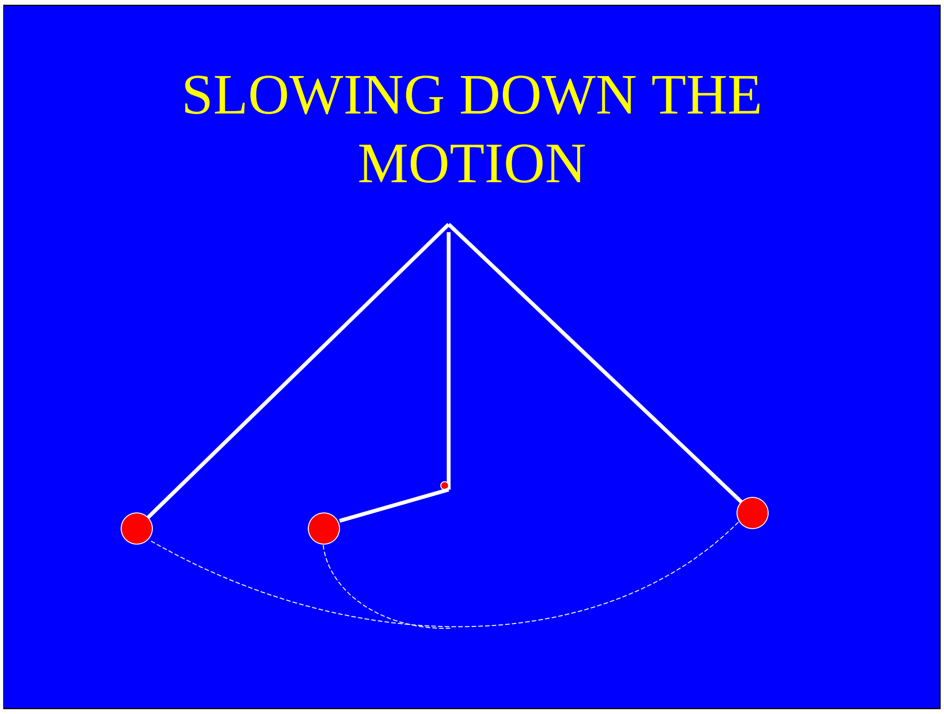
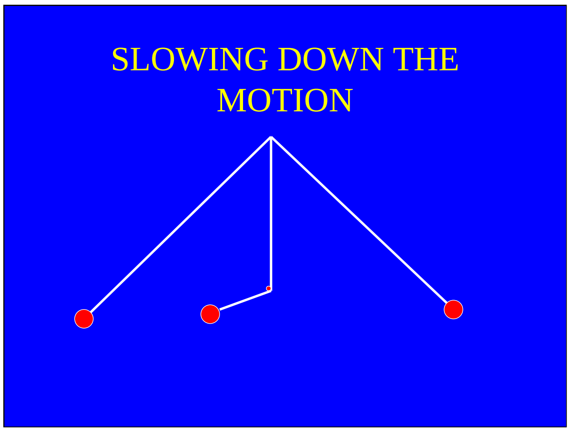


Galileo's hypothesis for falling bodies is that they gain equal amounts of speed in equal intervals of time as they fall. This is not the only possibility he considered. He suggests others in TNS, but rejects them with careful arguments. For example, what about saying that a falling body gains equal amounts of speed when falling through equal amounts of height? (According to this hypothesis, it would take a weight equal times to fall 4 feet as to fall 8 feet. This is not what is observed.)

Galileo is asking a new kind of question: How does the speed change as a body falls? His problem was, things fall too fast to be measured repeatedly as they fall using the methods available at the time. So he wanted to slow the motion down without changing its character. If he were to drop a stone in water, it would quickly reach terminal speed and continue at that same speed for the remainder of the fall.

Galileo used the pendulum sketched above to convince himself that he could slow the motion down without introducing friction. When a pendulum swings from one side to the other, it rises to nearly the same height. (A small amount of friction slows it down slightly so it does not rise back all the way). The height to which it rises is determined by the speed of its motion at the bottom of the trajectory.

