Quantum information processing with prethreshold superconducting qubits

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www.physast.uga.edu/~mgeller/group.htm

SES computing

n qubits

full Hilbert space

$$\psi \to U\psi$$

2^n variables

exponential storage capacity

but 2^{2n} gates

natural Hamiltonians are one- and two-body

efficient algorithms poly(n)

number of gates is large

error correction required

this is the bottleneck for superconducting architectures

single-excitation subspace (SES)

$$\psi \to U\psi$$

n variables

1 shot!

full controllability of projected H

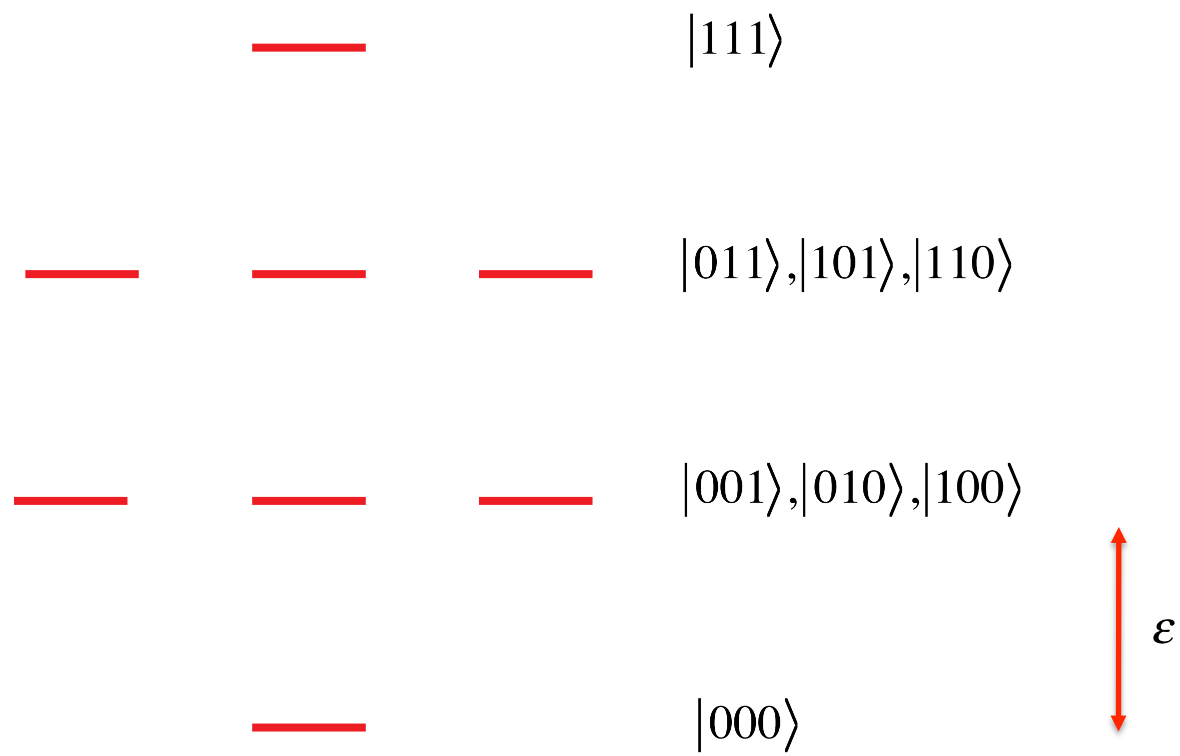
requires fully connected qubits $\left(\sim n^2 \text{ JJs}\right)$

can only run for time τ

more qubits required, but this is OK

single-excitation subspace

$$n = 3 \quad \{|000\rangle, |001\rangle, |010\rangle, |011\rangle, |100\rangle, |101\rangle, |110\rangle, |111\rangle\}$$



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1 shot!

full controllability of projected H

applications:

1. factoring n ~ 2^{100s} (not possible)

2. general purpose simulation

polynomial speedup for class of simulations

quantum simulation

Feynman, 1982

Lloyd, 1996

digital
(gate-based)

analog

Buluta &
Nori, 2009

Aspuru *et al*, 2005

Greiner *et al*, 2002

quantum simulation

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general-purpose simulator (UGA/UCSB)

