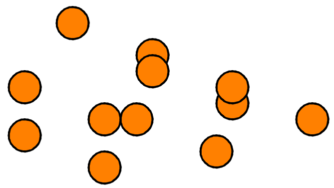
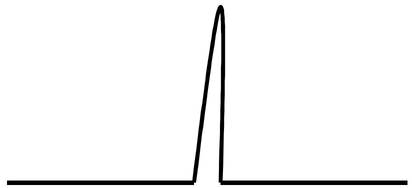
April 2011

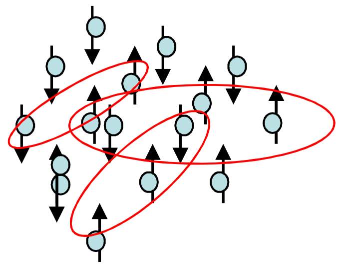
Elementary Particles of Superconductivity

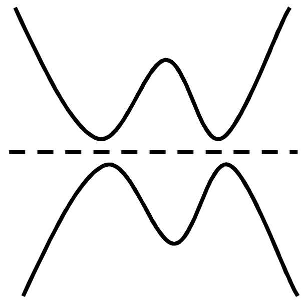
1. Schafroth's bosons ?





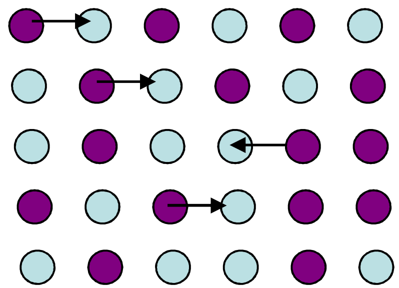
2. BCS paired electrons ?

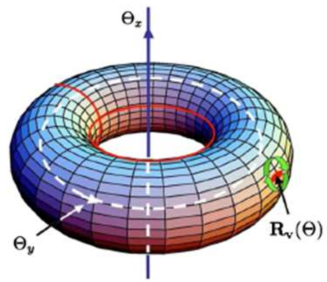




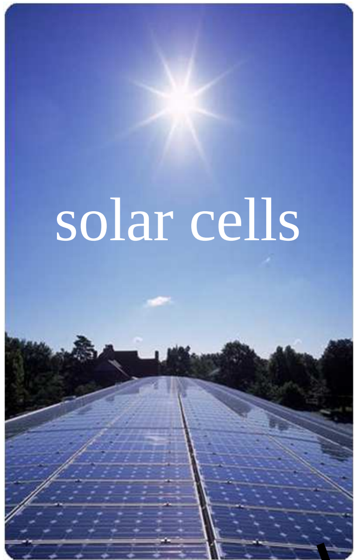
3. Lattice Bosons ?!

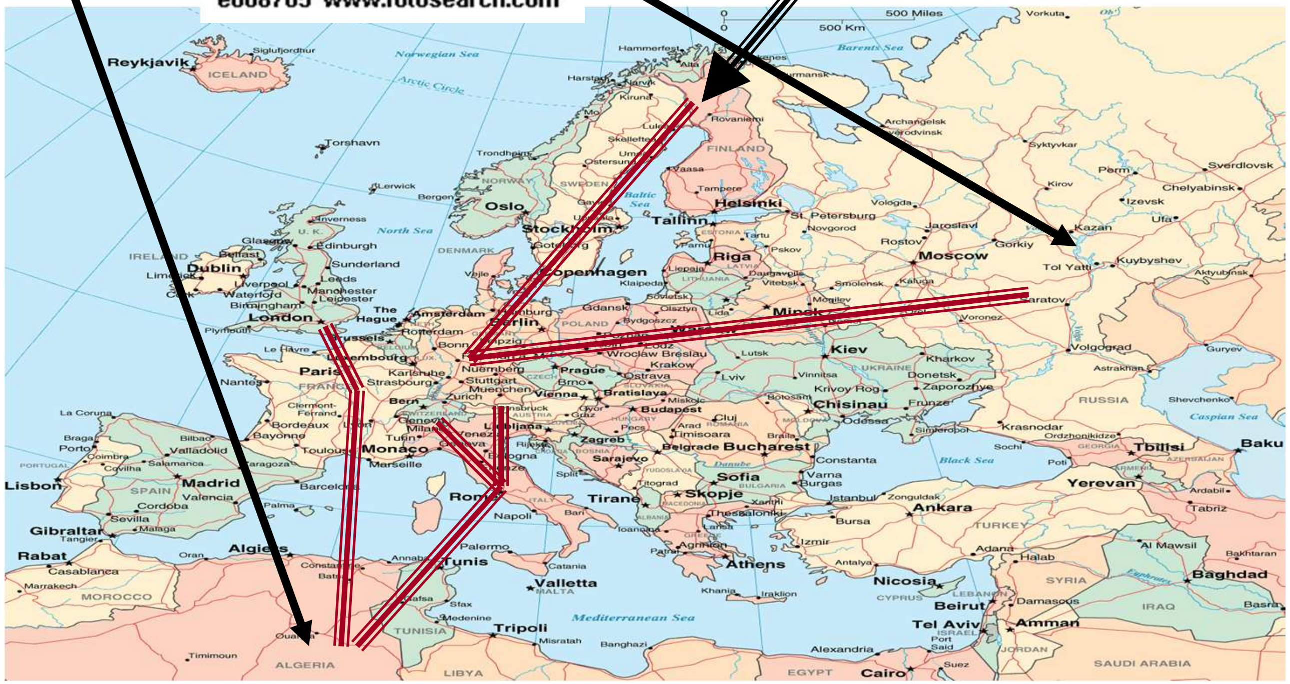
-- new paradigm of metallic conductivity…





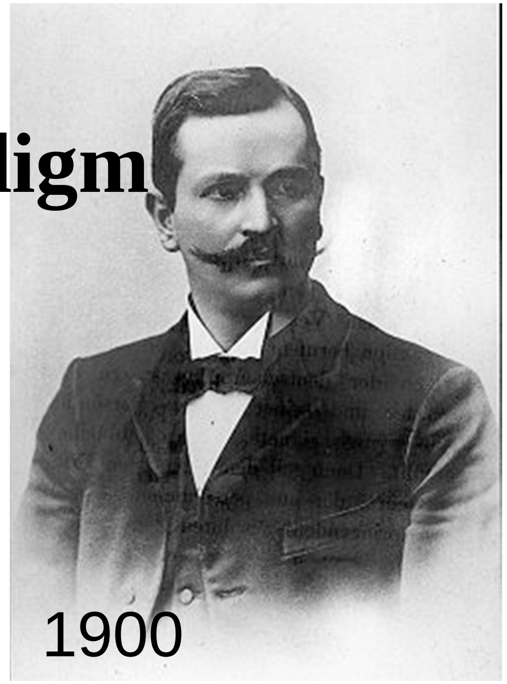
Energy transport





15% of electric power is wasted in wires resistance!

Resistivity: Boltzmann-Drude Paradigm

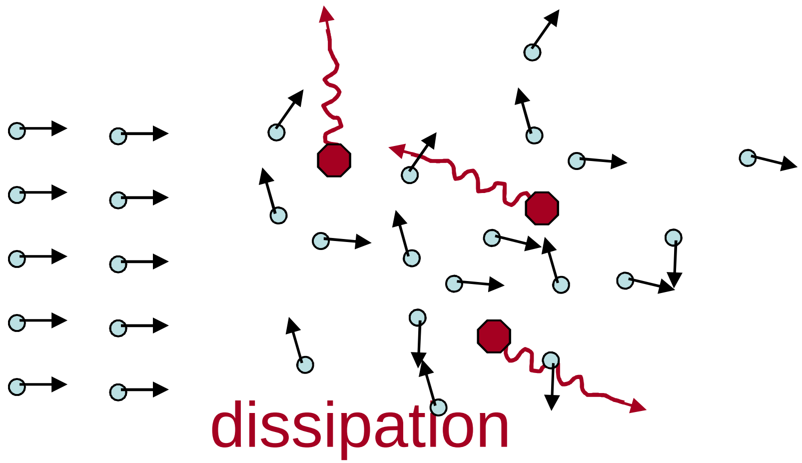


1. Electrons in metals behave like a free gas.
2. Uncorrelated scattering causes dissipation.

Ohm $en\vec{v} = \sigma \vec{E}$

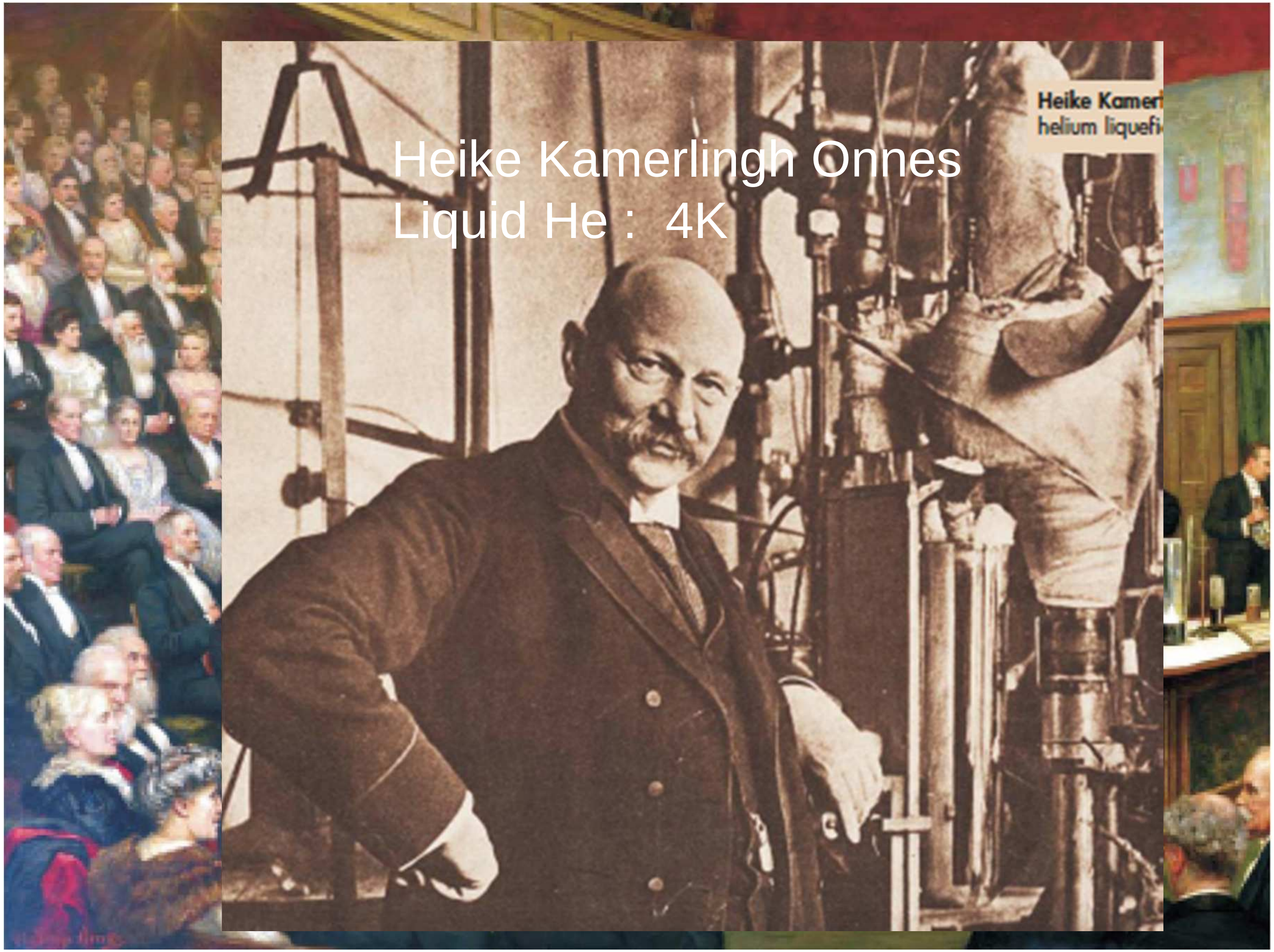
Newton $\vec{v} = -\frac{e\vec{E}\tau}{m}$

Kinetic theory of gases $\sigma = \frac{ne^2\tau}{m}$



'scattering time'

The race to absolute 0



Discovery of Superconductivity

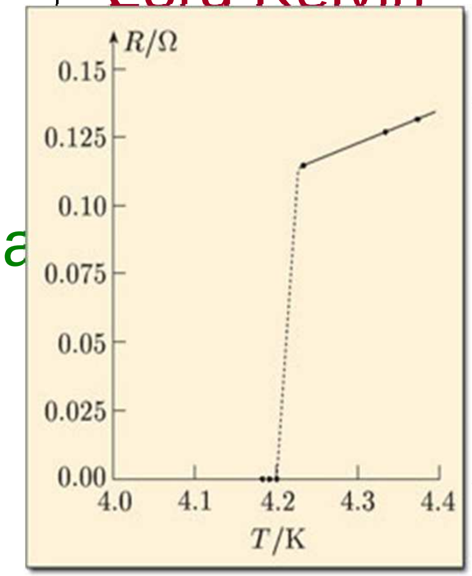
Heike Kamerlingh Onnes, Nov 1911

mercury

Lord Kelvin

R

Ma

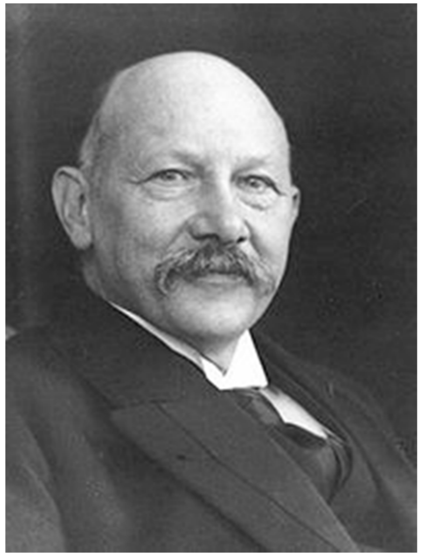


R=0 !

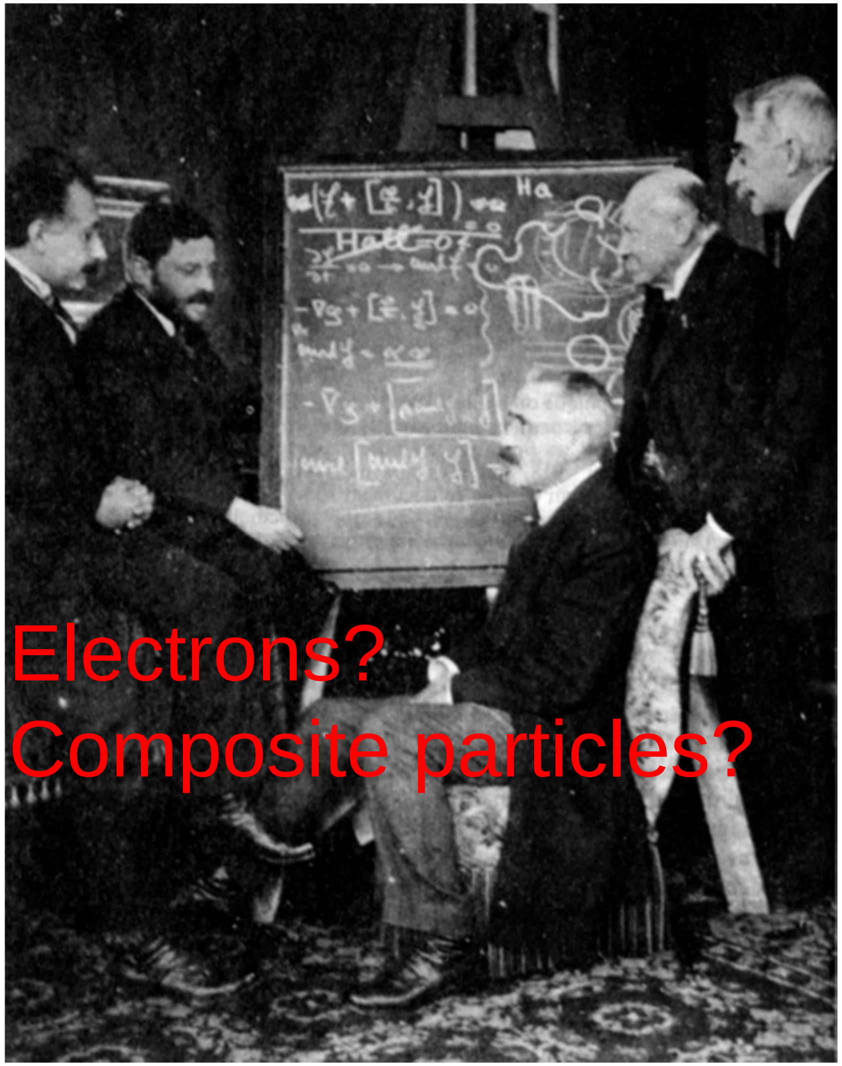
T

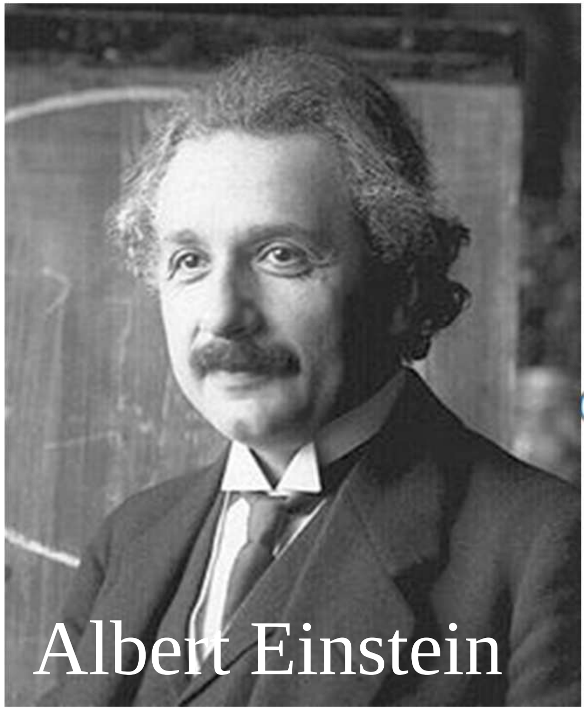
T_c = 4.2K

Liquid He (4.22K)