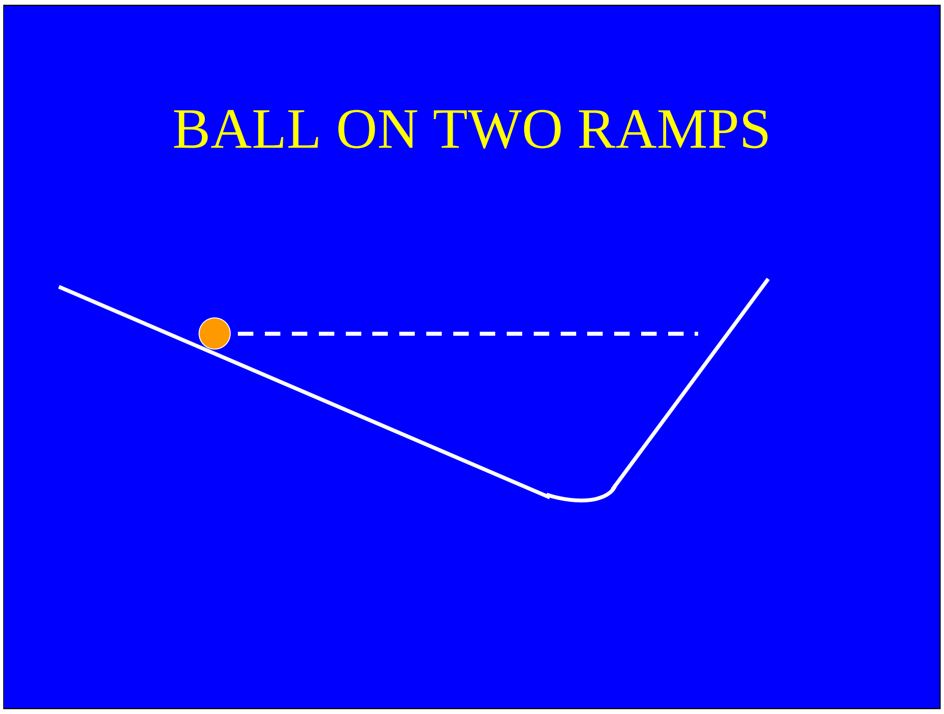


He then built a modified pendulum as shown above. A peg is inserted below the suspension point, so as the weight swings past it, the peg suddenly shortens the length of the pendulum causing it to swing up abruptly. What he found is that it still rises to the same height as before, and does so again back on the other side. Galileo argued that this shows that a falling pendulum reaches the same speed at the bottom of its motion regardless of how steep the descent was.

This is a remarkable insight. Let's state this more clearly. Starting with an ordinary pendulum without the peg, if we raise it up to a certain height, it will rise to the same height on the other side, ignoring a small amount of friction. Then it swings back to the original side, again nearly to the original height. As it moves through the bottom of its motion, its speed determines how high it will go. This can be determined by releasing it from different heights and noticing that the higher the release, the faster it goes at the bottom.

Now put the peg in. It still rises just as high as before on the side opposite the release, even though it rose on a steeper trajectory. And as it returns to the original side, it again rises to the original height. But on the way up once it gets past the peg, it is the full length pendulum we started with. So it must have been going just as fast at the bottom as if the peg had not been there.

Galileo then applied these ideas to a ball rolling down one ramp and then up another.



Galileo thought of his pendulum as being a succession of ramps of different slopes, and what he had just shown is that the slope doesn't matter in determining how far up the other side the ball will go, or how fast the ball will go at the bottom.

Of course there is a difference now between a rolling ball and a sliding (or swinging) ball, but Galileo's intuition told him this should not affect the basic argument, and he was right. He was a genius at choosing what difficulties to ignore.

So let's apply the pendulum conclusions to two opposing ramps as shown. We expect then, that for very smooth ramps and a round hard ball, that it will roll up the other side to the same height from which it was released. And, it will have the same speed at the bottom no matter which side it was released from.

