Measurement of the Polarization Observables I^s and I^c for $\vec{\gamma} p \to p\, \pi^+ \pi^-$ using the CLAS Spectrometer

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Outline

1. Theory and Motivation
   - Theory: QCD
   - Constituent Quark Models and Missing Resonances
2. Experiment
   - Jefferson Lab
   - Hall B
3. Data Analysis
   - Polarization Observables
   - Kinematic Fitting
   - Extraction of Polarization Observables

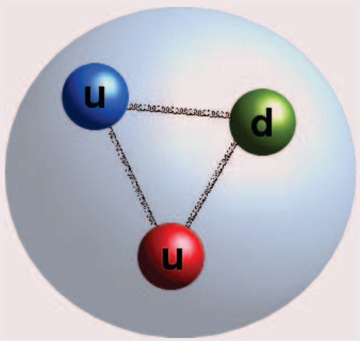
What is the big picture?

Hadrons:

- Particles that are held together and interact via the strong force.
- Exist as bound states of quarks.
- Are the most fundamental of systems found in nature.
- Members of this group are Baryons and Mesons.

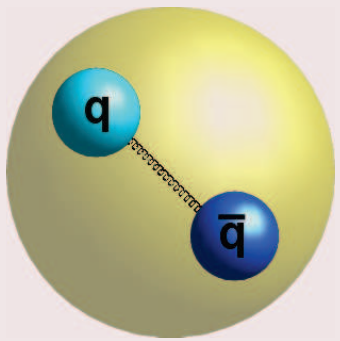
Quark Structure:

- Baryon: qqq



Quark Structure:

- Meson: $q\bar{q}$



Quantum Chromodynamics (QCD)

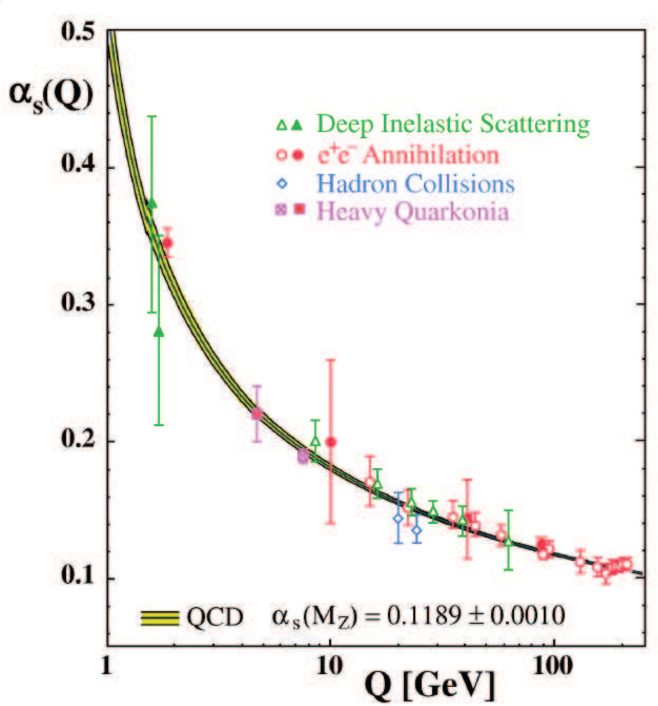
Properties of QCD

- Describes the strong force.
- Introduces a “color charge” analogous to the electric charge (red, blue, green).
- Gluons and quarks possess a color charge, antiparticles possess “anticolor”.
- All observed particles are “white” or colorless states.
- Interactions are mediated by gluons.
- Since gluons carry a color charge, they mediate AND participate in the interactions.

Quantum Chromodynamics (QCD): Features

Two main features of QCD

- Asymptotic Freedom: When the energy of the system is high (like in a high energy reaction $\rightarrow$ high momentum transfer), the quarks and gluons interact very weakly. Baryon can then be treated as being comprised of three non-interacting quarks $\rightarrow$ pQCD.



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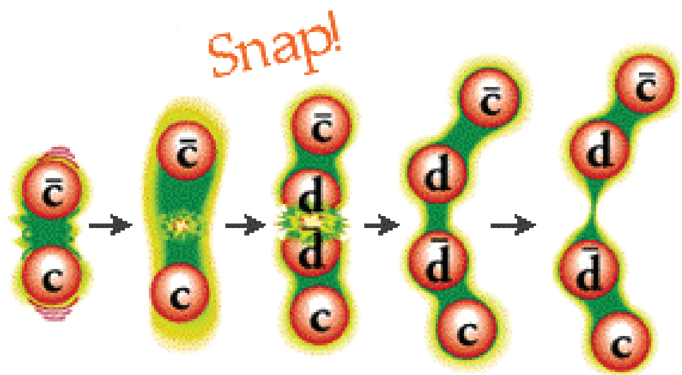
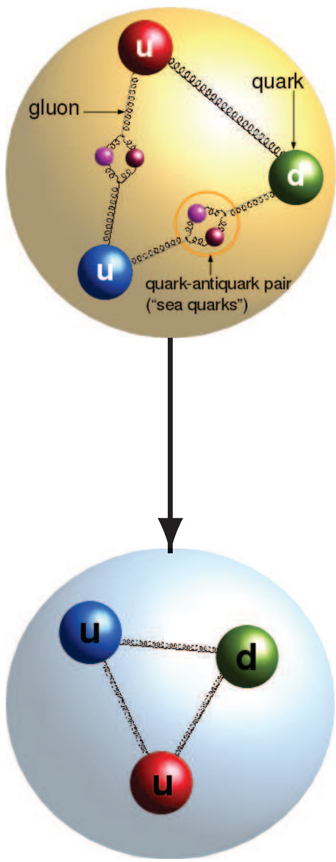
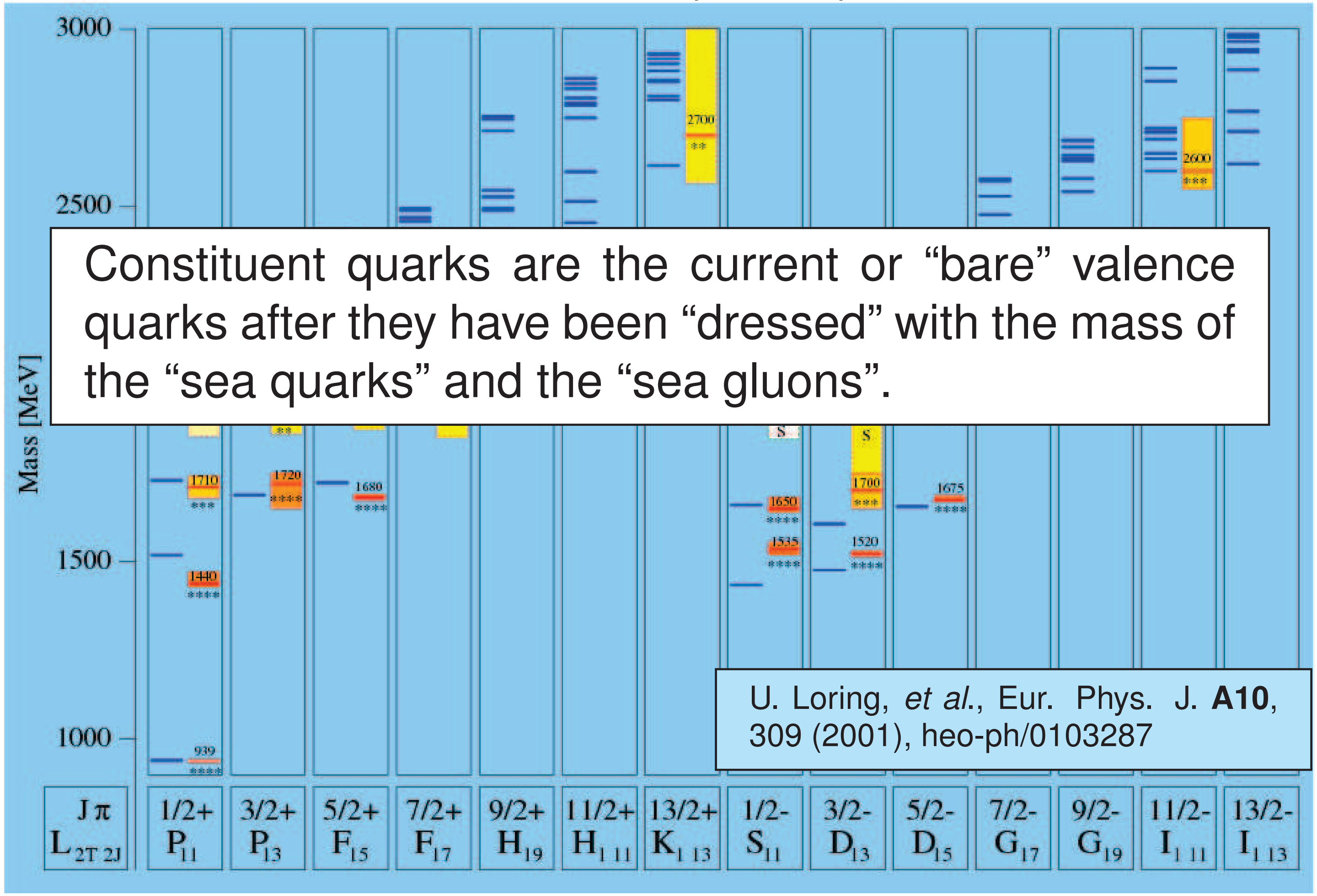
- Confinement: The attractive force between quarks grows with the distance between them. This means that the quarks are confined to be in the hadron $\rightarrow$ there are no free quarks (non-pQCD).



Constituent Quark Models (CQMs)



N* Resonances (I = 1/2)



Constituent quarks are the current or "bare" valence quarks after they have been "dressed" with the mass of the "sea quarks" and the "sea gluons".

U. Loring, *et al.*, Eur. Phys. J. **A10**, 309 (2001), heo-ph/0103287

⇒ Generated using predictions according to a model view of the baryon.
⇒ Assumes three quark degrees of freedom.
⇒ Various models differ in how they treat short-range interactions.

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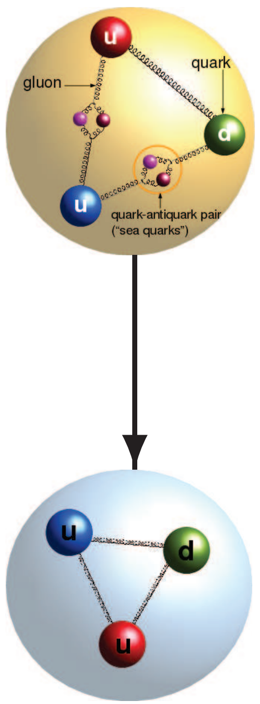

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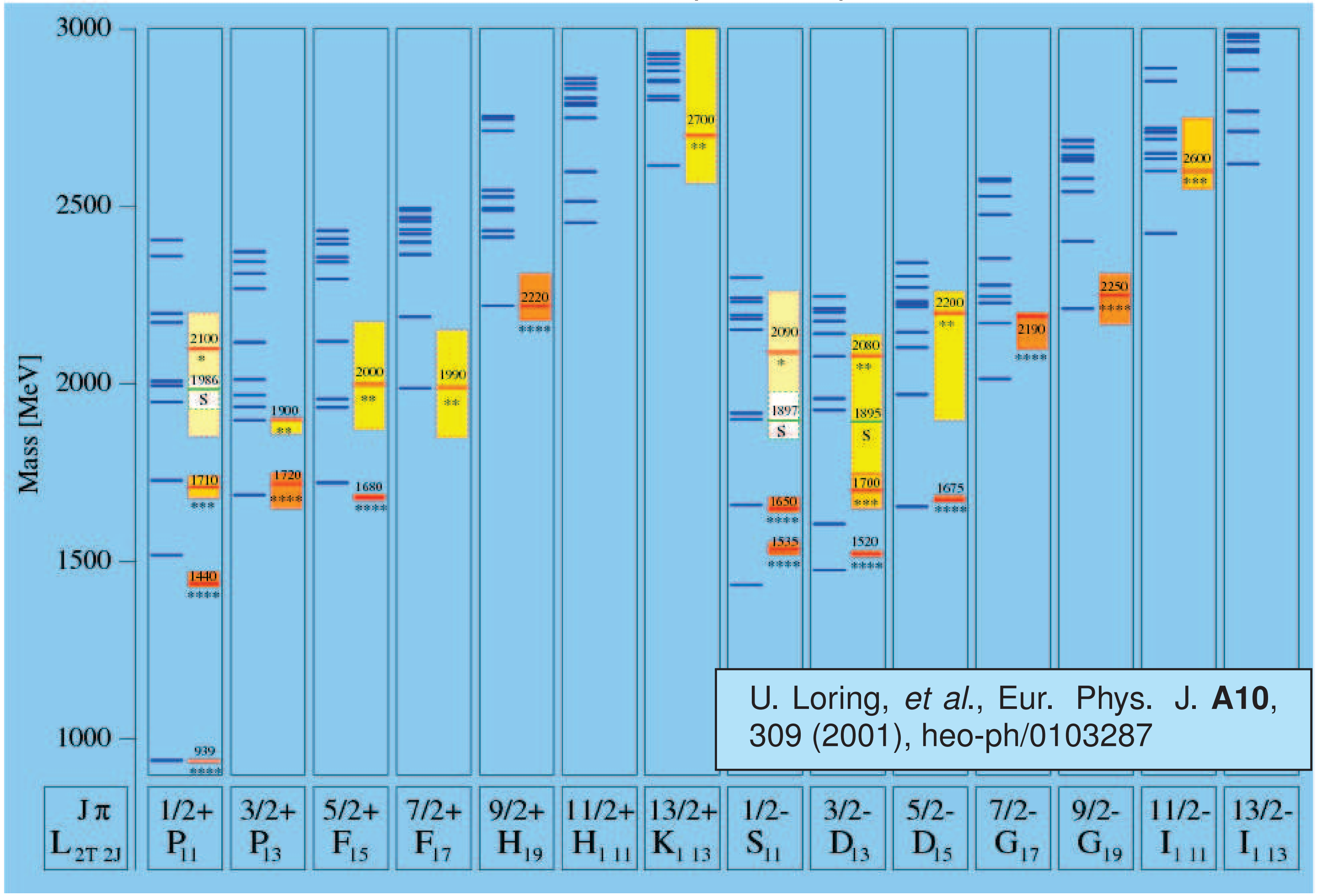

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CQMs: Missing Resonances

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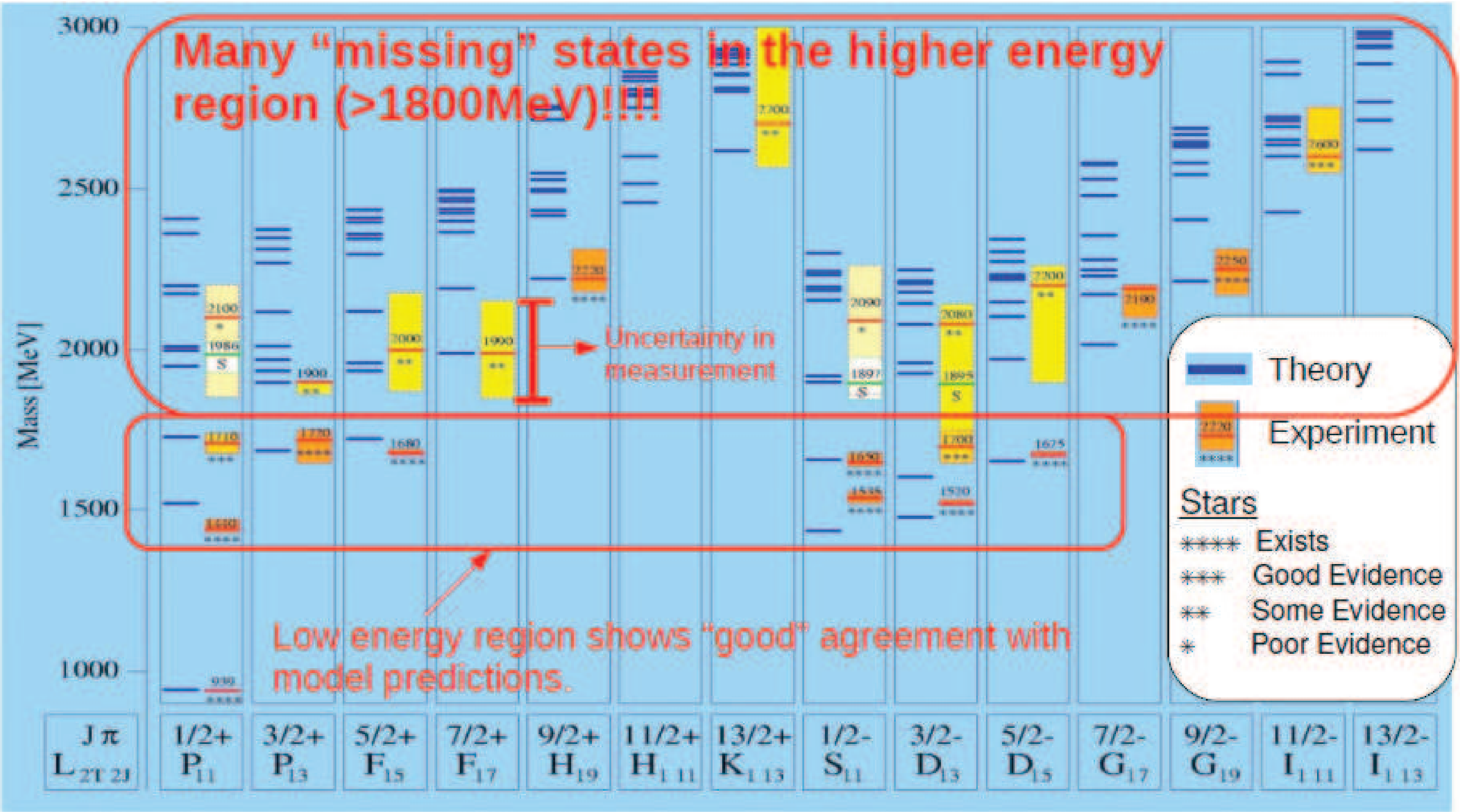

Why so many missing resonances?

Possible answers and reasons...

- Most of what is found in the PDG handbooks has come from πN and KN scattering $\rightarrow \gamma$N.
- Analyses involved a single meson final state (i.e. $K^+\Lambda$, $K^+\Sigma^0$, Nπ, Nη).
  - Analyze a double-meson final state $\rightarrow$ has the largest cross section.
  - Sequential decay to final ground state particles: ex. $\gamma p \rightarrow N^* \rightarrow \Delta^{++}\pi^- \rightarrow p\,\pi^+\pi^-$.
  - Particle mass widths expected to be $\approx$150 MeV $\rightarrow$ too big for a two-body (single meson) final state.
- Possible quark-diquark structure of the baryon.