What is a superconductor made of?





arXiv:Physics/0510251 – (Thanks to Zlatko Tesanovich)

Theoretical remark on the superconductivity of metals*

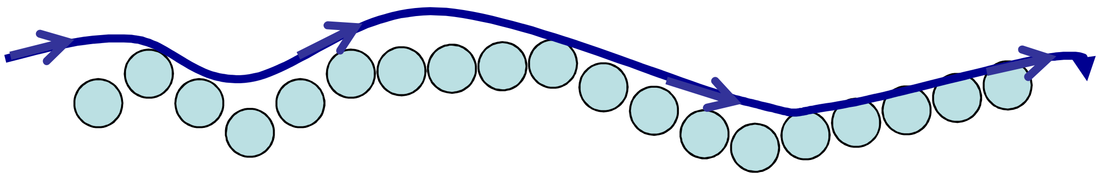
A. Einstein

Gedenkboek aangeb. aan H. Kamerlingh Onnes, eaz. Leiden, E. IJdo, 1922,

The theoretical oriented scientist cannot be envied, because nature, i.e. the experiment, is a relentless and not very friendly judge of his work. In the best case scenario it only says "maybe" to a theory, but never "yes" and in most cases "no". If an experiment agrees with theory it means "perhaps" for the latter. If it does not agree it means "no". Almost any theory will experience a "no" at one point in time - most theories very soon after they have been developed. In this paper we want to focus on the fate of theories concerning metallic conductivity and on the revolutionary influence which the discovery of superconductivity must have on our ideas of metallic conductivity.

Einstein’s ideas… (1922)

It seems unavoidable that superconducting currents are carried by closed chains of molecules (conduction chains) whose electrons endure ongoing cyclic exchanges. Therefore, Kamerlingh Onnes compares the closed currents in superconductors to Ampere’s molecular currents.

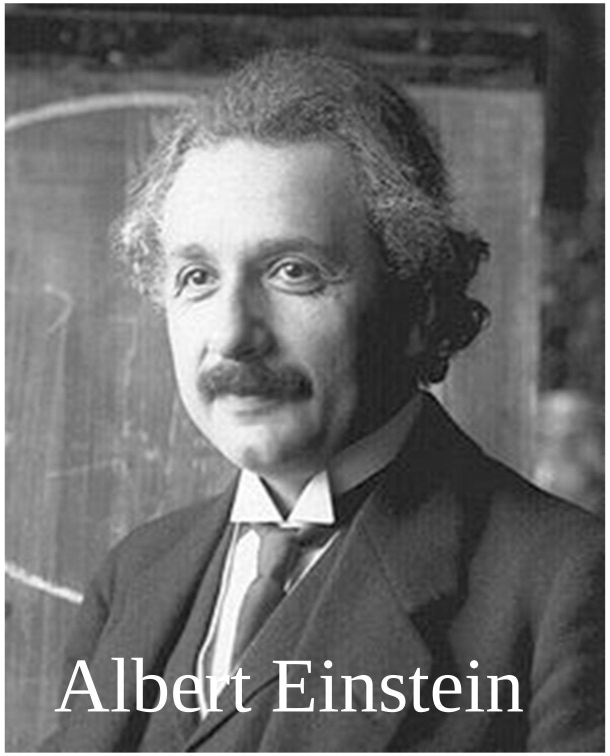


If this idea of elementary currents caused by quanta proves correct it will be evident that such chains can never contain different atoms.

P.S.

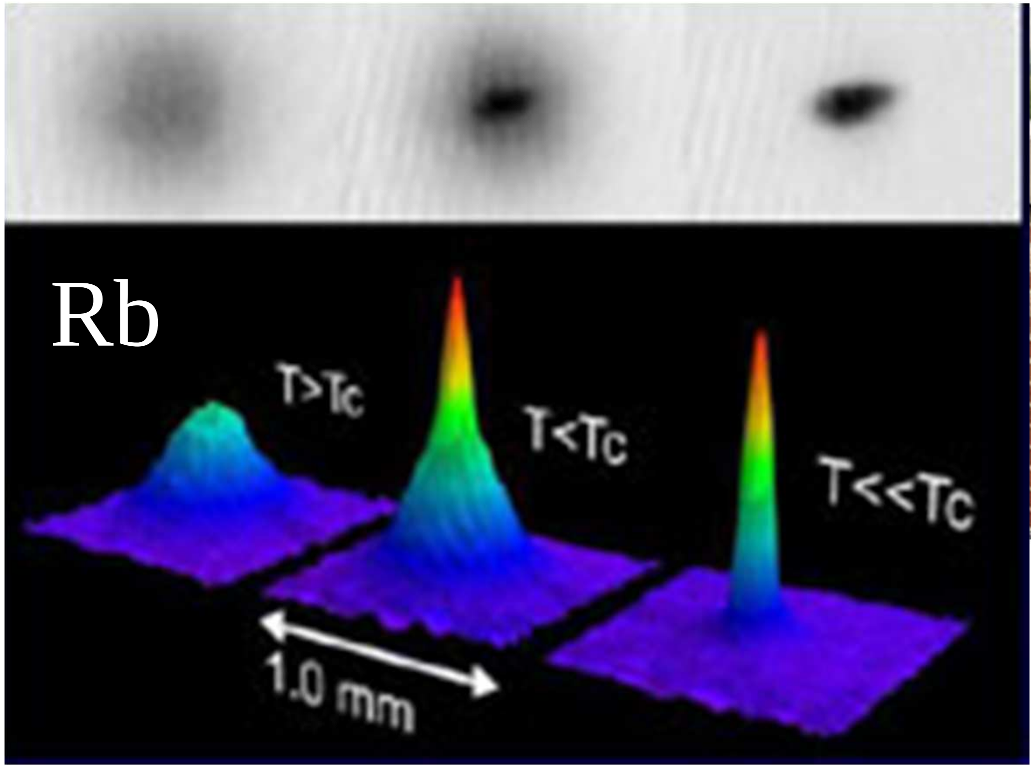
The last speculation (which by the way is not new[2]) is contradicted by an important experiment which was conducted by Kamerlingh Onnes in the last couple of months. He showed that at the interface between two superconductors (lead and tin) no measureable Ohm resistance appears.

Bose and Einstein, 1924...



At low temperatures, noninteracting bosons may condense into a single quantum wavefunction

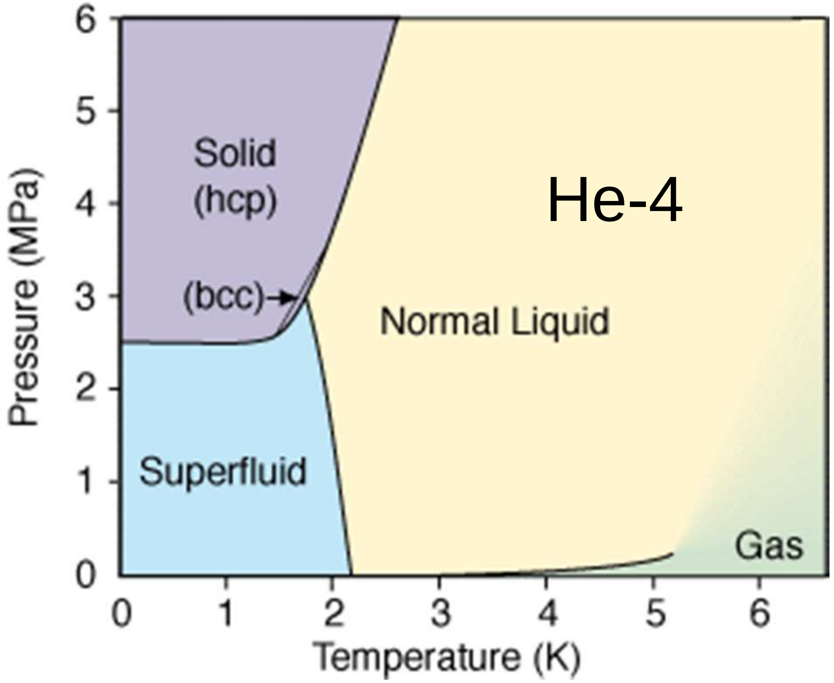
Discovery of Bose Condensation, 1995

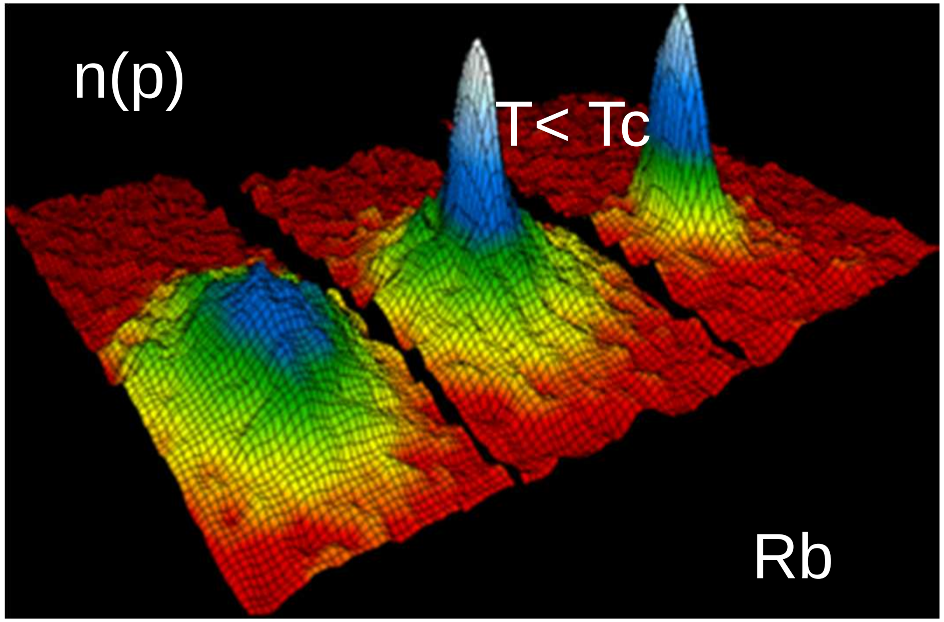


Weiman, Cornell, Ketterle
Nobel prize, 2001

Condensation at zero momentum

Superfluidity

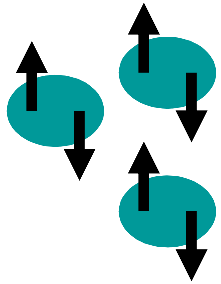




Order parameter $T < T_c$ $$\langle \psi^\dagger(x) \rangle = \sqrt{\rho_s(x)} e^{i\phi(x)}$$

Phase fluctuations $$H = E_0 + \frac{\hbar^2 \rho_s}{2m} \int d^3x (\nabla \phi)^2$$

Theory of Ogg pairs, 1945:
real-space “molecules” that underwent
Bose-Einstein condensation in Amonia solutions… (Ted Geballe)



Superconductivity of a Charged Boson Gas

M. R. SCHAFROTH

F. B. S. Falkiner Nuclear Research Laboratory, School of Physics, The University of Sydney, Sydney, Australia*

(Received September 7, 1954)

IT is the purpose of this note to point out that there exists a relatively simple physical system which exhibits the essential equilibrium features of a superconductor,[1] namely a phase transition of the second kind at a critical temperature T_c and the occurrence of a Meissner-Ochsenfeldt effect below it. This system is the ideal gas of charged bosons. The existence of a transition point is well known.[2]

One would then have to show that in a metal at low temperatures charge-carrying bosons occur, e.g., because of the interaction of electrons with lattice vibrations.[5]

Early demise of the BEC theory of superconductivity

Theoretical shortcomings:

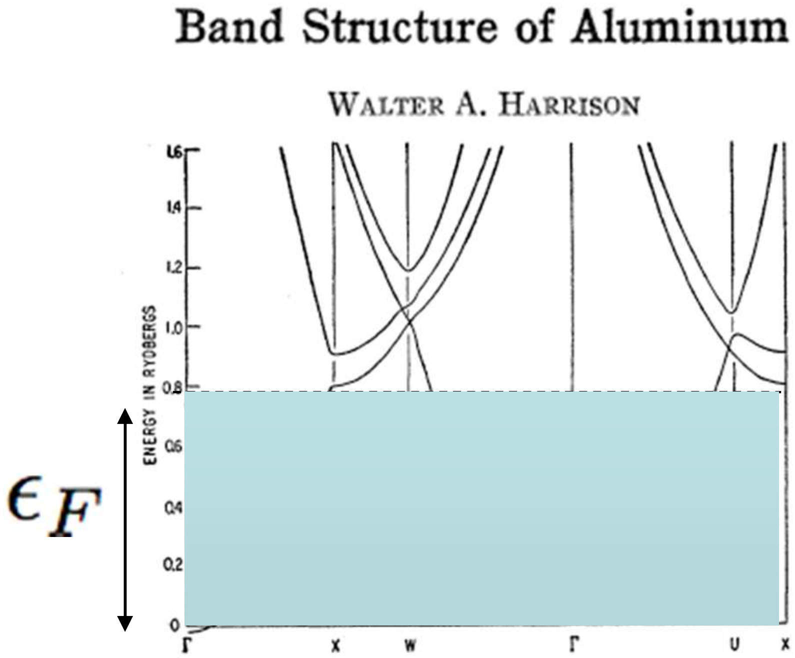
1. Poor microscopic understanding of pairing.
2. No determination of 'boson density' n_b...

Disagreement with experiments:

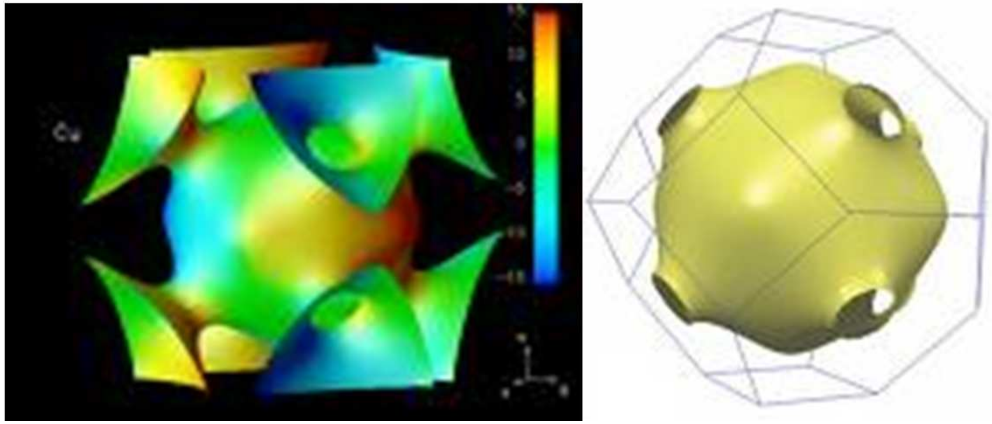
1. BEC cannot explain excitation gap of magnitude ~Tc.
2. BEC relations between order parameter, superfluid stiffness, and Tc are inconsistent with experiments.
3. BEC cannot explain normal phase above Tc, which is well described by Fermi liquid theory.