Now imagine the right ramp being made steeper and steeper. If it is steep enough we can think of the ball as simply falling. He concludes that for a ball rolling down a ramp, the speed at various heights is the same as if it had simply fallen vertically from the starting point to that height. He further concludes that the ramp has allowed him to slow down the motion of free fall, allowing him to make measurements of it.

GALILEO'S ACCELERATION EXPERIMENT

From Two New Sciences p 178:

A piece of wooden moulding or scantling about 12 cubits long(!), half a cubit wide, and three finger-breadths thick was taken; on its edge was cut a channel a little more than one finger in breadth; having made this groove very straight, smooth, and polished, and having lined it with parchment, also as smooth and polished as possible, we rolled along it a hard, smooth, and very round bronze ball. Having placed this board in a sloping position by raising one end above the other we rolled the ball along the channel noting the time required to make the descent. We then rolled the ball only one-quarter of the length of the channel, and found it precisely one-half of the former. Next we tried other distances, and always found the spaces traversed were to each other as the squares of the times.

Above is Galileo's description of his inclined plane experiment so carefully justified by the experiments we have described that he knew he was studying diluted "naturally accelerated motion" (i.e. freely falling bodies).

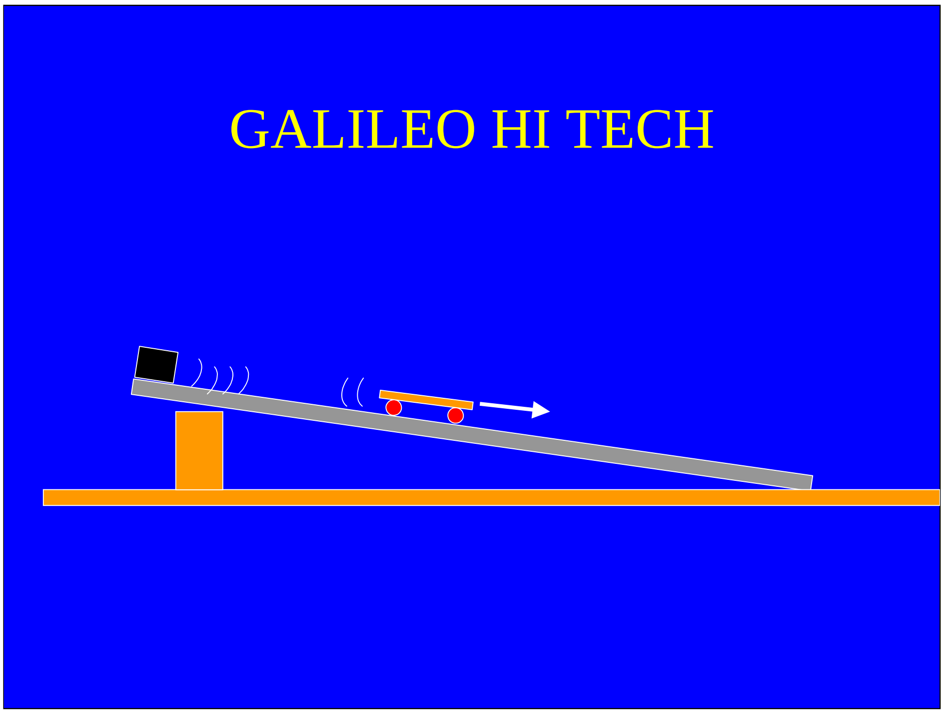
Galileo measured the time of descent using a water clock. We will do the same. This is about as accurate as human reaction times allow. The amount of water that flows, determined by its weight, is a measure of the elapsed time. Galileo states in TNS that a given experiment could be repeated many times without observing time differences greater than a tenth of a pulse beat.

Carry out experiment measuring times for the ball to roll 1/9, 4/9, 9/9 of the length of the board. From our result for uniform acceleration we expect

$$x = 1/2at^2 \text{ so } x^{1/2} = kt$$

As a simple test, plot $x^{1/2}$ vs t to see if it is a straight line.