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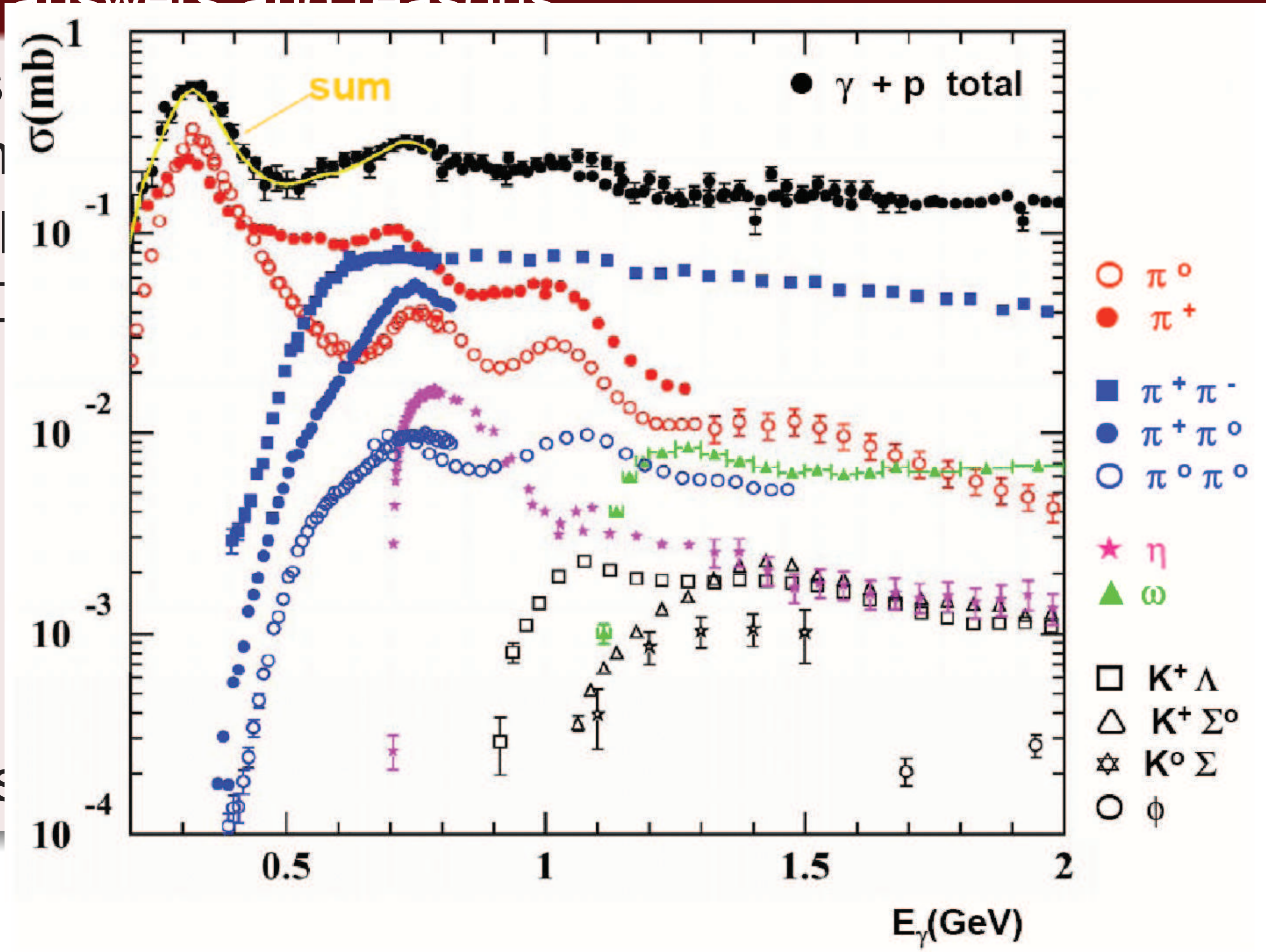
- Mos… e
  from…
- Anal… cross
  $K^+\Sigma$…
  -
  -
  - … big
- Poss…



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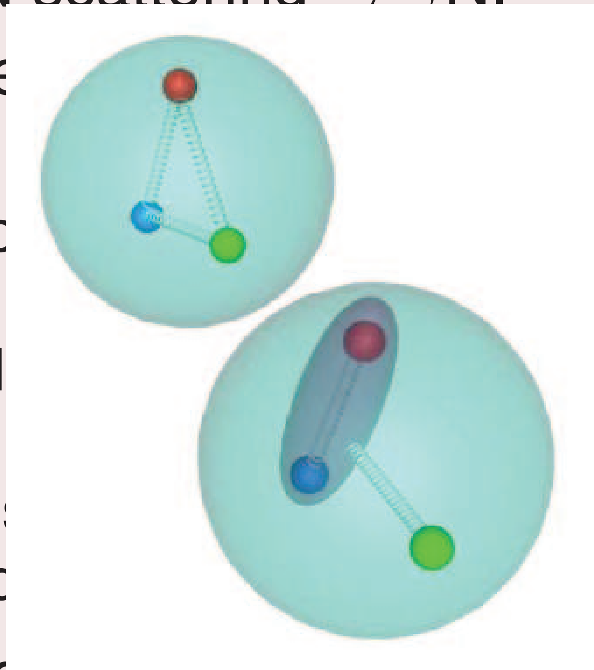

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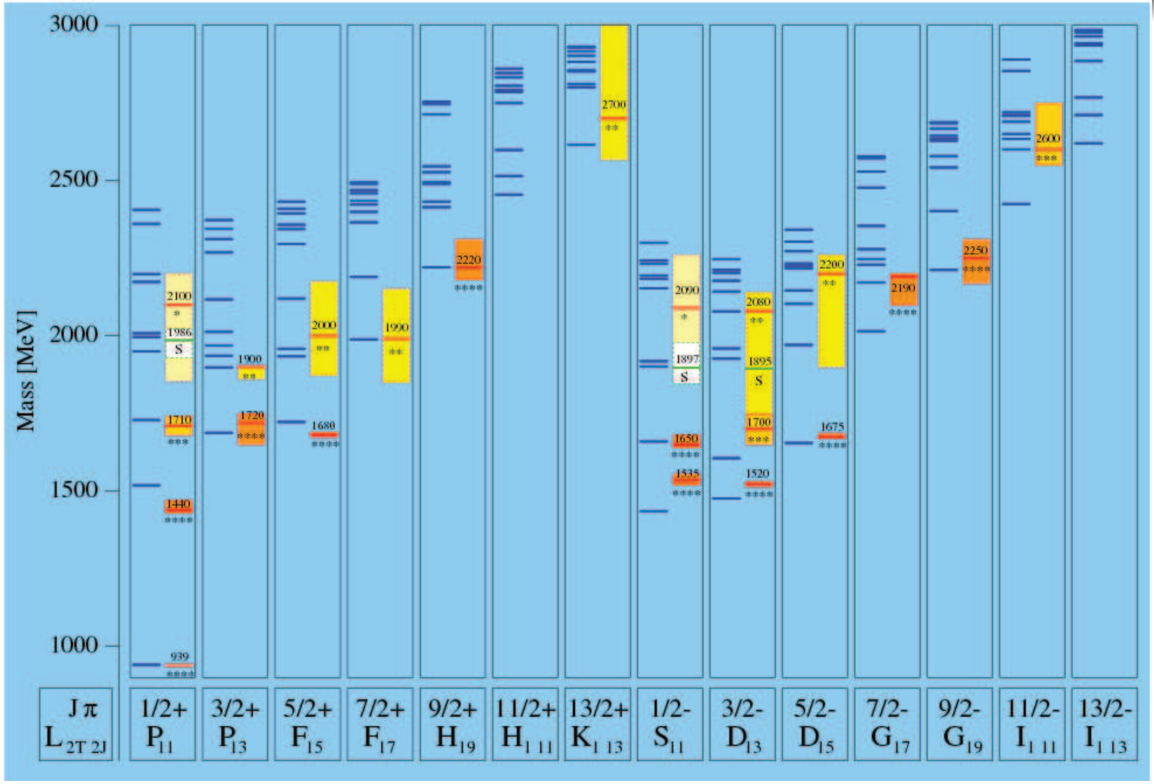
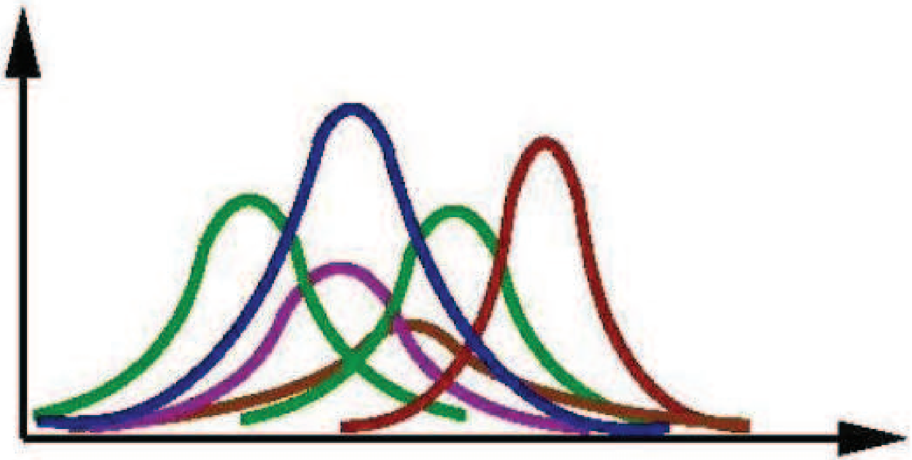


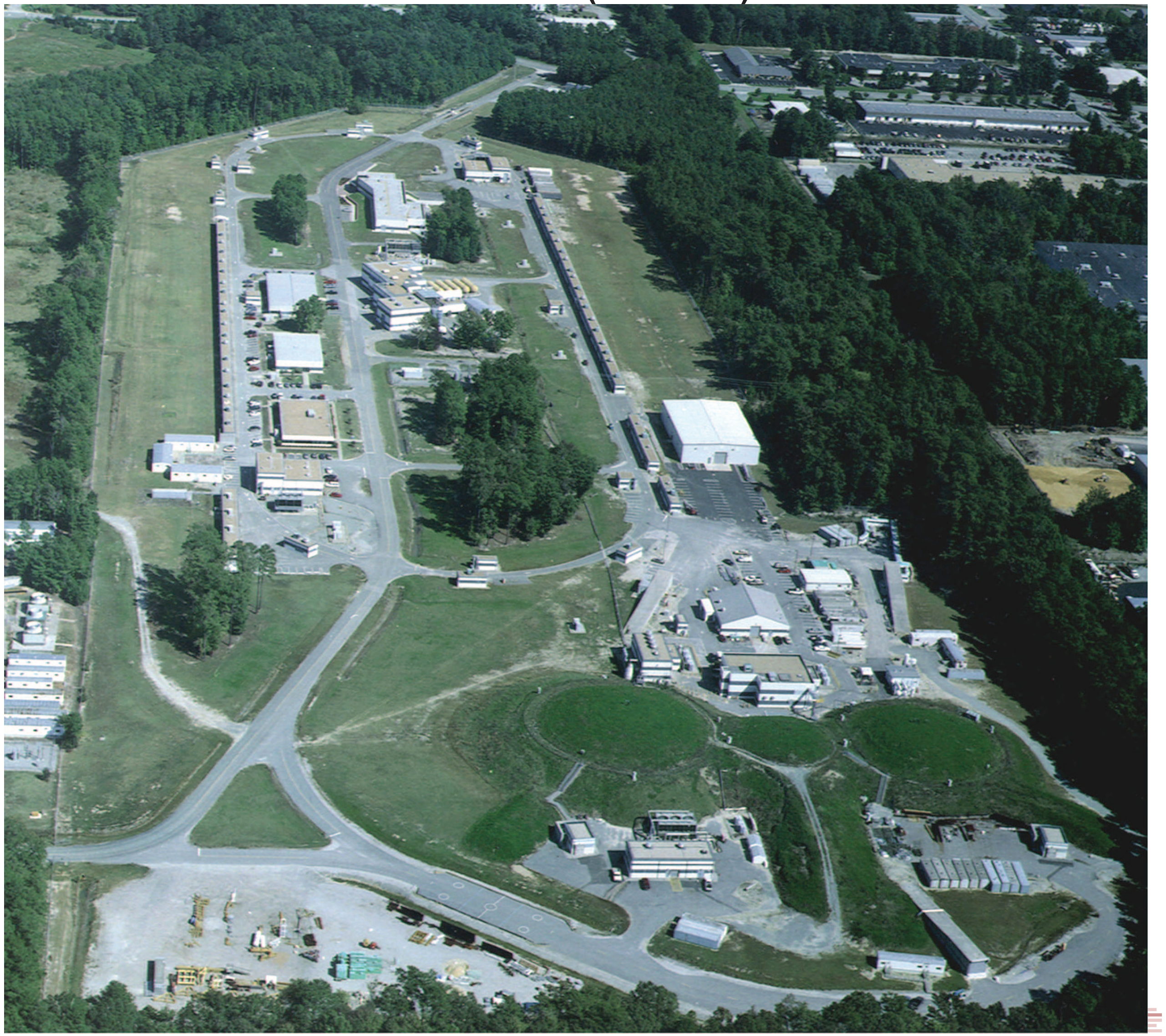
- Resonances are both broad and overlapping!
- Makes it very difficult to determine singular resonance contributions.

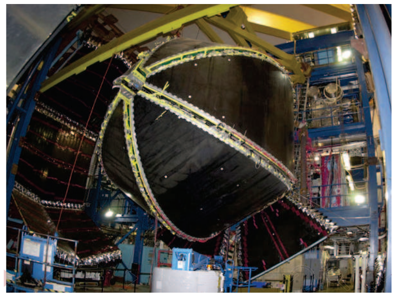
We need a way to disentangle these resonances and determine which resonances are produced!



The Facility

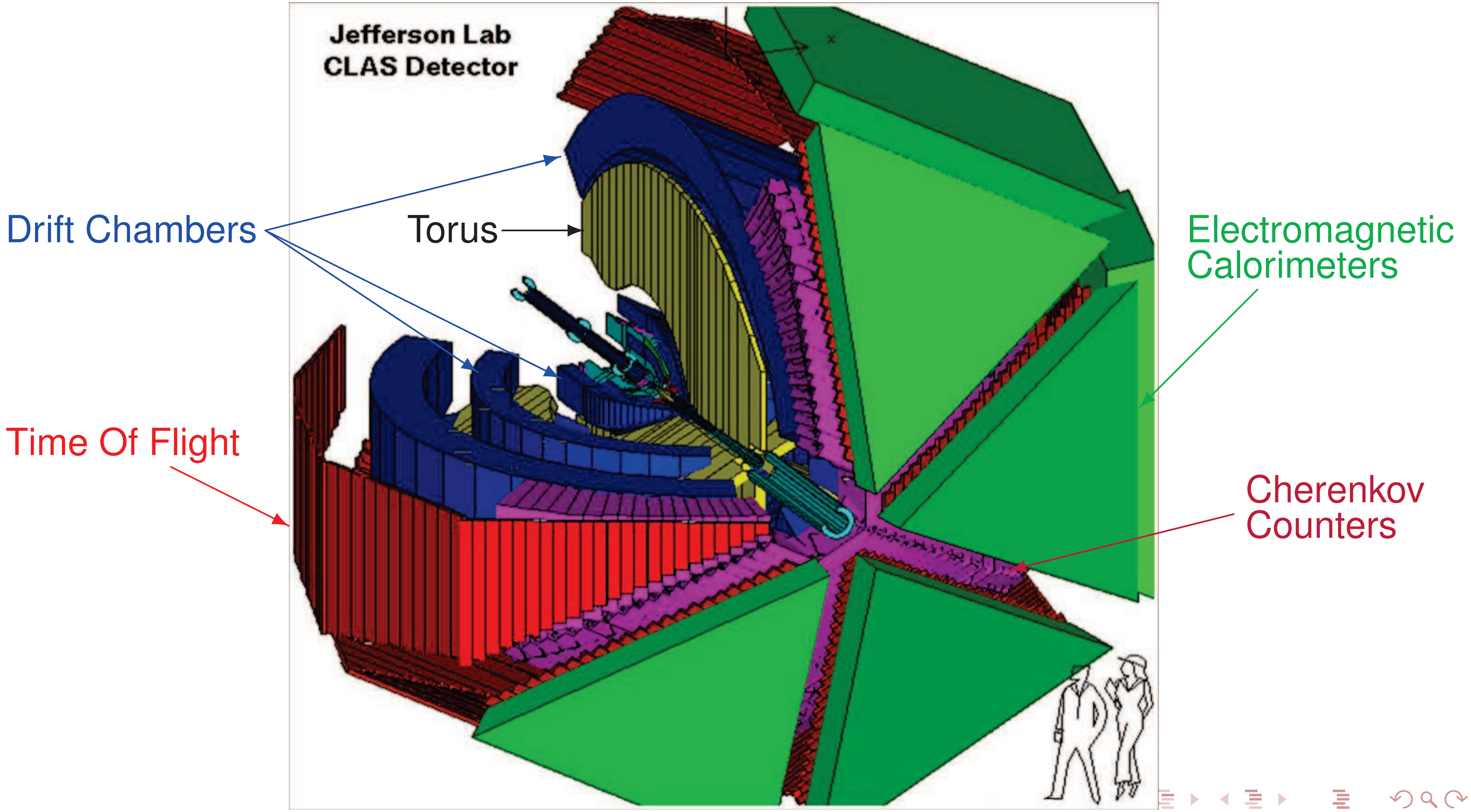
THOMAS JEFFERSON NATIONAL ACCELERATOR FACILITY (JLAB)





