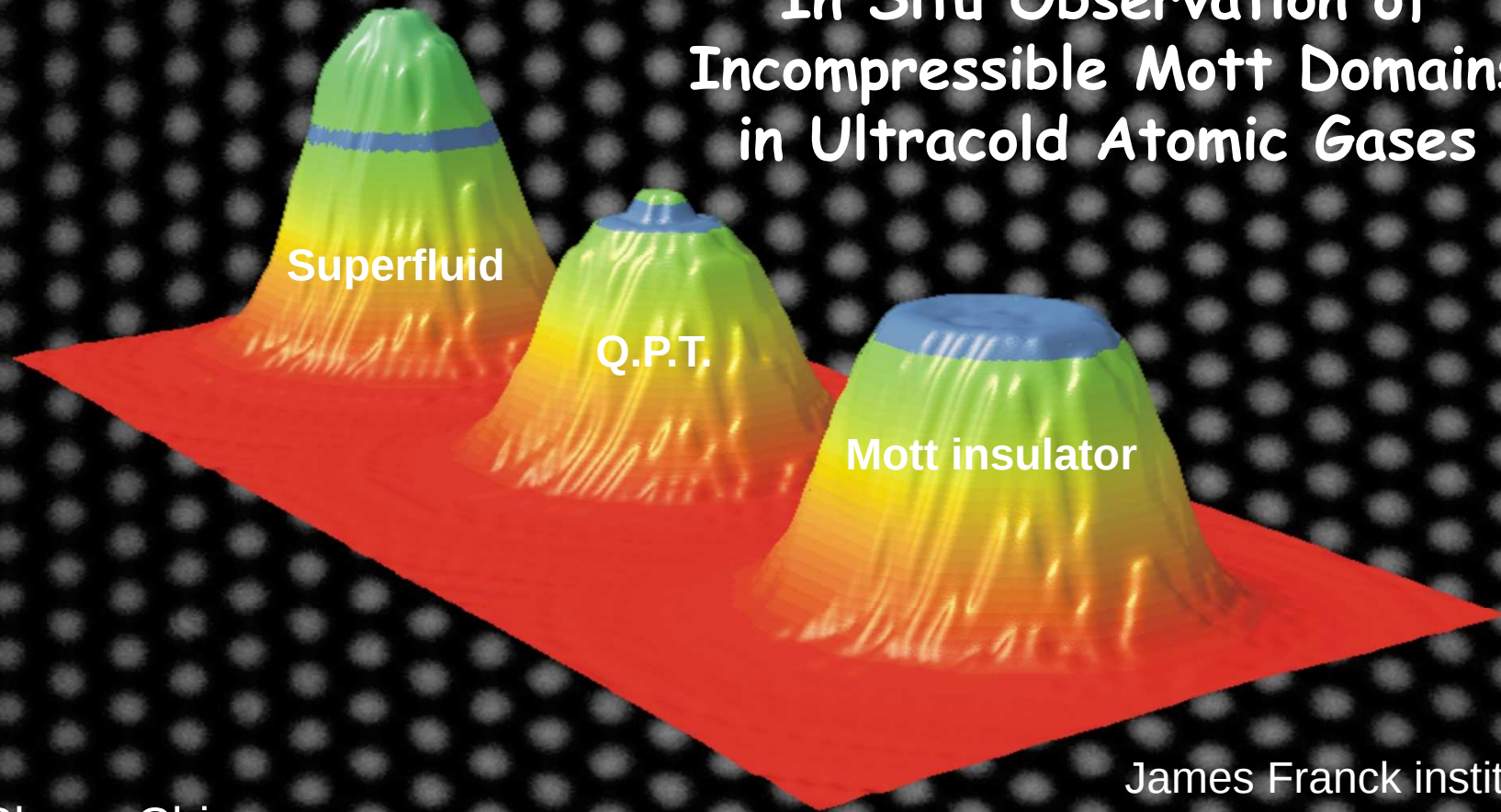


Colloquium, Physics Department at the University of Virginia, 9/02/2011

Having your cake and seeing it too -

In Situ Observation of Incompressible Mott Domains in Ultracold Atomic Gases



Cheng Chin

James Franck institute
Physics Department
University of Chicago

Funding:



