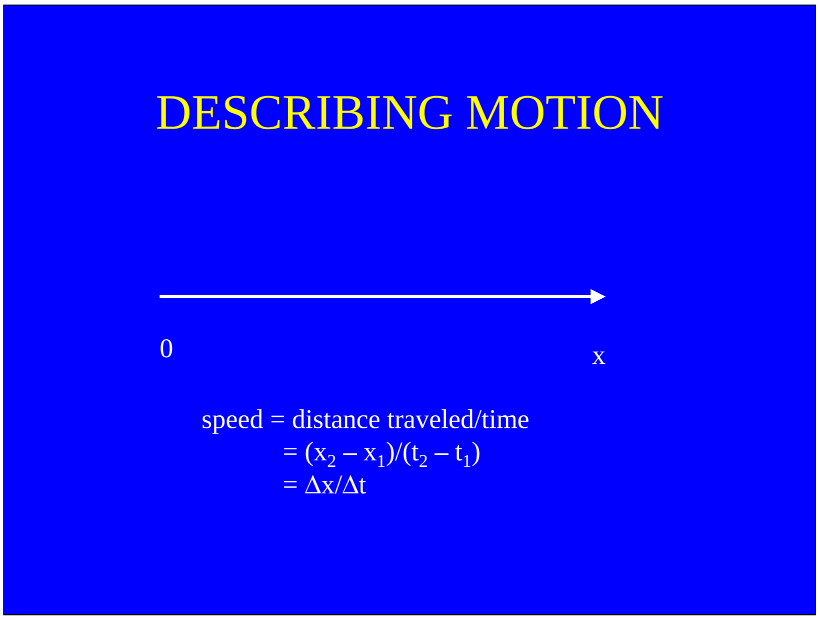


Our goal is to learn what causes motion. We will do this by breaking the problem into two parts. The first step is to develop a way to describe motion. After we have that in place we will ask what causes the motion we have learned how to describe. Some of this may seem very simple – but remember, the Greeks, who did so much so well, did not get this right. Galileo took the first steps in the right direction, then Newton went farther using the same approach, and finally, Einstein showed us how to do this right.

Let's imagine we want to describe the motion of something that is very small that is moving in one direction only. Then all we need to do to specify its location is give its position x along that direction. If we are dealing with something larger, say a car on a road, then we can put a small marker on it like a spot of paint, and specify its location.

The idea of speed has been understood since ancient times. If you had asked a traveler how fast he was going 2000 years ago, he might have said "I have gone 30 miles in the last two days." Speed is the rate at which position is changing. The traveler's answer gives his average speed, since part of the time he was presumably sleeping.

If someone asks how far it is from Charlottesville to Washington DC, the answer usually given is two hours. We use time to specify distance when the speed is understood.

TYPICAL SPEEDS

Motion	v(mph)	v(m/s)	v/c
Light	669,600,000	300,000,000	1
Earth around sun	66,600	29,600	10^{-4}
Moon around Earth	2300	1000	$3*10^{-6}$
Jet fighter	2200	980	$3*10^{-6}$
Sound in air	750	334	10^{-6}
Commercial airliner	600	267	10^{-6}
Cheetah	62	28	10^{-7}
Falcon diving	82	37	10^{-7}
Olympic 100m dash	22	10	$3*10^{-8}$
Flying bee	12	5	10^{-8}
Walking ant	0.03	0.01	$3*10^{-11}$
Swimming sperm	0.0001	0.000045	10^{-13}
Cock roach			

Speeds are specified in units of distance divided by time. Many different combinations of units are used to make the order of magnitude of the speed convenient. For a walking ant, cm/s would be convenient.Car speeds use mph or km/hr. The standard scientific notation is m/s.

Later we will learn that the speed of light is a special speed in our universe. For future reference, shown also are these speeds divided by the speed of light. All of these “normal” speeds are clearly very small compared with c, the speed of light.

Light travels about one foot in a nanosecond. So it will be convenient when we talk about speeds approaching c to have our clocks read in nanoseconds, and measure distances in feet.

Sound in air travels about 1 mile in 5 seconds. Light takes only about 5 microseconds to travel the same distance. So when you are out walking and a thunderstorm approaches, you can count off the seconds between seeing the flash of a lightning stroke, and hearing the thunder clap. Each 5 seconds means one mile of distance between you and the lightning bolt.