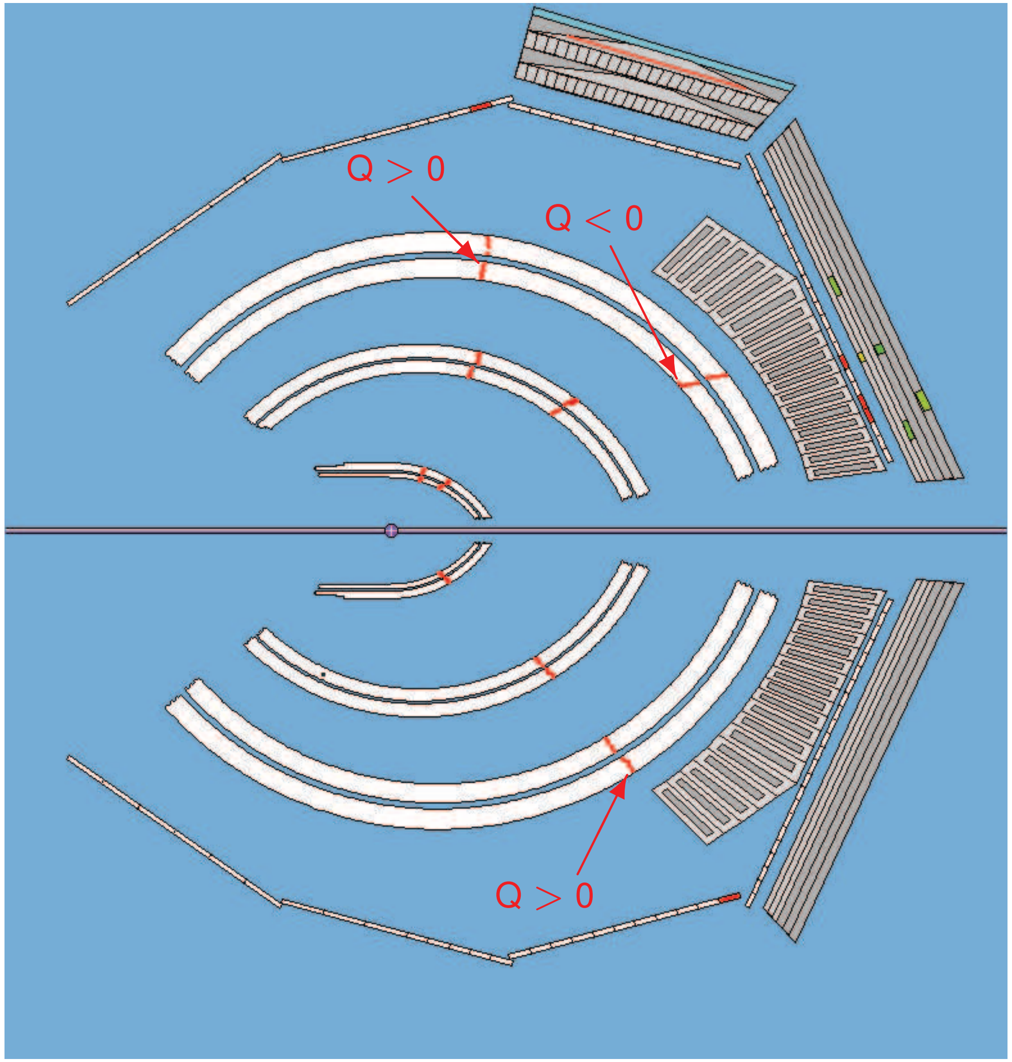
Hall B: The CLAS Detector

CEBAF Large Acceptance Spectrometer



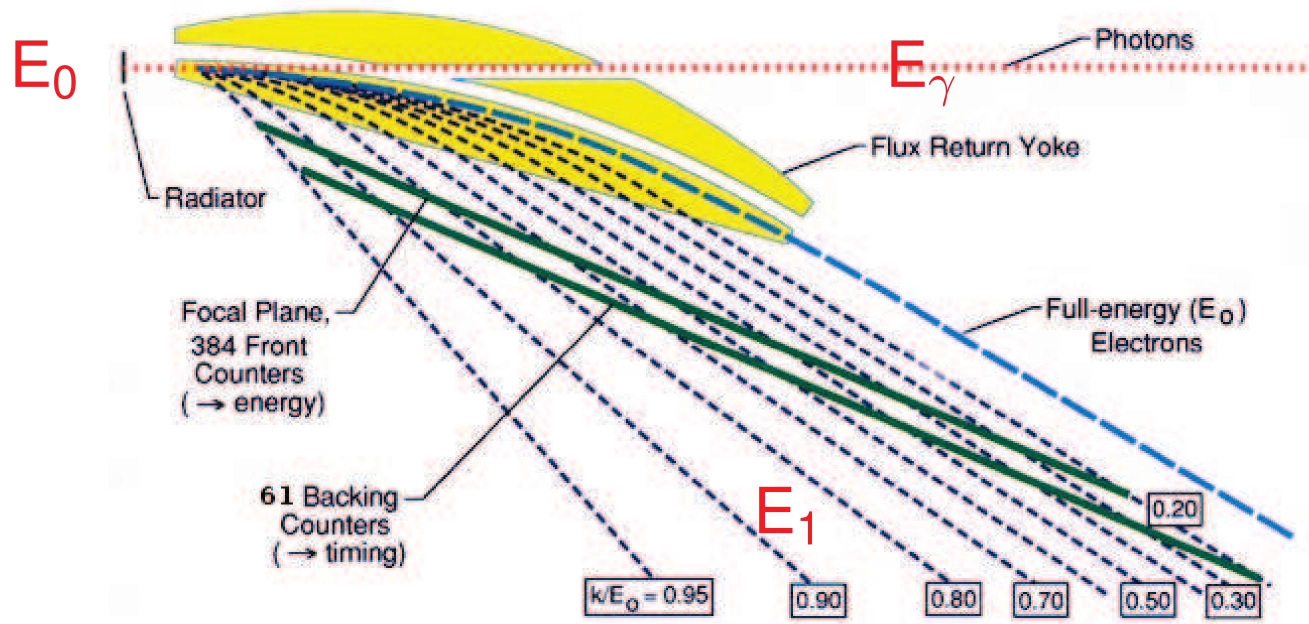
Hall B: the CLAS Detector

- Magnetic field of Torus affects the particle's trajectory (bending it toward or away from the beamline).
- Particle's path through CLAS is tracked.
- If trigger requirements are met, event is recorded.
- Reconstruction code determines particle's trajectory through CLAS.
- Information regarding the incident particle and final state particles are determined.



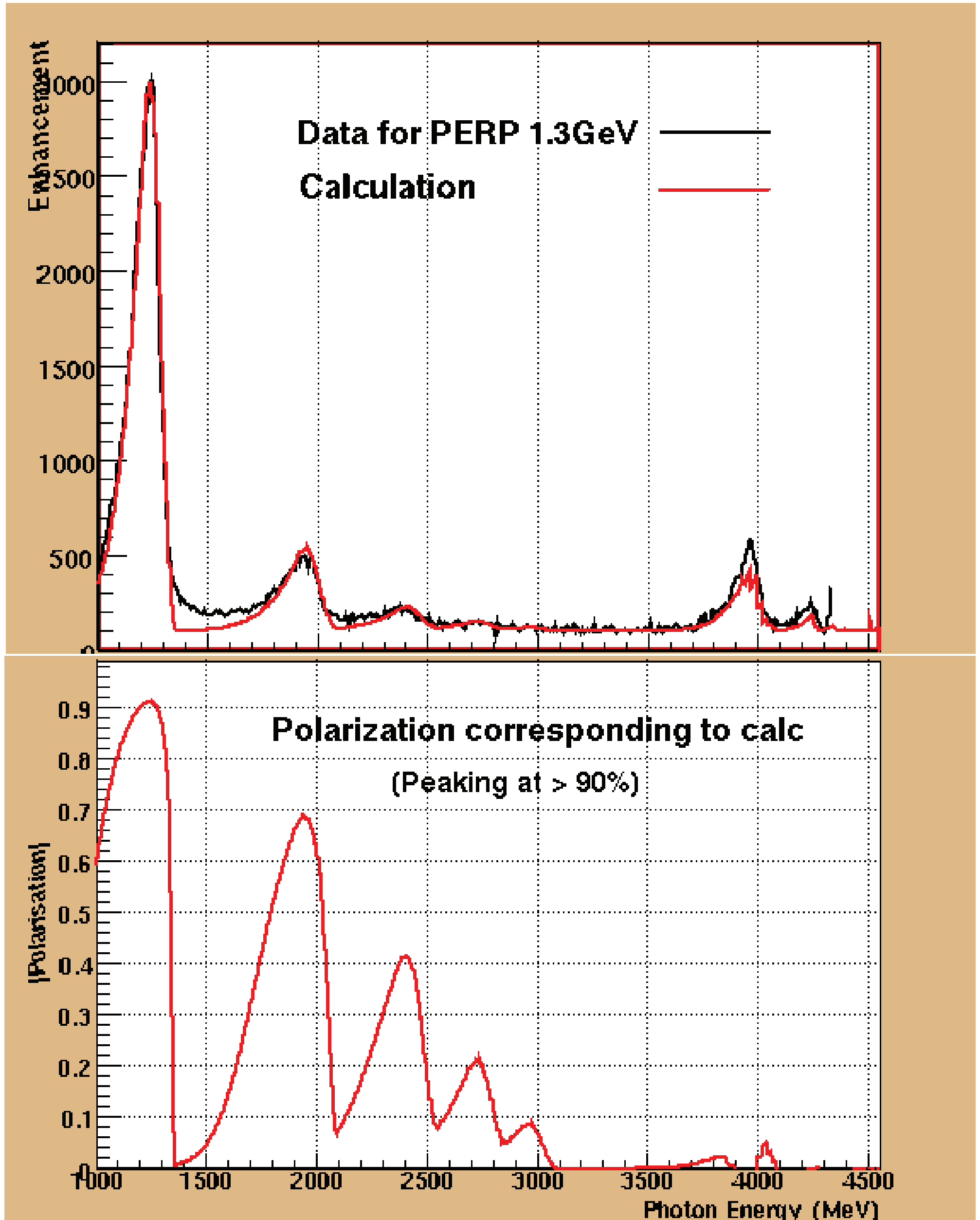
Hall B: Photon Tagger

- Electrons from accelerator interact with the radiator, producing photons via bremsstrahlung.
- Electrons are bent into hodoscope via the Tagger Magnet.
- Time stamp and energy of electron is measured.
- Photon energy determined via $E_\gamma = E_0 - E_1$
- Can tag photons with energies ranging from $(0.2)E_0$ to $(0.95)E_0$.

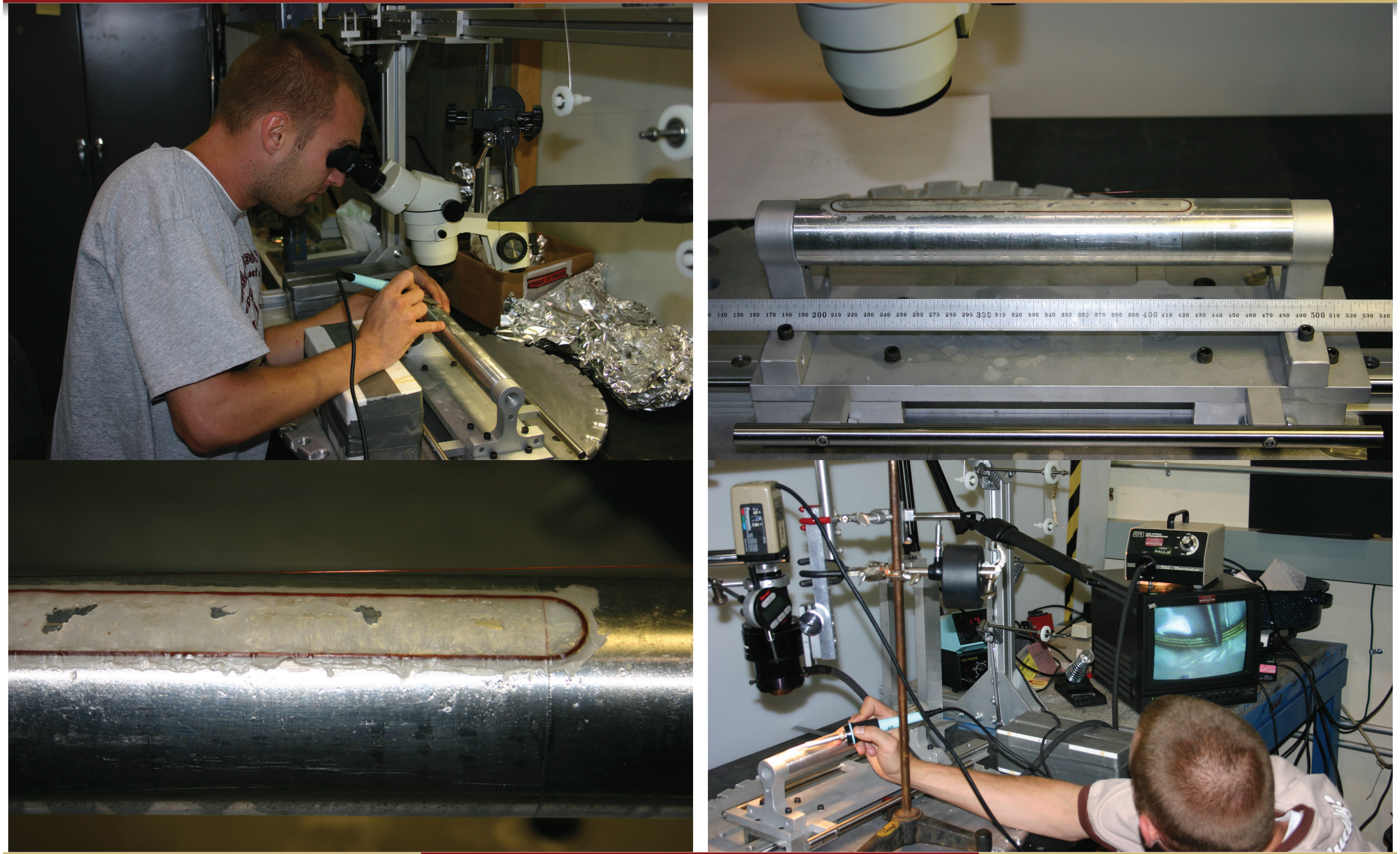


Polarized Photons

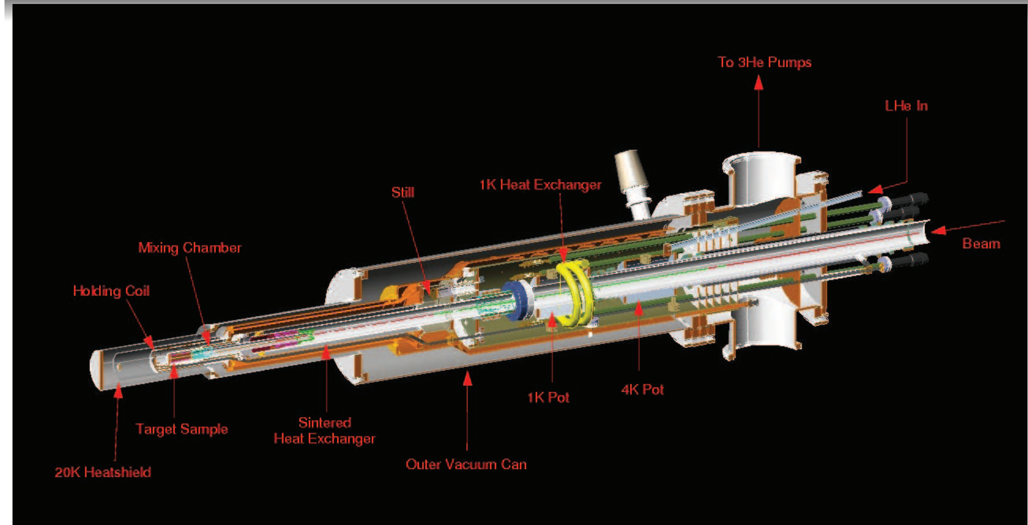
- Circularly polarized beam:
  - longitudinal electron beam + amorphous radiator
- Linearly polarized beam:
  - unpolarized electron beam + oriented diamond radiator
  - Can obtain 90% polarization
- g8b $\rightarrow$ highly polarized photons at five energies: 1.3 GeV, 1.5 GeV, 1.7 GeV, 1.9 GeV and 2.1 GeV.