the David &
Lucile Packard
FOUNDATION



Cake Recipe

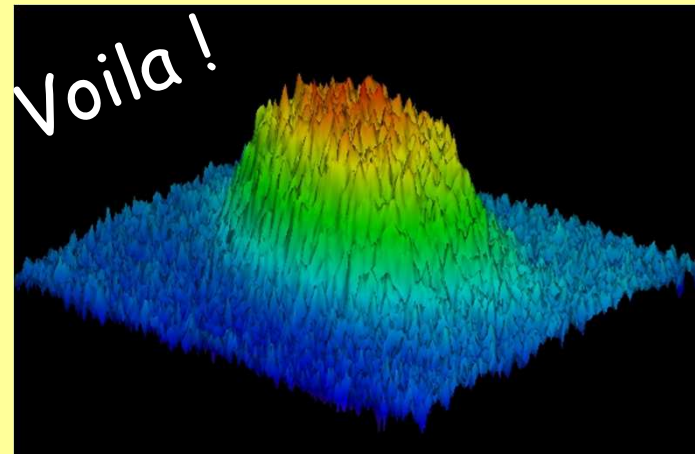
(0th order approximation)

1. Mix flour and baking powder
2. Heat it to 350 F in an oven
3. Bake for 30~40 minutes



Our "Ice Cream" Cake Recipe

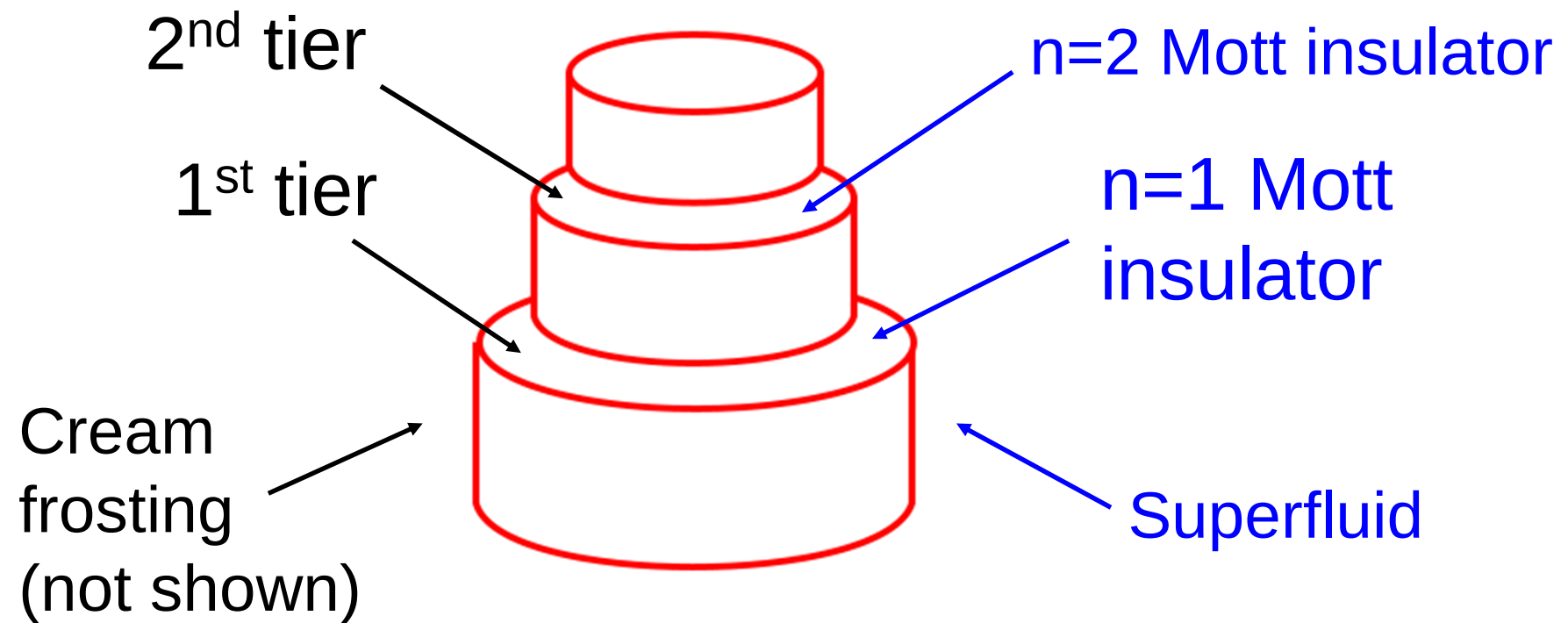
1. Prepare some **atoms** in a **bowl**
2. Cool it to 10 nano-K
3. "**Bake**" for 200ms