Greiner *et al*, 2002

- tied to UCSB phase qubit architecture
- simulates arbitrary time-dependent H's
- uses and interfaces with conventional QC

mapping

$$H = \frac{p_1^2}{2m_1} + \frac{P_1^2}{2M_1} - \frac{Z_1 e^2}{|r_1 - R_1|} + \frac{p_2^2}{2m_2} + \frac{P_2^2}{2M_2} - \frac{Z_2 e^2}{|r_2 - R_2|} + \frac{Z_1 Z_2 e^2}{|R_1 - R_2|} + \frac{e^2}{|r_1 - r_2|} - \frac{Z_2 e^2}{|r_1 - R_2|} - \frac{Z_1 e^2}{|r_2 - R_1|}$$

?

$$H_{\text{ucsb}} = \sum_{i=1}^{n} \varepsilon_i c_i^\dagger c_i + \frac{1}{2} \sum_{\substack{i,j=1 \\ i \neq j}}^{n} g_{ij} K^{\alpha\beta} \sigma_i^\alpha \otimes \sigma_j^\beta$$

what we solve

given a d-dimensional Hilbert space

$$\{|m\rangle\}_{m=1}^{d}, \quad \langle m|m'\rangle = \delta_{mm'}$$

and time-dependent Hamiltonian

$$H_{mm'}(t), \quad t \in (t_{\mathrm{i}}, t_{\mathrm{f}})$$ (real?)

time-evolution operator

$$U \equiv T\, e^{-(i/\hbar)\int_{t_{\mathrm{i}}}^{t_{\mathrm{f}}} H(t)\, dt}$$

$$= \lim_{\Delta t \to 0} e^{-(i/\hbar)H(t_{\mathrm{f}})\Delta t} \times \cdots \times e^{-(i/\hbar)H(t_{\mathrm{i}}+\Delta t)\Delta t} e^{-(i/\hbar)H(t_{\mathrm{i}})\Delta t}$$

$$|\psi(t_{\mathrm{f}})\rangle = U|\psi(t_{\mathrm{i}})\rangle$$ we compute U

n-qubit simulator

UCSB phase qubits with tunable inductive coupling

$$H_{\mathrm{ucsb}} = \underbrace{\sum_{i=1}^{n} \varepsilon_i(t)}_{n} c_i^\dagger c_i + \frac{1}{2} \sum_{\substack{i,j=1 \\ i \neq j}}^{n} \underbrace{g_{ij}(t)}_{\frac{n(n-1)}{2}} K^{\alpha\beta} \sigma_i^\alpha \otimes \sigma_j^\beta \qquad (g \ll \varepsilon)$$

$\frac{n(n+1)}{2}$ controls

$$\dim(H_{\mathrm{ucsb}}) = 2^n \quad \text{not controllable}$$

R. C. Bialczak *et al*, arXiv:1007.2209

single-excitation subspace

$$\left|m\right) \equiv c_m^\dagger \left|00\cdots0\right\rangle = \left|00\cdots1_m\cdots0\right\rangle$$

this has dimension n
(number of qubits)

complete control over real Hamiltonians
in this subspace

energy/time rescaling

$$U = e^{-(i/\hbar)Ht} = e^{-(i/\hbar)\left(\frac{H}{\lambda}\right)(\lambda t)} = e^{-(i/\hbar)H_{qc}t_{qc}}$$