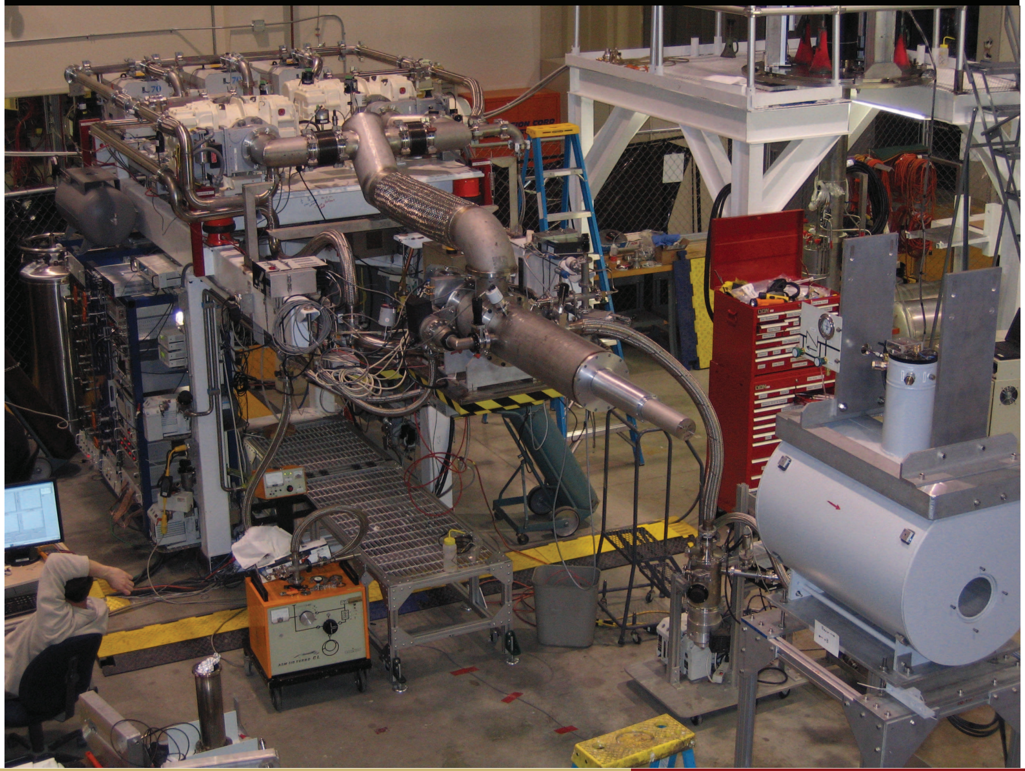


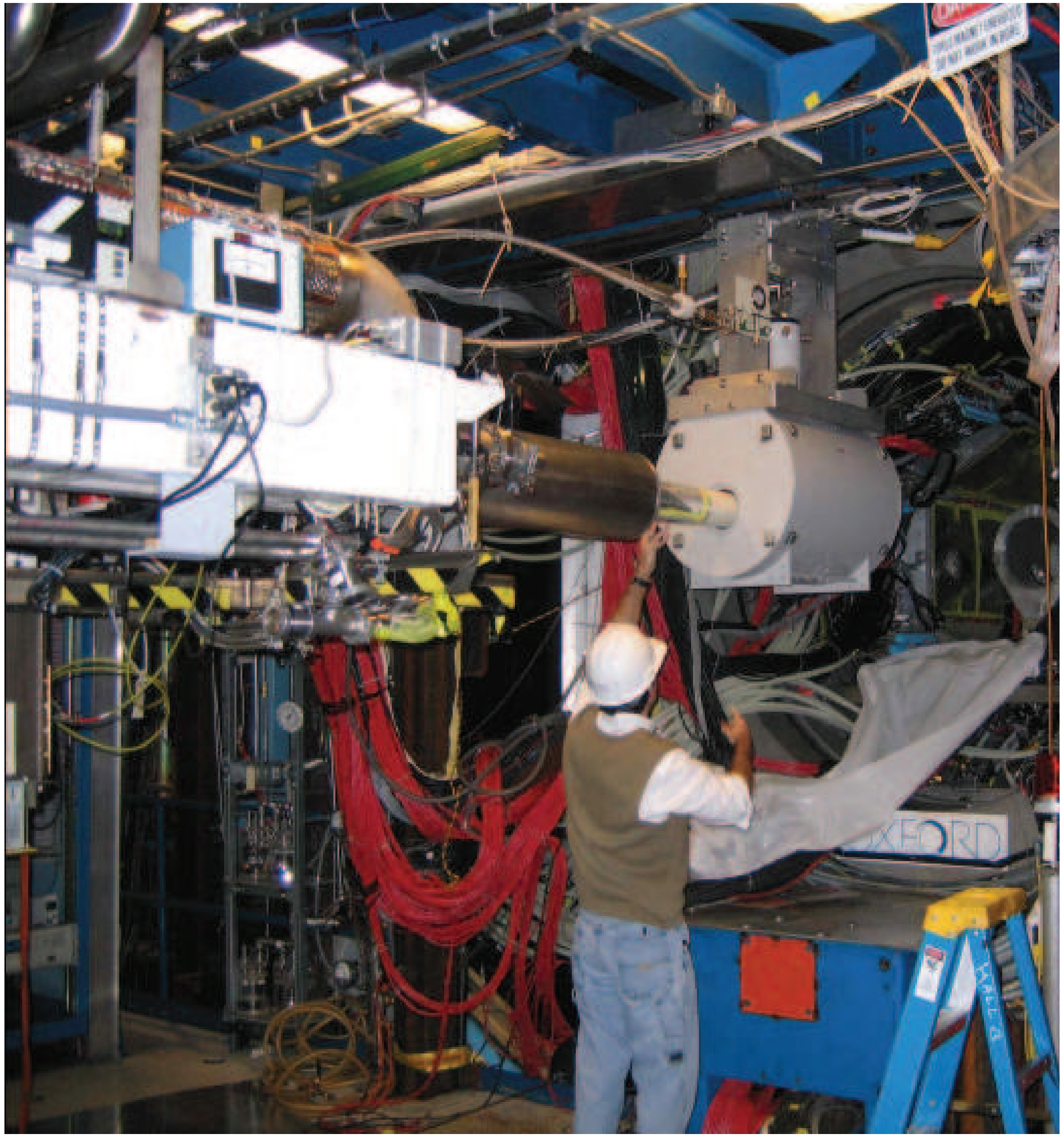
FROST: Transverse Holding Coil



FROST: Target Assembly, Testing, Installation



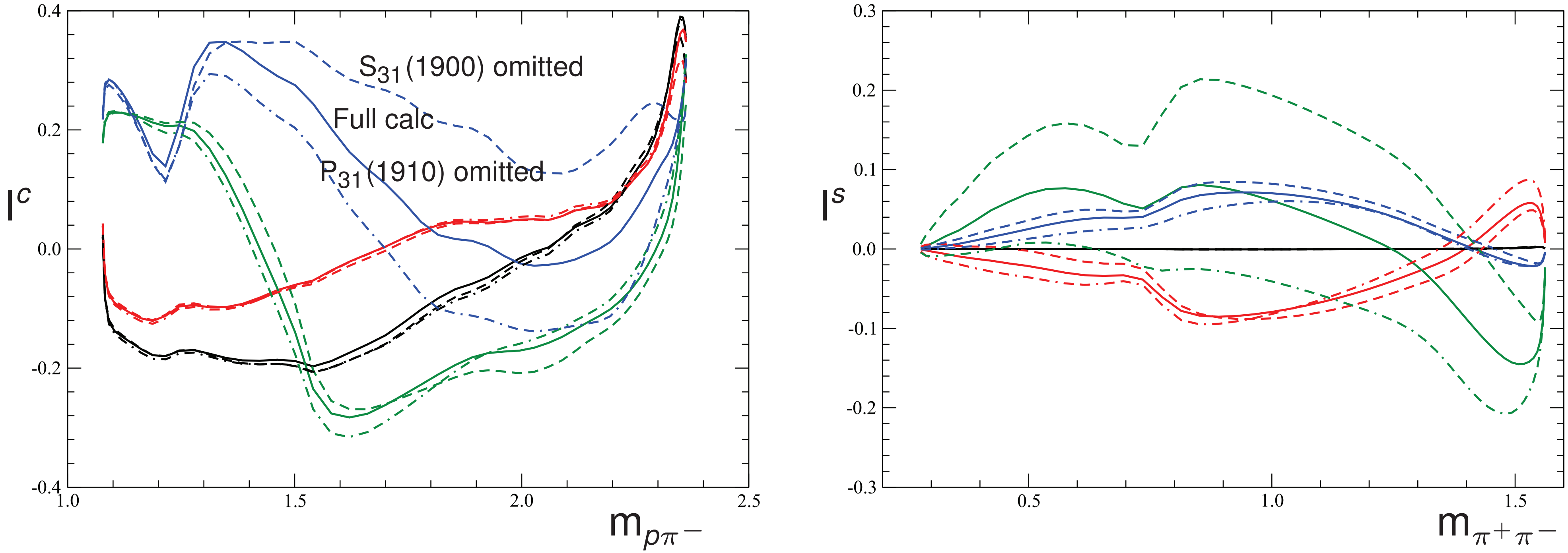




Polarization Observables

- Measurable quantities called Polarization Observables are sensitive to resonance contributions.
- Occur when the constraint of polarization (beam and/or target) is imposed on the reaction(s).
- Can be predicted according to model calculations.

Dr. W. Roberts



ϕ^* = 0.00356138 rad ϕ^* = 0.56052 rad ϕ^* = 2.0876 rad ϕ^* = 3.0439 rad

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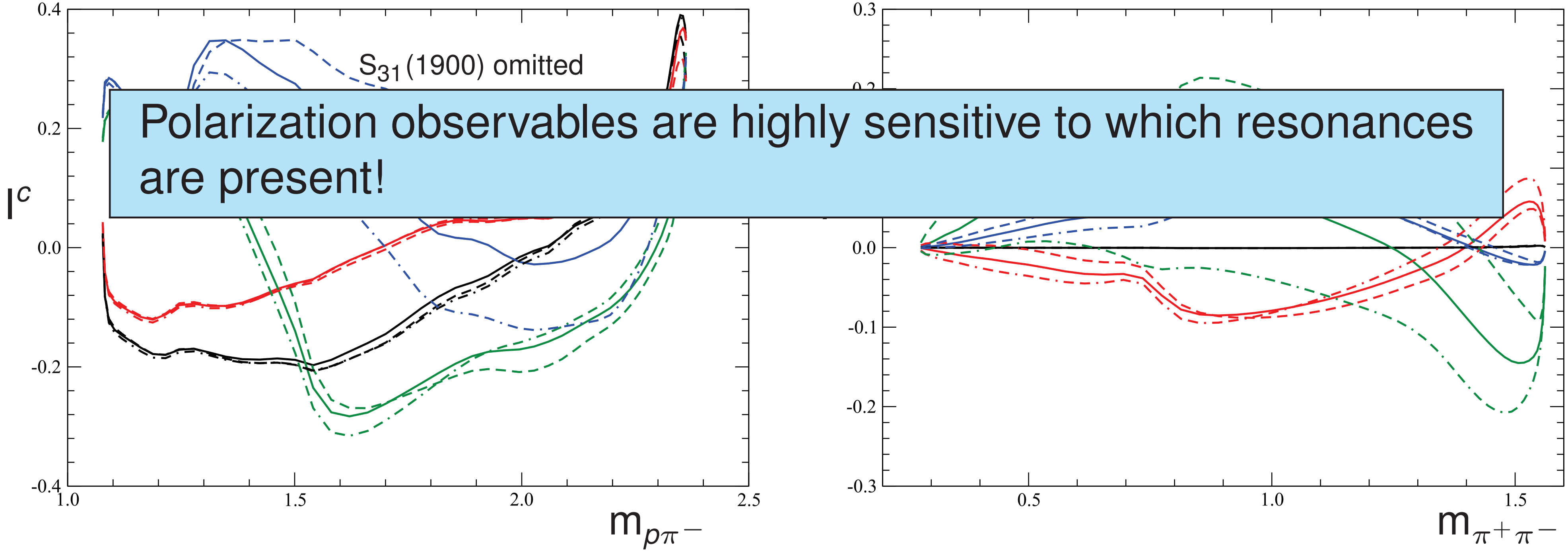

Polarization observables are highly sensitive to which resonances are present!

ϕ^* = 0.00356138 rad ϕ^* = 0.56052 rad ϕ^* = 2.0876 rad ϕ^* = 3.0439 rad

Polarization Observables: Single Meson Final State

- The final state equation relates the differential cross section, $\frac{d\sigma}{d\Omega}$, to the polarization observables.
- For a final state with one meson, there are a total of 7 observables.

$$
\begin{aligned}
\frac{d\sigma}{d\Omega} = \sigma_0 \{ 1 - \delta_l \, \Sigma \cos(2\phi) \\
&+ \Lambda_x \, (-\delta_l \, \mathbf{H} \sin(2\phi) + \delta_\odot \, \mathbf{F}) \\
&- \Lambda_y \, (-\mathbf{T} + \delta_l \, \mathbf{P} \cos(2\phi)) \\
&- \Lambda_z \, (-\delta_l \, \mathbf{G} \sin(2\phi) + \delta_\odot \, \mathbf{E}) \}
\end{aligned}
$$

Definitions

- $\frac{d\sigma}{d\Omega}$ = differential cross section
- σ_0 = unpolarized cross section
- $\delta_{l,\odot}$ = degree and orientation of photon beam polarization
- $\Lambda_{x,y,z}$ = direction and degree of polarization of the target
- Σ, **H**, **F**, **T**, **P**, **G**, **E** = polarization observables.

Polarization Observables: Double Meson Final State

- For a final state with two mesons, there are a total of 15 observables!

$$I = I_0 \{ (1 + \vec{\Lambda}_i \cdot \vec{\mathbf{P}}) + \delta_\odot (\mathbf{I}^\odot + \vec{\Lambda}_i \cdot \vec{\mathbf{P}^\odot}) + \delta_l [\sin(2\beta) (\mathbf{I^s} + \vec{\Lambda}_i \cdot \vec{\mathbf{P^s}}) + \cos(2\beta) (\mathbf{I^c} + \vec{\Lambda}_i \cdot \vec{\mathbf{P^c}})] \}$$

Definitions

- I_0 = unpolarized reaction rate
- $\vec{\Lambda}_i$ = direction and degree of polarization of the target
- $\delta_{l,\odot}$ = degree and orientation of photon beam polarization
- $\vec{P}$ = observable arising from the use of a polarized target
- $\mathrm{I}^{\odot,s,c}$ = observables arising from the use of polarized photons
- β = orientation of polarization w.r.t. a final state particle ($\beta = \phi_{lab} + \phi_{polarization}$)

Reducing the final state equation

- Applying the run conditions of g8b simplifies the final state equation and reduces the number of observables.
- Linearly polarized photon beam incident on an unpolarized LH_2 target. ($\overrightarrow{\Lambda_i} = 0$, $\delta_\odot = 0$)

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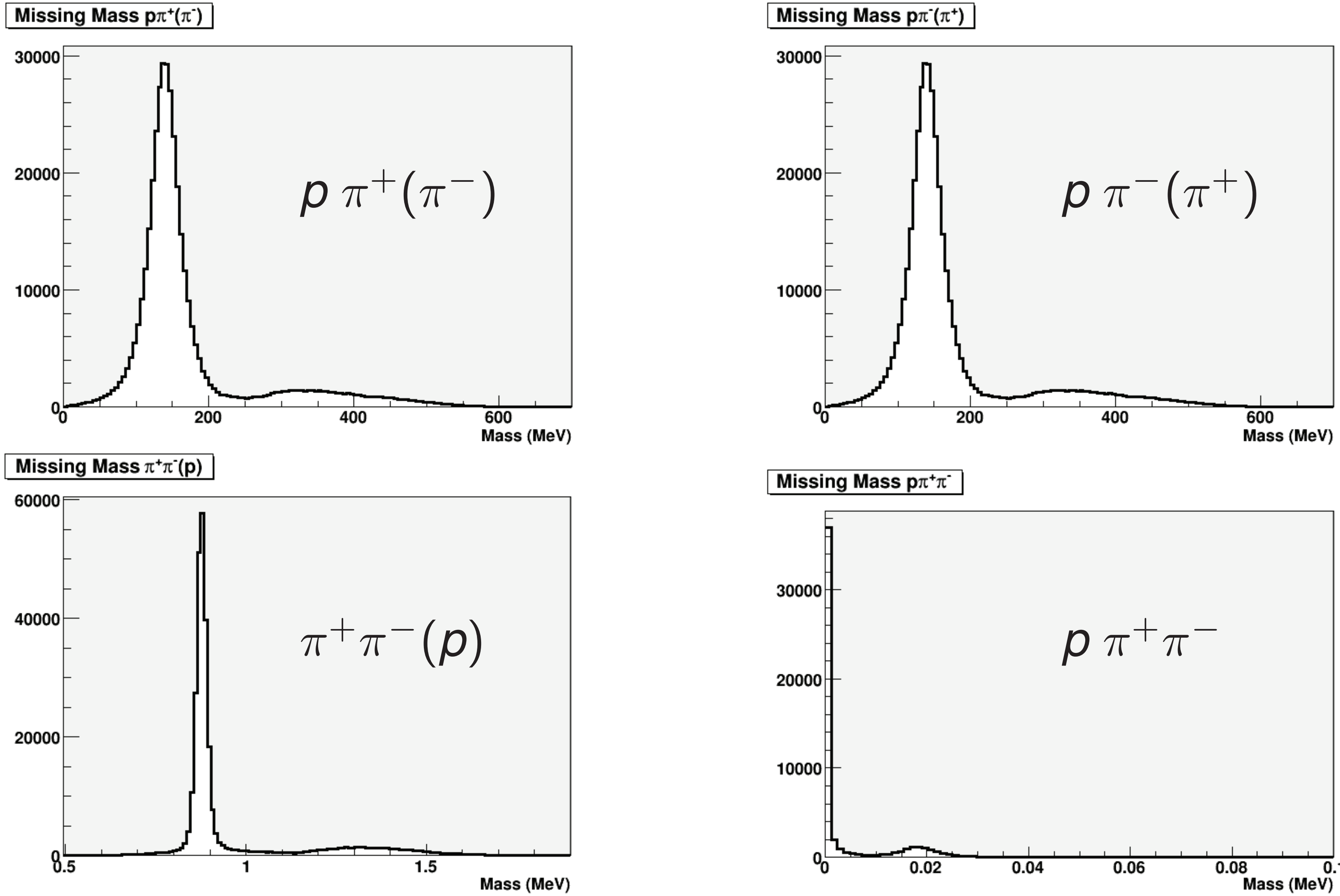
$$I = I_0 \{ 1 + \delta_l [\mathbf{I^s} \sin(2\beta) + \mathbf{I^c} \cos(2\beta)] \}$$

- I^s
- I^c (Σ in the single-meson final state equation)
- Measuring both for $\vec{\gamma} p \to p \pi^+ \pi^-$

Analysis of $\vec{\gamma}p \to p\,\pi^+\pi^-$

- While $\vec{\gamma}p \to p\,\pi^+\pi^-$ is studied, a total of four topologies are kinematically fitted.



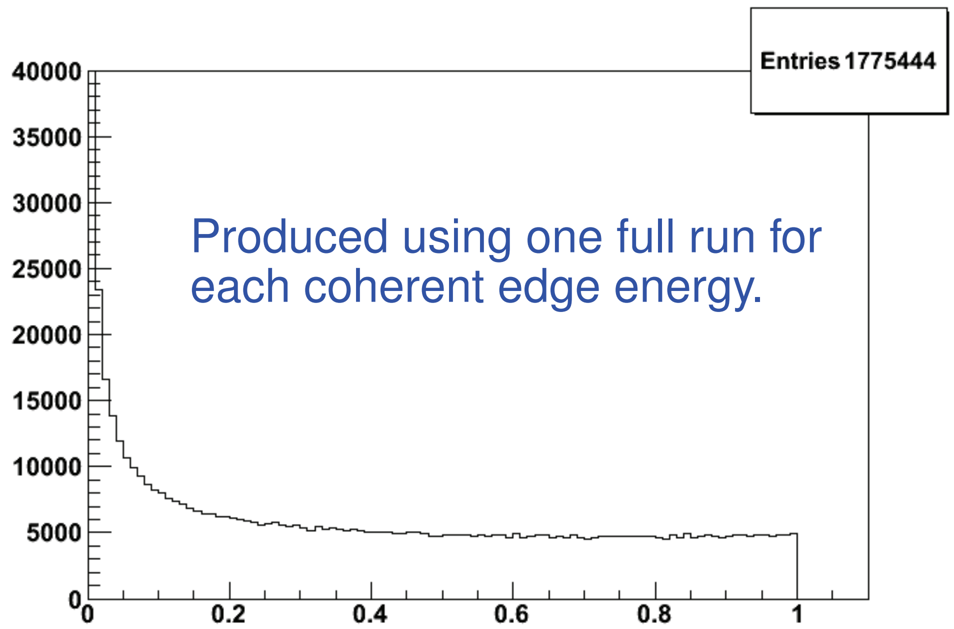
Kinematic Fitting

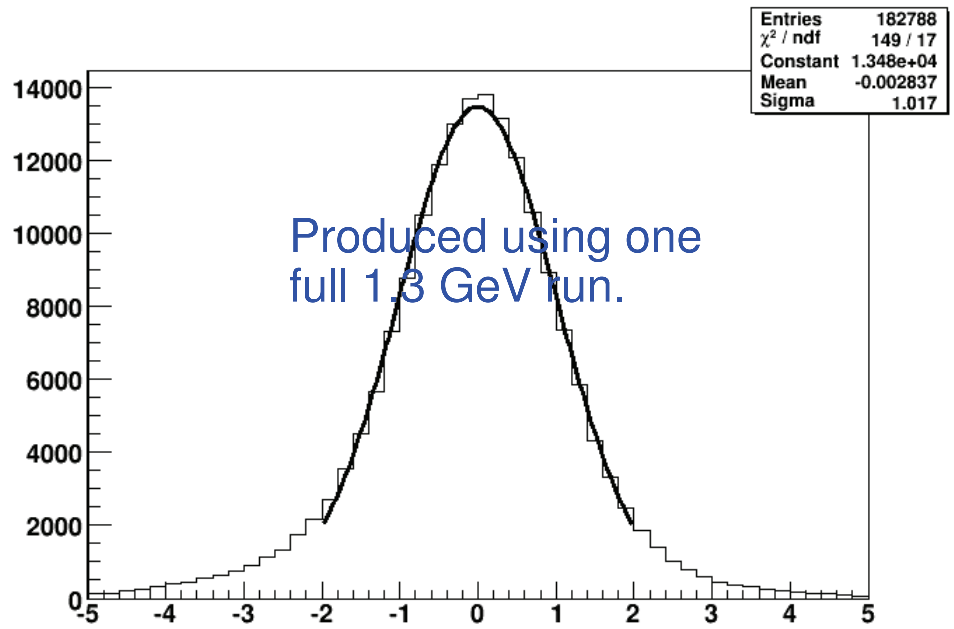
- Enforces energy-momentum conservation.
- Varies final state measurements (4-vectors) within resolution limits (errors) of said measurements.
  - Errors (as calculated by the reconstruction code) are extracted from data and used to build the tracking covariance matrix (event by event).
  - Errors are then scaled to actual run-dependent resolutions to make the full covariance matrix.

$$\begin{pmatrix} \sigma^2_{E_\gamma} & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & C_1^{pp} & C_1^{p\lambda} & C_1^{p\phi} & \cdots & 0 & 0 & 0 \\ 0 & C_1^{p\lambda} & C_1^{\lambda\lambda} & C_1^{\lambda\phi} & \cdots & 0 & 0 & 0 \\ 0 & C_1^{p\phi} & C_1^{\lambda\phi} & C_1^{\phi\phi} & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & C_k^{pp} & C_k^{p\lambda} & C_k^{p\phi} \\ 0 & 0 & 0 & 0 & \cdots & C_k^{p\lambda} & C_k^{\lambda\lambda} & C_k^{\lambda\phi} \\ 0 & 0 & 0 & 0 & \cdots & C_k^{p\phi} & C_k^{\lambda\phi} & C_k^{\phi\phi} \end{pmatrix}$$

Kinematic Fitting

- Quality of the fit is judged by examining the confidence level and pull distributions.
- Every diagonal element of the covariance matrix has a pull value associated with it.
- Systematics affect pull means (μ's), errors contained in the covariance matrix affect pull widths (σ's) and the slope of confidence level plots.