Then came BCS theory…

Theory of (good) Metals



Fermi surfaces



Fermi liquid theory of excitations

Boltzmann transport:

Fermi energy $\epsilon_F >> \frac{\hbar}{\tau}$ scattering rate

Bardeen Cooper Schrieffer,1957

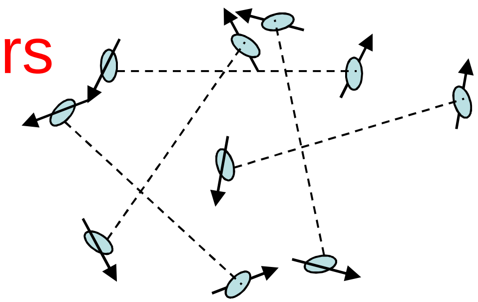
1972

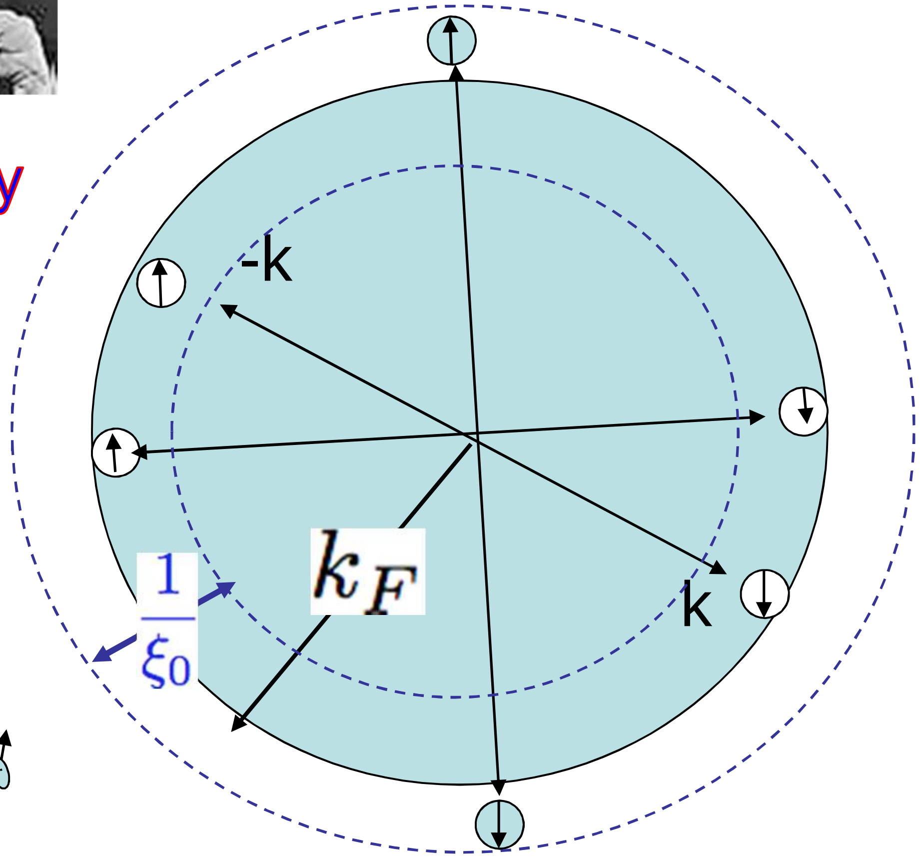
k-space pairing instability

$$\langle c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow} \rangle = \Delta$$

$$k_F \xi_0 \sim \epsilon_F / \Delta >> 1$$

large pairs



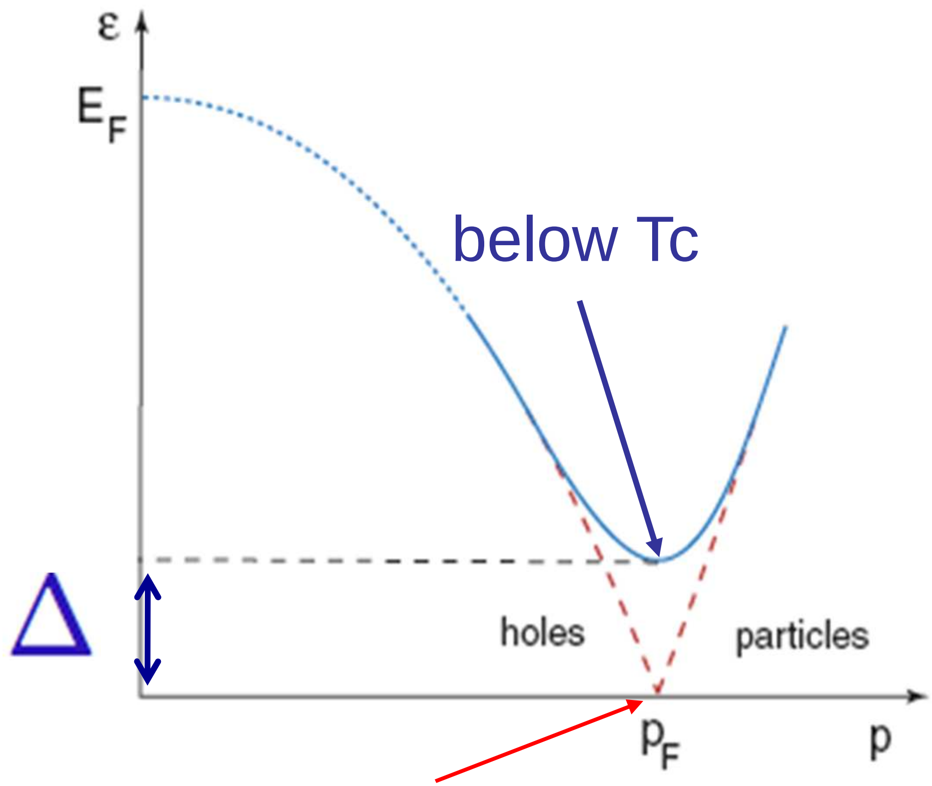


Schrieffer’s ballroom dance

BCS Gap

quasiparticle excitations

$$E_{\mathbf{k}} = \sqrt{(\epsilon_{\mathbf{k}} - \mu)^2 + \Delta^2}$$



$$\Delta << E_F$$

BCS vs BEC

order parameter excitation gap transition temperature

BCS theory: $V^{\mathrm{el-ph}}\langle c_\uparrow^\dagger c_\downarrow^\dagger\rangle \sim \Delta \sim T_c \qquad \rho_s \propto E_F$

supressed phase fluctuations

superfluid stiffness

BEC theory $\langle c_\uparrow^\dagger c_\downarrow^\dagger\rangle \propto T_c^{3/4} \propto \sqrt{\rho_s} \qquad \Delta = \infty$

order parameter 3D transition temperature

ignores pair breaking

MACROSCOPIC QUANTUM PHENOMENA FROM PAIRING IN SUPERCONDUCTORS

Nobel Lecture, December 11, 1972

by

J. R. SCHRIEFFER

A clue to the nature of the states Φ_n entering strongly in ψ_0 is given by combining Pippard's coherence length ξ with Heisenberg's uncertainty principle

$$\Delta p \sim \hbar/\xi \sim 10^{-4} p_F \tag{4}$$

where p_F is the Fermi momentum. Thus, Ψ_0 is made up of states with quasi-particles (electrons) being excited above the normal ground state by a momentum of order Δp. Since electrons can only be excited to states which are initially empty, it is plausible that only electronic states within a momentum $10^{-4} p_F$ of the Fermi surface are involved significantly in the condensation, i.e., about 10^{-4} of the electrons are significantly affected. This view fits nicely with the fact that the condensation energy is observed to be of order 10^{-4}. $k_B T_c$. Thus, electrons within an energy $\sim v_F \Delta p \simeq kT_c$ of the Fermi surface have their energies lowered by of order kT_c in the condensation. In summary, the problem was how to account for the phase transition in which a condensation of electrons occurs in momentum space for electrons very near the Fermi surface. A proper theory should automatically account for the perfect conduc-

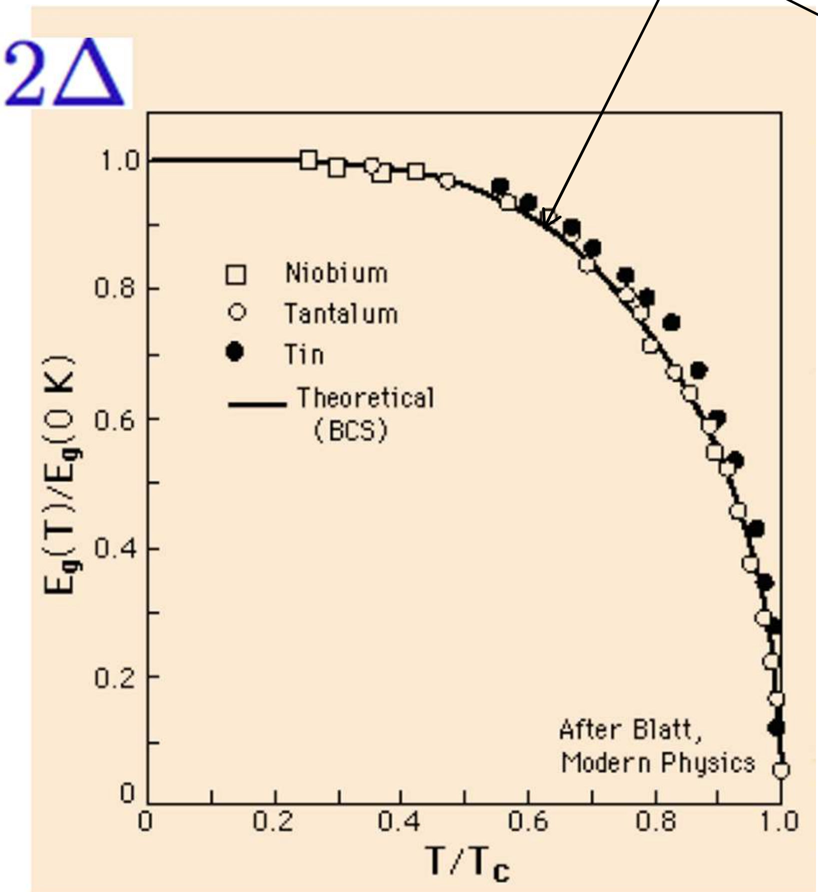
Schrieffer continued...

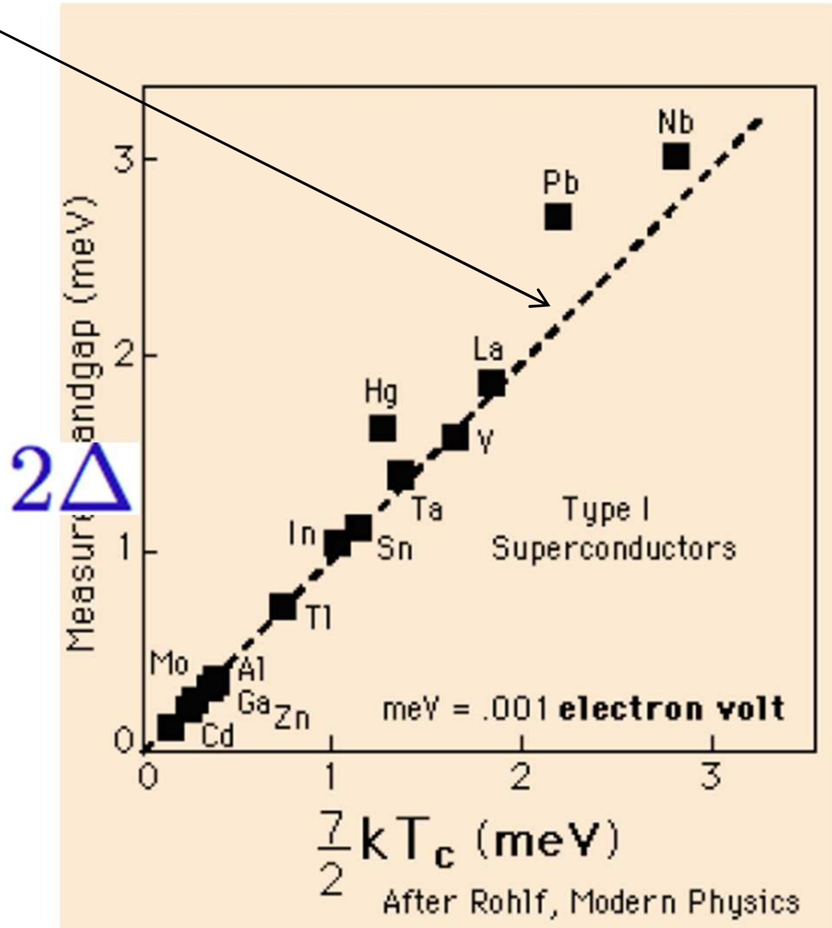
The idea that electron pairs were somehow important in superconductivity has been considered for some time (16, 17). Since the superfluidity of liquid He^4 is qualitatively accounted for by Bose condensation, and since pairs of electrons behave in some respects as a boson, the idea is attractive. The essential point is that while a dilute gas of tightly bound pairs of electrons might behave like a Bose gas (18) this is not the case when the mean spacing between pairs is very small compared to the size of a given pair. In this case the inner structure of the pair, i.e., the fact that it is made of fermions, is essential; it is this which distinguishes the pairing condensation, with its energy gap for single pair translation as well as dissociation, from the spectrum of a Bose condensate, in which the low energy exictations are Bose-like rather than Fermi-like as occurs in acutal superconductors. As London emphasized, the condensation is an ordering in occupying momentum space, and not a space-like condensation of clusters which then undergo Bose condensation.

In summary:
BCS Superconductivity = large Cooper pairs,
→ very little phase fluctuations

Early success of BCS theory

Predictions of BCS gap equation





However: superconductivity is more general than that described by BCS....

Phil Anderson:

1. R=0

2. $$\mathbf{j} = -\frac{e^2 \rho_s}{\hbar c}\mathbf{A}$$

London Eq.

3. No need for a gap...

conditions:

1. Spontaneously broken gauge symmetry
2. Wave function rigidity

Superconductors and superfluids

New classes of superconductors

The “glue” mechanism

Transition temperatures of well-known superconductors (Boiling point of liquid nitrogen for comparison)

Transition Temperature (in Kelvin)	Material	Class
138	$Hg_{12}Tl_3Ba_{30}Ca_{30}Cu_{45}O_{127}$	Copper-oxide superconductors
110	$Bi_2Sr_2Ca_2Cu_3O_{10}$(BSCCO)	
92	$YBa_2Cu_3O_7$ (YBCO)	
77	Boiling point of liquid nitrogen	
18	Nb_3Sn	Metallic low-temperature supercondu
10	NbTi	
4.2	Hg (Mercury)	

electron-electron (?)

el-el + el-ph (?)

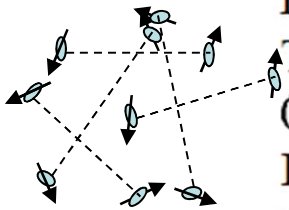
electron-phonon

What is the condensation mechanism?

Classification by Coherence Lengths

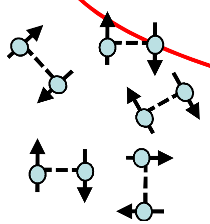
ξ = vortex core radius > Cooper pair size $\quad k_F \xi_{BCS} \sim \dfrac{\epsilon_F}{2\pi\Delta} >> 1$

Guy Deutscher & Bok 1993



	T_c (K)	ξ (μm)
Aluminium (1)	1.19	1.20
Indium (1)	3.40	0.33
Tin (1)	3.72	0.26
Callium (1)	5.90	0.16
Lead (1)	7.20	0.080
Niobium (1)	9.25	0.035
$PbMoS_8$ (2)	15	0.0025
Nb_3Sn (1)	17	0.0040
$C_{60}K_3$ (3)	19	0.0030
$C_{60}Rb_3$ (3)	31	0.0023
$Pr_{.4}Y_{.6}Ba_2Cu_3O_7$ (4)	40	0.007
$YBa_2Cu_3O_7$ (1)	93	0.0015
$BaFe_{1.8}Co_{0.2}As_2$ Yi Yin et. al. 2009		0.0027

BCS regime



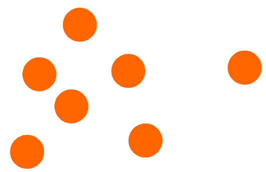
tightly bound pairs – interacting bosons

Small Cooper pairs Versus Schafroth bosons

Schafroth:

‘electron pairs are weakly interacting bosons’—theory

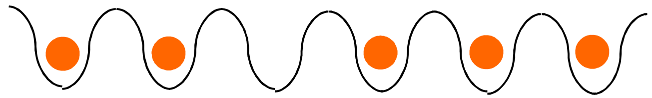
didn’t work….