$$H_{\mathrm{qc}} \equiv \frac{H}{\lambda}, \quad t_{qc} \equiv \lambda t$$

molecular collisions

$\lambda \sim 10^6$

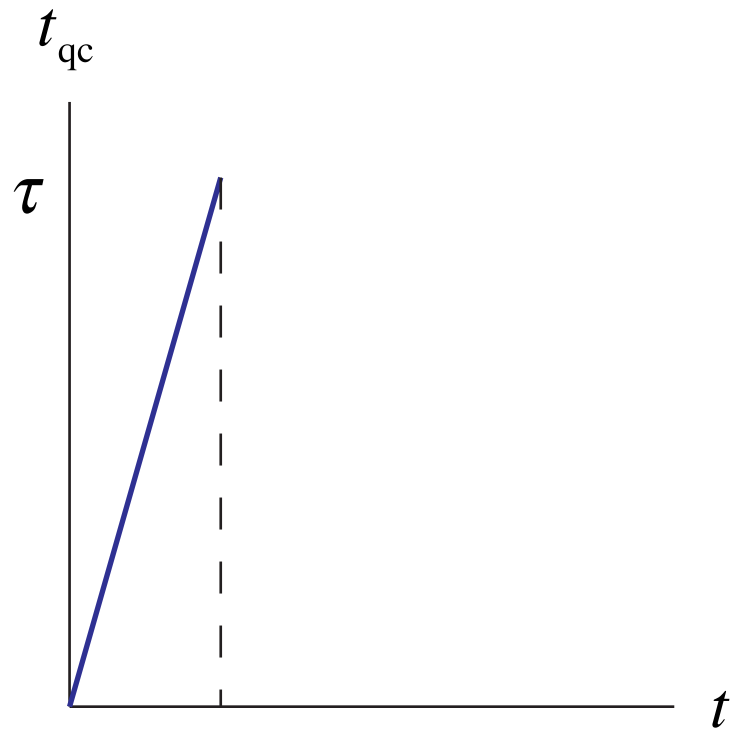
$\sim$ GHz

coherence limits length
of process simulated

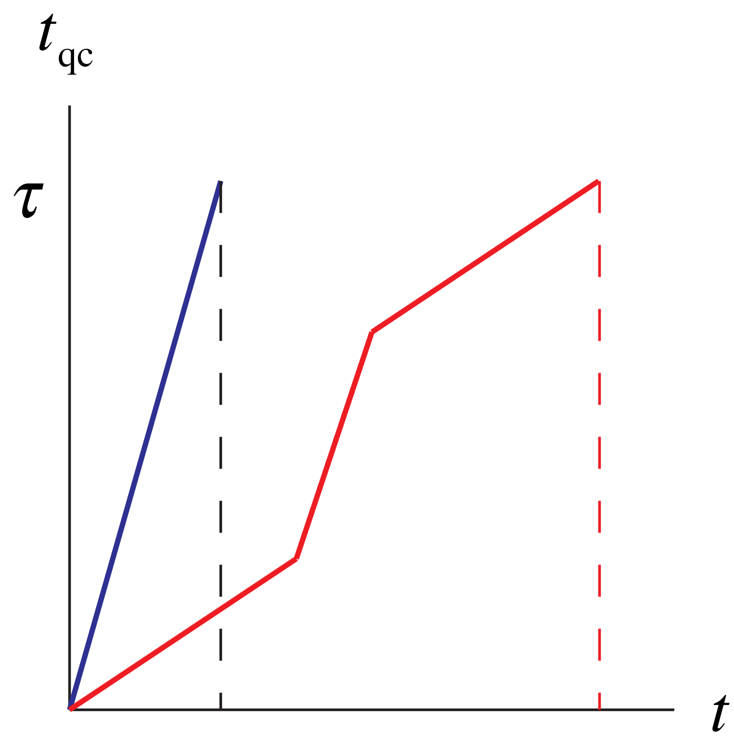
we will have to emulate H_{qc}

(can also subtract time-dependent c-number)