MODERN VERSION: SHOW gliders on level and tilted air track.

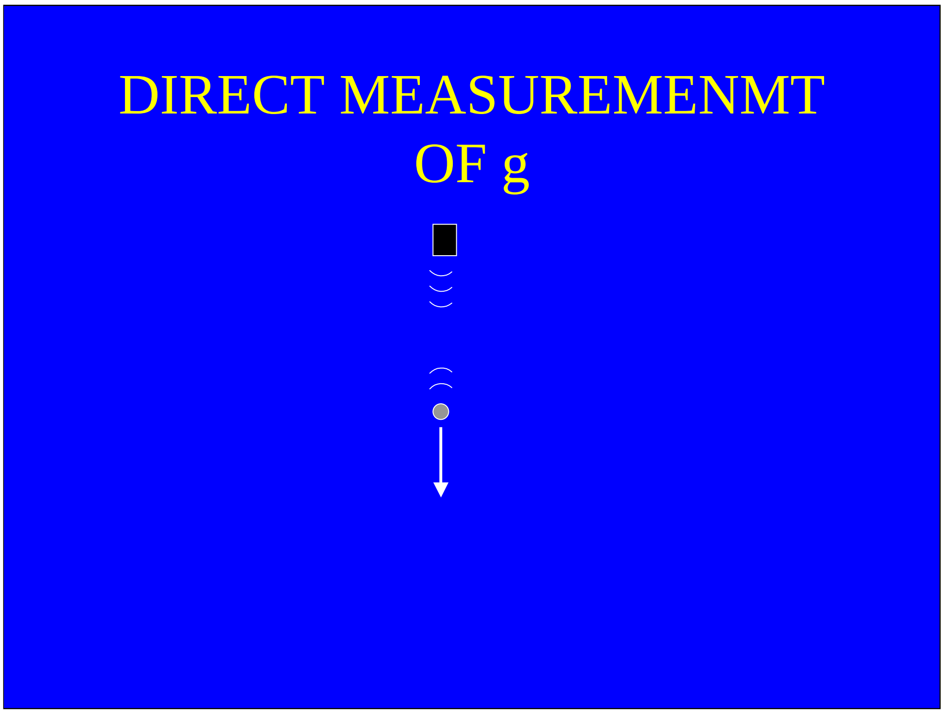


Here is a modern version of Galileo's experiment that he would have loved to use. It not only takes the data faster and more precisely but does all the data analysis and presents the results to us graphically.

We have an Al inclined track with a cart with very low friction wheels. The wheels require bearings that were not available until last few decades. The cart will roll down the track and its position recorded by a sonar device. This is much like the police radar gun that tells the traffic cop your car's speed, only it uses sound waves instead of radio waves. You will not hear the sound because the frequency is well above the human audio range. The sound waves bounce off the cart and are reflected back to the box at the top of the track that acts as both source and receiver. The position of the cart is determined by the time delay between transmission and reception.

Since the speed of sound is about 334 m/s, a round trip distance of 3m for the sound pulse causes a time delay of about 0.01s. The sensor can do this with considerable accuracy and measures the cart's position several times per second.

SHOW cart rolling from rest down track. SHOW graph of position vs time, then speed vs time, then acceleration vs time.