However:

Small Cooper pairs are hard core Lattice Bosons

1. Underlying lattice periodicity:

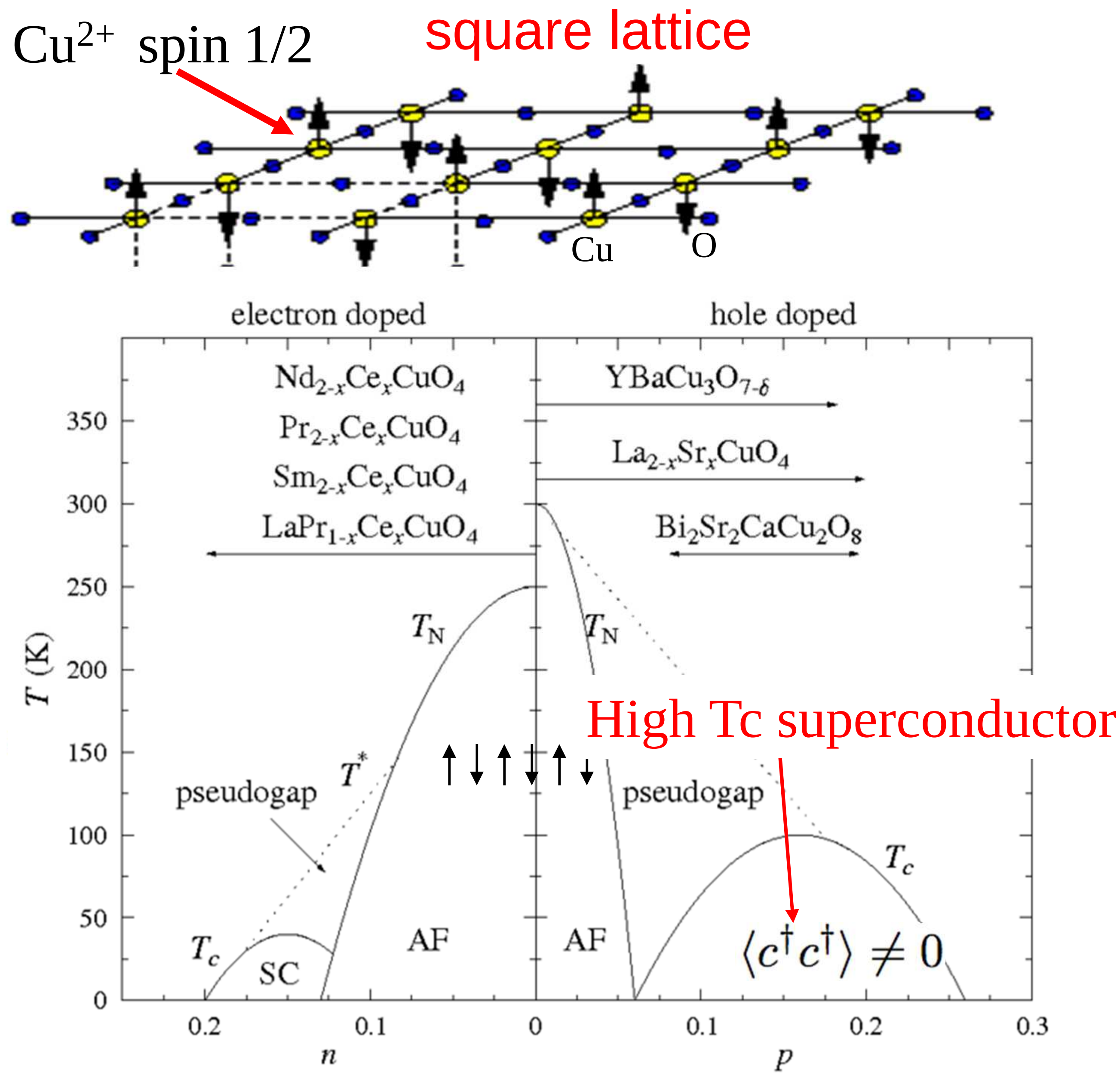


2. Hard Core constraints $(b_i^\dagger)^2 = (c_{i\uparrow}^\dagger c_{i\downarrow}^\dagger)^2 = 0$

→ Mott phases, current is scattered by lattice potential,

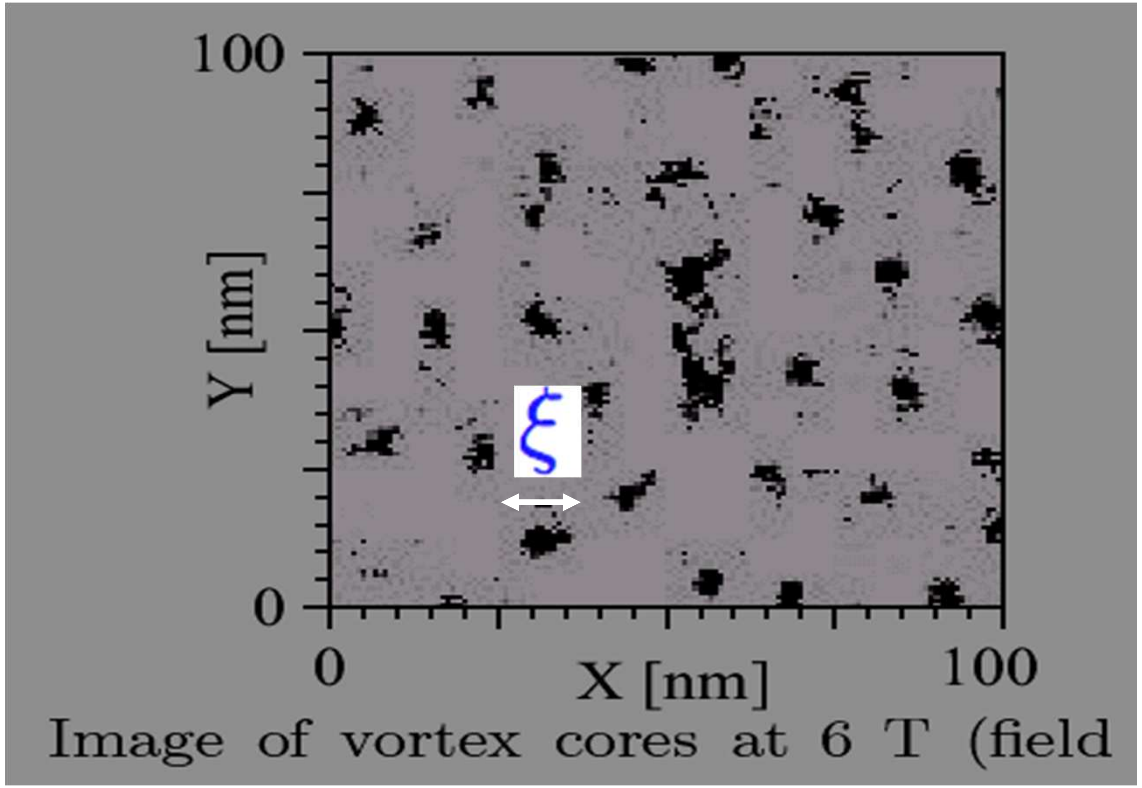
Lattice induced Berry phases, modified vortex dynamics

High Tc Cuprates, 1987



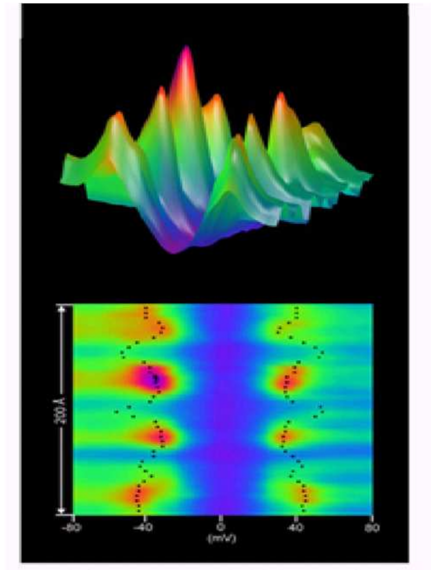
Cuprates: Coherence Length ξ

Hoogenboom et.al



Howald et.al (Stanford 00')

Pan et.al (Berkeley)



30 A

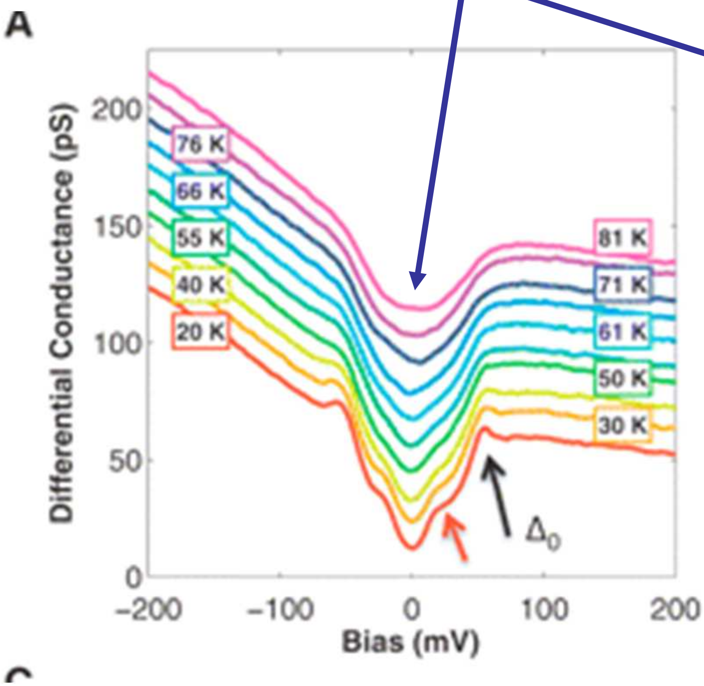
$$\xi \approx 20\ \mathring{A} \approx 3a$$ **small pairs**

$$H_{c2} \equiv \frac{hc}{2e\xi_0^2} \sim 100T$$

The “Pseudogap Problem”

Density of states suppression in the normal phase Loeser et. al ‘96

Gomes et.al, STM



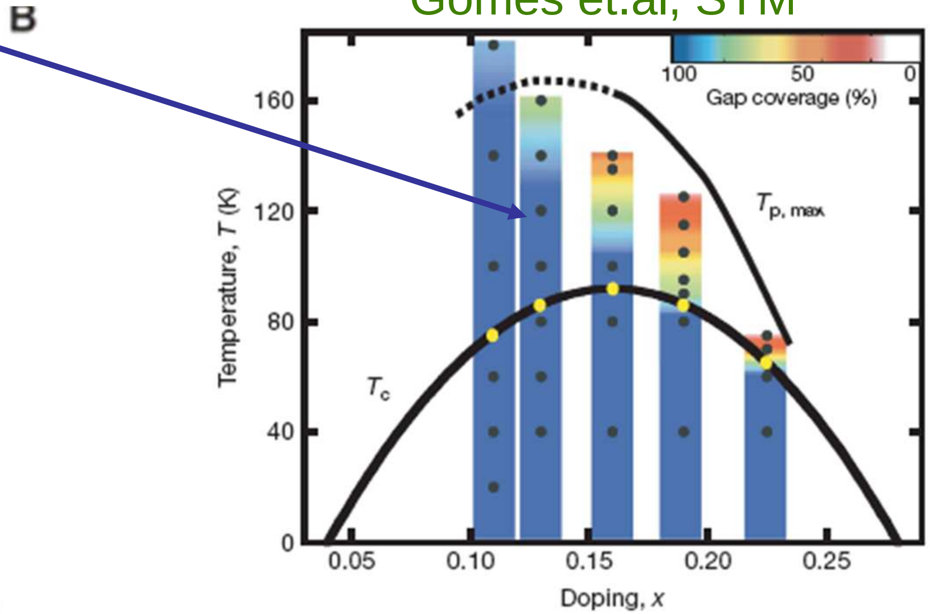
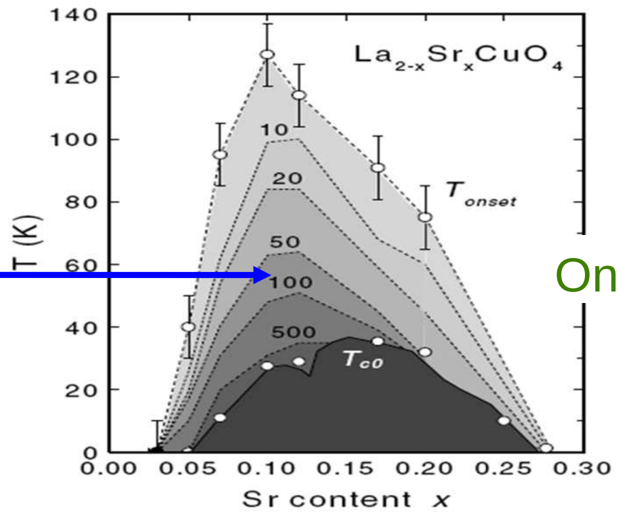


Figure 5 | Schematic phase diagram for $Bi_2Sr_2CaCu_2O_{8+\delta}$. Temperatures



Diamagnetism and Nernst signal:
short-range phase correlations
above Tc

Ong, et. al.

Cooper pairs (bosons) above Tc?

Cuprates Boson – Fermion phenomenology

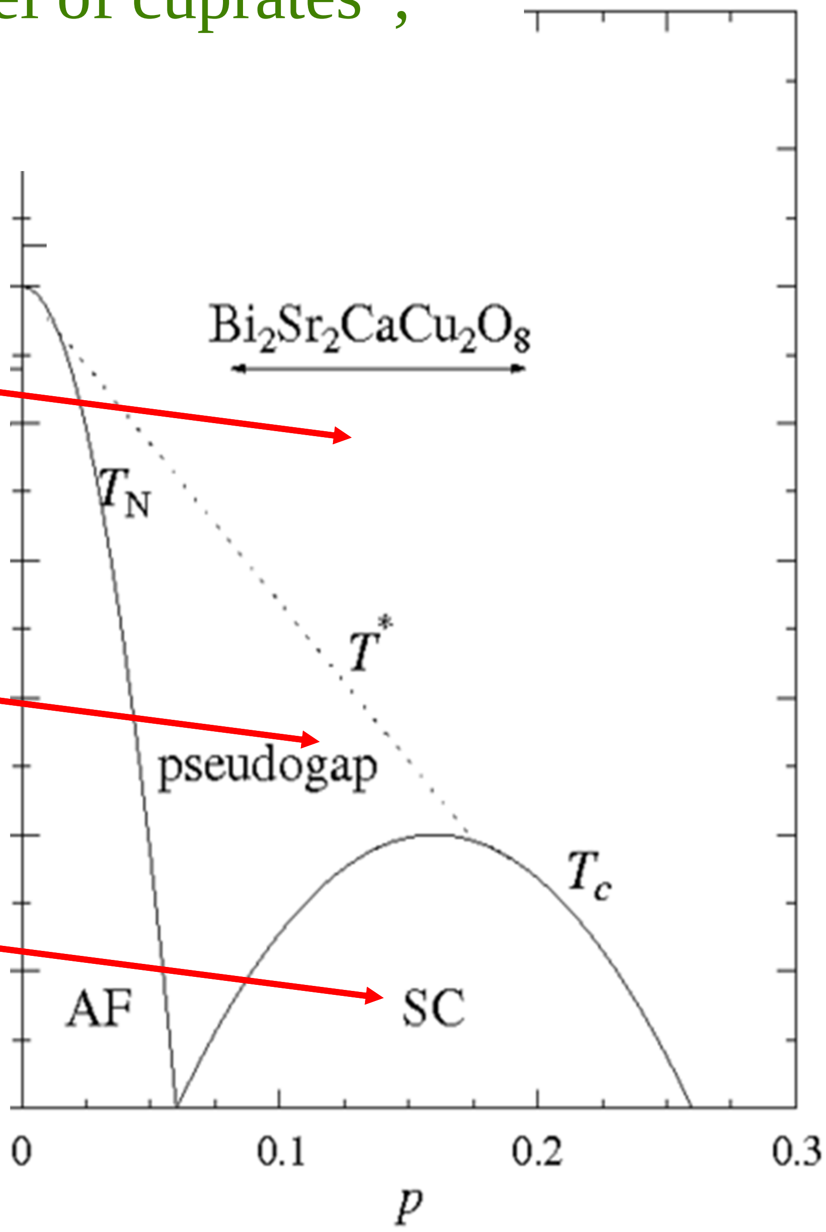
"Plaquette Boson-Fermion model of cuprates",