continuous rescaling



continuous rescaling



$$\lambda \equiv \frac{dt_{\text{qc}}}{dt} > 0$$

smaller λ is better

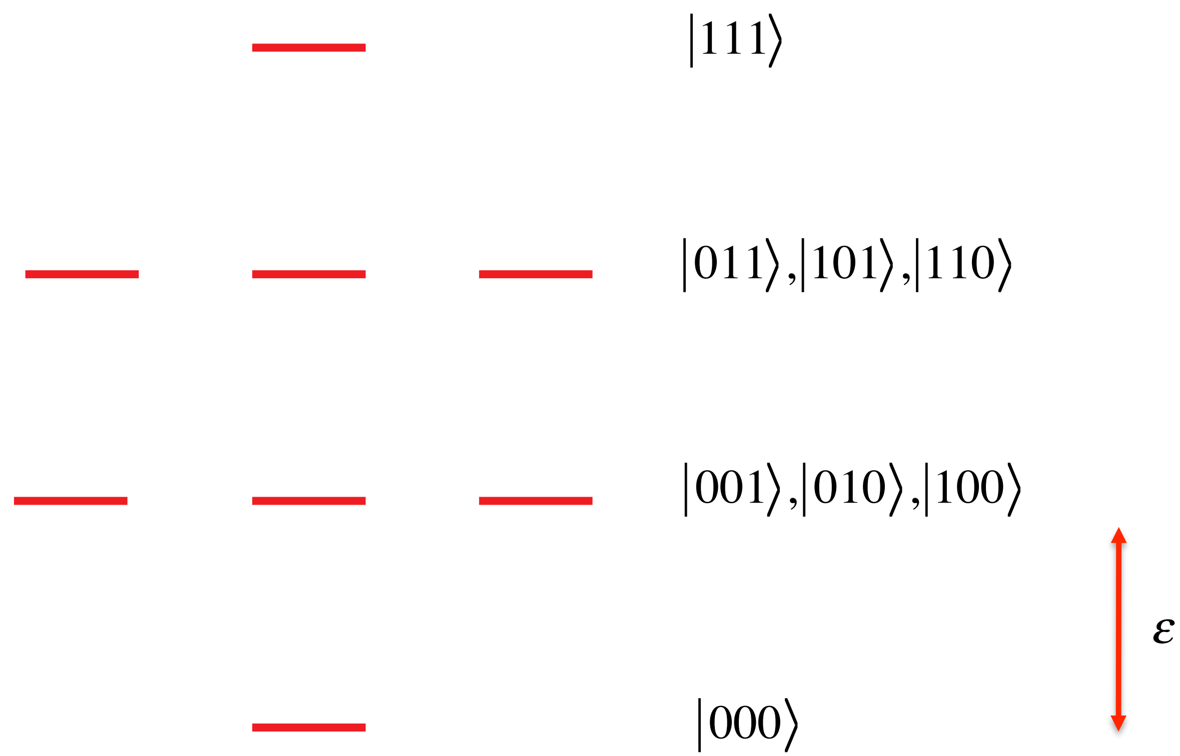
$t_{\text{qc}}(t)$ is nonlinear

$$U = Te^{-(i/\hbar)\int_{t_i}^{t_f} H(t)\,dt} = Te^{-(i/\hbar)\int_{t_{qc}(t_i)}^{t_{qc}(t_f)} H_{\text{qc}}(t_{\text{qc}})\,dt_{\text{qc}}} \qquad H_{\text{qc}}(t_{\text{qc}}) \equiv \left.\frac{H(t)}{\lambda(t)}\right|_{t=t(t_{\text{qc}})}$$

exact symmetry of U

main error: leakage

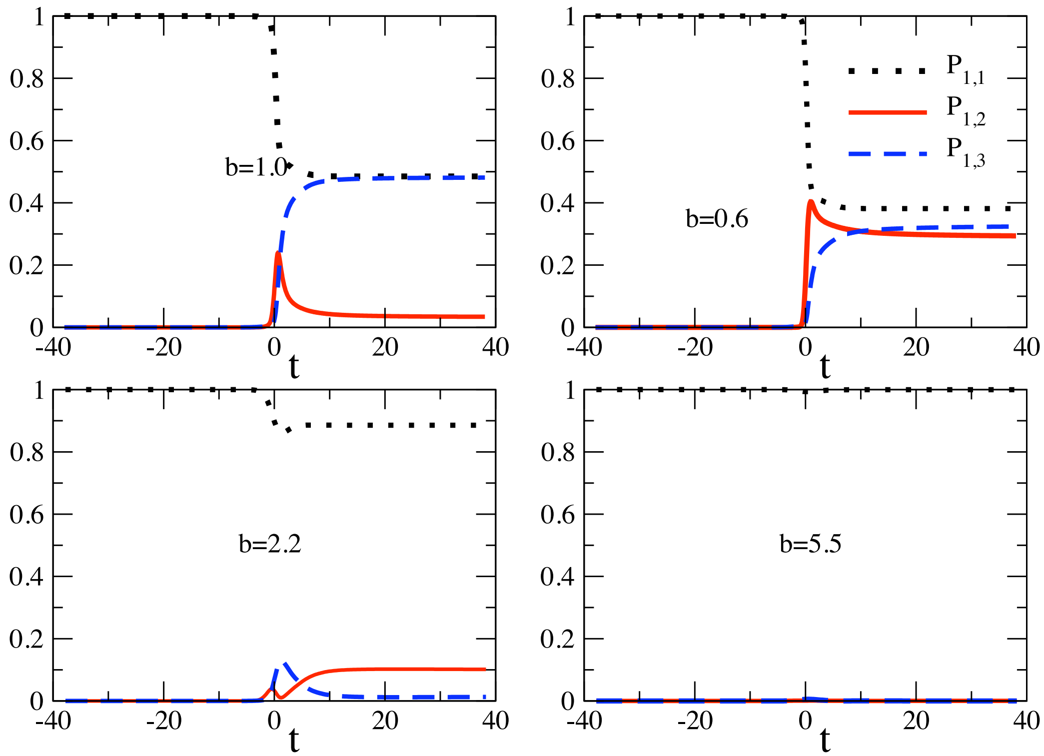
$$\text{leakage prob} \sim \left(\frac{g}{\varepsilon}\right)^2 \approx 10^{-4} \text{ (UCSB)}$$