Comments

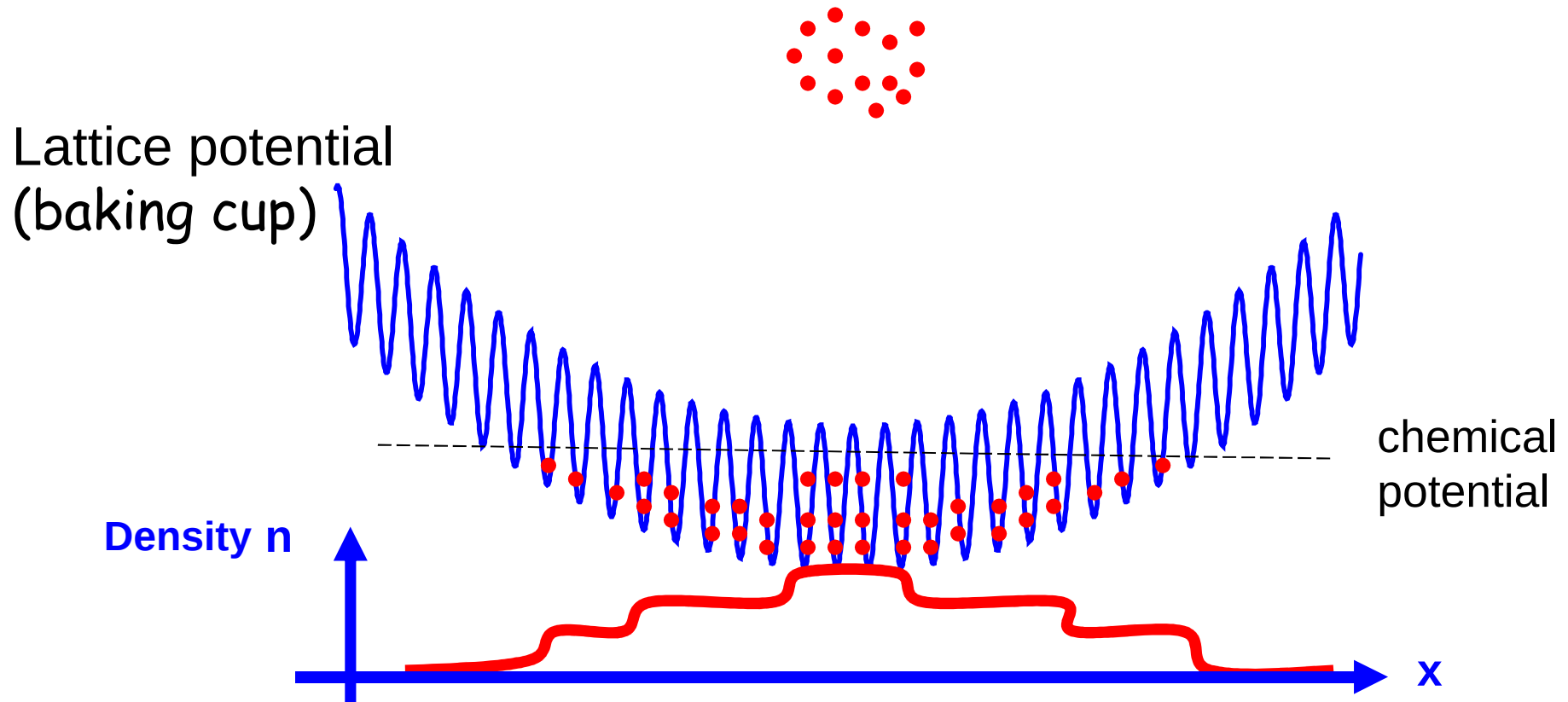
- We are not the first one to make the cake, but the first one to see it.
- We can see it because we make only one cake.
- All physics occurs in the baking.