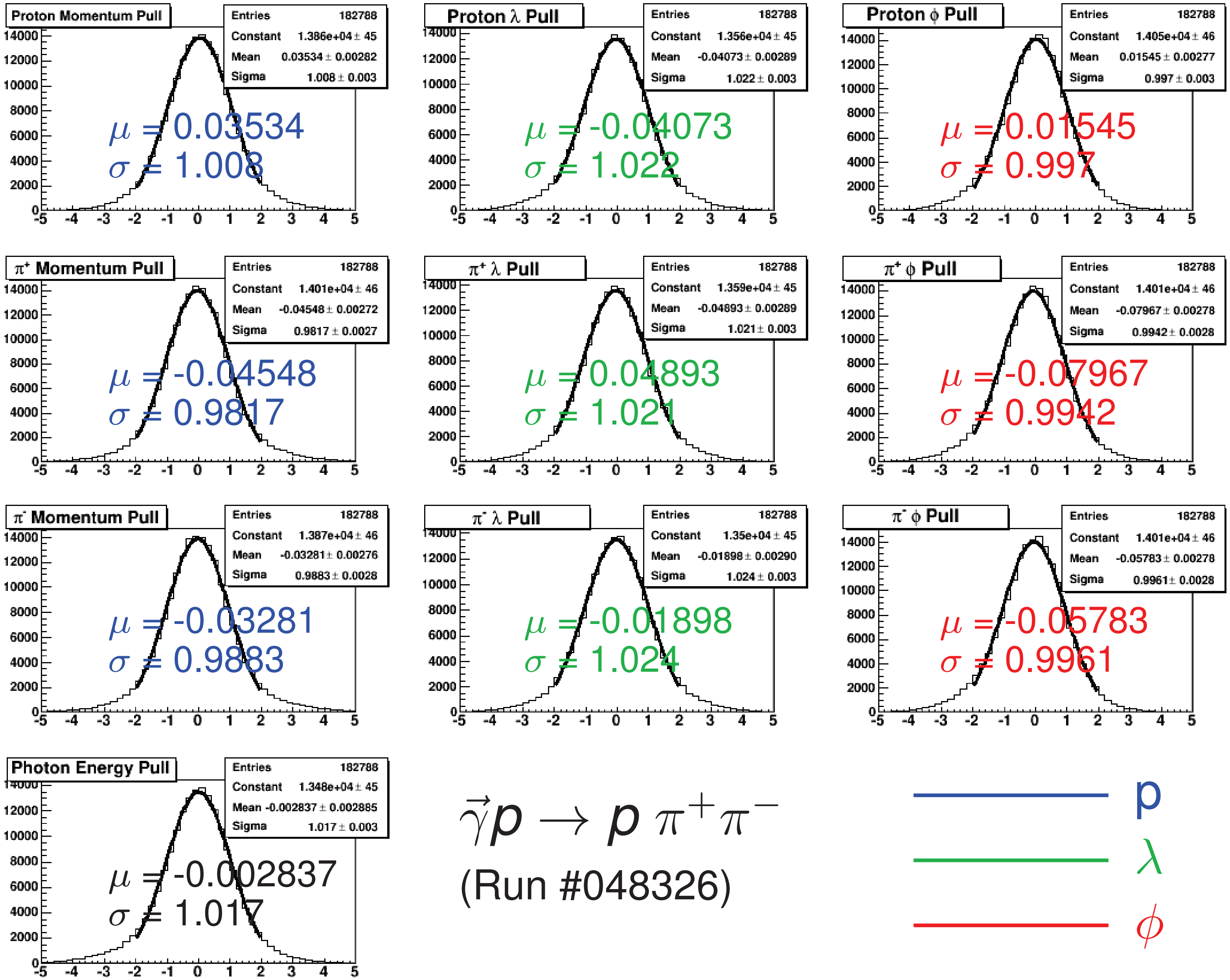




Momentum Corrections and Tagger Sag

- Momentum Corrections:
  - Kinematic fitter is highly sensitive to systematic effects $\rightarrow$ good tool for determining momentum corrections.
  - Corrections are needed to account for variations in torus $\vec{B}$ field and misalignments of Drift Chambers.
  - Determined by examining momentum pull distributions for the proton for $\vec{\gamma}p \rightarrow p\,\pi^+\pi^-$ fits/events.
  - Correction factor and pulls binning:
    - 7 bins in θ_{lab} ($10^\circ \leq \theta_{lab} \leq 70^\circ$), 18 bins in ϕ_{lab}
    - 6 bins in momentum (0.2 GeV $\leq |\vec{p}| \leq 1.7$ GeV)
- Tagger Sag
  - Occurs due to a physical sagging of the support structure for Tagger Hodoscope.
  - Affects the determination of photon energy.
  - Has been seen in several runs in Hall B.
  - Requires energy-dependent correction $\rightarrow$ M. Dugger, C. Hanretty, CLAS-Note 2009-030

Check the pull distributions

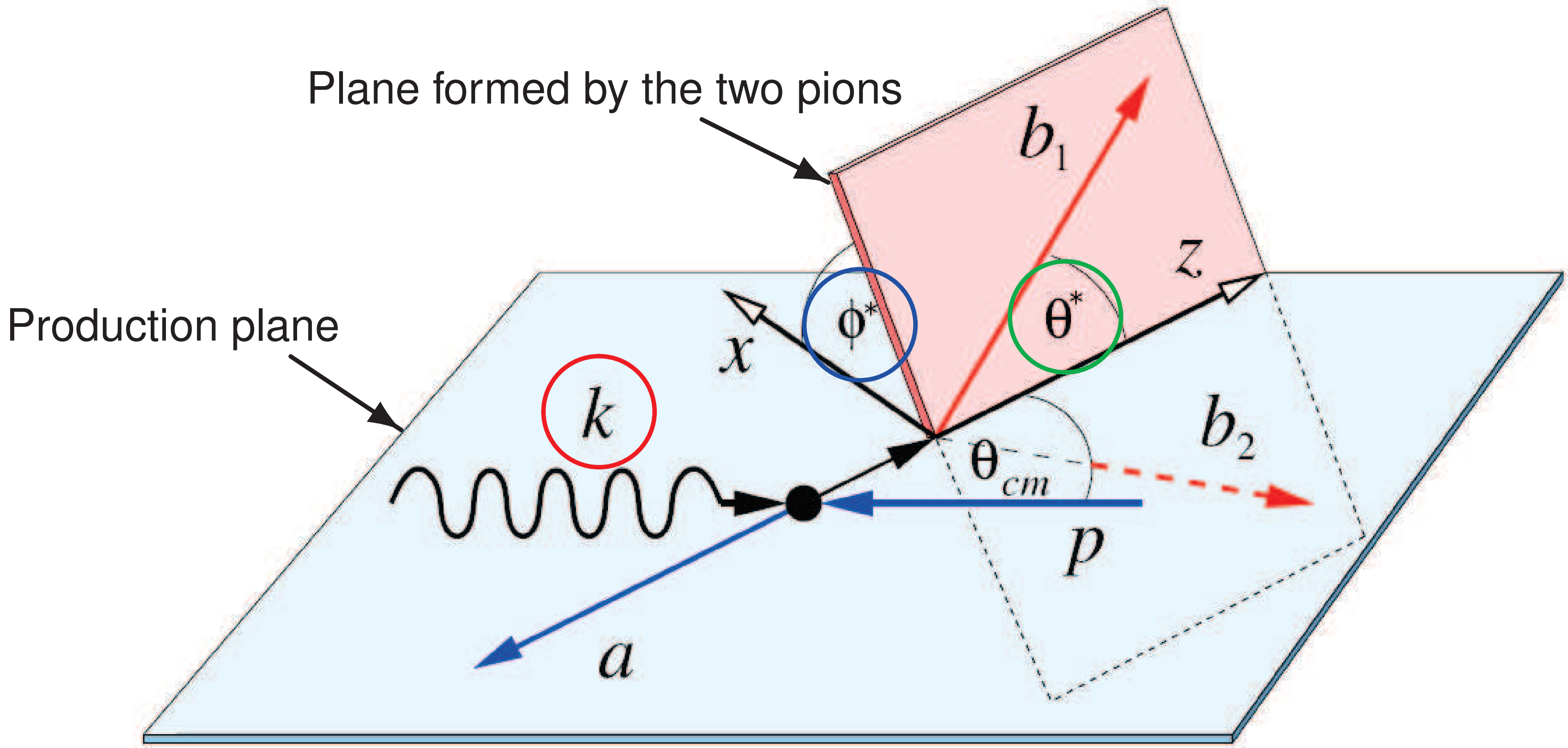


$\vec{\gamma} p \to p\, \pi^+ \pi^-$

(Run #048326)

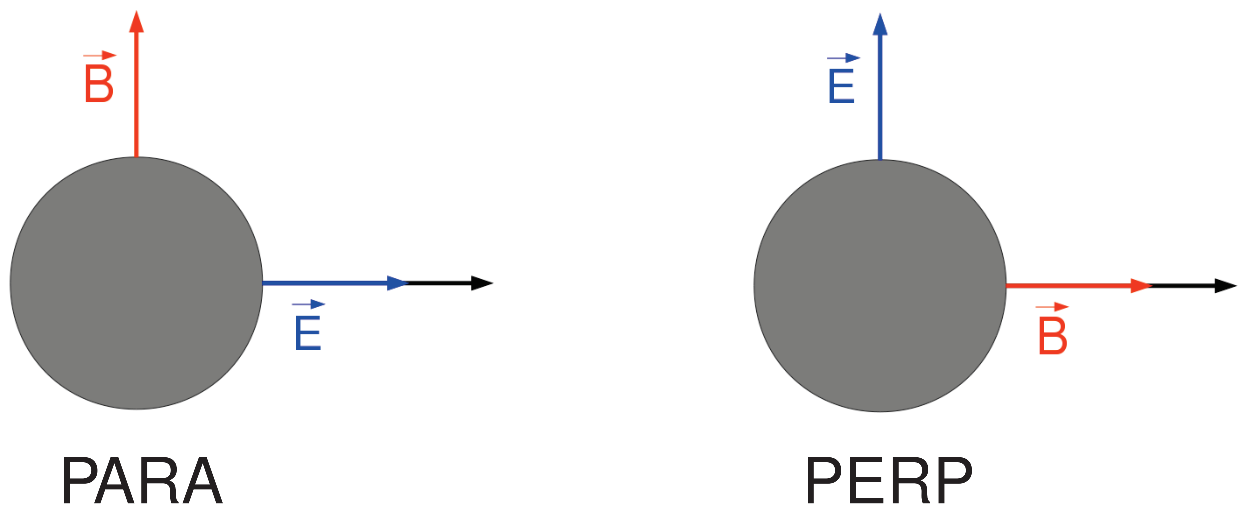
Kinematics

- The analysis of the (double-meson) $\vec{\gamma}p \to p\,\pi^+\pi^-$ channel requires the use of 5 independent kinematic variables: ($m_{p\pi}$ or $m_{\pi\pi}$), $\cos\theta_p^{CM}$, k, $\cos\theta^*_{\pi^+}$, $\phi^*_{\pi^+}$.



Extracting I^s and I^c: ϕ-distributions

- Three types of (photon) polarization settings were used:
  - PARA
  - PERP
  - AMO



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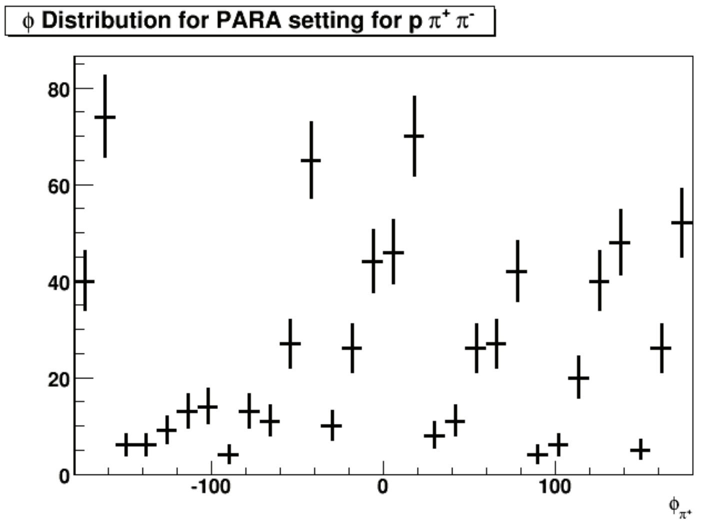
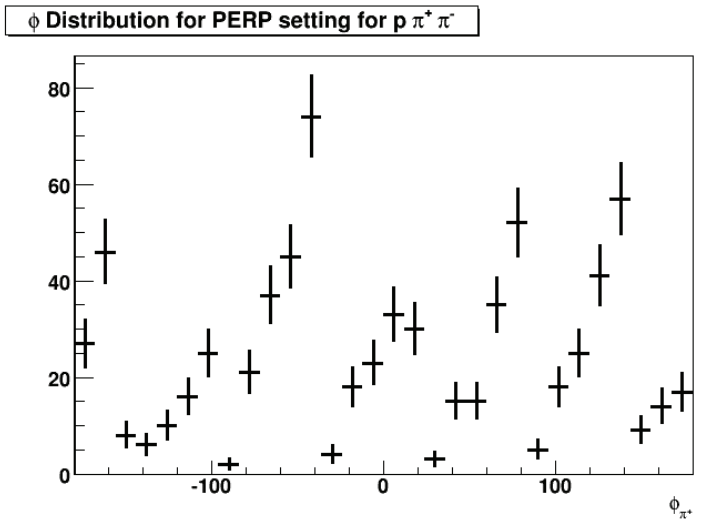
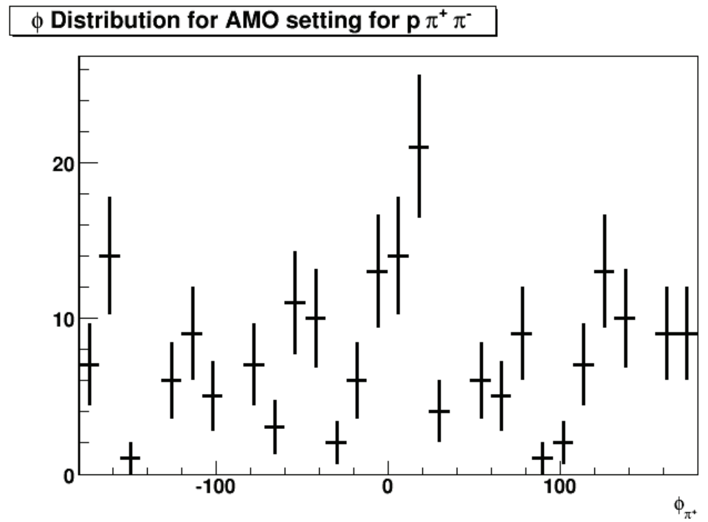
- The polarization of the photon beam breaks the usual ϕ symmetry.
- Events are plotted as a function of lab angle ϕ_{π^+} for each polarization setting.







