# **Final Review Sheet**

The best strategy is to read through my web notes, review all the answers to homework problems, then read your own notes from class and/or Shankar on the Hydrogen atom and the later lectures on angular momentum.

## Physics 751: Midterm Study Guide

I will ask no questions on the historical stuff at the beginning, except for the purely physics content. For example, you should have a good understanding of the Bohr atom, including the classical limit of a very large atom and how that fixes the quantum of angular momentum, and know how the Uncertainty Principle can be used to estimate the size of bound states, such as the atom on the table.

You should know Schrödinger's equation by heart, the probability interpretation of the wavefunction, the conservation of probability current, and the expectation value of an operator. Be able to find the time-dependent probability and current distribution for a superposition of states of different energies.

Understand the connection between particle speed and wave group velocity.

Know the definition of the delta function as a limit of a Gaussian wavepacket, and be able to Fourier transform a Gaussian wavepacket. (Know the integral you need by heart.)

You must know the bra and ket notation, be able to construct the first few elements of an orthogonal basis, the definition of adjoint, and be able to diagonalize a (small) Hermitian matrix. You should also feel comfortable going back and forth between the Schrödinger wavefunction notation and the Dirac bra ket notation.

Know the normalization conventions for the eigenkets of the momentum operator, and those of the position operator. Know the identity in terms of these kets, and be able to use it to express a Schrödinger wave function as an integral over  $|x\rangle$  or  $|k\rangle$  kets.

I expect you to be able to solve simple boundary-matching problems for square barriers, steps, square wells, and delta function potentials.

## **Physics 751 Midterm II Review Sheet**

## Simple Harmonic Oscillator: know

$$x = \sqrt{\hbar/2m\omega} \left( a^{\dagger} + a \right), \quad p = \sqrt{m\omega\hbar/2} \left( a^{\dagger} - a \right), \quad \left\langle n + 1 \middle| a^{\dagger} \middle| n \right\rangle = \sqrt{n+1}, \quad \left\langle n - 1 \middle| a \middle| n \right\rangle = \sqrt{n}.$$

Know the definition of the **propagator**, and be able to derive the free particle propagator.

Be familiar with the **Heisenberg representation**, know how to find the equation of motion for an operator in that representation, and know the connection with Ehrenfest's theorem.

Memorize the result  $e^{A+B} = e^A e^B e^{-\frac{1}{2}[A,B]}$ , know for what operators it is valid, and be able to use it for normalizing coherent states, etc.

Know what the *P*-basis is, and when it's useful.

### **Angular Momentum**

From class notes: be able to show, if the operator  $\vec{J}$  is defined by  $|\psi\rangle \rightarrow e^{-\frac{i}{\hbar}\vec{J}\cdot\vec{\theta}}|\psi\rangle$  under a rotation defined by  $\vec{\theta}$ , then from our knowledge of *classical* rotations,  $[J_i, J_j] = i\hbar\varepsilon_{ijk}J_k$ , and know this formula by heart! Be able to derive from this the commutation relations among  $J^2, J_z, J_{\pm}$ . Be able to derive the matrix elements of these operators for the common eigenkets of  $J^2, J_z$ , and know how to prove that 2j is an integer. In fact, you should *memorize*:

$$J_{\pm}|j,m\rangle = \hbar\sqrt{j(j+1) - m(m\pm 1)}|j,m\pm 1\rangle.$$

**Orbital angular momentum**: know the formulas for the components  $L_i$  in Cartesian coordinates. From the formula you've memorized above, be able to construct the matrix operators corresponding to rotations of an orbital angular momentum equals one state about the *x*, *y* or *z* axes.

Know what  $L_z$  is in spherical polar coordinates, and what its eigenstates are. (We won't do Legendre polynomials on this test.)

### **Material since Midterm II:**

### Read Shankar (also covered in class):

Chapter 12: Rotational Invariance and Angular Momentum: page 334 to 343.

Chapter 13: The Hydrogen Atom: read Sections 13.1, 13.3, 13.4.

Chapter 14: Spin: read all except negative temperatures section.

Chapter 15: Addition of Angular Momenta: read up to page 420.