Cake Terminology



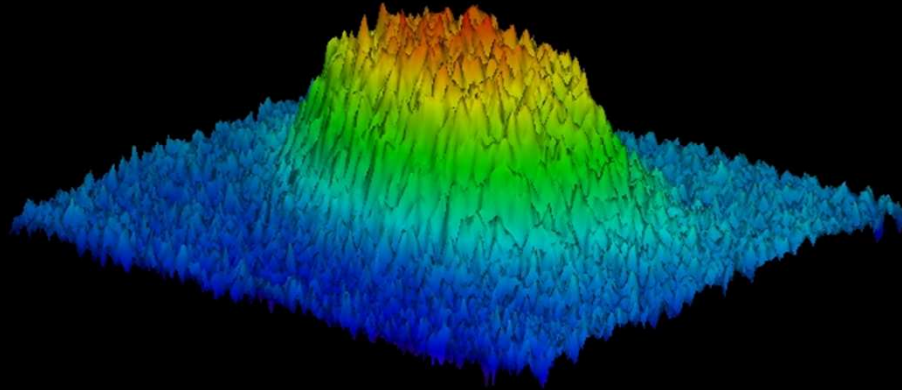
How does cake form in the lab?

(*Bose-Hubbard process*)



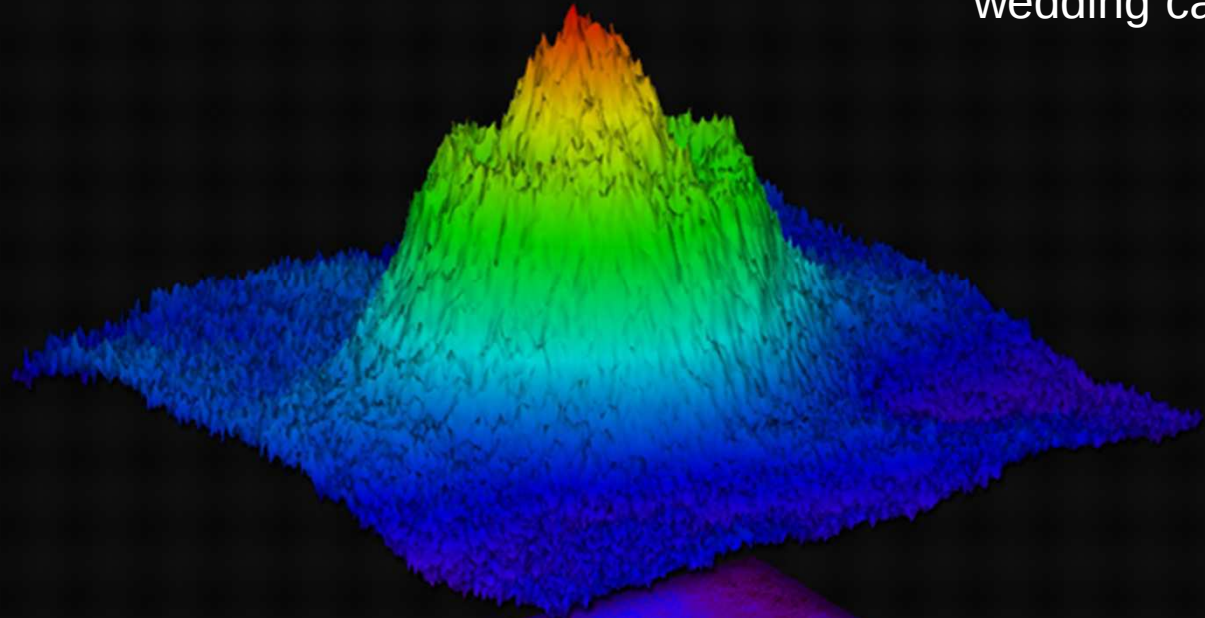
N=8000

Poor man's
wedding cake



N=13000

Middleclass
wedding cake



Bose-Hubbard model (*Fisher et al., PRB 1989, Greiner et al., Nature 2002*)

$$\hat{H} = -t \sum_{\langle i,j \rangle} (\hat{b}_i^\dagger \hat{b}_j + \hat{b}_j^\dagger \hat{b}_i) + U \sum_i \frac{\hat{n}_i(\hat{n}_i - 1)}{2} - \sum_i \mu_i \hat{n}_i$$