EXAMPLE

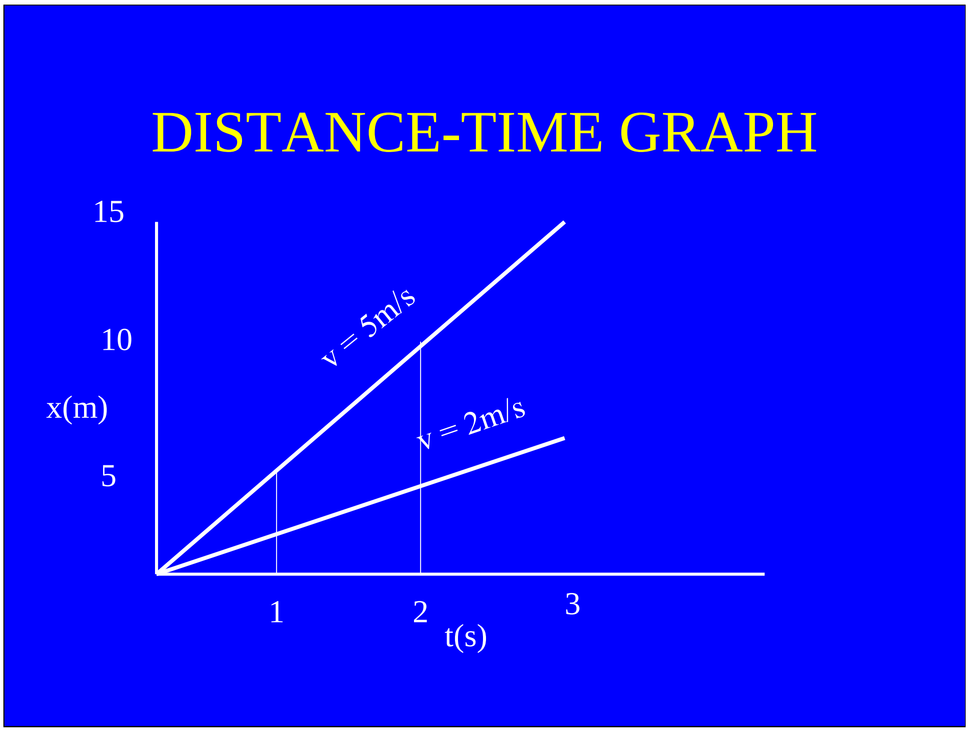
Speed of moon around Earth

v = distance/time = $2\pi R/T$

R = 240,000 mi
T = 27.3 days

$v = (6.28 * 2.4 * 10^5)/(27.3 * 24) = 2300$ mph

Here is a simple example. The Moon moves in a nearly circular orbit about Earth. Its distance away is 240,000 mi, so this is the radius of the circle. The period of the orbit, measured relative to the fixed stars, is 27.3 days. To get the speed in mph, we need to convert days to hours.

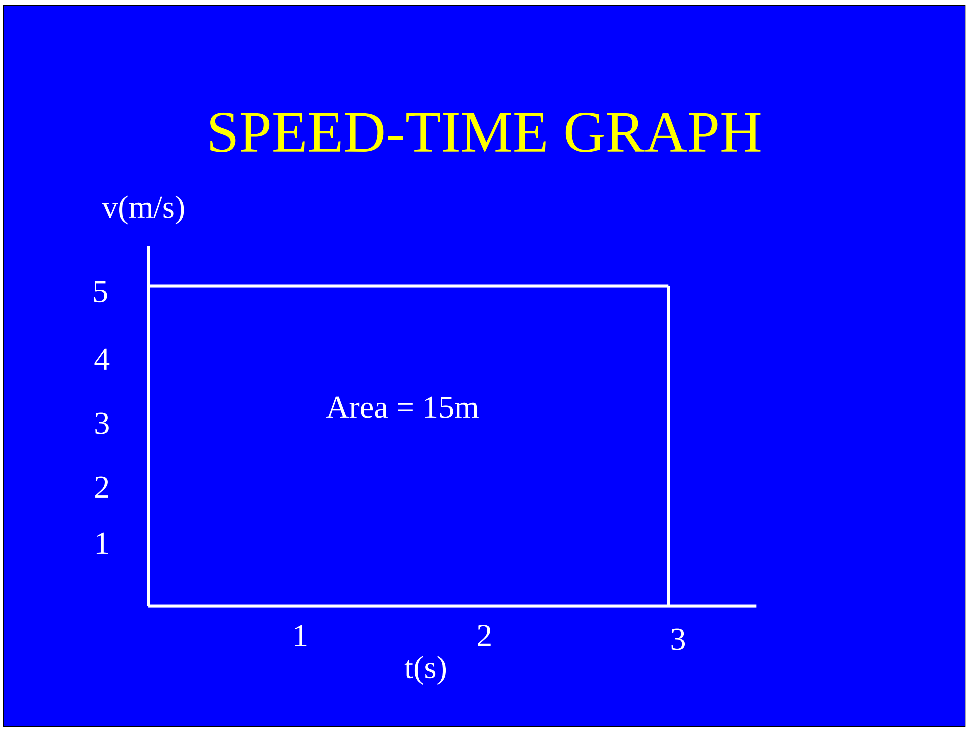


It is often useful to show the motion of an object graphically. A graph of distance versus time shows how the object's position changes with time. The graphs above show the flights of two different bees. One bee moves 15 m in 3 s. The other flew 6 m in 3 s.

Let's ask how far the first bee flew in the first second. Drawing a vertical line up to the graph at the one-second point, we see that it went 5 m. So its speed in the first second was 5 m/s. And at the end of 2 s it had gone 10 m. So it is clear that the speed of the bee remained constant during its flight.

The distances we have just measured divided by the corresponding time intervals, are just equal to the slope of the graph. And for these straight line graphs, the slope is constant, so the speed of the bees remained constant during their flights.