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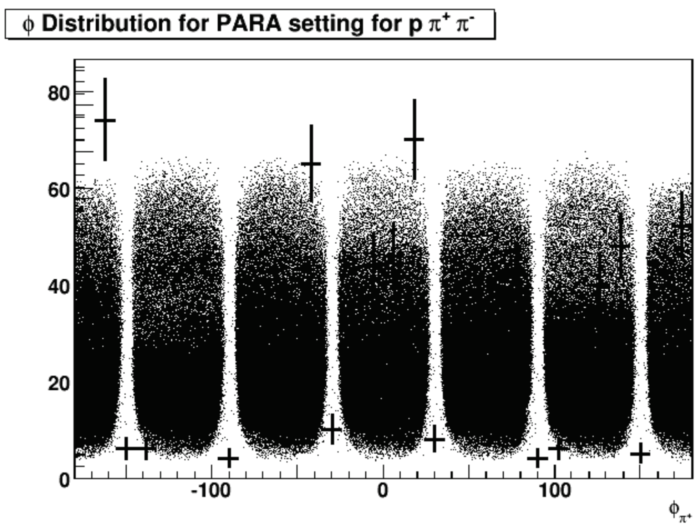
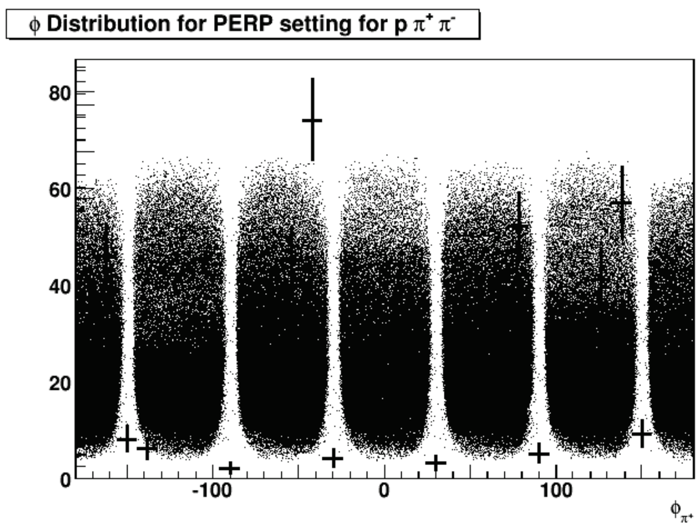
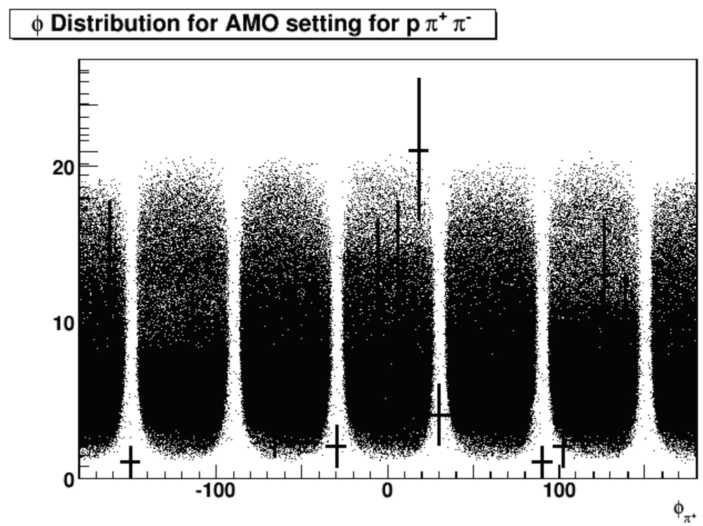
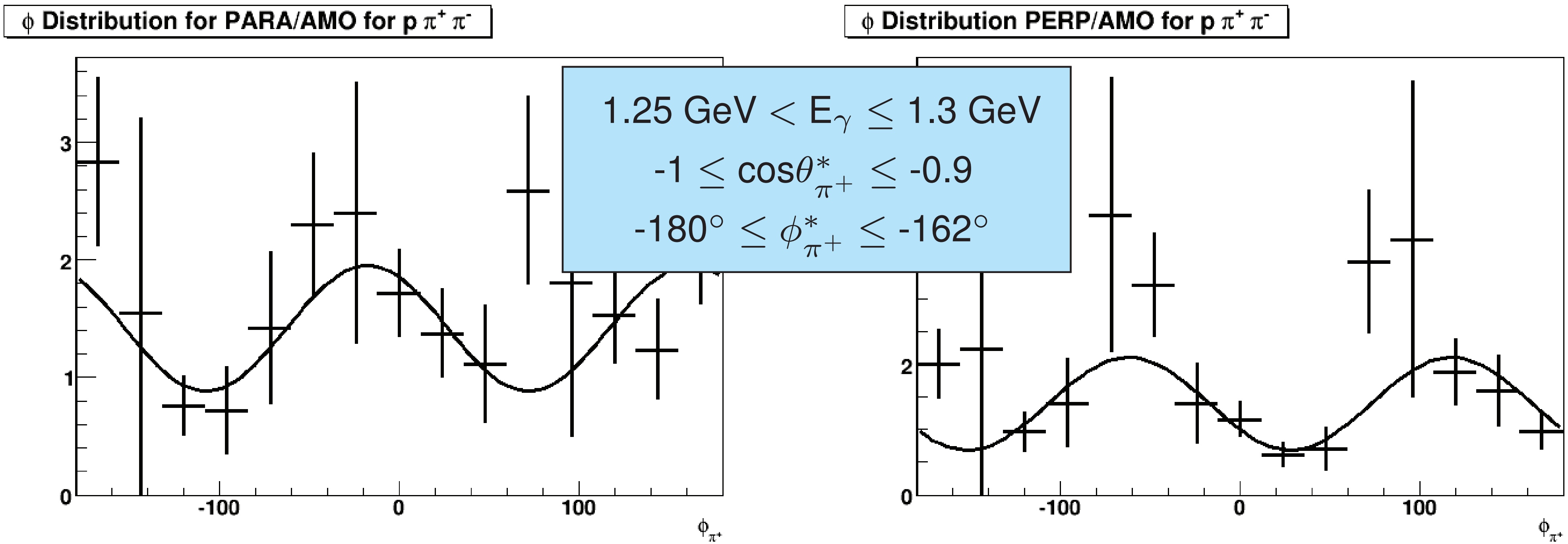






Extracting I^s and I^c: Method 1

- Characteristic: use of AMO ϕ-distributions.
- To remove (divide out) the effects of the experimental setup and acceptance, the ϕ-distributions for PARA and PERP are divided by the AMO ϕ-distributions (for matching bin combinations).
- Fit to $I = I_0 \{ 1 + \delta_l [\mathbf{I}^s \sin(2\beta) + \mathbf{I}^c \cos(2\beta)] \}$
  where $\beta = \phi_{lab} + \phi_{polarization}$



Extracting I^s and I^c: Method 2

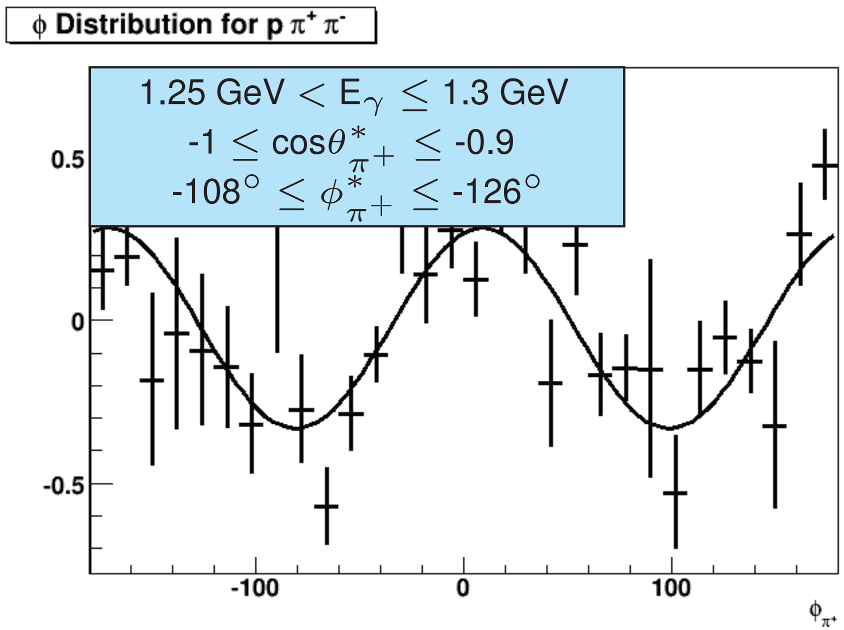
- Characteristic: No use of AMO data $\Rightarrow$ uses asymmetry between PARA and PERP.
- Asymmetry between PARA and PERP is formed for matching bin combinations.
- Fit to:

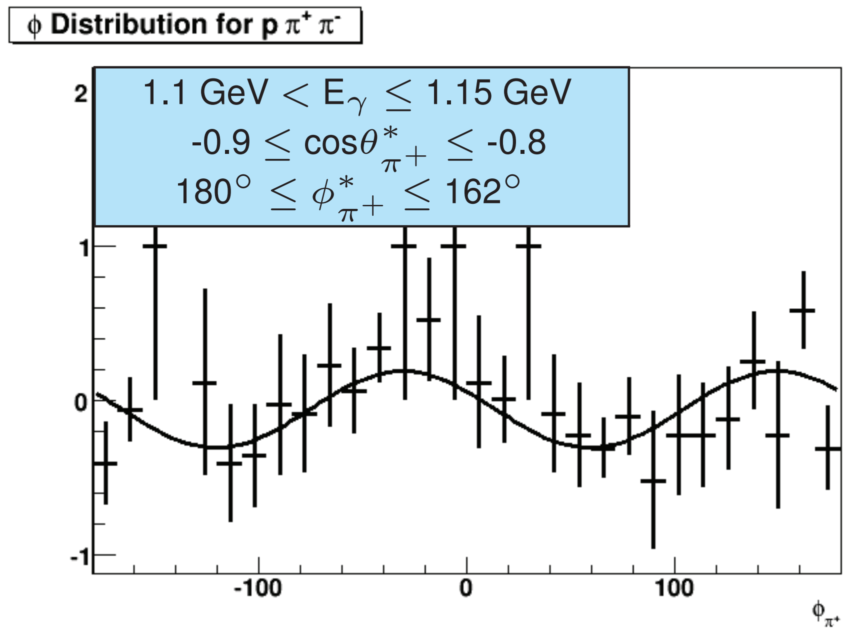
$$A(k,\cos\theta^*_{\pi^+},\phi^*_{\pi^+}) = \frac{PARA - PERP}{PARA + PERP} = \frac{I_{PARA} - I_{PERP}}{I_{PARA} + I_{PERP}}$$

Extracting I^s and I^c: Method 2

- Characteristic: No use of AMO data $\Rightarrow$ uses asymmetry between PARA and PERP.
- Asymmetry between PARA and PERP is formed for matching bin combinations.
- Fit to:

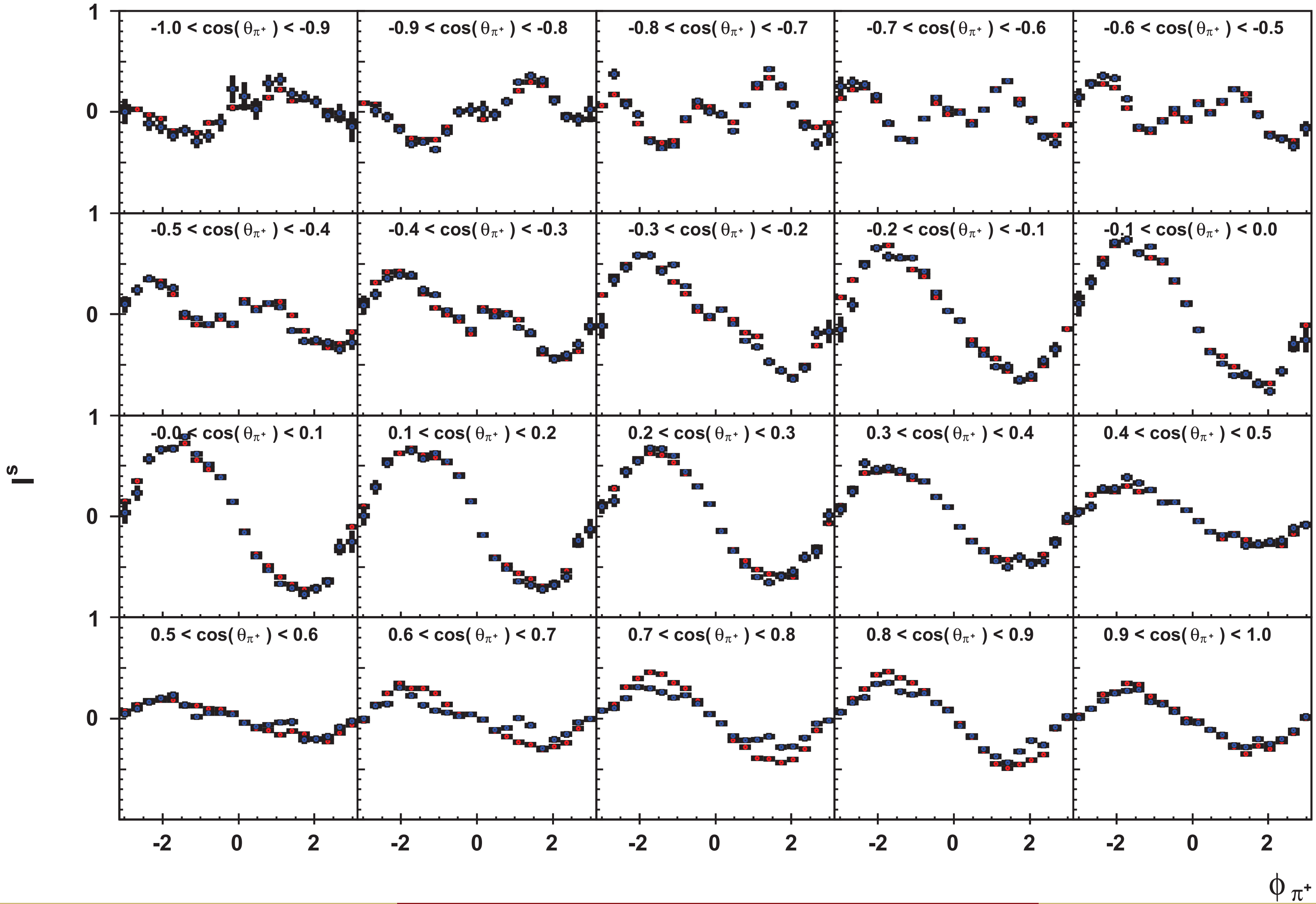
$$A(k,\cos\theta^*_{\pi^+},\phi^*_{\pi^+}) = \frac{(\delta_l^{PARA} + \delta_l^{PERP})\ \mathbf{I^s}\ \sin(2\beta) + (\delta_l^{PARA} + \delta_l^{PERP})\ \mathbf{I^c}\ \cos(2\beta)}{2 + (\delta_l^{PARA} - \delta_l^{PERP})\ \mathbf{I^s}\ \sin(2\beta) + (\delta_l^{PARA} - \delta_l^{PERP})\ \mathbf{I^c}\ \cos(2\beta)}$$





Preliminary Measurement: I^s

$1.2\ \text{GeV} < E_\gamma < 1.25\ \text{GeV}$



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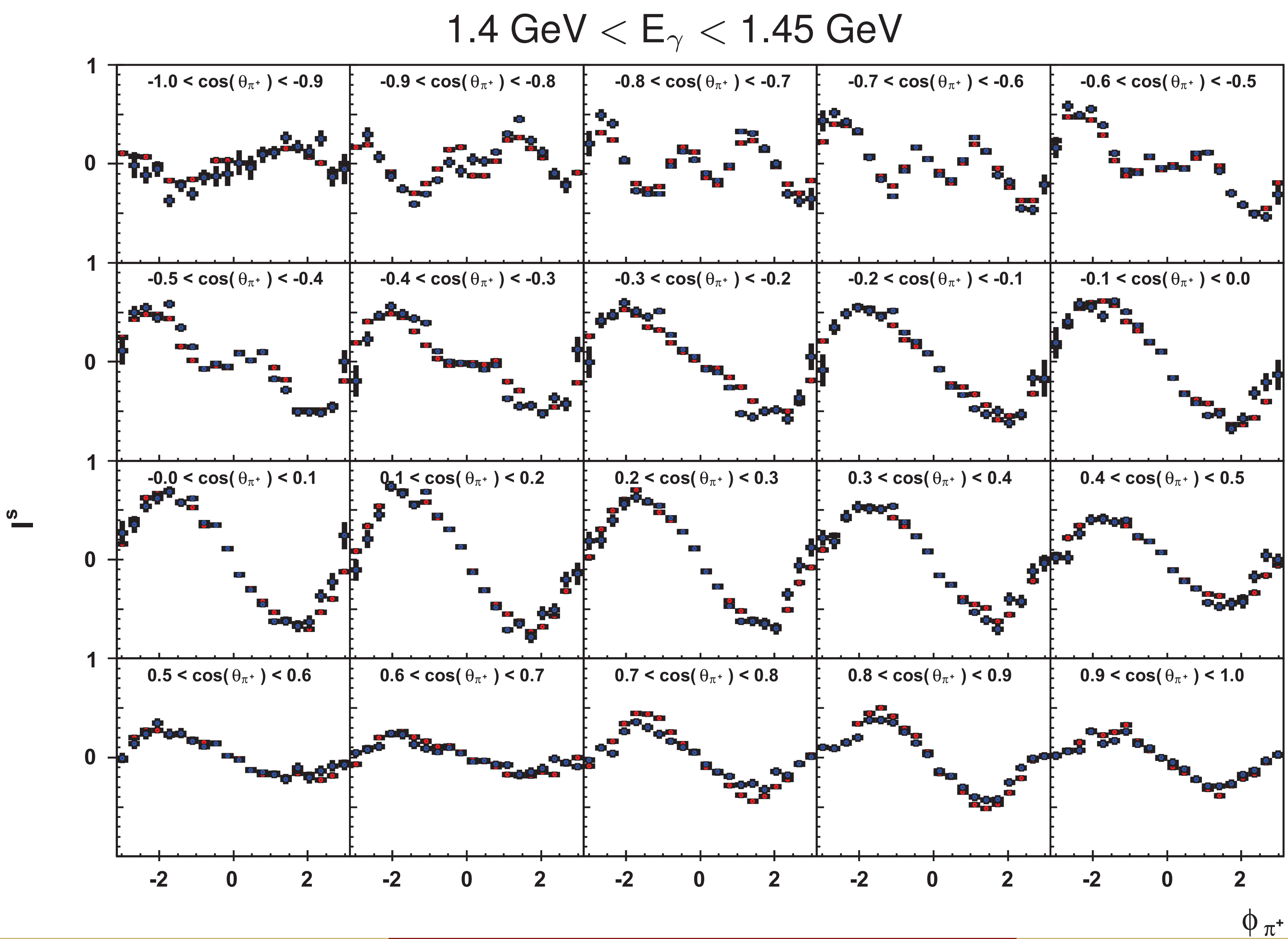
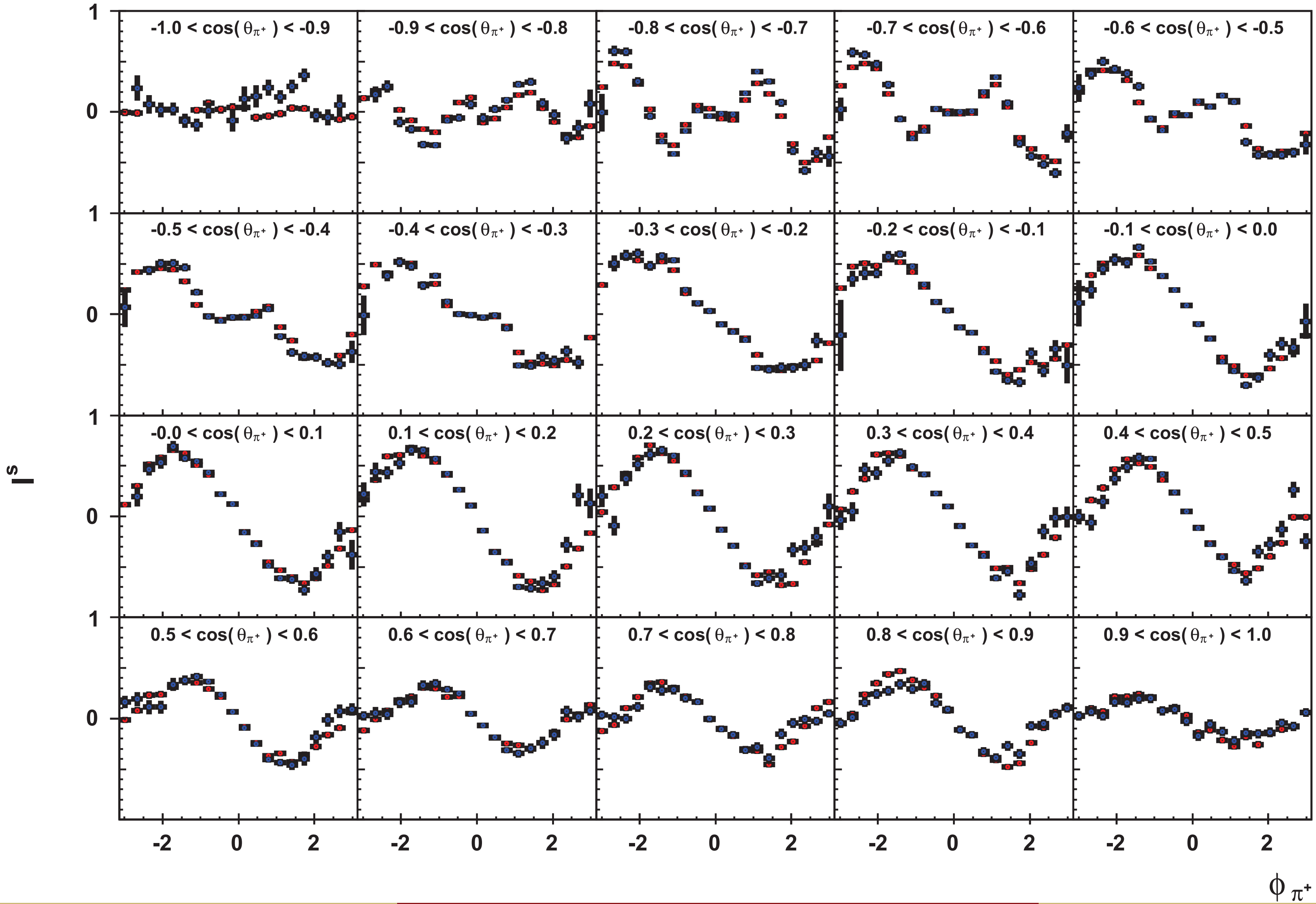
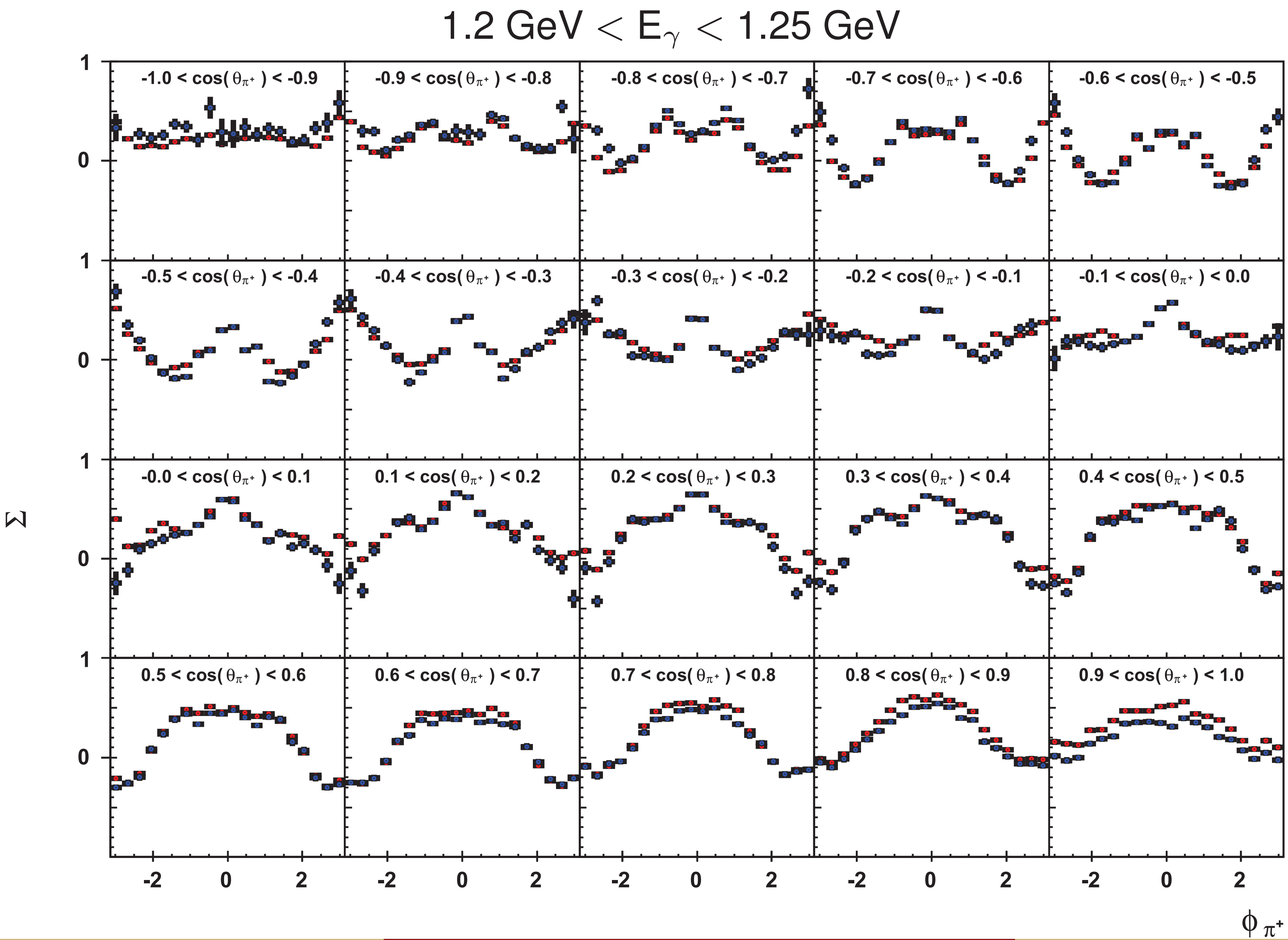


