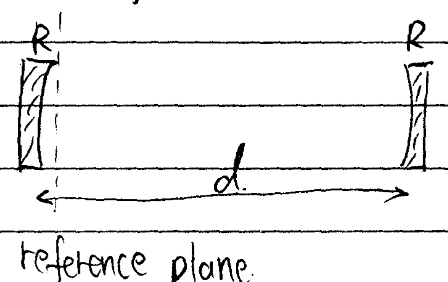
"−" sign, if $C<0$

Also useful to have $\frac{1}{q}$, specifies beam at reference location.

$$\frac{1}{q} = \frac{1}{B}\left[\frac{D-A}{2} \mp i\sqrt{1-\left(\frac{A+D}{2}\right)^2}\right]$$

$\pm$ depends on sign of B

Example: symmetric two-mirror cavity



$R < 0$

start just after light bounces off first mirror.

(since form of solution depends on choice of reference plane, so it's important to specify it)

$$M = \begin{bmatrix} 1 & 0 \\ \frac{2}{R} & 1 \end{bmatrix} \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{2}{R} & 1 \end{bmatrix} \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 + \frac{2d}{R} & 2d + \frac{2d^2}{R} \\ \frac{4d}{R^2} + \frac{4}{R} & 1 + \frac{6d}{R} + \frac{4d^2}{R^2} \end{bmatrix}$$

So $$\frac{A+D}{2} = 1 + \frac{4d}{R} + \frac{2d^2}{R^2}$$

$$\frac{A-D}{2} = -\frac{2d}{R} - \frac{2d^2}{R}$$

So
$$q = \frac{1}{\frac{4d}{R^2} + \frac{4}{R}} \left[-\frac{2d}{R}\left(1 + \frac{d}{R}\right) - i\sqrt{1 - \left(1 + \frac{4d}{R} + \frac{2d^2}{R^2}\right)^2} \right.$$

$$= -\frac{R}{4\left(\frac{d}{R}+1\right)} \left[\frac{2d}{R}\left(1+\frac{d}{R}\right) + i\sqrt{1 - \left(1 + \frac{16d^2}{R^2} + \frac{4d^4}{R^4} + \frac{8d}{R} + \frac{4d^2}{R^2} + \frac{16d^3}{R^3}\right)} \right]$$

$$= -\frac{R}{4\left(1+\frac{d}{R}\right)} \left[\frac{2d}{R}\left(1+\frac{d}{R}\right) + i\sqrt{-\frac{4d}{R}\left(2+\frac{d}{R}\right)\left(1+\frac{d}{R}\right)^2} \right]$$

$$q = -\frac{R}{2}\left[\frac{d}{R} + i\sqrt{\frac{-d}{R}\left(2+\frac{d}{R}\right)} \right]$$

$$= -\frac{d}{2} + \frac{i}{2}\sqrt{-d(2R+d)}$$

$$= z + i z_0$$

* need $2R+d < 0$ for stability

Since $z = -\frac{d}{2}$, focus is $\frac{d}{2}$ in front of ref. plane at center of cavity.

$$z_0 = \frac{\pi W_0^2}{\lambda} = \frac{1}{2}\sqrt{-d(2R+d)}$$

$$W_0 = \left(\frac{\lambda}{2\pi}\right)^{1/2} \left[-d(2R+d)\right]^{1/4} \quad \text{beam waist.}$$

Note: for this example, could have used symmetry better.

$$M_1 = (M_{1/2})^2 \qquad M_{1/2} = \begin{bmatrix} 1 & 0 \\ \frac{2}{R} & 1 \end{bmatrix} \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix}$$

Looks like two passes of simpler cavity.
Could have demanded $q_{out} = q_{in}$ after single pass.

Get same result, math is easier.