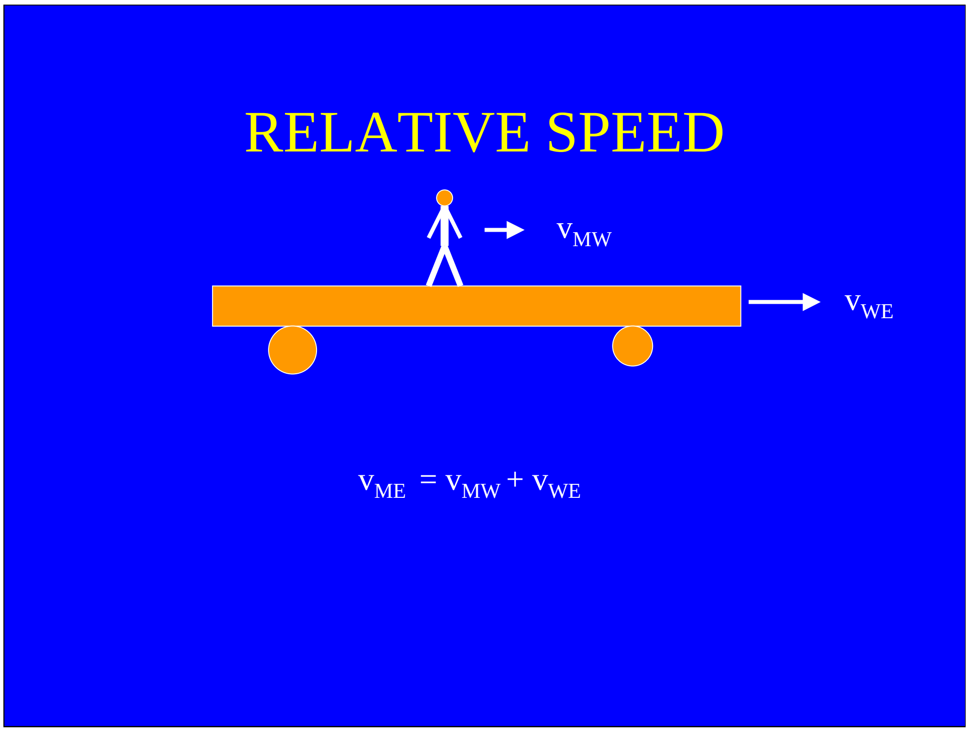
So the slope of the distance-time graph is equal to the speed.



A graph of speed versus time is very simple when the speed is constant. Why do we bother to make such a graph at all? A graph of something versus time is normally enlightening only if the something is changing with time.

We can learn something interesting from this graph however. The area under the graph is just the area of a rectangle whose height is the speed, 5m/s times the width, 3 s. This area is equal to 15 m which is the distance the bee went.

So the area under a speed-time graph is the distance traveled, at least when the speed is constant.



Speeds are always measured with respect to an origin of coordinates, or a reference frame. Sometimes that reference frame might itself be moving with respect to something else. We saw an example of this in the Ptolemaic model of the universe – the reference frame for that model is the Earth which is itself moving with respect to the Sun.

How do we take a moving reference frame into account? The Galilean method is illustrated here. A wagon moves along the ground with a speed v_{WE}, the speed of the wagon with respect to the Earth. A man walks on the wagon with a speed v_{MW}, the speed of the man with respect to the wagon. What is the man's speed with respect to the Earth? It is simply the sum of the above two speeds.

The man could of course be walking backwards along the wagon, in which case v_{MW} would be negative. Labeling the speeds this way provides a mnemonic to help remember the result, and to help keep signs straight. The moving reference frame labeled by W in this case, appears twice on the right side of the equation, sandwiched between M and E which appear on the left side. The first and last subscripts on the left side are the same as the first and last on the right.

EXAMPLE: FLEA ON DOG ON TRAIN

A train travels east with a speed v_{TE} = 20 km/hr

A dog walks down the aisle eastward with speed v_{DT} = 2 km/hr

A flea walks rumpward on the dog's back with speed v_{FD} = 0.01 km/hr

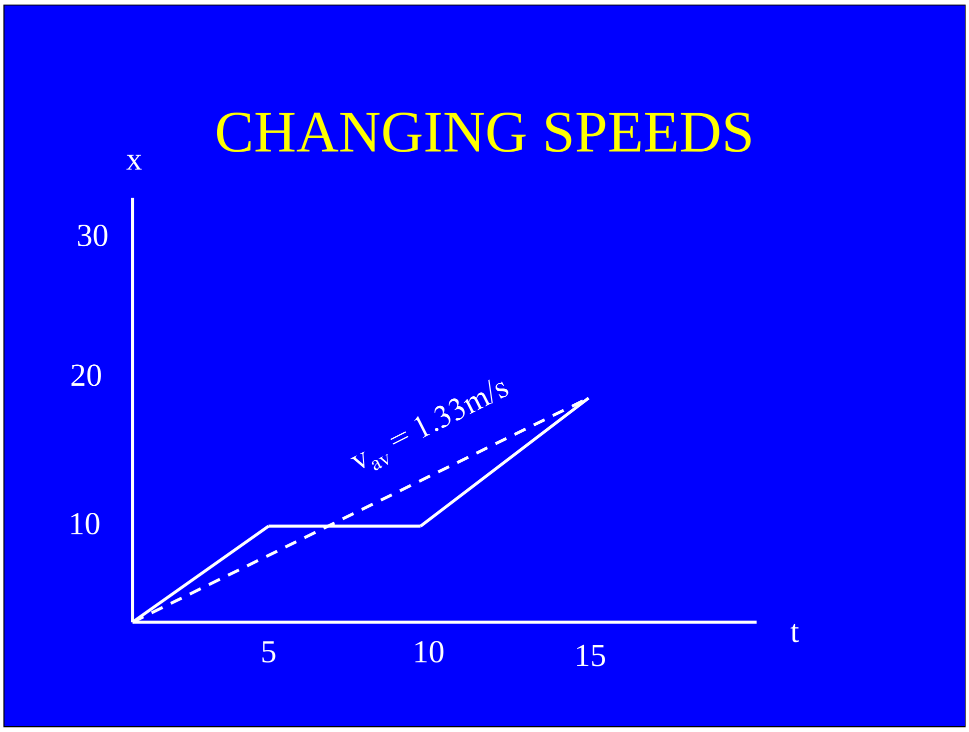
What is the speed of the flea with respect to the Earth?

$$v_{FE} = v_{FD} + v_{DT} + v_{TE} = -0.01 + 2 + 20 = 21.99 \text{ km/hr}$$

Here is an example with three moving things. The train moves eastward with speed 20 km/hr. A dog walks down the aisle forward in one of the cars with a speed of 2 km/hr. A flea walks down his back (to the west) with a speed of 0.01 km/hr.