Altman & AA, PRB (2002)

Unpaired electrons

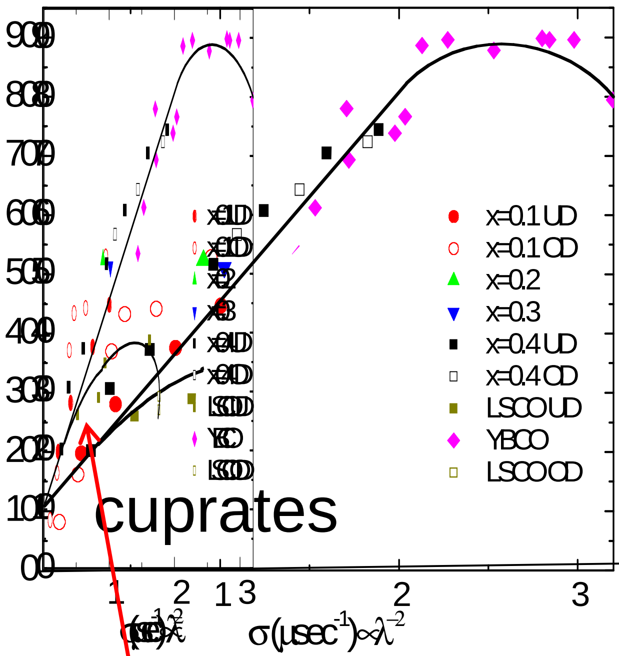
uncondensed boson liquid
+ nodal fermions

Condensed hole-pair bosons



The “Superfluid Stiffness problem”

Uemura’s scaling



T_c

cuprates

BCS $\rho_s \sim E_{Fermi}$

Al, Pb,Nb

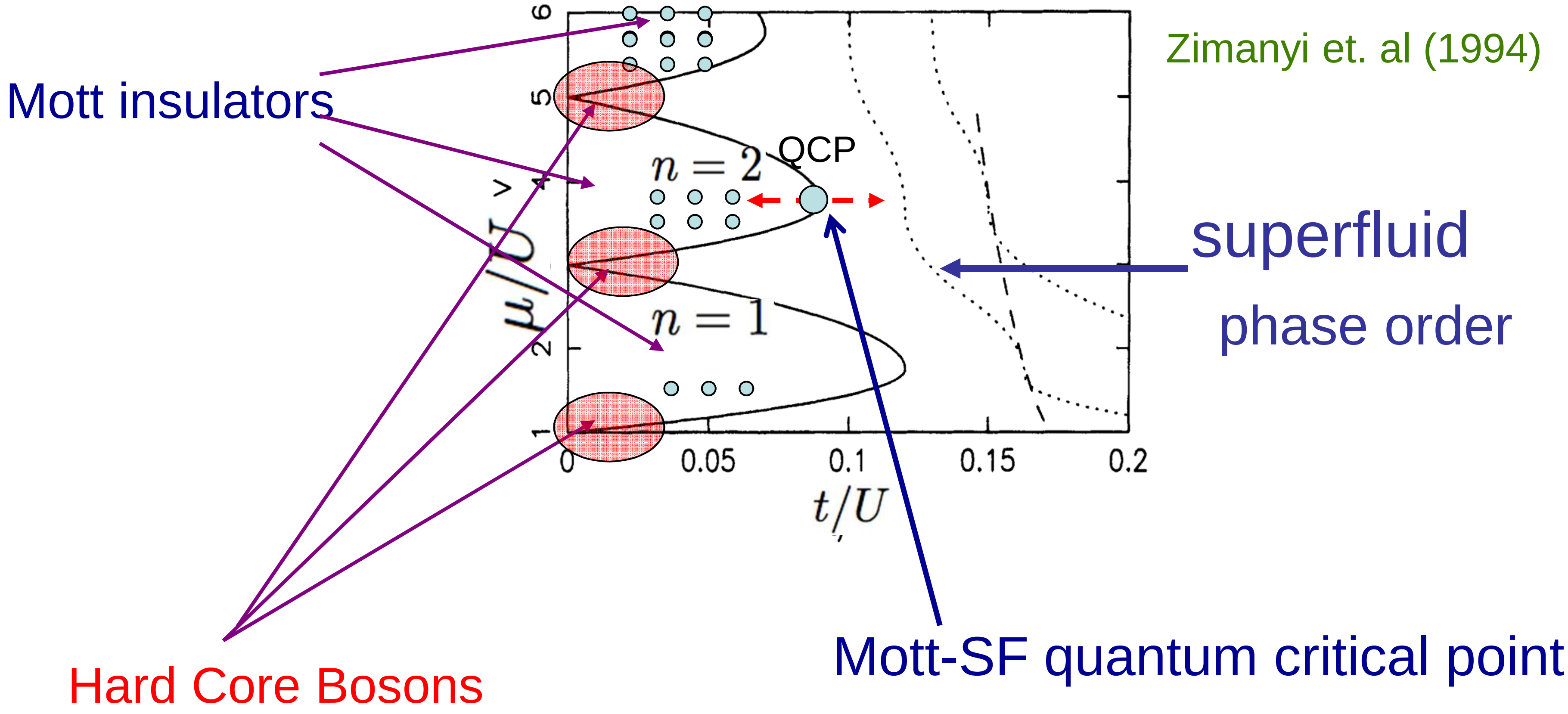
ρ_s

$\rho_s \propto n_{holes}$

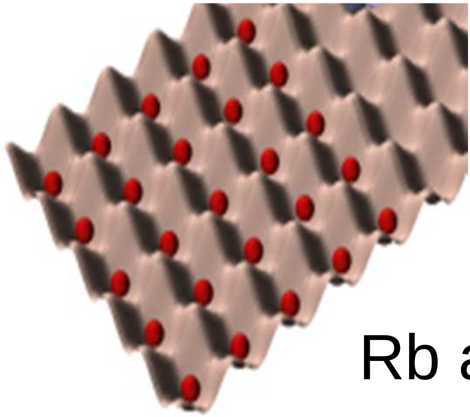
Non-BCS, Bosonic relation

Phases of lattice bosons

$$\mathcal{H} = -t\sum_{ij} a_i^\dagger a_j + U\sum n_i{}^2 - \mu\sum_i n_i$$

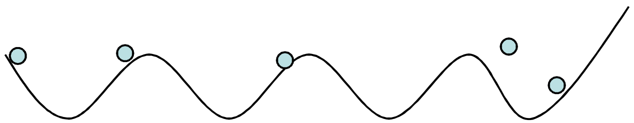


Mott-SF transition

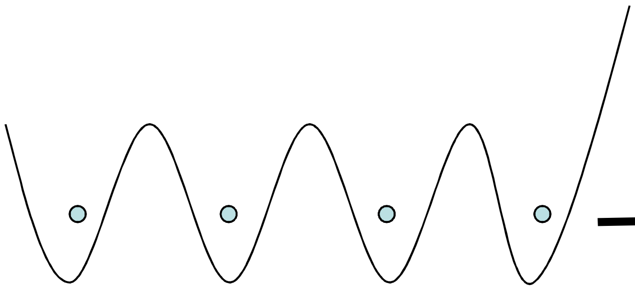


Bloch
Ketterle
Kasevich

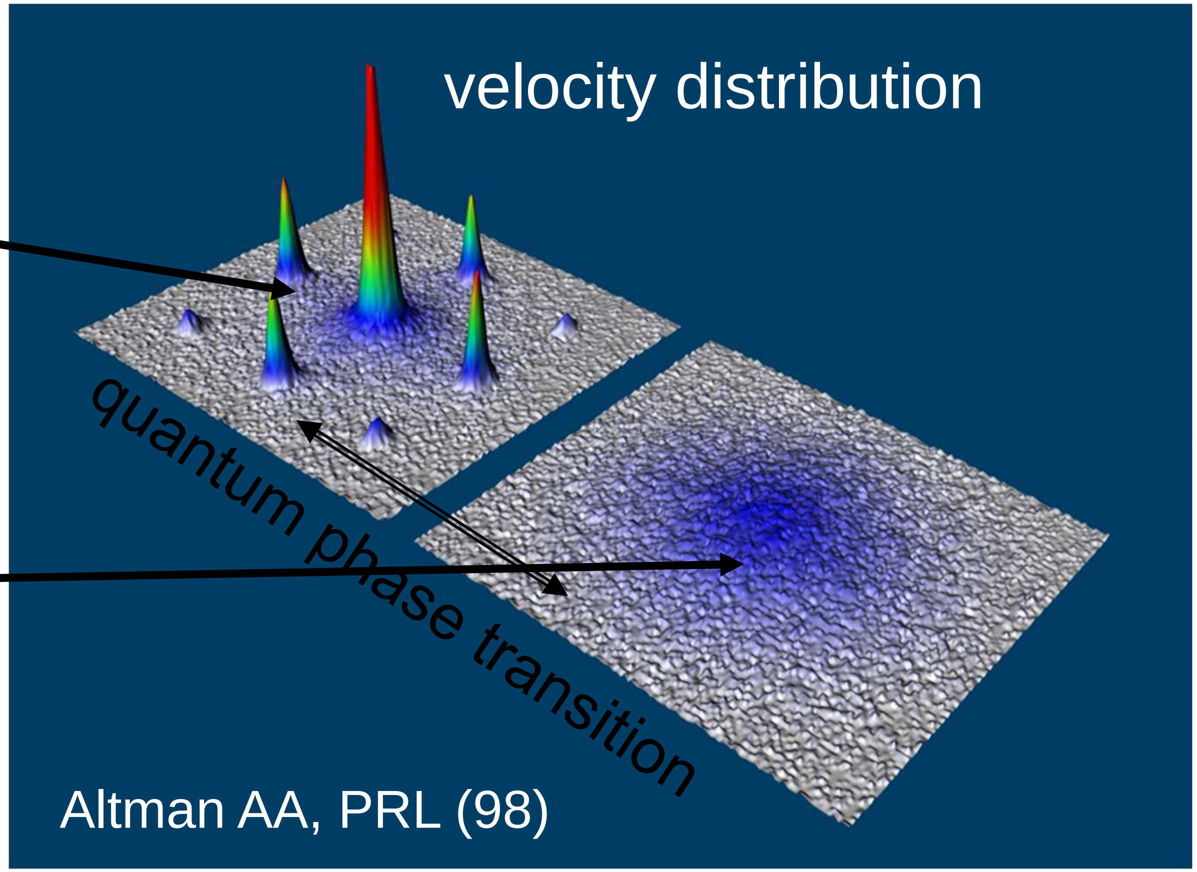
Rb atoms in optical lattices



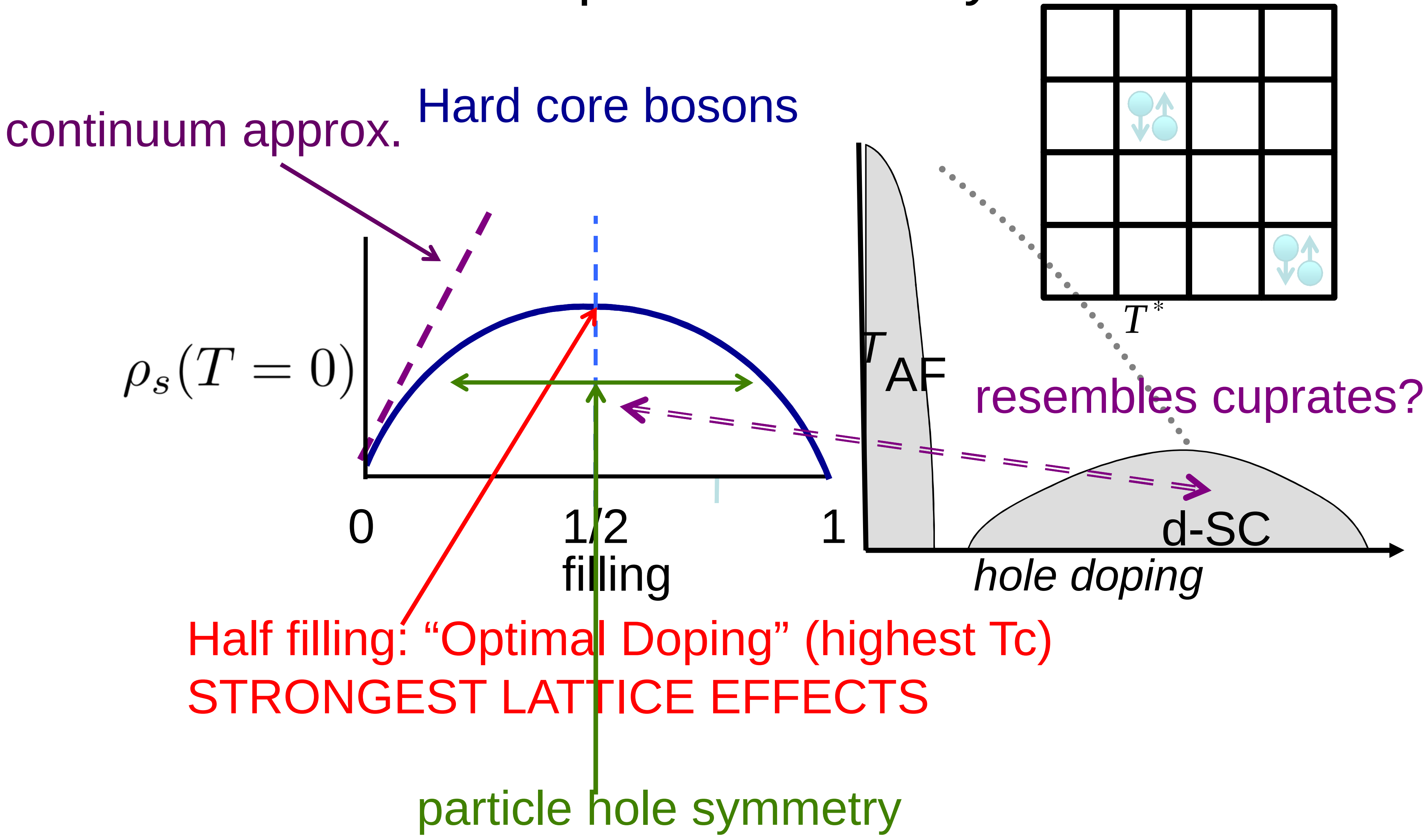
superfluid



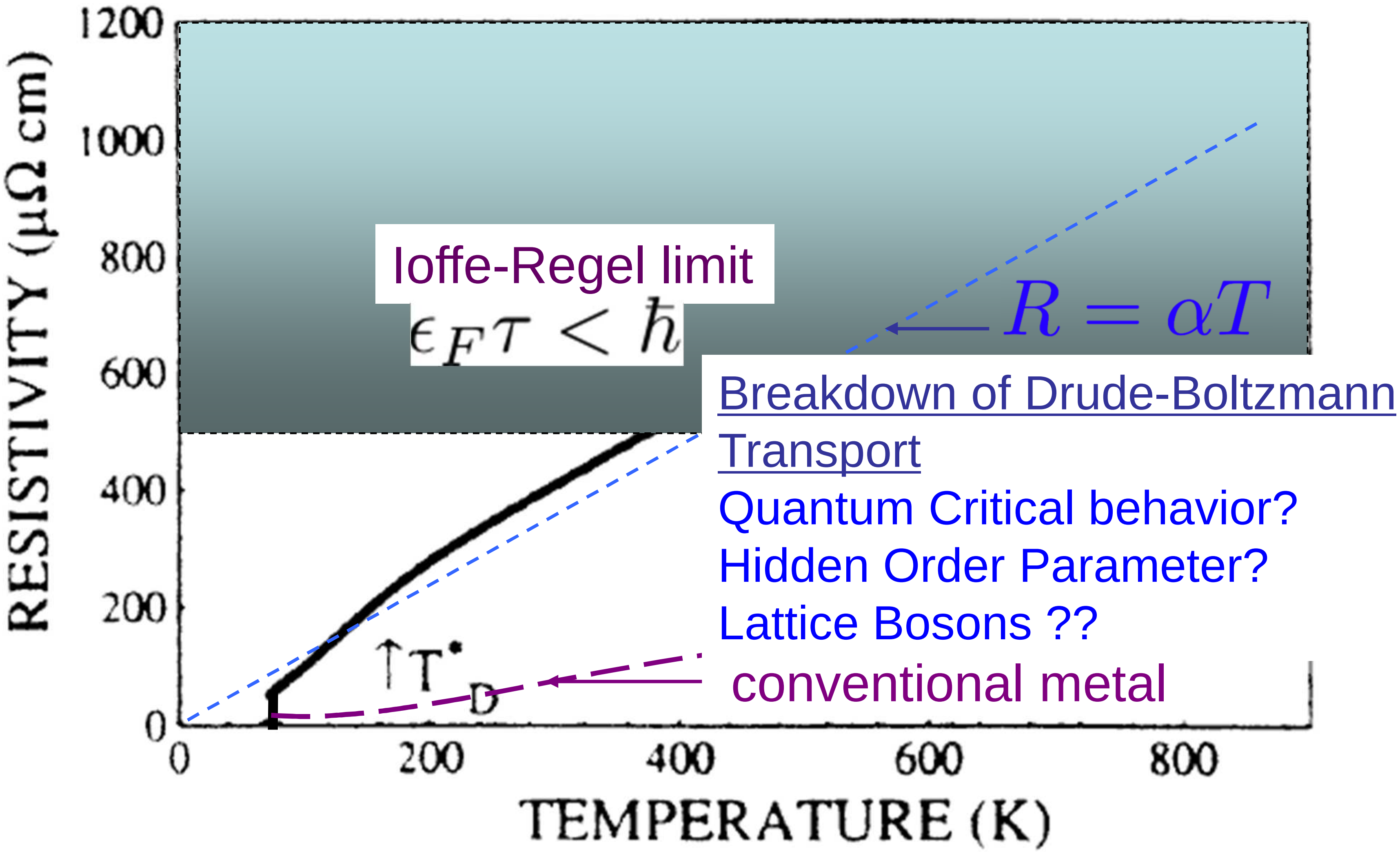
Mott insulator



Reentrant Superfluid Density



The “Linear Resistivity Problem”



