Phys 532/822 Assignment 6 Due April 15

1. **Kerr Modulator:** Suppose an intensity modulator is constructed by placing an isotropic Kerr medium between two polarizers, as in Saleh and Teich Fig.18.1-6. Calculate the output intensity as a function of applied voltage, in terms of the medium length ℓ, medium thickness d, index of refraction n, Kerr coefficient s, and light wavelength λ_0. (If you want to worry about polarization details, assume that the light propagates along z and is polarized along x, the electric field is applied along x, and you are using the s_{xxxx} Kerr coefficient. The polarizers are at $\pm 45°$ to x.)

2. **Designing a Modulator:** In section 18.1-B (see figure 18.1-5), Saleh and Teich describe how the electro-optic effect can be used in a Mach-Zehnder interferometer to construct an intensity modulator integrated with a fiber-optic system. Design a modulator of this type using the material $LiNbO_3$. Select the orientation of the crystal and the polarization of the guided wave so as to obtain the smallest possilbe half-wave voltage V_π. Note that for an integrated modulator, it is not easy to apply an electric field along the direction of light propagation, so you can assume a transverse field configuration.
   Example 18.2-1 has some discussion of the properties of $LiNbO_3$. However, to be complete, you should consider all possible system configurations, and not just the one discussed in the example.
   If the active region has length $\ell = 1$ mm and width $d = 5$ μm, and the wavelength $\lambda_0 = 850$ nm, calculate V_π. The refractive indices for $LiNbO_3$ are $n_0 = 2.29$ and $n_e = 2.17$, and the non-zero electro-optic coefficients are $r_{33} = 30.9$ pm/V, $r_{13} = 8.6$ pm/V, $r_{22} = 2.4$ pm/V, and $r_{51} = 28$ pm/V.

   822 students only

3. **Electro-optic Phase Modulation:** The electro-optic effect can also be used for frequency modulation. For example, suppose a $LiNbO_3$ crystal of length ℓ is oriented with a laser beam propagating along x and polarized along z. An oscillating electric field $E_1 \cos \Omega t$ is applied along the z direction. The electric field of the laser itself oscillates as $E_L = E_0 \exp i(\omega_0 t - kx)$.
   (a) Show that after exiting the crystal, the laser field has a time dependence of $\exp i(\omega_0 t + \delta \cos \Omega_0 t)$, and determine δ.
   (b) If the "instantaneous" frequency of the laser $\omega(t)$ is defined by

$$\frac{dE_L}{dt} = i\omega(t)E_L, \tag{1}$$

   find the range of instantaneous frequencies sampled by the laser output.
   (c) In the limit $\delta \ll 1$, show that the laser electric field can be expressed as a sum

of three components oscillating at $\omega_0, \omega_0 + \Omega, \omega_0 - \Omega$, and find their relative amplitudes.

Frequency modulation can thus be thought of either as a variation of the instantaneous frequency, or as the generation of additional frequency components at $\omega_0 \pm \Omega$. (This is a little counter-intuitive when δ is small, since the instantaneous frequency never actually equals $\omega_0 \pm \Omega$!)