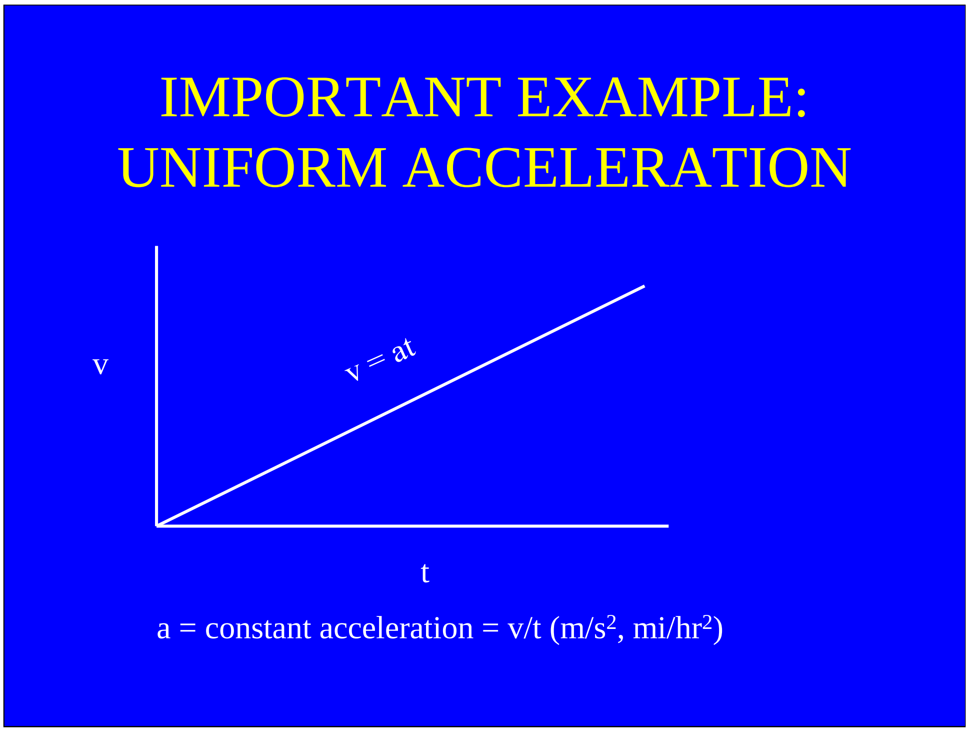
The equation for the speed of the flea with respect to the Earth must include three terms. The subscripts for the two intermediate reference frames, the dog and the train, appear in succession. The subscripts for the flea and the earth appear first and last.

Question: What is v_{ET}? This is the speed of the earth with respect to the train. Since the train is going east at 20 km/hr with respect to the earth, then the earth is going west at 20 km/hr with respect to the train. We have all experienced this. When you are in a car going north, and you look down at the pavement, you see it going south. Interchanging the two subscripts changes the sign of the velocity.



Speeds of moving objects seldom remain constant for very long. Here is a graph showing our bee visiting flowers in a garden. It flies 10 m in 5 s during which time its speed is 2 m/s. Then it lands on a flower and stays there for 5 s. During this time its speed is zero, so the graph is horizontal for 5 s. Its position did not change during this time so the graph has zero slope. Then it takes off and flies on for 5 more seconds at a speed of 2 m/s.

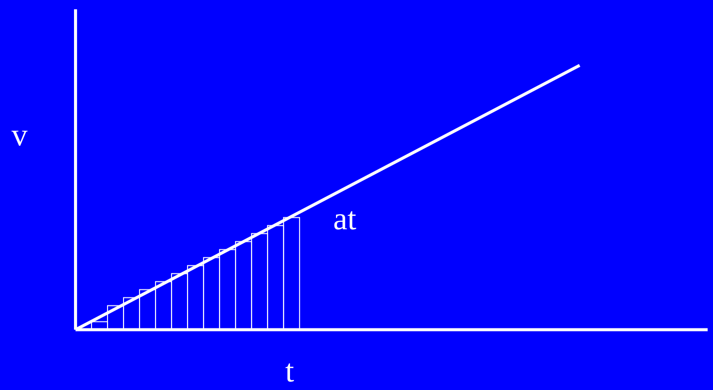
We could define an average speed over this 15 s interval. It traveled 20 m in 15 s, so its average speed is 1.33 m/s. Note that the bee's speed during this time was never equal to 1.33 m/s even though that is its average speed.



Acceleration is the rate of change of speed, and so has units of m/s^2 or mi/hr^2, etc. Here we see the speed increasing uniformly so a = constant. This is an important example for two reasons. First, we will encounter familiar situations for which the acceleration really is constant.

In addition, in many circumstances we can approximate the acceleration as being constant even if it is not. For example, a hammer hitting a nail, or a baseball bat hitting a ball. In these cases the acceleration is not constant, but the collision is so brief that all that counts is the average acceleration and its duration. The details of how the acceleration increased and then decreased with time do not matter.

