Preface

The physicist's description of the universe is most naturally expressed in the language of mathematics[1]. The scientist or engineer who needs the ideas of physics must therefore learn as much of this mother tongue as he can absorb—subjects that seem arcane to one generation become the routine mathematics of the next. Demand for mathematical proficiency has called forth myriad texts on "Mathematical Methods of Physics" or "Applied Mathematics for Physics and Engineering". Most such books cover applied analysis (what used to be called multi-dimensional differential and integral calculus); the theory of functions of a complex variable; areas of advanced algebra such as linear equations and group theory; and sometimes differential geometry and tensor analysis. Some books of this genre cover topics in numerical mathematics and approximate methods such as perturbations, asymptotic approximations, *etc.*

The changing needs of new generations of students demand careful selection of the materials covered in a one- or two semester course of mathematical methods of physics. A survey of a dozen instructors will yield twelve distinct views as to what subjects simply *must* be included. Courses offered by first-rank institutions vary widely, even idiosyncratically, in what they cover. This book therefore presents far more material than can be taught even in a fast-paced one-year course[2], in the hope that the range of topics provides adequately for most types of course.

[1] Why this should be so is hardly obvious—see, *e.g.*, E. Wigner, *Symmetries and Reflections.*

[2] Many institutions have, for reasons that seem inadequate, even frivolous, reduced the mathematical methods requirement from two semesters to one. In my opinion this reduction badly scants the needs of the typical student.

An undergraduate one-semester course might cover the following topics:

- *Multivariate calculus.*
- *Linear equations and finite vector spaces.*
- *Introduction to complex variables and analytic functions.*
- *Fourier series, Fourier and Laplace transforms.*
- *Ordinary and partial differential equations.*
- *Linear operators and eigenvalue problems.*

An introduction to tensors, group theory, or probability theory could be substituted for the study of linear operators. I have arranged the order of presentation so that the listed topics appear in the first portion of the book.

A graduate one-semester course might include

- *Complex variables and analytic functions.*
- *Ordinary differential equations.*
- *Linear vector spaces and Hilbert space.*
- *Partial differential equations.*
- *Transform methods.*
- *Linear operators and eigenvalue problems.*
- *Perturbation theory.*
- *Variational methods.*

For students who lack mathematical preparation (as is often the case) the chapters on infinite series and multivariate calculus, should be added as preliminary review.

A graduate two-semester course would cover most of the chapters, depending on the instructor's tastes.

In addition to what topics to cover, the author of a text in mathematical methods of physics must decide the appropriate level of mathematical rigor. Pure mathematics texts often read like legal tomes, whose technicalities will relieve the stubbornest insomnia. At the opposite extreme, texts like *Mathematical Methods of Physics* by Mathews and Walker assume a strong background in formal mathematics and therefore adopt an informal style. In this book I have sought the middle ground. Many students enrolled today in graduate physics programs have not been exposed to a rigorous course in modern analysis, taught in a Department of Mathematics. Because it seems to me pedagogically unsound to pull theorems from a magician's hat, I prove the more important results with more amplification than might be found in a typically laconic mathematics text. The proofs emphasize the geometric ideas that underlie both analysis and the theory of linear vector spaces.

The advent of powerful, inexpensive, general-purpose computers has affected mathematical methods courses in several ways. The more benign has been the reduction of emphasis on "special functions of mathematical physics". A large part of the courses of years past was devoted to studying the properties of Legendre polynomials, Bessel functions, and related solutions of second order linear differential equations of Stürm-Liouville[3] type. These functions arise naturally in that (small) subset of problems for which we can find solutions in closed form. While it is undeniably useful to know about special functions, it is no longer quite so urgent, since it is often faster to solve the corresponding differential equation numerically, than to look up and interpolate its known solution in a table of special functions. Thus, this book discusses special functions more as illustrations of ideas than for their own sake, leaving more detailed expositions to specialized monographs.

The second major effect of cheap, powerful computers has been to make computer-aided mathematics available to all. Such programs as muMath®, MathCad®, Maple®, Mathematica®, MACSYMA®, *etc.* have changed mathematical education irrevocably. On the plus side, a program like MathCad can provide immediate visual feedback about the geometric meanings of integrals and derivatives, say. However, many students—and, sadly, some of their professors and deans—have become convinced that quantitative disciplines like chemistry, physics, or engineering no longer require mathematical proficiency because "the computer can do whatever I need". But this is a dangerous misconception. Computers cannot replace human knowledge and insight, nor can they replace critical thinking, creativity or intuition.

For example a student turned in the following

$$\frac{d}{dx}\left(\int^{x} ds\,\frac{s^4}{\sqrt{s^2+1}}\right) = \frac{3}{4}x^2\sqrt{x^2+1} + \frac{1}{4}\frac{x^4}{\sqrt{x^2+1}}$$

[3]See §??, p. ??.

$$-\frac{3}{8}\sqrt{x^2+1}-\frac{3}{8}\frac{x^2}{\sqrt{x^2+1}}+\frac{3}{8}\frac{1}{\sqrt{x^2+1}}\,,$$

as part of a homework solution in a course in astrophysics. Inquiry revealed that he obtained this result with a computer algebra program—evidently the fundamental theorem of calculus[4,5] did not occur to this student. Although the failures of computer-aided mathematics are generally more subtle than this, my colleagues and I have encountered them with depressing frequency.

Followers of the USENET newsgroup **comp.math.num-analysis** will be amused (or depressed) by shoals of frantic requests for help, by users of canned mathematical or numerical routines who lack the mathematical background to understand why their integrals fail to converge, why the solutions ground out by their differential equation solvers are unstable, or even why they cannot solve for two unknowns with a single equation.

The Luddite view, that computer-aided mathematics should be done away with entirely, is as extreme (and wrong) as the view that such programs can substitute for knowledge. In the hands of experts they can be very useful. Unfortunately such powerful tools pose a danger to the unsophisticated.

The hazard extends beyond bad pedagogy or loss of homework credit. Designers of safety-critical structures or vehicles routinely employ computer-aided mathematics, so an error can involve loss of life and property. A case in point was the expansion of a football stadium to add additional tiers of seats in the nose-bleed section. The supports for these tiers were prefabricated from pre-stressed concrete. Sadly, the structural engineer was a recent civil engineering graduate who misused a CAD/CAM program, through inexperience did not recognize the error, and thereby specified 10× less steel than the expected loads demanded. Luckily, the error was so extreme that the faulty supports began to crack as soon as they were installed. The supports were replaced before they could be crushed by the excesses of 5,000 rabid rooters performing a victory dance. But it would be foolish to count on luck to avert the effects of ignorance.

We can reduce the likelihood of dangerous errors by learning to recognize

[4]... which states that for reasonable functions, $\frac{d}{dx}\left(\int_a^x f(s)\,ds\right)=f(x)$.

[5]... and yes, I am aware that most algebra programs can simplify the above expression; this does not vitiate my objection to substituting learning how to use such programs for learning how to do mathematics.

when a computer-generated answer is ridiculous. To do this requires us to understand what lies within the “black box” presented by even the best of computer-aided mathematics programs. To integrate this course with any of the popular symbolic programs would, in my view, vitiate the aim of providing a student with a thorough grounding in applied mathematics. Thus, while an instructor may permit students to check their work using such programs, he should insist that they tackle the homework problems using the computers between their ears.