Emery & Kivelson `Bad Metal’ behavior

“Homes Law”

A universal scaling relation in high-temperature superconductors

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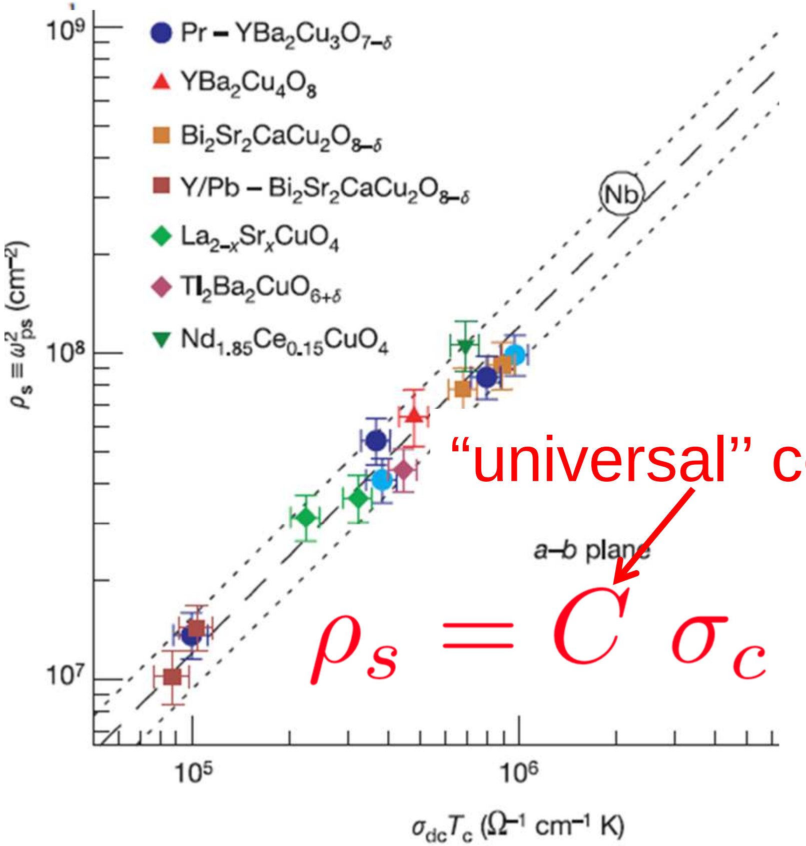
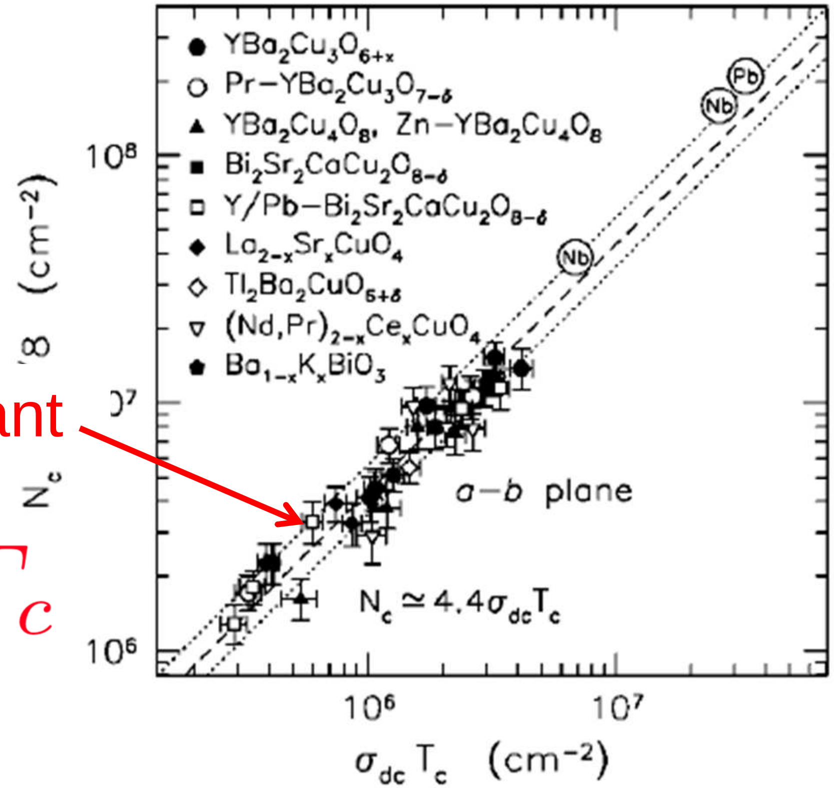


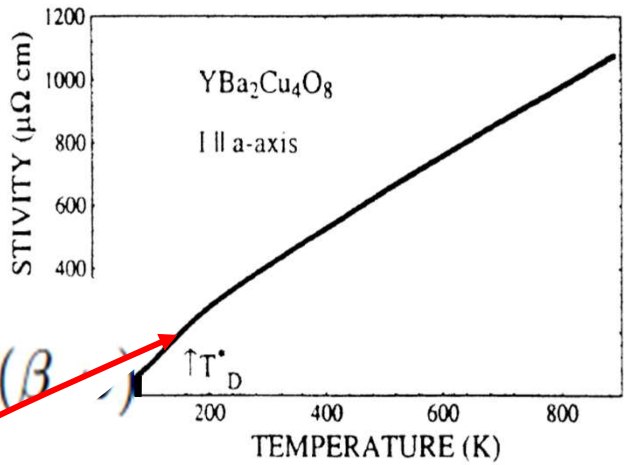
“universal” constant

$$\rho_s = C\ \sigma_c \times T_c$$



Conductivity of hard core bosons

Lindner & AA, PRB (2010)

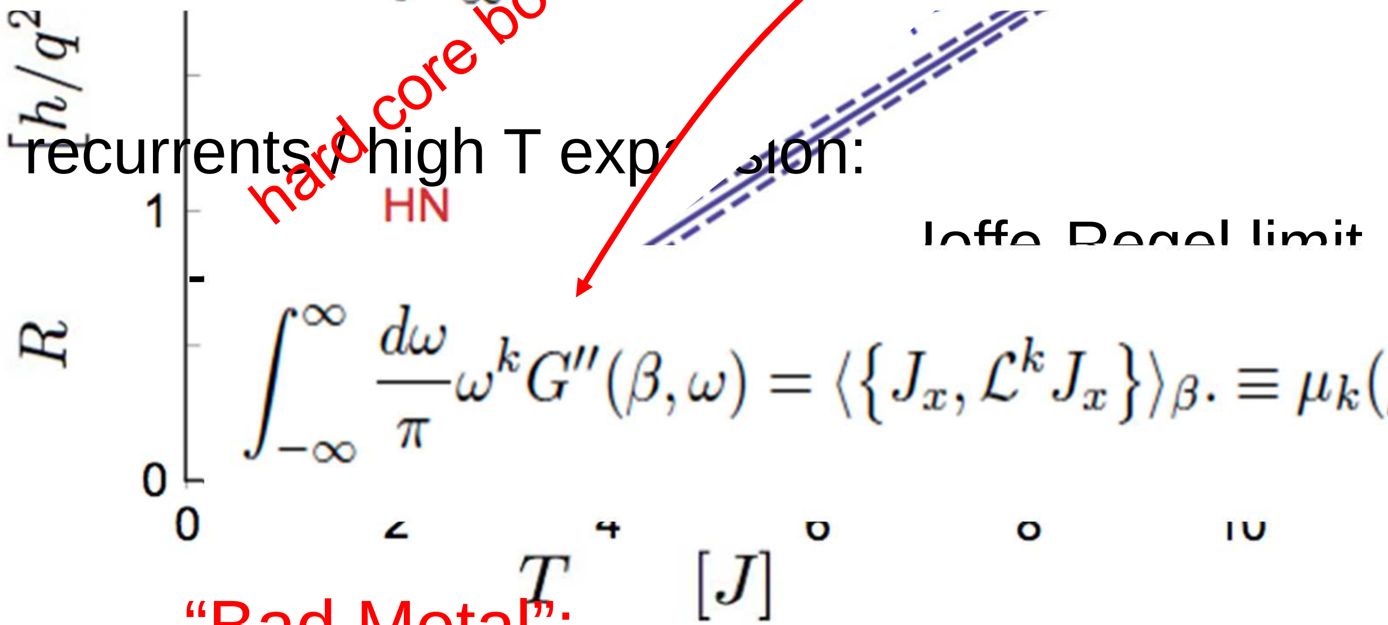


hard core bosons contribution ?

$$\sigma(\beta,\omega) \;=\; q^2\pi\rho_s(\beta)\delta(\omega) + \frac{\tanh(\beta\omega/2)}{\omega}G''(\beta,\omega)$$

$$G''(\beta,\omega) \;=\; \frac{1}{2}\int_{-\infty}^{\infty} dt e^{-i\omega t}\langle\{J_x(t),J_x\}\rangle_\beta.$$

recurrents/high T exp... sion:



...ry

$$\int_{-\infty}^{\infty}\frac{d\omega}{\pi}\omega^k G''(\beta,\omega) = \langle\{J_x,\mathcal{L}^k J_x\}\rangle_\beta. \equiv \mu_k(\beta)$$

“Bad Metal”:

linear increase, no resitivity saturation

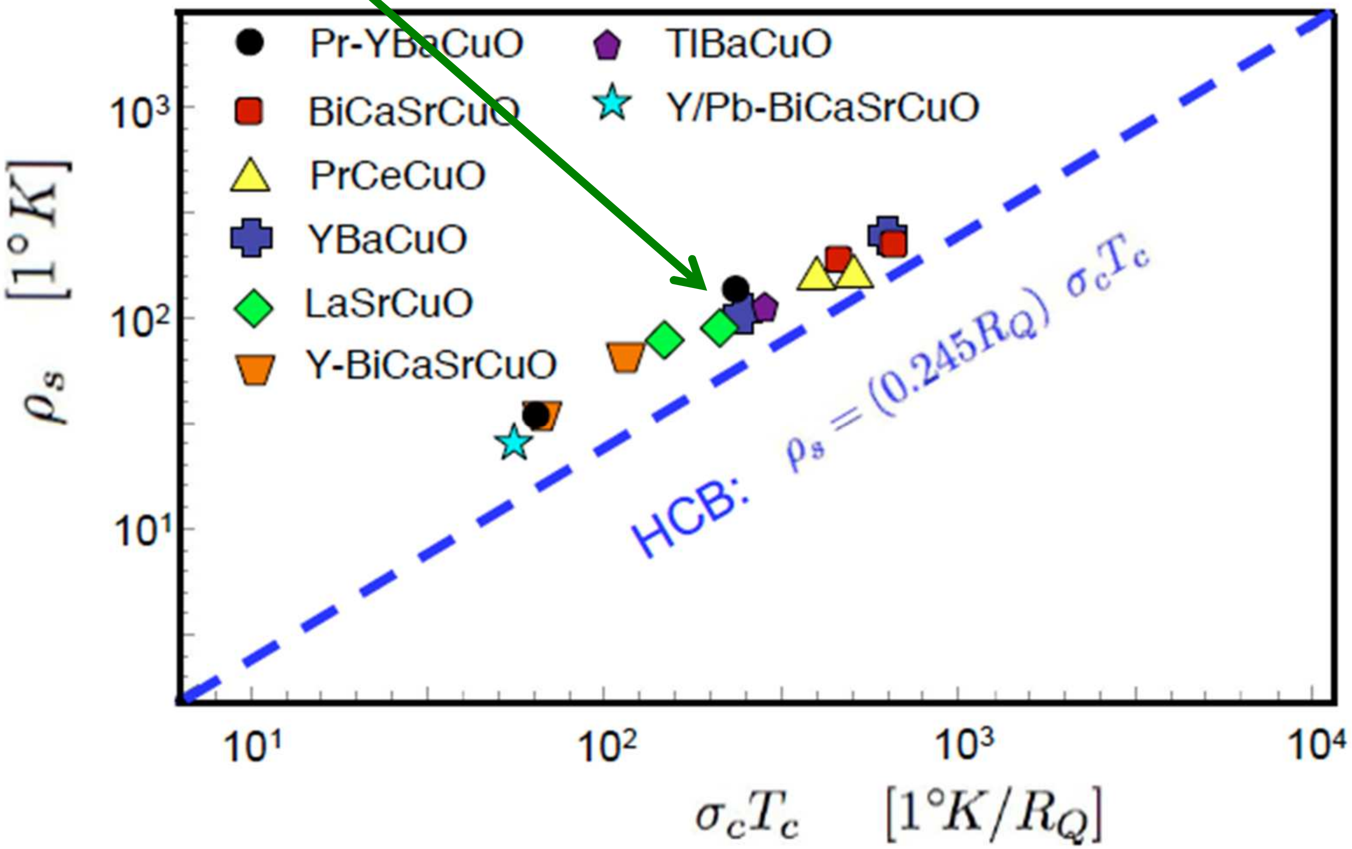
“Homes law” of HCB

Lindner & AA, PRB (2010)