HOW FAR DOES IT GO?

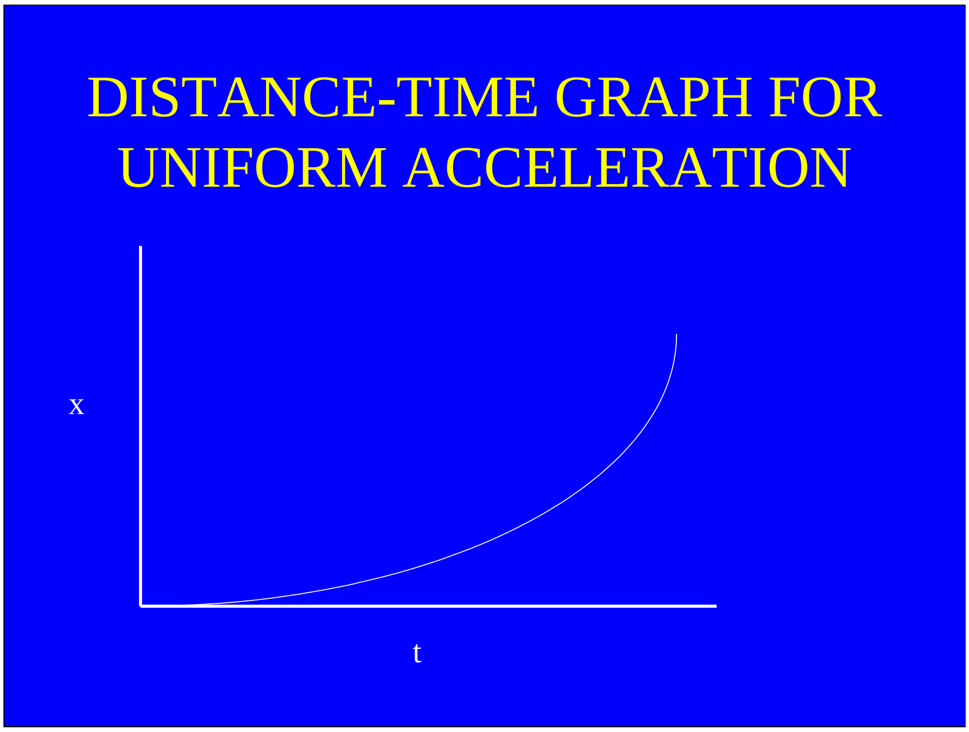


Distance equals area under speed graph regardless of its shape

Area = x = ½(base)(height) = ½(t)(at) = $1/2at^2$

For the case of constant acceleration, how far does the moving object go in a specified time. We have seen that the area under the speed vs time graph equals the distance gone for constant speed, i.e.when the graph has the shape of a rectangle. But we can approximate any shape graph by a series of narrow rectangles. If we center the rectangles on the linearly rising line for constant acceleration we make no errors in the total area. So the distance gone is always equal to the area under the speed vs time graph.

For constant acceleration what we are evaluating is the area of a triangle, which we know is ½ the base times the height. This is just x = ½(t)(at) = $1/2at^2$.



We have seen that the distance-time graph for constant speed is a straight line. For constant acceleration we have just shown that the distance gone increases as the square of the time. This relation, when shown graphically, has the shape of a parabola.

Note that this means the distance traveled starts out very slowly, but as time passes, x increases more and more rapidly with t. This is because the speed is increasing linearly with t as we have seen.

EXAMPLE: SUPERTANKER

Accelerates from 0 to 40 mph in 1 hour.
so $a = 40$ mi/hr^2

Then $x = ½(40)(1) = 20$ miles

If it continues for 2 hours,
then $x = ½(40)(4) = 80$ miles

Supertankers accelerate and decelerate very slowly. This just means that a is small. In this example $a = 40$ mi/hr^2. As the calculation shows, it goes 20 miles during the first hour while getting up to speed.

If it could continue accelerating another hour it would have gone 80 miles. The travel time has doubled, and so has the speed, so it goes four times as far in two hours as in one hour.

This is not likely however, since then it would be going 80 mph – way above a supertanker’s top speed.

HORIZONTAL & VERTICAL MOTION

Aristotle

Horizontal motion: v = kF where F = applied force

Vertical motion: v = kW where W = body’s weight

So now that we know something about how to describe motion, let us ask what kinds of motion do we observe in nature? Aristotle wrote that for horizontal motion, objects, such as an oxcart, move in the direction of an applied force with a speed that is proportional to that force. This does indeed describe the motions of many objects in everyday life. A car does not move along a horizontal road unless the engine pushes it along. When you stop pushing a book across a desk, it stops moving.

What Aristotle did not realize was that friction plays a role in the motion of most everyday objects. Not only is the ox pulling the cart forward, but friction is pulling it back.

Aristotle’s rule for vertical motion is very similar to that for horizontal motion. The speed of a falling body is proportional to its weight. If we recognize the body’s weight as the applied force, then this is the same as the rule for horizontal motion.

Aristotle’s rule works for objects falling through the air from great height. They accelerate at first until they reach a terminal velocity. The next slide shows some examples.

TERMINAL SPEEDS

Object `	Speed (m/s)	Speed (mph)
Feather	0.4	0.9
Snowflake	1	2.2
BB	9	20
Mouse	13	29
Sky diver	60	134
Cannonball	250	560

As the above objects fall through the air they accelerate. As their speed increases, so does their air resistance. Eventually the upwards force of air resistance becomes equal to the weight of the object, and there is no net force on the falling body, and it's speed becomes constant.