Preliminary Measurement: I^s

1.6 GeV $< E_\gamma <$ 1.65 GeV

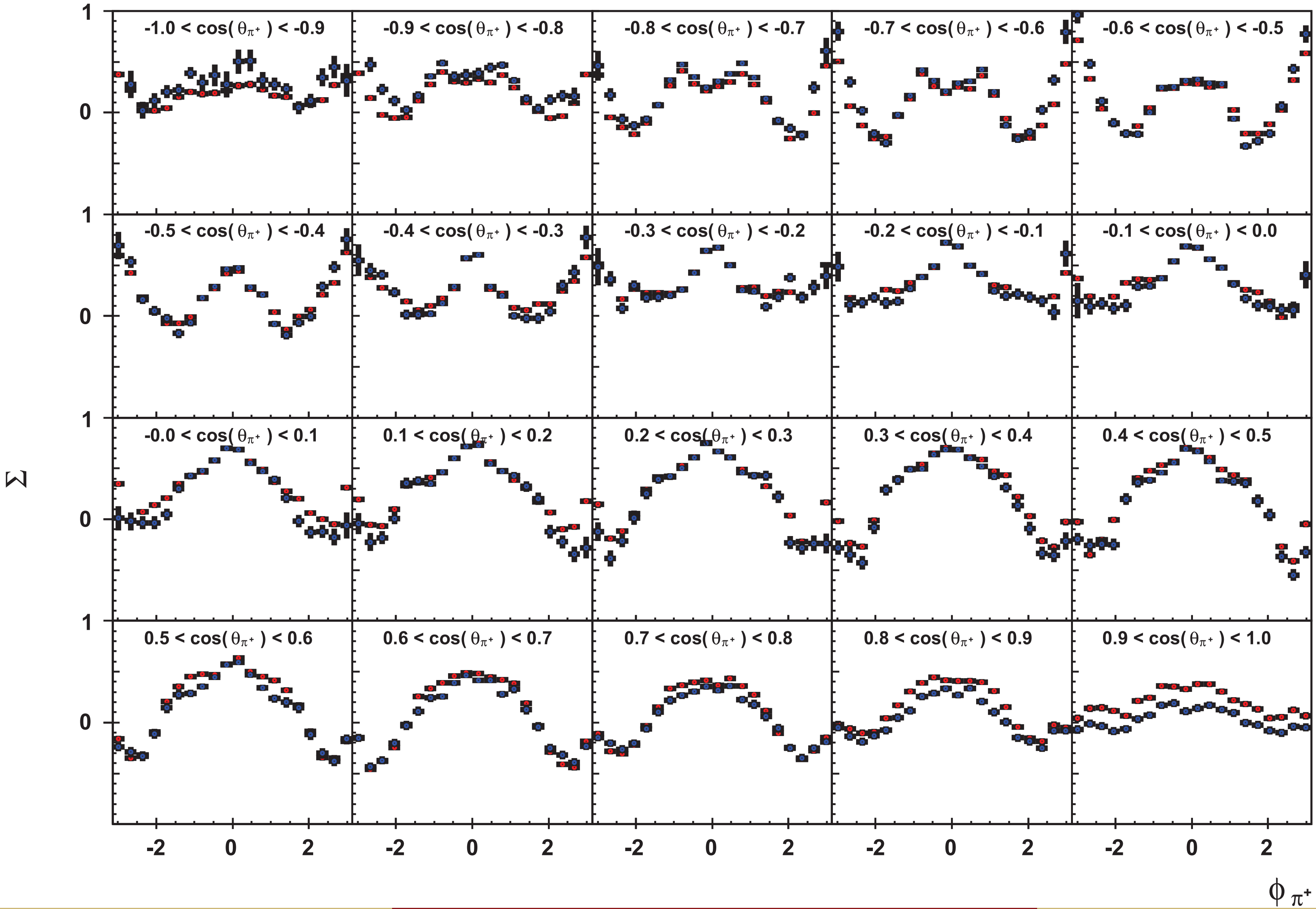


Preliminary Measurement: I^c



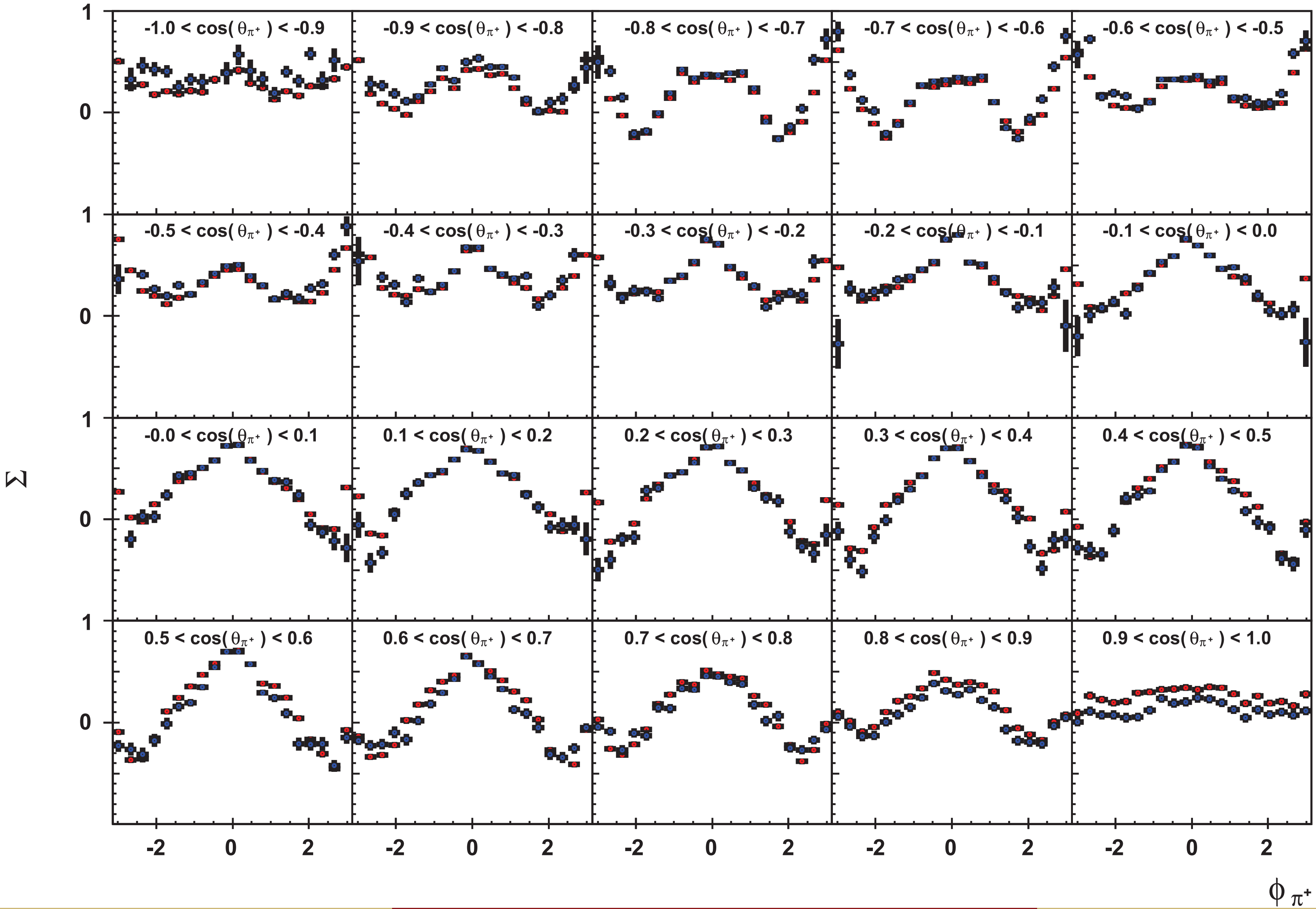
Preliminary Measurement: I^c

1.4 GeV $< E_\gamma <$ 1.45 GeV



Preliminary Measurement: I^c

1.6 GeV $< E_\gamma <$ 1.65 GeV



Summary

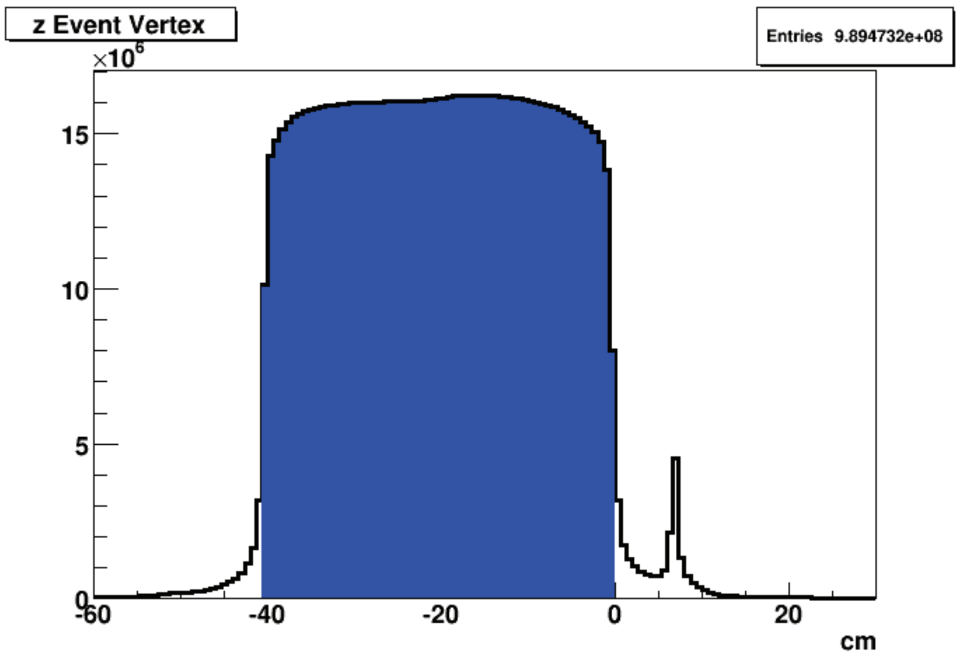
- Hadrons form the basis of most matter.
- These hadrons are systems containing quarks and gluons.
- Through the study of these systems when excited, the internal dynamics of the hadron may be demystified.
- Polarization observables are sensitive to these dynamics.
- Measured for $\vec{\gamma} p \to p\, \pi^+ \pi^-$:
  - I^c (first for a two-charged-pion final state)
  - I^s (first for a two-pion final state)
- These observable measurements as well as future measurements will aid in the refinement of CQMs as well aid in the revelation of the internal dynamics of the baryon.
- The FROST experiment will provide access to all 15 observables for a double-meson final state.

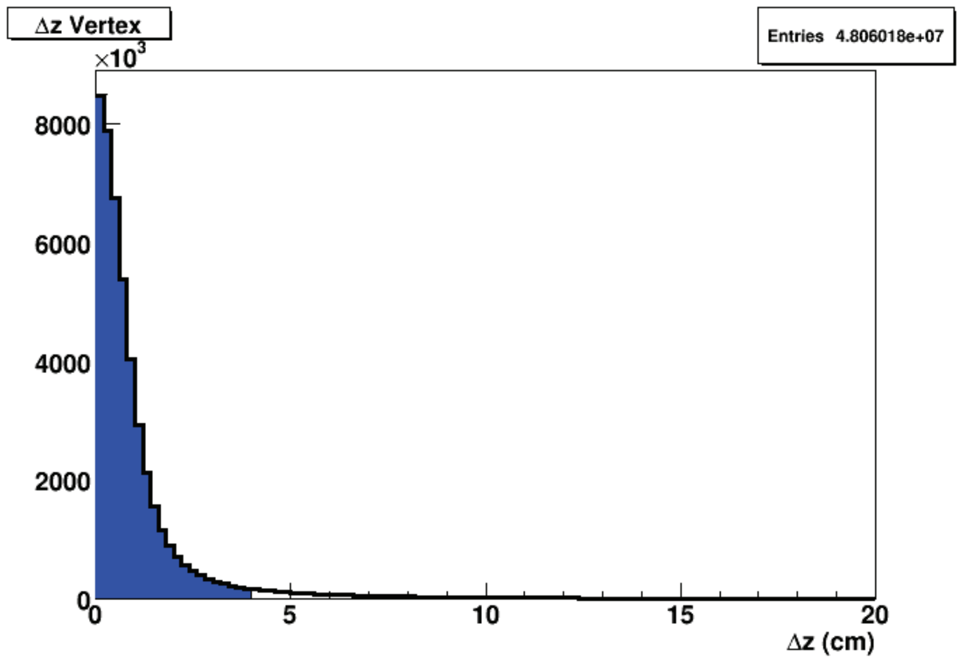
END

Backup Slides

Cuts

- Vertex
  - -40 cm $< z_{vertex} <$ 0 cm (for all detected particles)
  - All z-vertices must be within 4 cm of each other.
  - Cut on (event) x,y-vertex requiring it originates in the target cell.
  - 0 ns $< t_{vertex} <$ 20 ns (for all detected particles)





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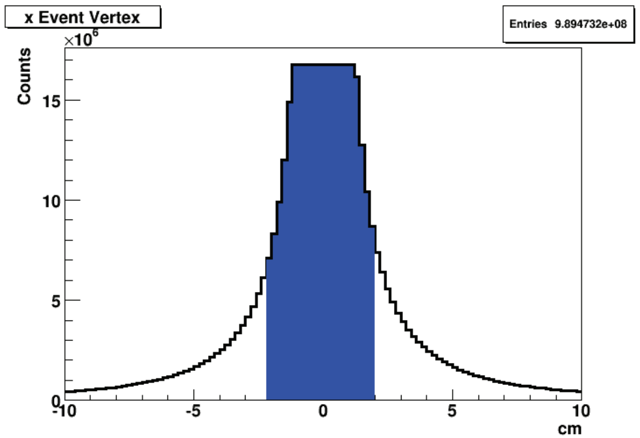