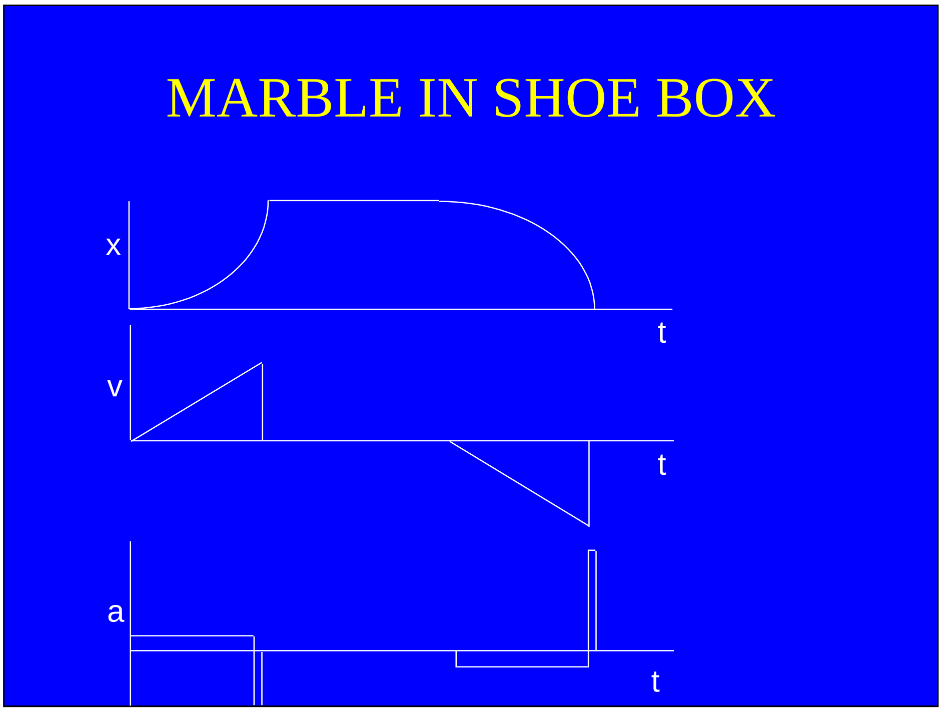
Since we have this sonar position sensor, why bother with an inclined plane at all? Let's just drop a ball and measure its position as it falls. This, after all, is what Galileo wanted to do, but could not. He substituted ingenuity for technology, arguing as we have seen that an inclined plane gave him a true picture of "naturally accelerated motion", or the acceleration due to gravity.

SHOW dropped ball measured by sonar position sensor. SHOW graph of position, speed, acceleration vs time. Use fitting programs to find g for each. $g = 9.8$ m/s^2 is quite accurate over most of the earth. We will use $g = 10$ m/s^2 for most problems for simplicity.

SHOW video of dropped ball as a visual, qualitative presentation of falling body motion.

SHOW reaction time measurement for hand motion using a dropped meter stick. 5 cm drop corresponds to 0.1 s reaction time. 10 cm to 0.14s, 20 cm to 0.2s.

SHOW dropped timed steel ball. A descent of 1.23m corresponds to a time of 0.50s using $g = 9.8$ m/s^2.

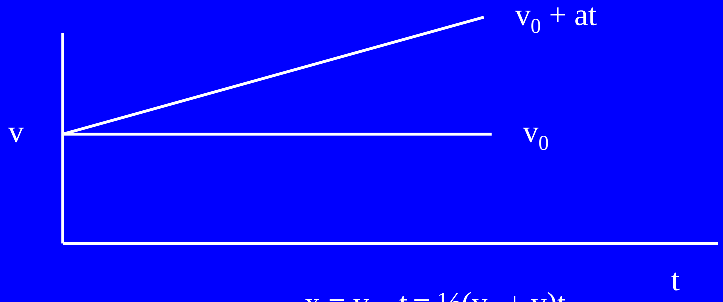


Suppose we put a marble in a shoe box, and tilt it back and forth. The marble will roll from one end to the other. We assume when it hits the end of the box it sticks without bouncing. Since the marble is a ball rolling down an inclined plane, we know how to describe its motion. The displacement increases quadratically with time until it abruptly stops at the other end. Then when the box is tilted the other way, it rolls back.

What does the speed v look like for this motion? When the marble is stuck at the ends, the speed is clearly zero. As it rolls, what does the speed graph look like?

Now what about the marble's acceleration? As it rolls, how does its acceleration behave with time? Galileo showed it is a constant. Then the marble hits the other end of the box and its speed suddenly stops. The acceleration must be large and negative when this collision takes place. Acceleration is the rate at which speed is changing, and in a very short time all its speed goes to zero.