The sky diver's terminal speed of 60 m/s (134 mph) is for a spread-eagle position. If the diver tucks his body into a spherical shape, his terminal speed will roughly double.

So the terminal speed of a falling body depends on its weight and shape. A feather falls slowly partly because it is light, but also because it is spread out and intercepts a great deal of air.

HORIZONTAL & VERTICAL MOTION

Galileo

Horizontal motion: v = constant provided F = 0

Vertical motion: acceleration = constant
(neglecting air friction)

In his book *Two New Sciences* Galileo begins by ignoring friction. This creates a completely different situation than was described by Aristotle. He wrote: “Imagine any particle projected along a horizontal plane without friction; then we know that this particle will move along this same plane with a motion which is uniform and perpetual, provided the plane has no limits.”

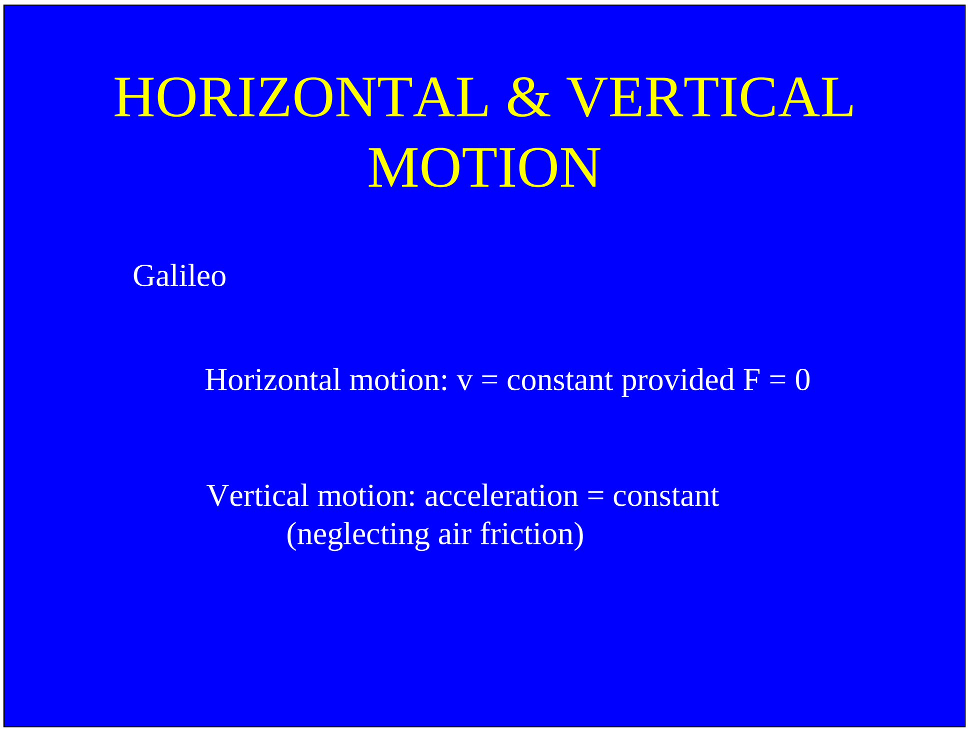
Galileo worked with rolling round metal balls on surfaces. He must have observed that as the surface is made smoother and harder, the ball, once given a push, will roll farther and farther. In the limit of perfection, it will roll forever.

This idea is often called Galileo’s principle of inertia. That if an object is initially at rest, it will remain so, and if moving, it will continue unless acted upon by a force. This is also called Newton’s first law of motion.

Galileo’s rule for vertical motion is that the acceleration of falling bodies is constant, regardless of their weight.

In TNS, the three characters discuss falling bodies on p 62:

SALVIATI: I greatly doubt that Aristotle ever tested by experiment whether it be true that two stones, one weighing ten times as much as the other, if allowed to fall from a height of, say, 100 cubits, would so differ in speed that when the heavier had reached the ground, the other would only have fallen 10 cubits.



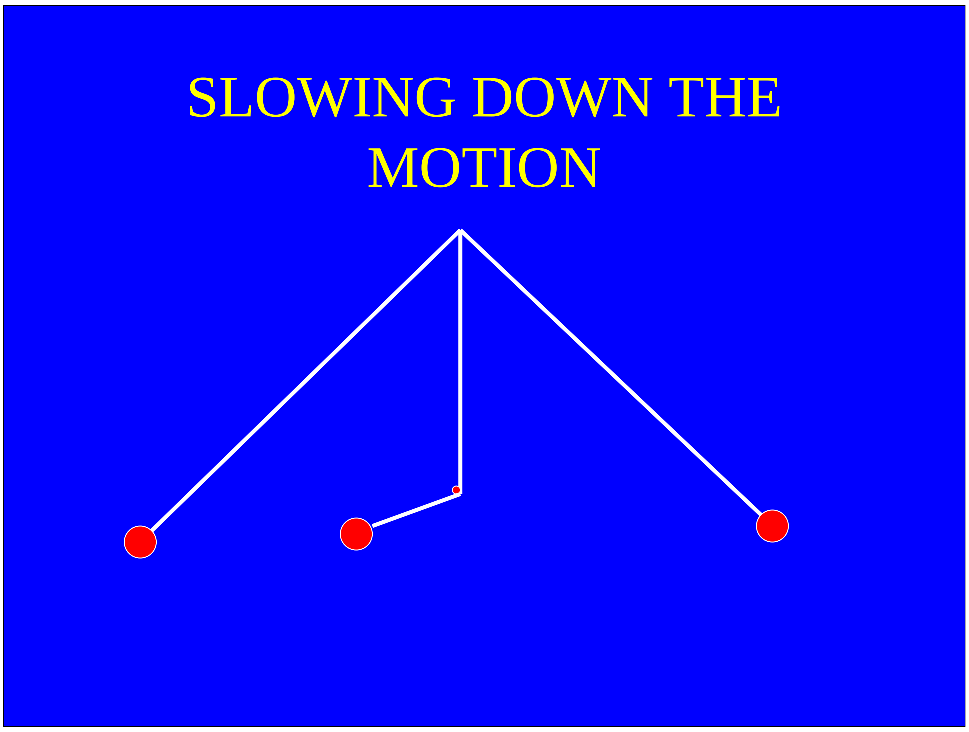
Simplicio's response is not to think of doing the experiment himself, but to examine more carefully the words of the great Authority.