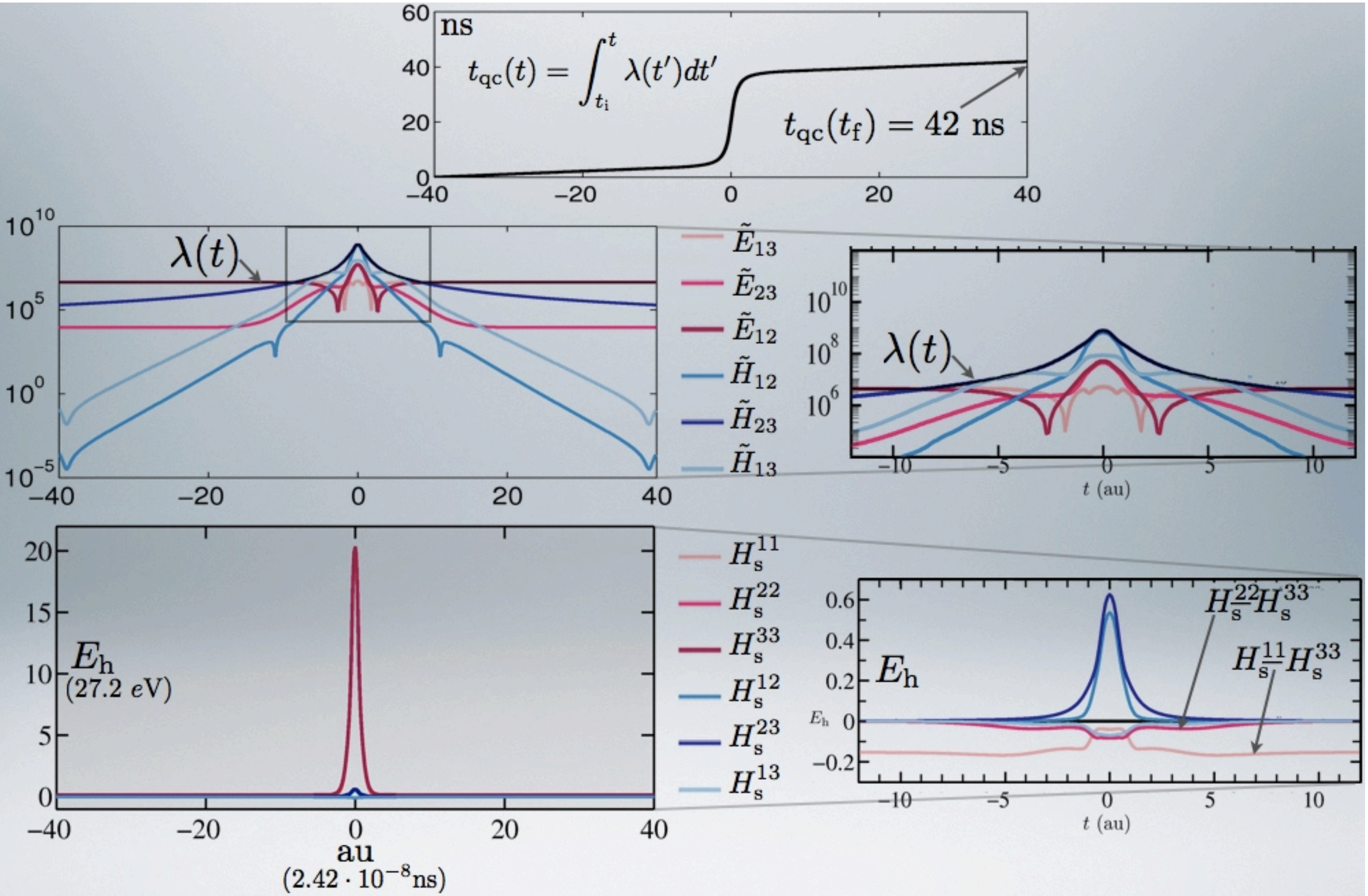


3-channel molecular collision

uses classically computed potential energy surface

$$\mathrm{Na(3s)} + \mathrm{He} \rightarrow \mathrm{Na}(m) + \mathrm{He} \qquad (m = 1,2,3)$$