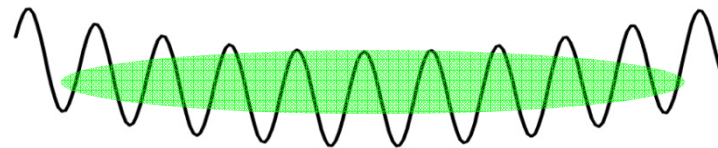
Tunneling

Interaction

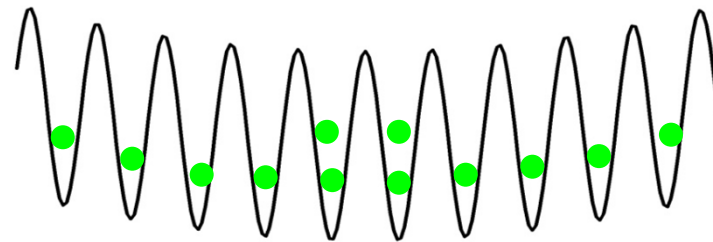
Trap potential

Density distribution

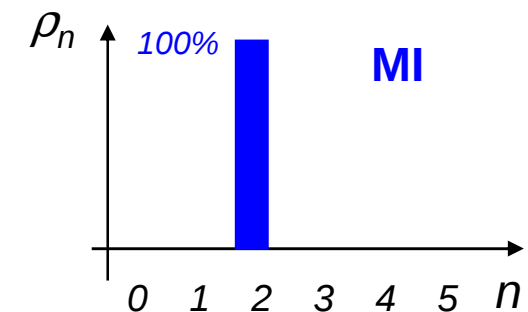
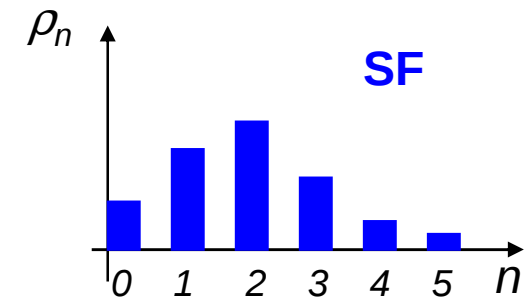
$t \gg U$
Superfluid
 compressible



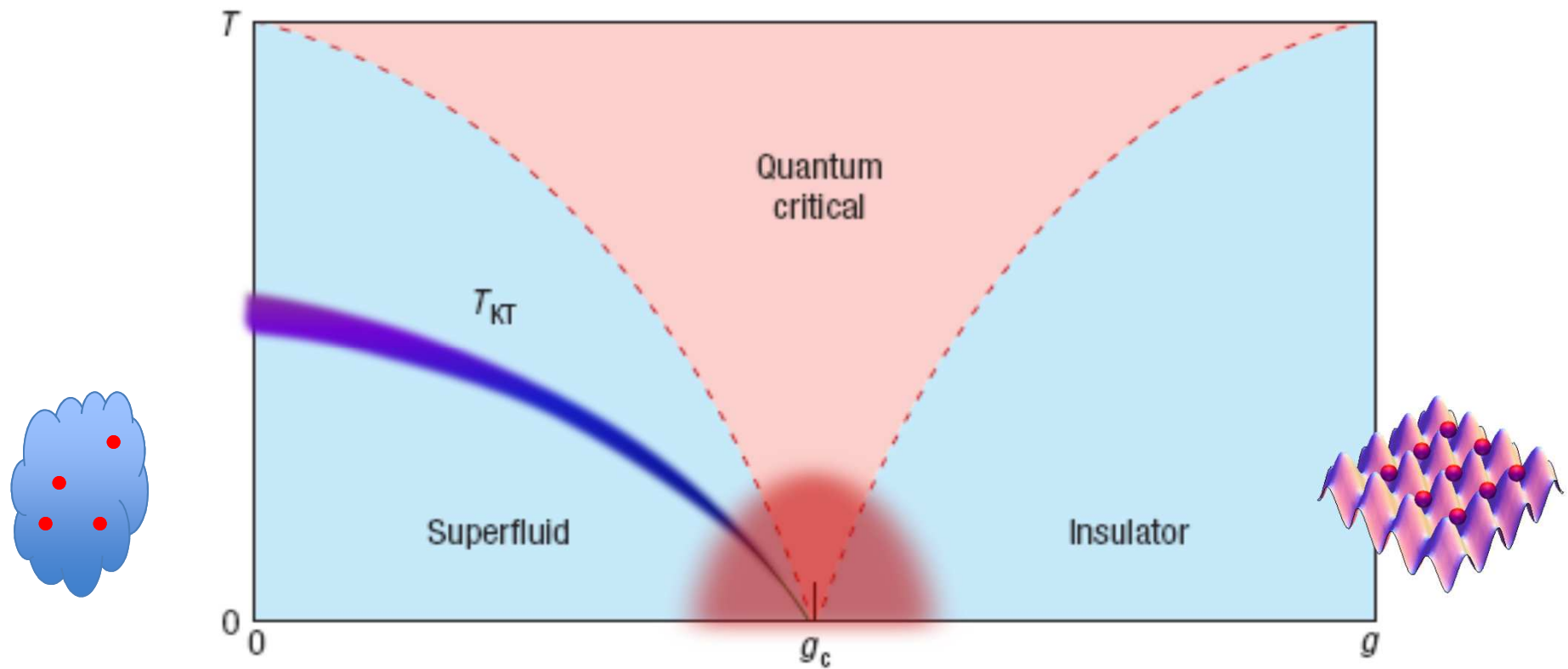
$U \gg t$
Mott insulator
 incompressible