SIMPLICIO: His language would seem to indicate that he had tried the experiment because he says: We see the heavier…; The word see shows he had made the experiment.

SAGREDO: But I, Simplicio, who have made the test, can assure you that a cannon ball weighing one or two hundred pounds, or even more, will not reach the ground by as much as a span ahead of a musket ball weighing only half a pound, provided both are dropped from a height of 200 cubits.

Here we see the beginnings of modern science. Here is the idea that the opinions about the physical world by revered authorities stand or fall by experimental test.

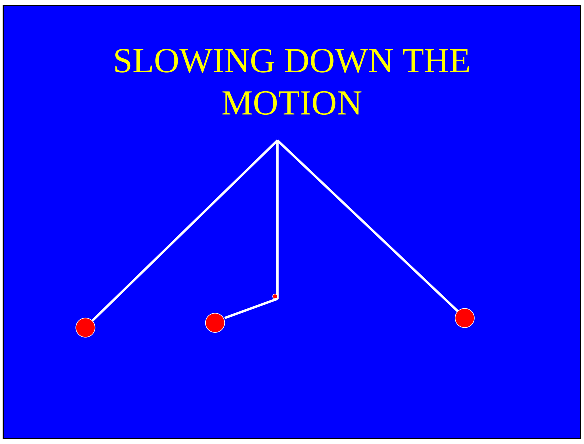
Having established that falling bodies fall at the same rate, he then goes on to assert his hypothesis that their speeds increase uniformly with time. In other words, they fall with constant acceleration.



Galileo's hypothesis for falling bodies is that they gain equal amounts of speed in equal intervals of time as they fall. This is not the only possibility he considered. He suggests others in TNS, but rejects them with careful arguments. For example, what about saying that a falling body gains equal amounts of speed when falling through equal amounts of height? (According to this hypothesis, it would take a weight equal times to fall 4 feet as to fall 8 feet. This is not what is observed.)

Galileo is asking a new kind of question: How does the speed change as a body falls? His problem was, things fall too fast to be measured repeatedly as they fall using the methods available at the time. So he wanted to slow the motion down without changing its character. If he were to drop a stone in water, it would quickly reach terminal speed and continue at that same speed for the remainder of the fall.

Galileo used the pendulum sketched above to convince himself that he could slow the motion down without introducing friction. When a pendulum swings from one side to the other, it rises to nearly the same height. (A small amount of friction slows it down slightly so it does not rise back all the way). The height to which it rises is determined by the speed of its motion at the bottom of the trajectory.

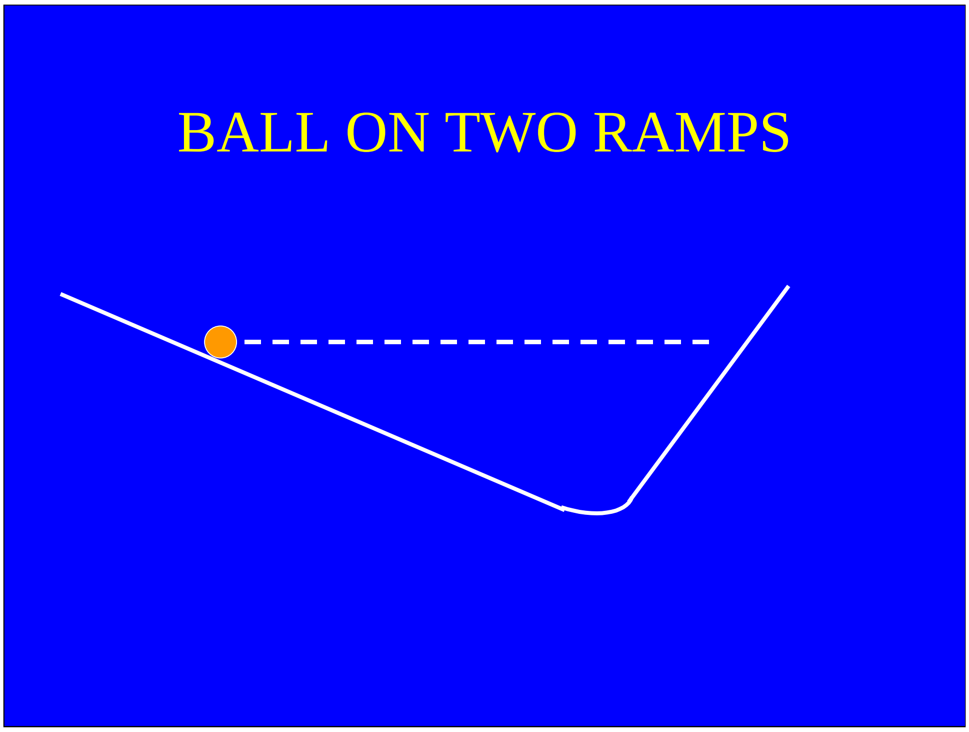


He then built a modified pendulum as shown above. A peg is inserted below the suspension point, so as the weight swings past it, the peg suddenly shortens the length of the pendulum causing it to swing up abruptly. What he found is that it still rises to the same height as before, and does so again back on the other side. Galileo argued that this shows that a falling pendulum reaches the same speed at the bottom of its motion regardless of how steep the descent was.