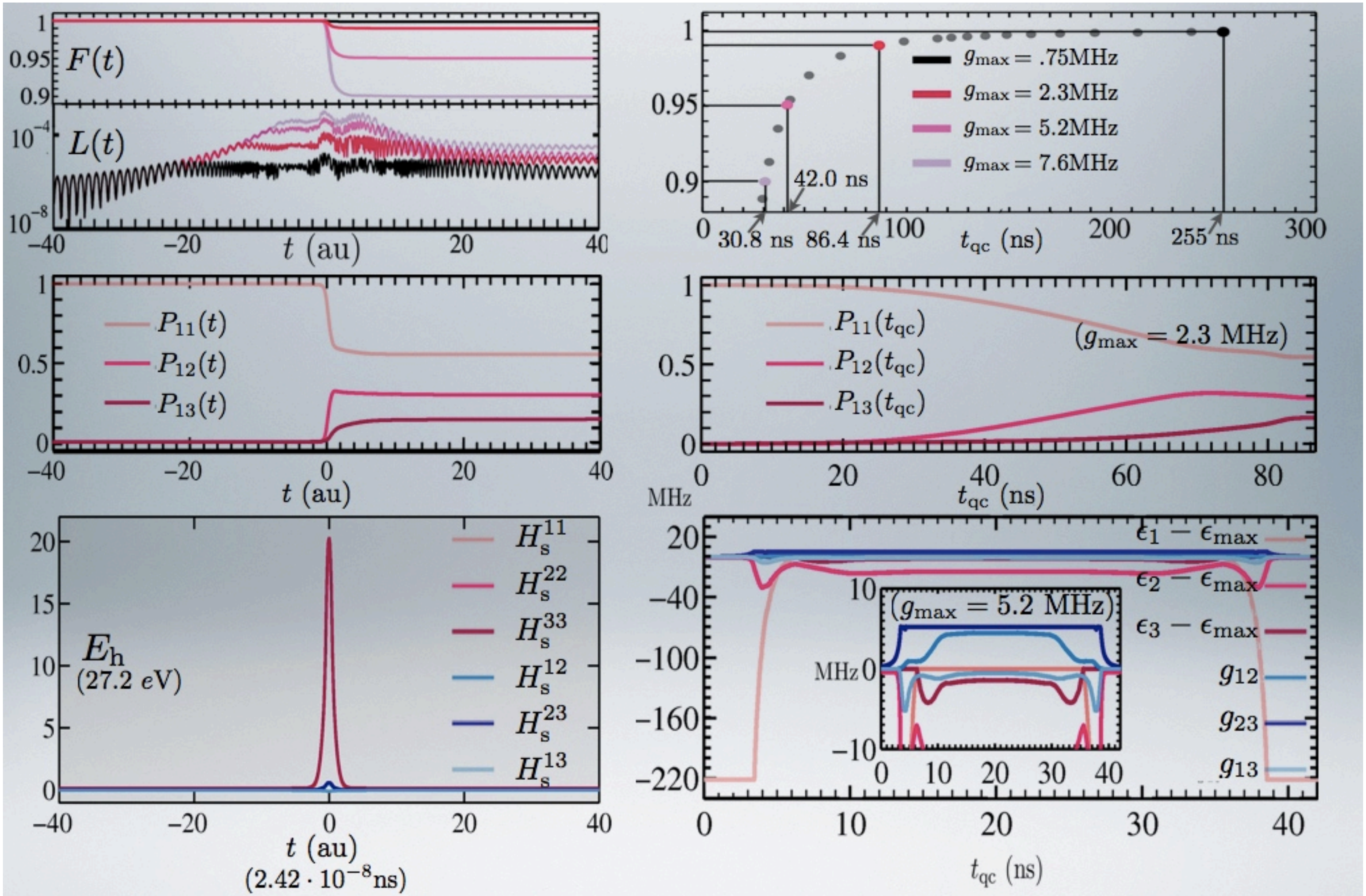




3 qubit quantum simulation

$\mathrm{Na}(3s) + \mathrm{He} \rightarrow \mathrm{Na}(3p) + \mathrm{He}$

C.Y. Lin, P.C. Stancil, H.-P.Liebermann, P. Funke, and R. J. Buenker, Phys. Rev. **A** 78, 052706 (2008)



3 qubit quantum simulation

$\mathrm{Na}(3s) + \mathrm{He} \rightarrow \mathrm{Na}(3p) + \mathrm{He}$

scaling

Hilbert space dimension d

classical (single fast processor)

time slice $t = 2d^3$ ns (large d)