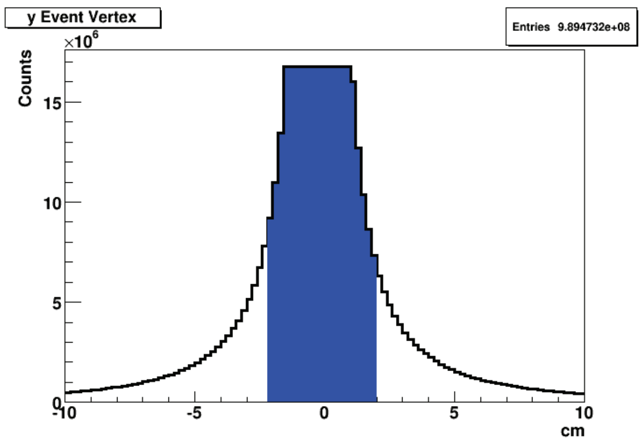

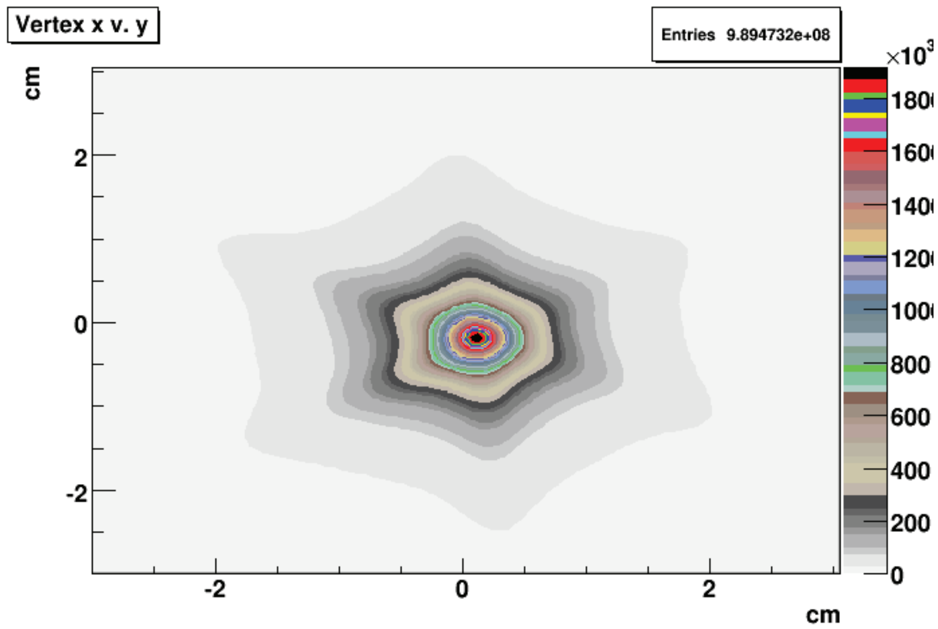

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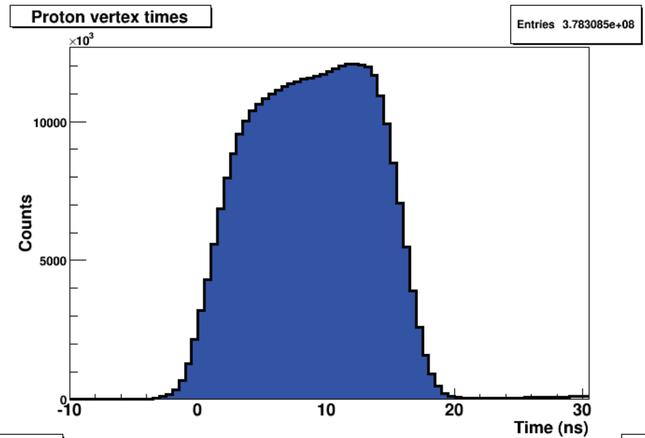
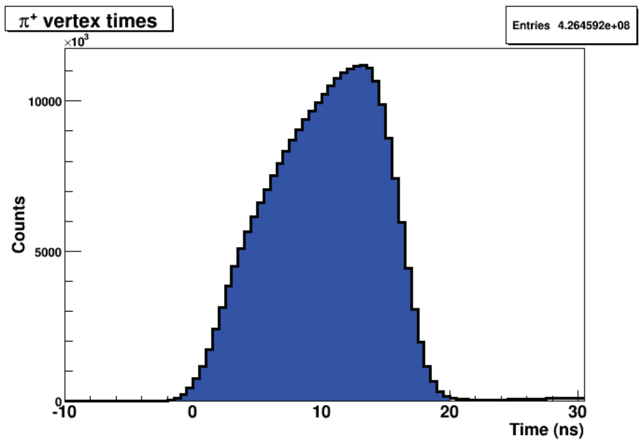
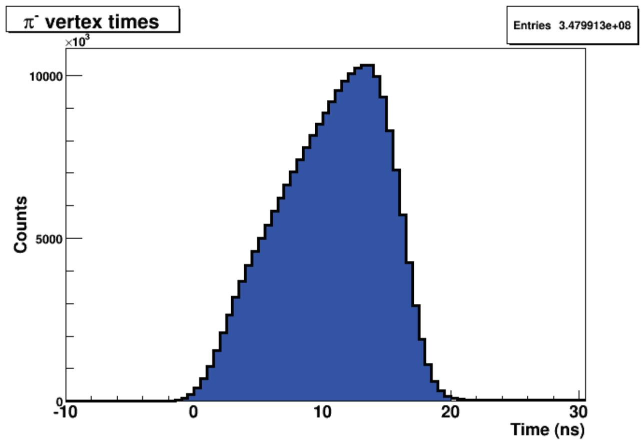
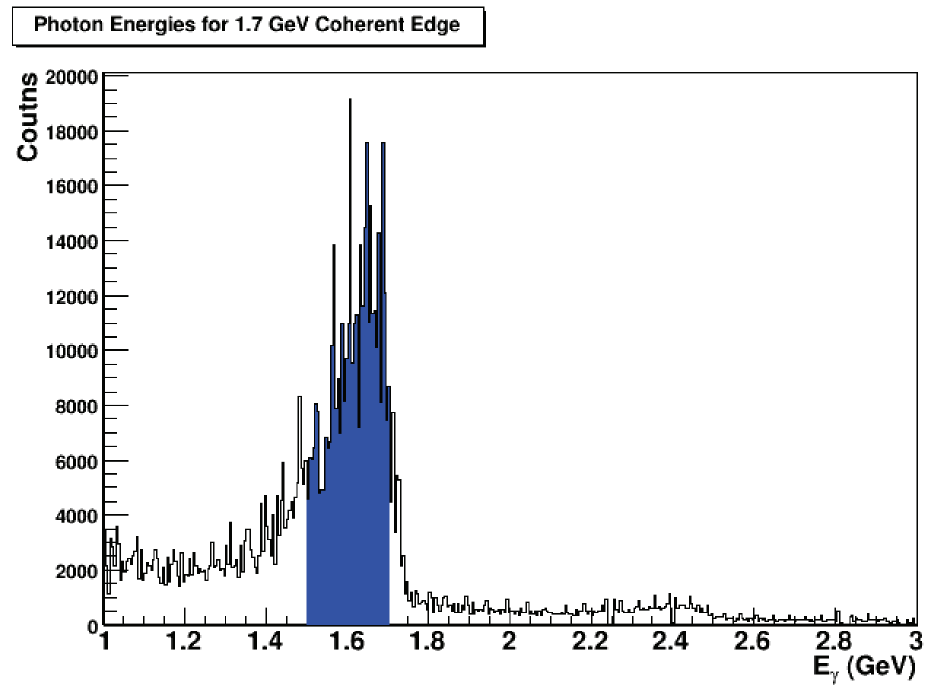






Cuts

- Photon Energy
  - 1.1 GeV $< E_\gamma <$ 1.3 GeV
  - 1.3 GeV $< E_\gamma <$ 1.5 GeV
  - 1.5 GeV $< E_\gamma <$ 1.7 GeV
  - 1.7 GeV $< E_\gamma <$ 1.9 GeV
  - 1.9 GeV $< E_\gamma <$ 2.1 GeV
- Momentum and Angle
  - $p_{proton} >$ 320 MeV/c
  - $p_{\pi^\pm} >$ 125 MeV/c
  - $\theta > 10^\circ$
  - $\theta_{\pi^\pm} < 120^\circ$
- Other Cuts
  - Confidence level cut of 5%.
  - ‘ngrf’ = 1 for all particles.
  - ‘tagrid’ same for all particles.



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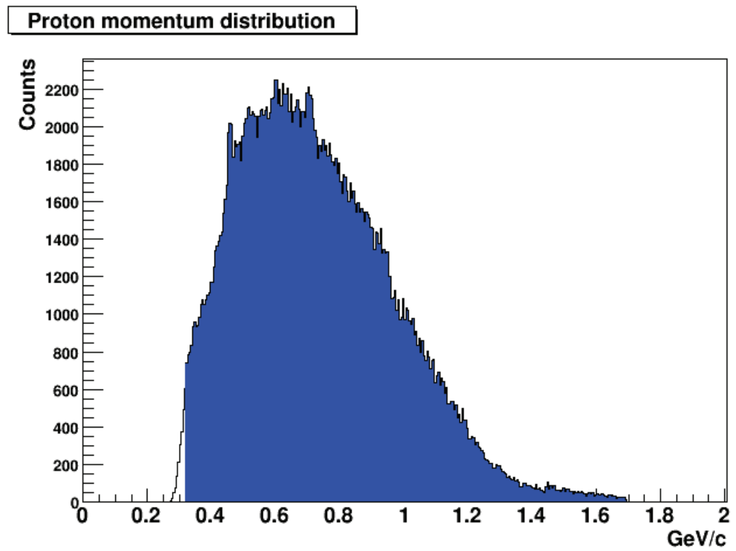

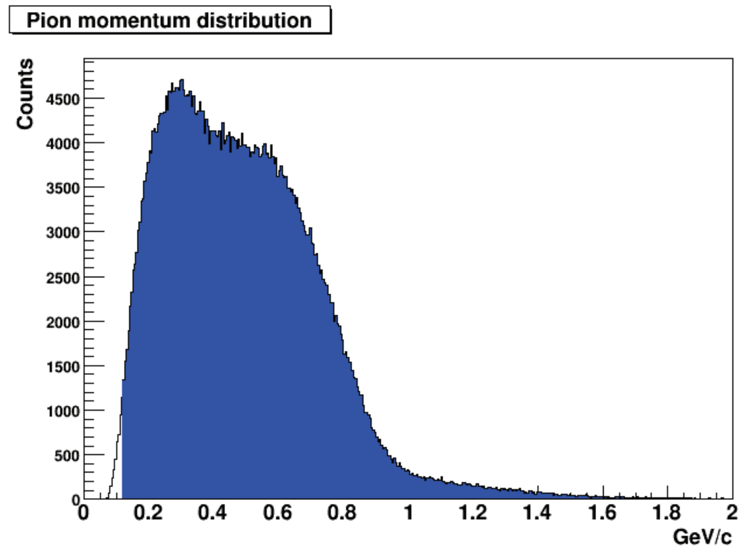

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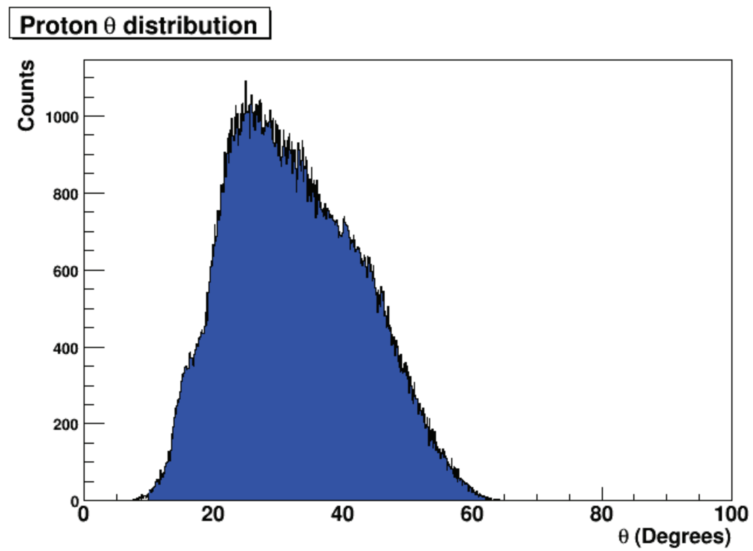

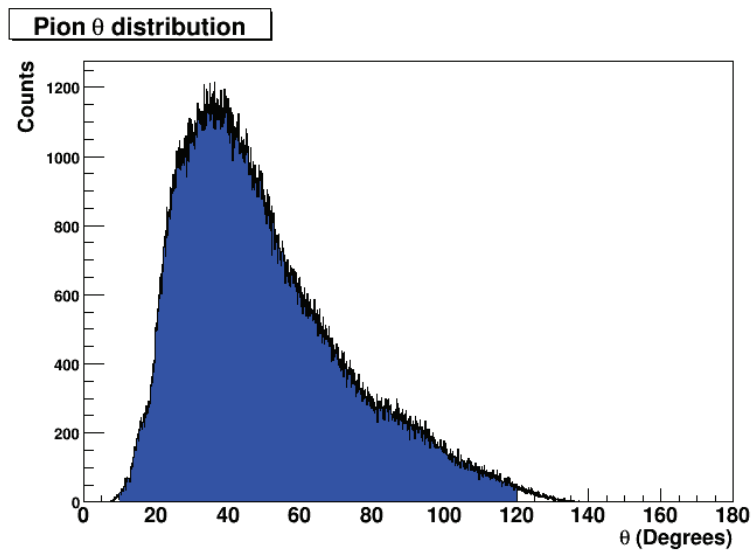

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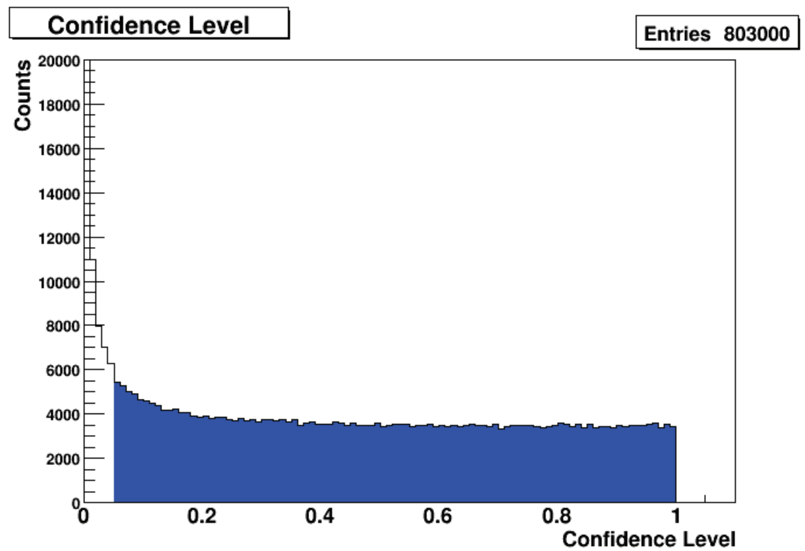
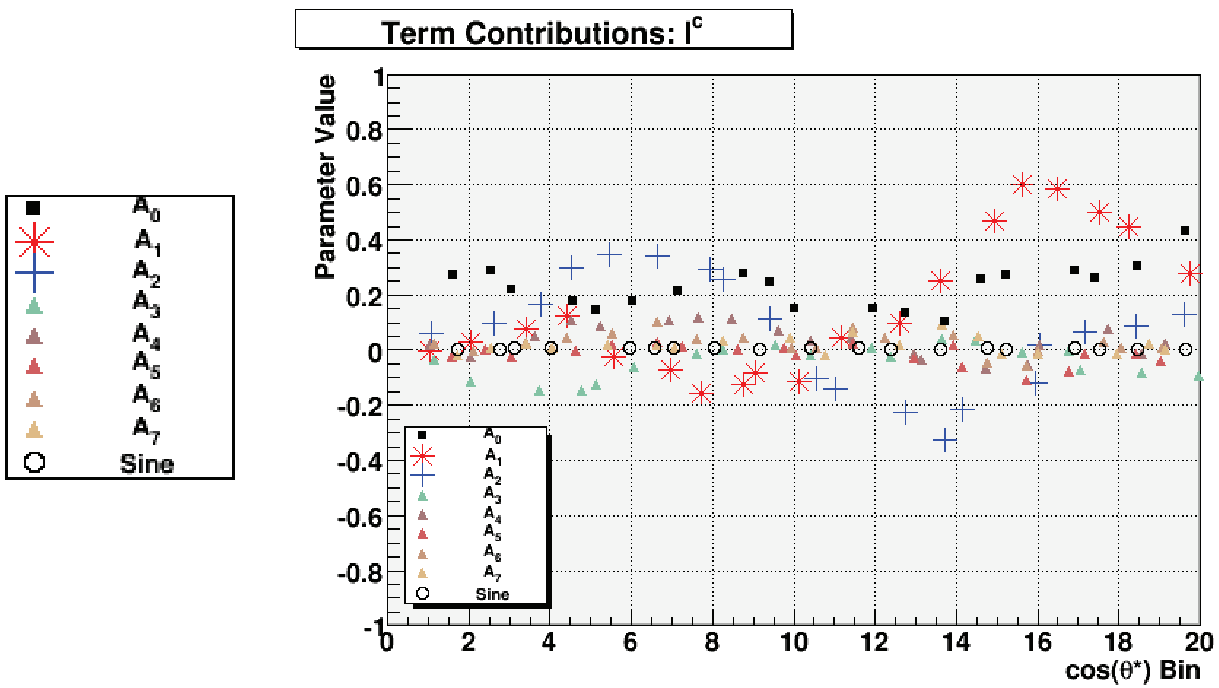


Examination of the behavior of I^s and I^c

- The behavior of the measured observables is predicted to be either even (I^c) or odd (I^s) as a function of ϕ^*.
- To check the behavior, measurements were fit with expansions of sine and cosine for each cosθ^* bin.
- Contributions from the different terms were examined.

$$f(\phi^*) = A_0 + A_1 cos(\phi^*) + A_2 cos(2\phi^*) + A_3 cos(3\phi^*) + A_4 cos(4\phi^*) + A_5 cos(5\phi^*) + A_6 cos(6\phi^*) + A_7 cos(7\phi^*) + A_8 sin(\phi^*)$$



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$$+ A_5 sin(5\phi^*) + A_6 sin(6\phi^*) + A_7 sin(7\phi^*) + A_8 cos(\phi^*)$$