NON-ZERO INITIAL SPEED



$v = v_0 + at$ $\qquad$ $x = v_{AV}t = ½(v_0 + v)t$

$x = ½(v_0 + v_0 + at)t = v_0t + 1/2at^2$

Suppose we are observing a motion in which the initial speed is not zero as we have assumed so far. For example we could throw a rock upwards in the air. How do we handle this?

We describe this graphically above. An initial speed v_0 exists, and acceleration begins at t = 0. This just means that the speed increases (or decreases) starting from its initial value.

How can we evaluate the distance traveled? The distance is always given by the average speed times the time. For uniform acceleration, the average speed is just the average between the initial and final values. Inserting our expression for the final speed, we have the result shown above.

A simple interpretation of this result is that the object moves as though its initial speed continued unchanged and in addition the acceleration occurred. It is as though the initial speed and the acceleration are unaware of each other.

EXAMPLE: THROW ROCK UP

Choose upward to be positive displacement and speed. (This choice is arbitrary)

Then $v = v_0 - gt$

Let $v_0 = 5\text{m/s}$ and $g = 10\text{m/s}^2$

When does the rock get to the top of its motion?

$v = 0 = 5 - 10t$ so $5 = 10t$ and $t = 0.5\text{s}$

How high did it go?

$y = v_0 t - 1/2gt^2 = 5(0.5) - 5(0.5)^2 = 2.5 - 1.25 = 1.25\text{m}$

Let's look at a simple example illustrating these results. This is something we have all done. Throw a rock in the air. To describe motion we must choose a coordinate system (think Ptolemy and Copernicus). Of course our coordinate system is fixed on the surface of Earth, but we must still choose whether up or down displacements will be labeled positive. This is arbitrary. The initial speed and g are opposed so one has to be negative.

When does the rock reach the top of its motion? How do we define the top of the motion. Try tossing your pencil in the air in front of you. What happens at the top? It stops and turns around. When you walk down the hall and turn around because you forgot something, you must stop in the process.

So to find t at the top, set v = 0 and solve for t. To find how high it went, we just use our expression (with correct signs) for displacement including an initial speed.

You could now find how long it stayed in the air by finding the time for it to fall from rest from a height of 1.25 m.

QUESTION

For the rock just thrown, what was its acceleration at the very top of its motion?

A. + 10 m/s^2
B. + 5 m/s^2
C. 0 m/s^2
D. - 5 m/s^2
E. - 10 m/s^2

Here is a short quiz. Think about this for a minute or two and then we will vote.

The question posed here is must the acceleration have any special value when the speed is zero? The answer is no. Acceleration is the rate of change of speed and does not depend on whatever value the speed has at the moment. The only question is whether the speed is changing or not, and of so, at what rate?

In this case, the acceleration due to gravity acts at all times for all unconstrained bodies. They always accelerate downward until they hit the ground.

SAME QUESTION USING INCLINED PLANE

Cart on track with motion sensor.

Start it initially moving upwards. In this case, with the sensor at the top of the track, downwards will be positive in the graphs shown.

SHOW cart on inclined track, starting it with an upward push from near the end of the track. Display x vs t. It first decreases, then increases. This is like measuring the position of the rock thrown up from up in a tree.

SHOW speed vs time. Note it goes through zero at the top of the motion as we said for the rock.

SHOW acceleration vs time. It remains constant throughout the motion except at the ends when other forces enter.

DROP ROCK FROM TOP OF BUILDING

Here we extend our earlier equations by adding an initial displacement

$y = y_0 - 1/2gt^2$

Let $y_0 = 50$m.

$y = 50 - 5t^2$

When does it hit? Answer: when $y = 0$

$0 = 50 - 5t^2$ so $5t^2 = 50$ and $t^2 = 10$ so $t = 3.2$ s

Instead of adding an initial speed, let's add an initial displacement. We drop a rock from the top of a building 50m high. We could use a coordinate system with y = 0 at the top of the building, but instead let's choose ground level as our zero of height. Then clearly we just add y_0 to our equation for displacement.

How long until the rock hits ground? This occurs when y = 0. Solving for t we find 3.2s.