$t_{\mathrm{cl}} = 100 \times 2d^3$ ns

unfavorable dimension dependence

(unrelated to exponentially large d problem)

d>1000 rare

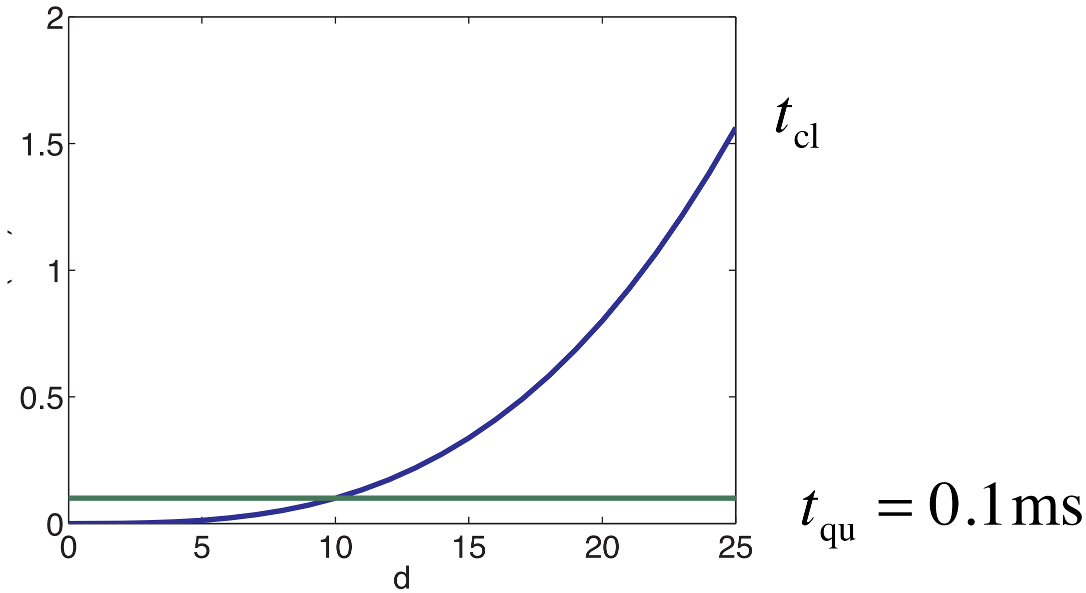
quantum simulator

each 100ns run $t = 1\mu$s

$t_{\mathrm{qu}} = 100 \times 1\mu\mathrm{s} = 10^5$ ns

independent of d!

scaling (continued)



100 qubit machine is 1000 times faster

1000 qubit machine is 10^6 times faster!

a practical QC?

E. J. Pritchett et al., arXiv:1008.0701