$$\rho_s = 0.245\,\frac{h}{4e^2}\left(\frac{dR}{dT}\right)^{-1}$$

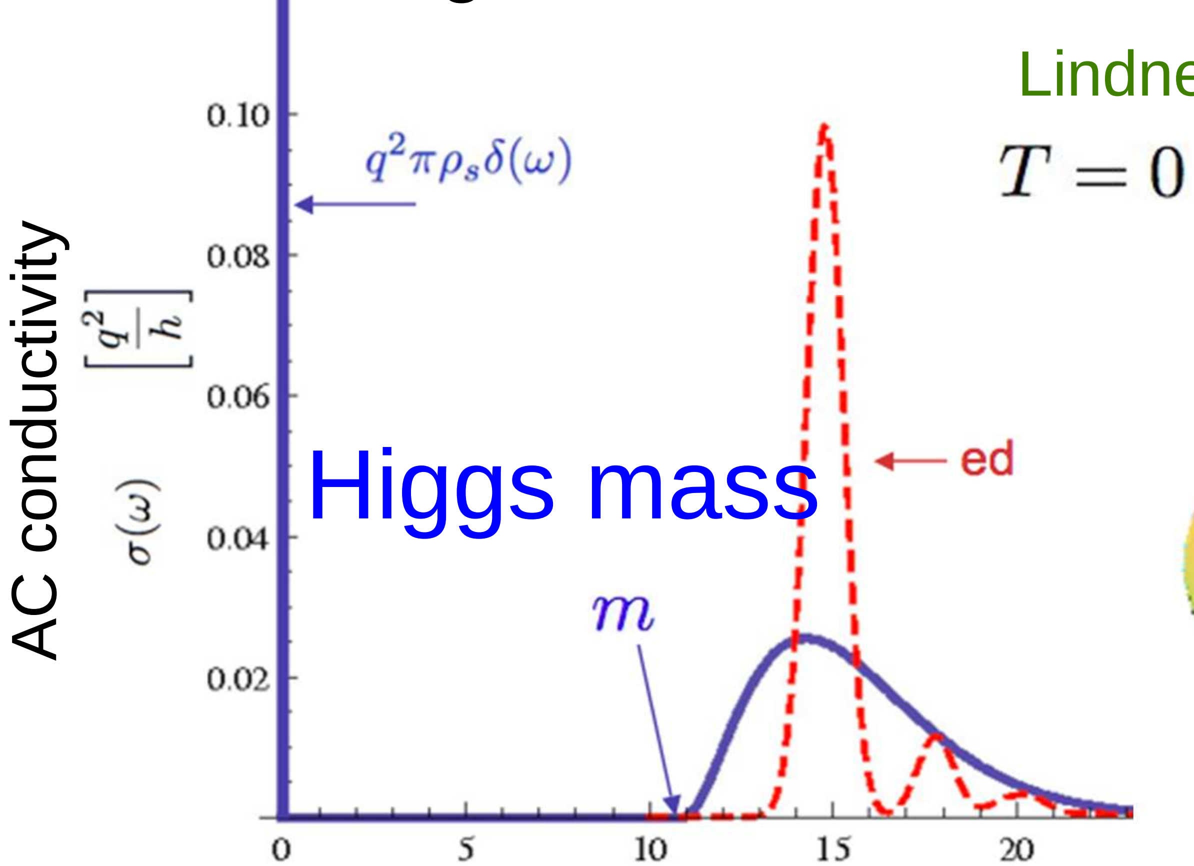
Boson quantum of resistance

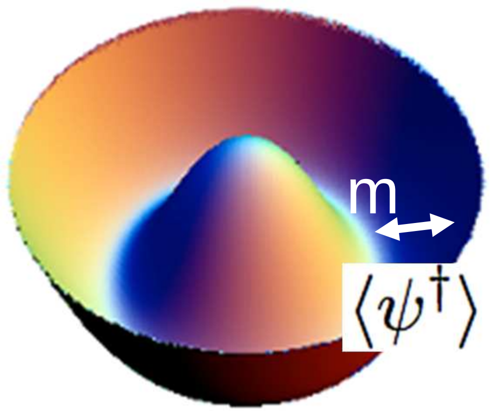
Data: Homes et. al.



Magnitude mode oscillations

Lindner, AA, PRB (2010)





Analogues: (w Daniel Podolsky)
Oscillating coherence near Mott phase of optical lattices
Magnitude mode in 1-D CDW's
2-magnon Raman peaks in O(3) antiferromagnets

Hall Conductance of Hard Core Bosons

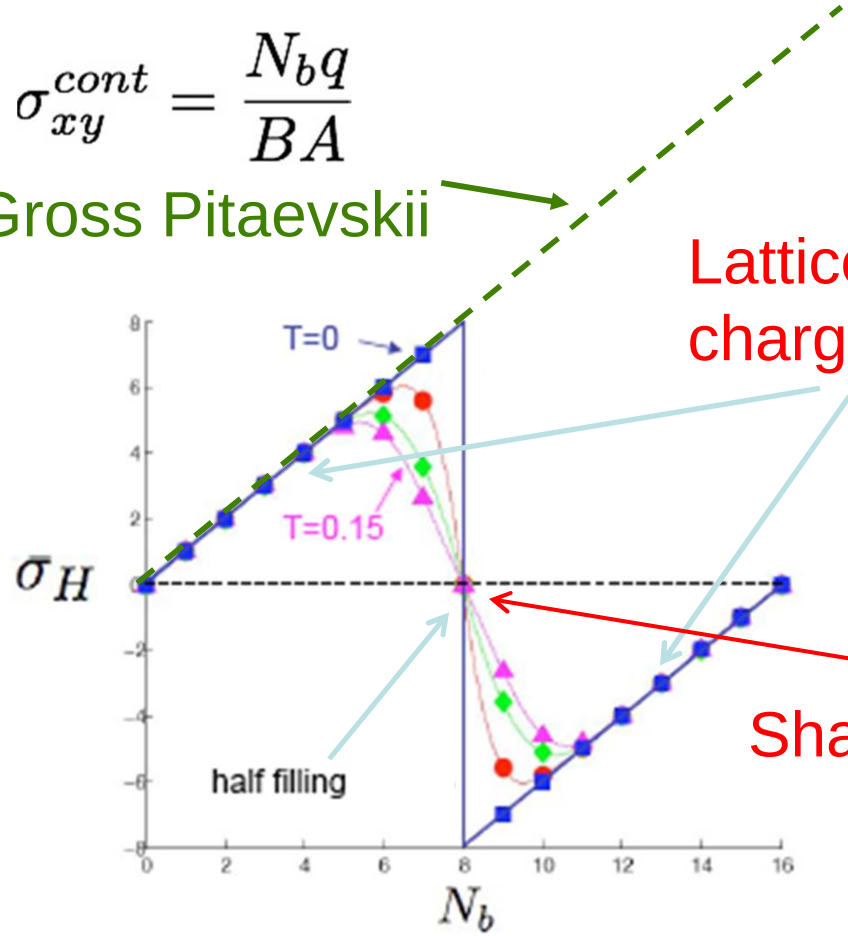
Thermally averaged Chern numbers

$$\sigma_{\mathrm{H}}(n_b,T) = \frac{1}{4\pi}\sum_{n=0}^{\infty}\int_0^{2\pi}\int_0^{2\pi} d^2\Theta \; \frac{e^{-E_n/T}}{Z} \; \mathrm{Im}\left\langle \frac{\partial\psi_n}{\partial\Theta_x}\middle|\frac{\partial\psi_n}{\partial\Theta_y}\right\rangle$$

$$\sigma_{xy}^{cont} = \frac{N_b q}{BA}$$

Gross Pitaevskii

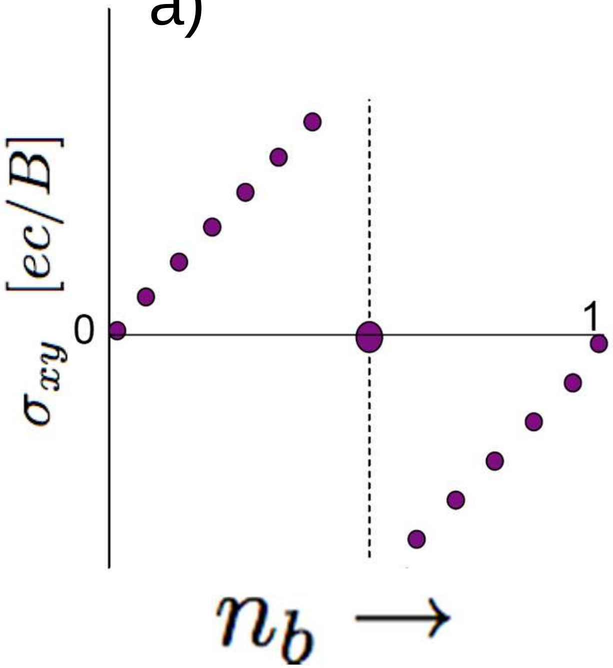
Lattice induced
charge conjugation antisymmetry



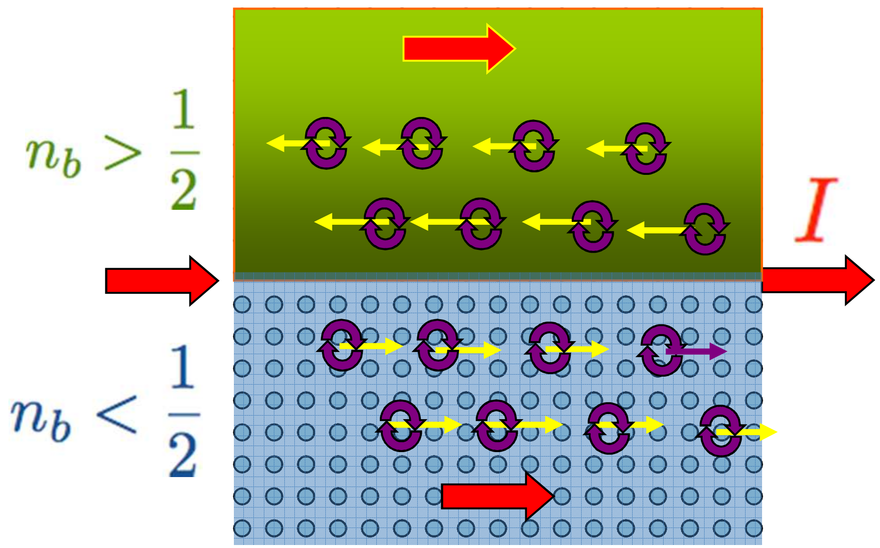
Sharp sign change at half filling

Drift direction reversal: proposed cold atoms experiment

a)

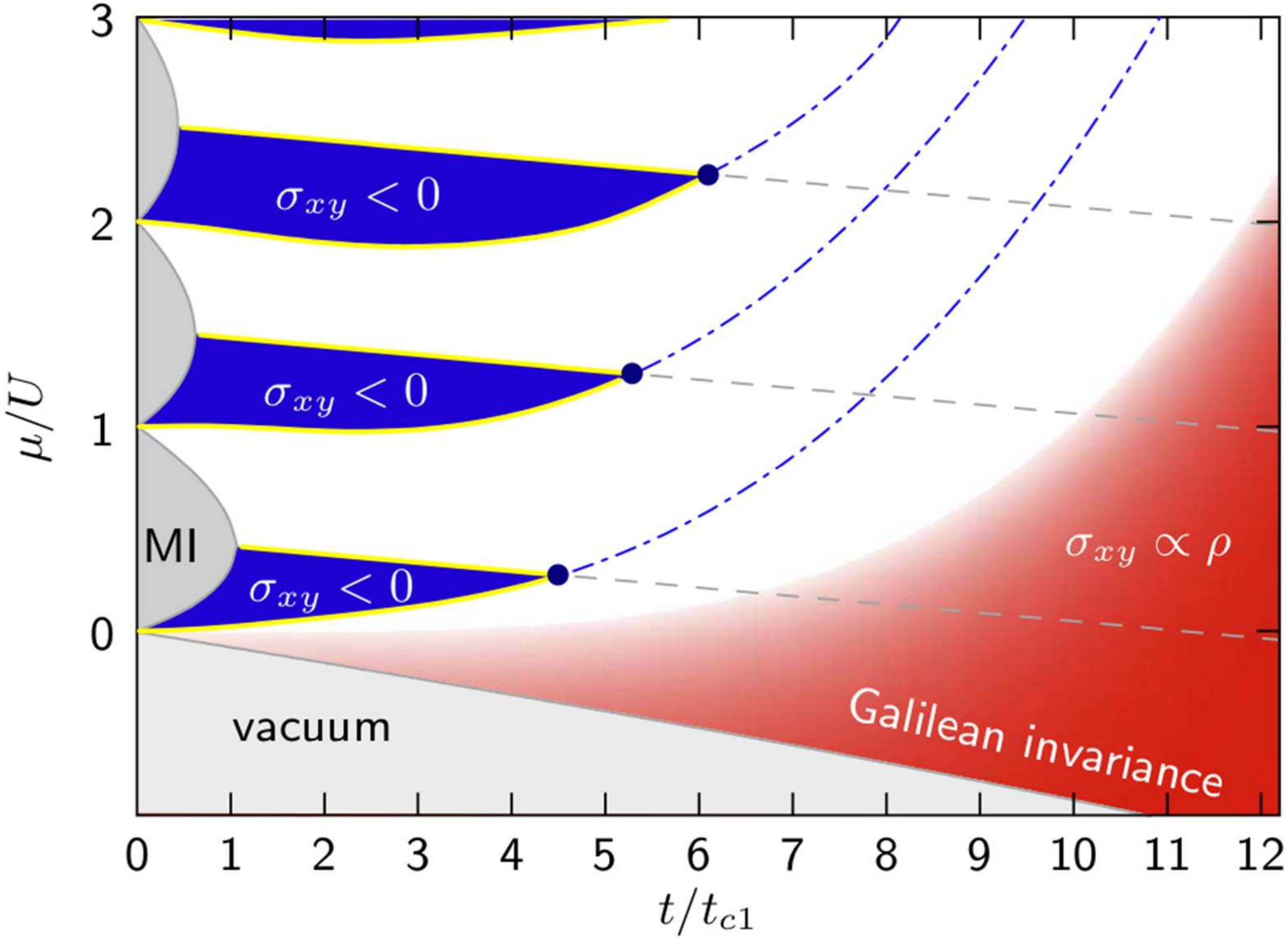


b)



Hall Coefficient Map

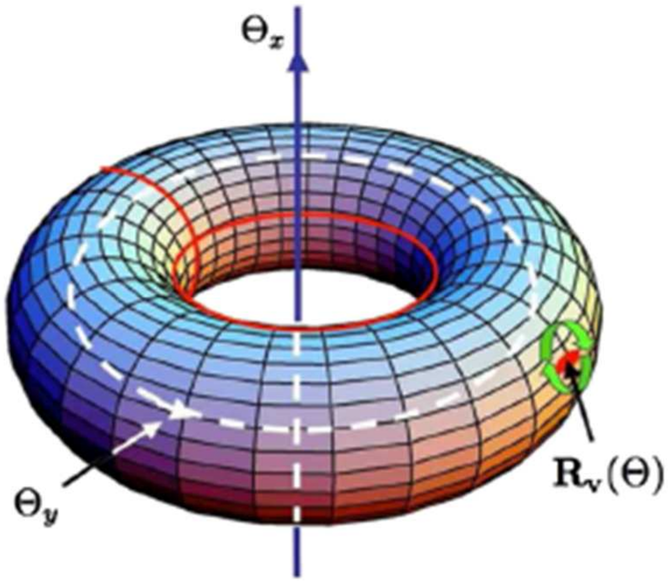
Podolsky, Lindner, Huber, unpublished

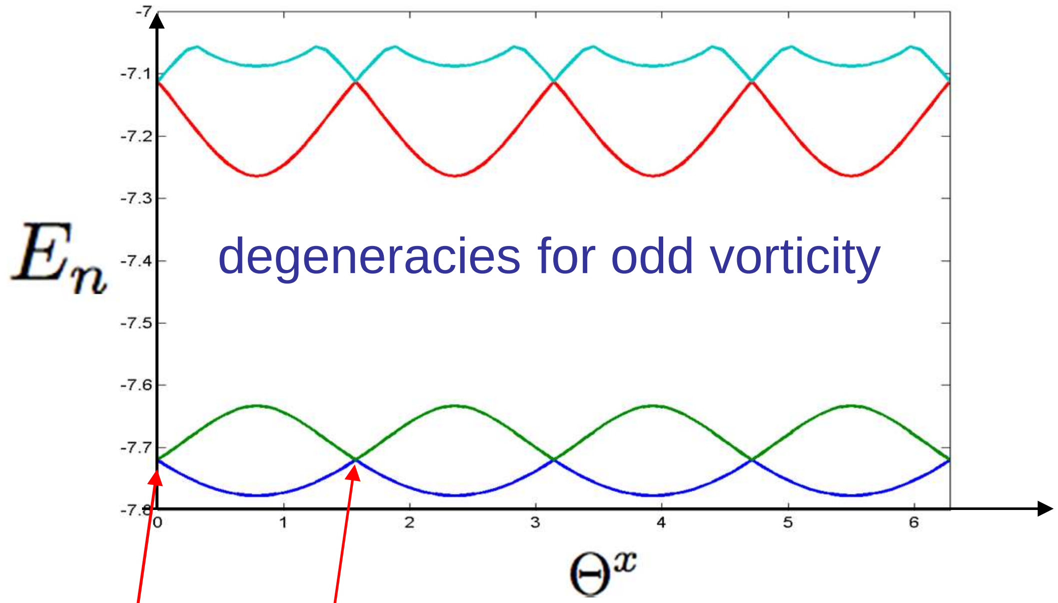


Lattice induced Hall coefficient variations

Emergence of "Vortex spin"

