Physics 751 Homework #8

Due November 7, 11:00 am.

1. Our definition of a coherent state, $|\lambda> = e^{-\lambda^2/2} e^{\lambda a^\dagger} |0>$, can be simply generalized to complex λ:

$$|\lambda> = e^{-|\lambda|^2/2} e^{\lambda a^\dagger} |0>.$$

(a) For two such in general complex states, find the overlap integral $<\mu|\lambda>$.

(b) Writing λ in terms of its real and imaginary parts, $\lambda = x + iy$, prove that

$$I = \iint \frac{dxdy}{\pi} |\lambda><\lambda|.$$

Hint: use polar coordinates.

2. Normalize the state $a^\dagger |\lambda>$. Can you give a physical interpretation of this state?

3. Write down the formulas for the three components of orbital angular momentum in Cartesian coordinates. Prove that they satisfy the angular momentum commutation relations with each other, and with L^2. Defining $L_\pm = L_x \pm iL_y$, prove that these act as ladder operators on L_z. Write down L^2 explicitly in terms of **r**, **p** in Cartesian coordinates. Classically, $p^2 = \frac{\mathbf{L}^2}{r^2} + \left(\frac{\mathbf{r}.\mathbf{p}}{r}\right)^2$. Is this still a correct statement in quantum mechanics? Give reasons.

4. Find the expectation values $<jm|J_y|jm>$ and $<jm|J_y^2|jm>$.