This is a remarkable insight. Let's state this more clearly. Starting with an ordinary pendulum without the peg, if we raise it up to a certain height, it will rise to the same height on the other side, ignoring a small amount of friction. Then it swings back to the original side, again nearly to the original height. As it moves through the bottom of its motion, its speed determines how high it will go. This can be determined by releasing it from different heights and noticing that the higher the release, the faster it goes at the bottom.

Now put the peg in. It still rises just as high as before on the side opposite the release, even though it rose on a steeper trajectory. And as it returns to the original side, it again rises to the original height. But on the way up once it gets past the peg, it is the full length pendulum we started with. So it must have been going just as fast at the bottom as if the peg had not been there.

Galileo then applied these ideas to a ball rolling down one ramp and then up another.

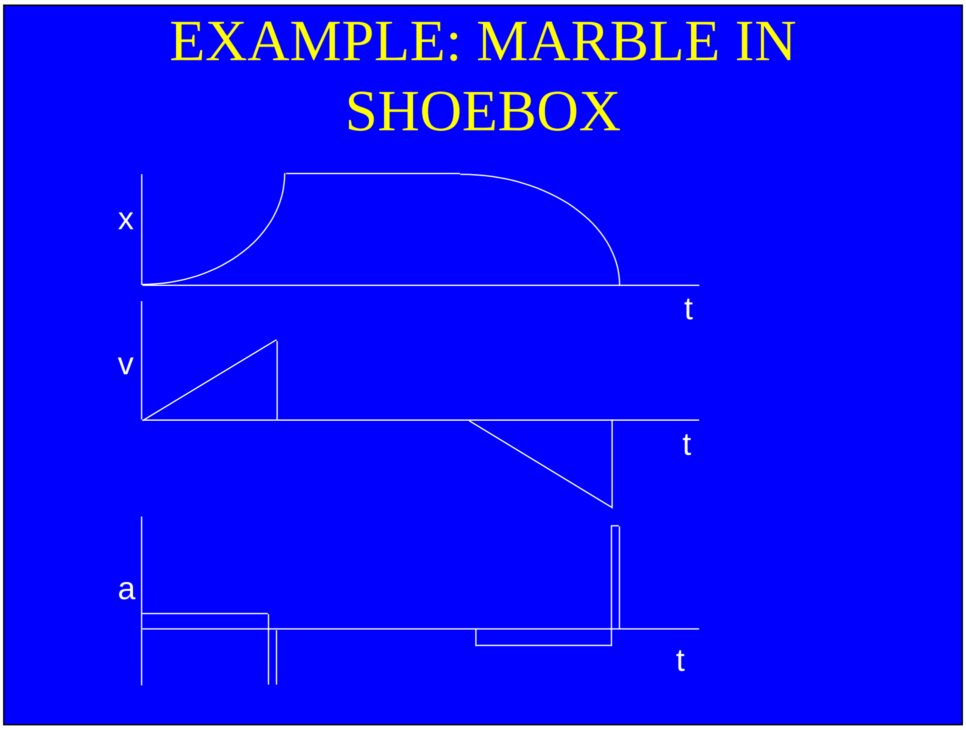


Galileo thought of his pendulum as being a succession of ramps of different slopes, and what he had just shown is that the slope doesn't matter in determining how far up the other side the ball will go, or how fast the ball will go at the bottom.

Of course there is a difference now between a rolling ball and a sliding (or swinging) ball, but Galileo's intuition told him this should not affect the basic argument, and he was right. He was a genius at choosing what difficulties to ignore.

So let's apply the pendulum conclusions to two opposing ramps as shown. We expect then, that for very smooth ramps and a round hard ball, that it will roll up the other side to the same height from which it was released. And, it will have the same speed at the bottom no matter which side it was released from.

Now imagine the right ramp being made steeper and steeper. If it is steep enough we can think of the ball as simply falling. He concludes that for a ball rolling down a ramp, the speed at various heights is the same as if it had simply fallen vertically from the starting point to that height. He further concludes that the ramp has allowed him to slow down the motion of free fall, allowing him to make measurements of it.



Suppose we put a marble in a shoe box, and tilt it back and forth. The marble will roll from one end to the other. We assume when it hits the end of the box it sticks without bouncing. Since the marble is a ball rolling down an inclined plane, we know how to describe its motion. The displacement increases quadratically with time until it abruptly stops at the other end. Then when the box is tilted the other way, it rolls back.

What does the speed v look like for this motion? When the marble is stuck at the ends, the speed is clearly zero. As it rolls, what does the speed graph look like?

Now what about the marble's acceleration? As it rolls, how does its acceleration behave with time? Galileo showed it is a constant. Then the marble hits the other end of the box and its speed suddenly stops. The acceleration must be large and negative when this collision takes place. Acceleration is the rate at which speed is changing, and in a very short time all its speed goes to zero.