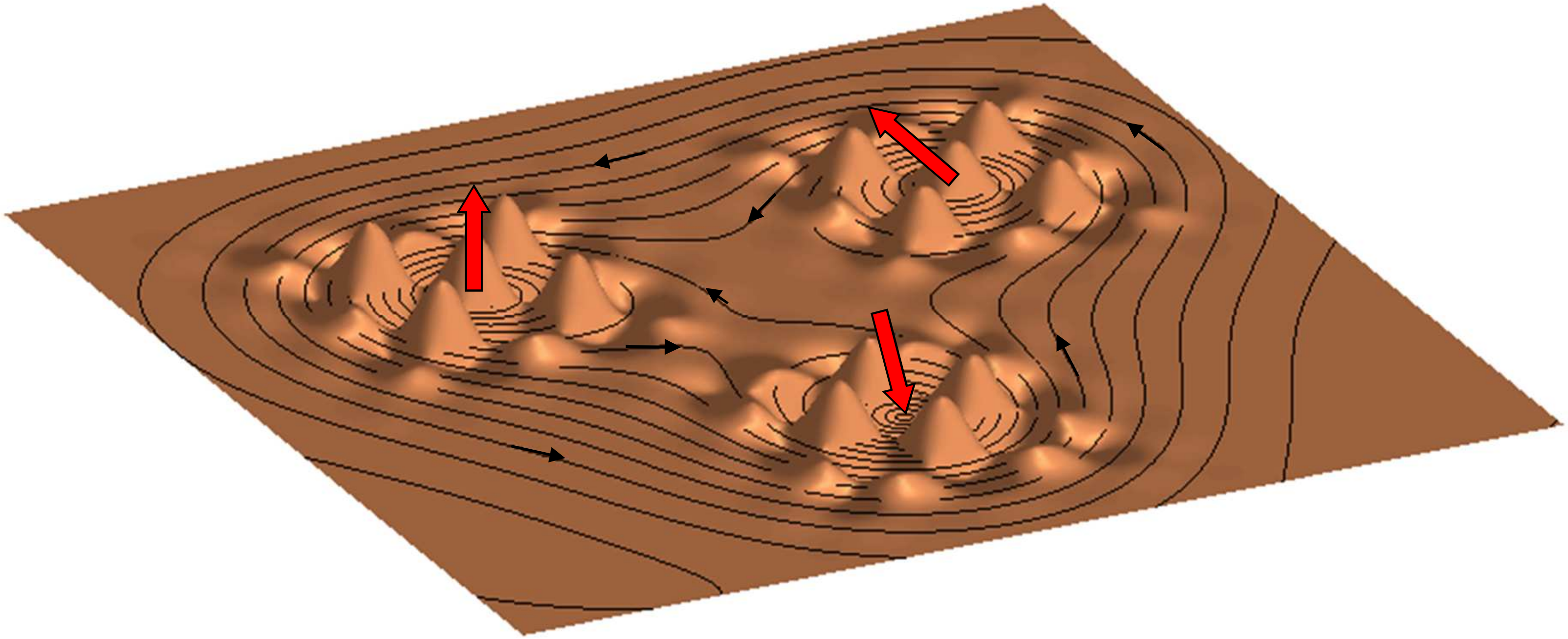
Lindner, AA, Arovas, PRL (2009); PRB 2010





Non abelian symmetry operators about vortex position:
Each vortex carries local spin half ('V-spin').

Illustration of 3 vortices with v-spin



Implications of v-spins:

1. order: CDW (supersolid) in the vortex lattice
2. Low temperature entropy of v-spins

Summary

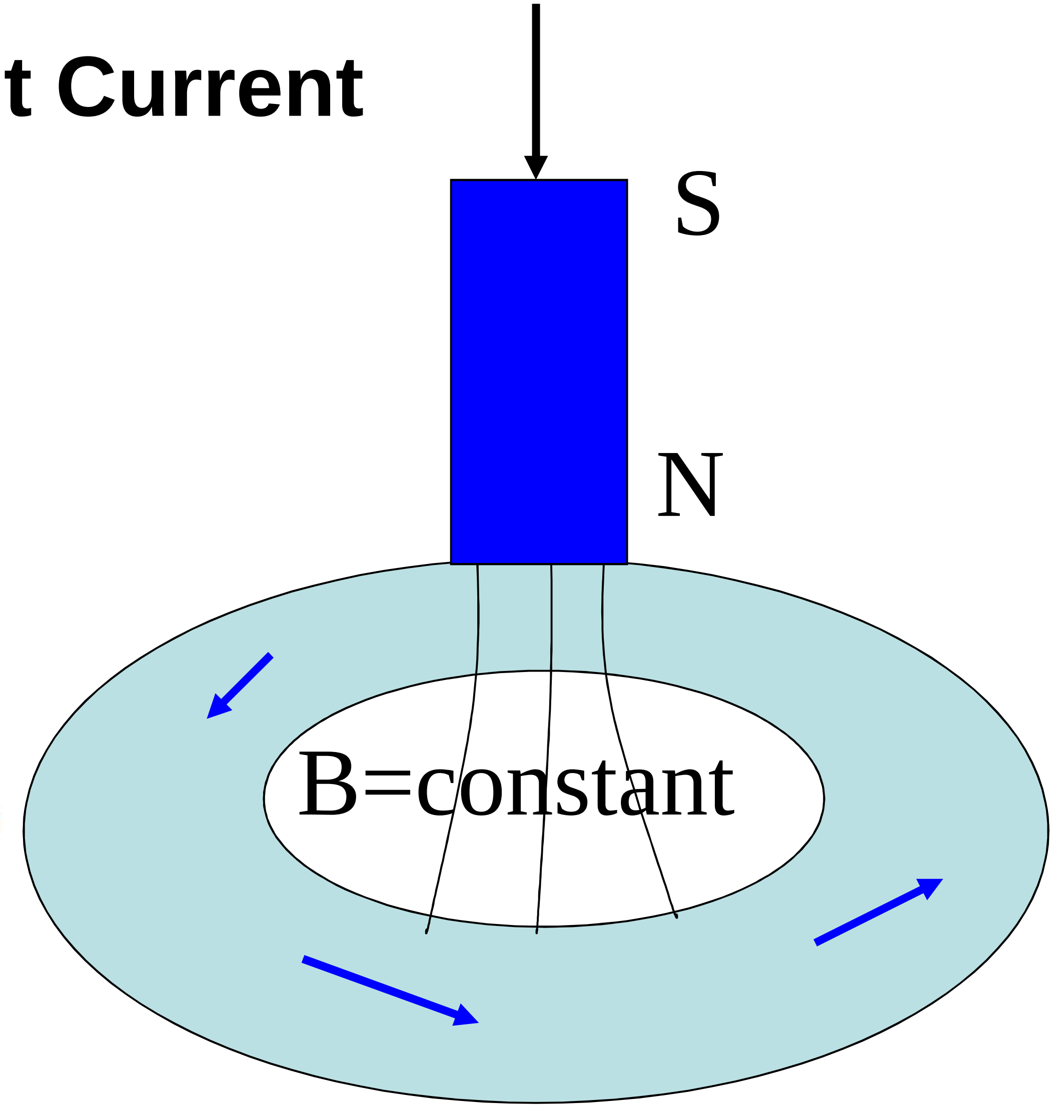
1. “Conventional superconductors” : large superfluid density and long coherence length.
   BCS Superconductivity, Normal phase = Fermi liquid.

2. “High Tc” superconductors exhibit low superfluid density and short coherence lengths – bosonic phenomenology.

3. Schafroth’s BEC superconductivity fails to include interactions and lattice effects on the bosons.

4. Hard core bosons exhibit reentrant SF density, “bad metal” resistivity, Hall sign changes, and vortex degeneracies -

Persistent Current



$$\varepsilon = -\frac{d\Phi}{dt} = 0$$

$$I(t) = I_0 > 0$$ Perpetum Mobile

R=0 !

The “Superfluid Stiffness problem”

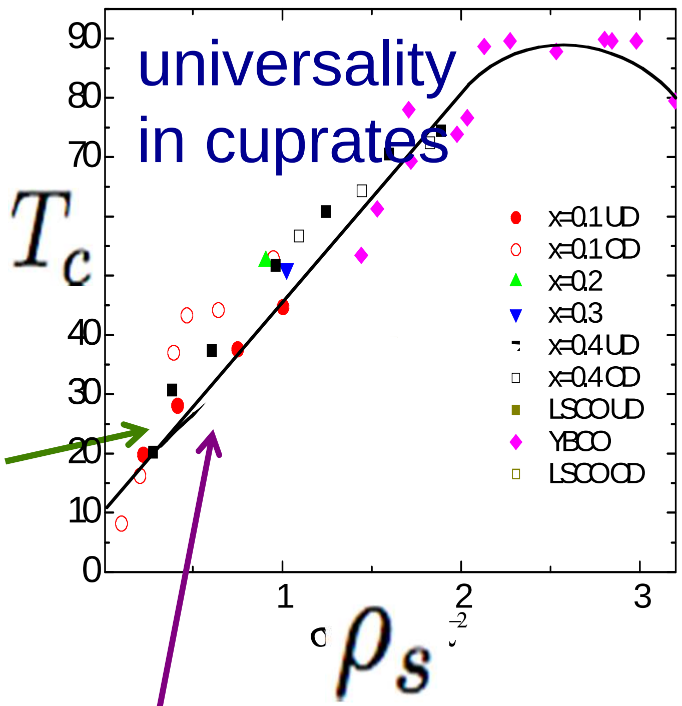
Letters to Nature

Nature **374**, 434-437 (30 March 1995) | doi:10.1038/374434a0; Receiv
March 1995

Importance of phase fluctuations in superconductors with small superfluid density

V. J. Emery* & S. A. Kivelson

Uemura’s Plot



Bosonic superfluids $T_c \sim \rho_s \sim \dfrac{\hbar^2 n_b}{m_b}$

BCS: $\rho_s \sim E_F >> T_c$

Was Schafroth right for cuprates:

Superconductivity=Bose condensation ?

Continuum bosons vs Lattice bosons

Gross-Pitaevskii theory

$$S_{\rm GP} = \int d^2x \int dt \left[\psi^*(\partial_t - \mu)\psi + \frac{1}{2m^*}\left|(-i\nabla - qA)\psi\right|^2 + \tfrac{1}{2}g|\psi|^4 \right]$$

Gauged Spin ½ XXZ model

$$\mathcal{H} = -2J \sum_{\langle ij \rangle} \left(e^{iqA_{ij}} S_i^+ S_j^- + e^{-iqA_{ij}} S_i^- S_j^+ \right) + 4V \sum_{\langle i,j \rangle} S_i^z S_j^z - \mu \sum_i (S_i^z + \tfrac{1}{2}).$$

Dynamics determined by:

Galilean symmetry

<u>At fillings n/2, n=1,2,…</u>
Relativistic dynamics,
Charge conjugation symmetry,
Berry phases

Hall conductance

$$\sigma_{\rm Hall} = \frac{nec}{B}$$

$$\sigma_{\rm Hall} \neq \frac{nec}{B}$$

Continuum bosons vs Hard Core bosons

Continuum bosons	Hard Core bosons
^{4}He, Cold atoms in large traps	JJ arrays, Cold atoms in Optical lattices, short coherence length SC

Galilean invariance

Order Parameter

Continuum bosons	Hard Core bosons
$\langle b^\dagger \rangle = \sqrt{n}$	$\langle b^\dagger \rangle \neq \sqrt{n}$

Superfluid density

Continuum bosons	Hard Core bosons
$\rho_s = \frac{\hbar^2 n}{m_b}$	$\rho_s \propto n(1-n)$

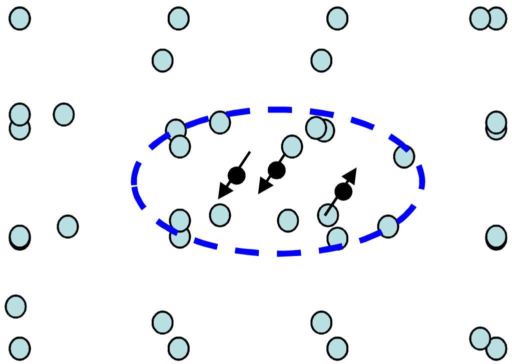
Transition temperature

Continuum bosons	Hard Core bosons
$T_c = \frac{2\pi\hbar^2}{m}\left(\frac{n}{\zeta_{3/2}(1)}\right)^{2/3}$	$T_c \propto \rho_s(T=0)$

Cooper pairing

Electron phonon interactions:

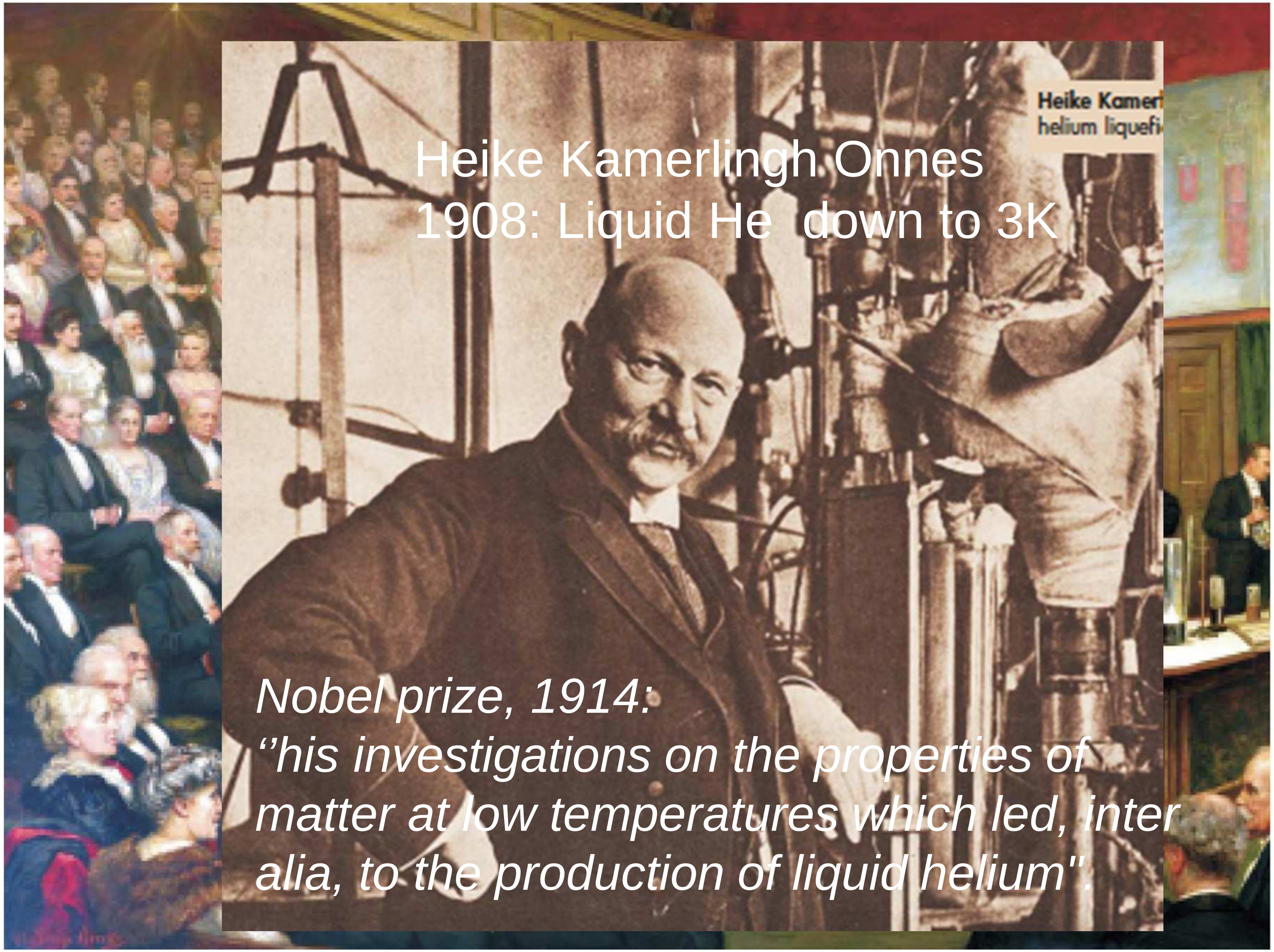
1. 2 electrons share lattice deformations
2. Attraction is local in space and retarded in time



Pairing in the many electron state

➔ Instability of the Fermi surface.

The race to absolute 0



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