Number statistics



Phase transition and quantum phase transition



Phase diagram and AdS4-CFT3 duality: Sachdev, Nature physics (2007)

Synopsis

How do we see the wedding cake?

Science Project 1:

Properties of SF and Mott insulator

Quantum phase transition

Science Project 2:

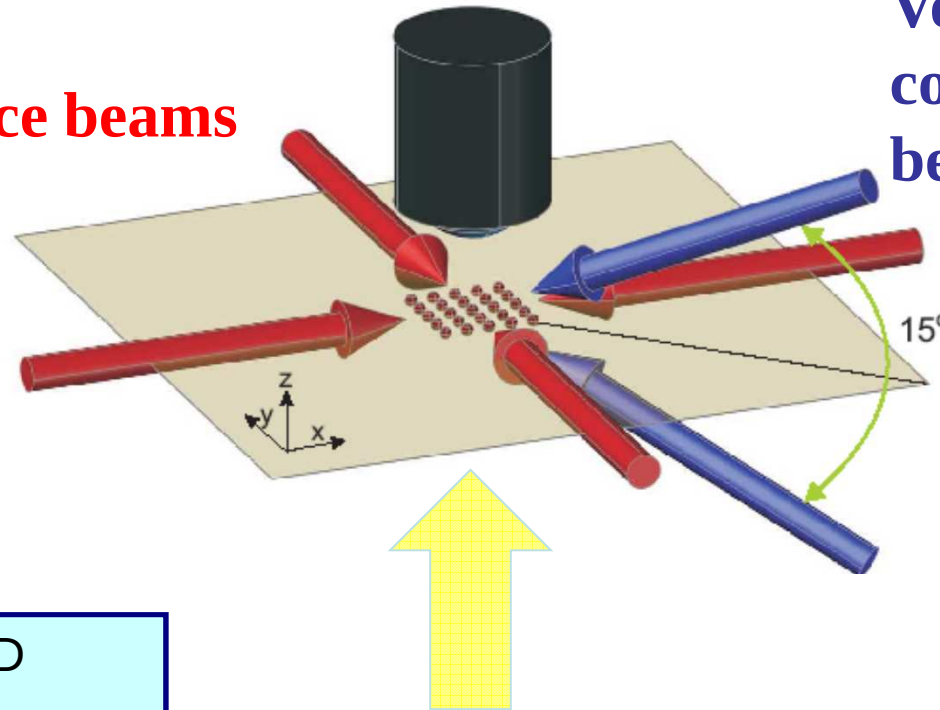
Quantum microscopy and quantum information

In situ Imaging a single layer of 2D gas

Microscope objective

Trap/lattice beams

Vertical
compress.
beams



Cesium atoms in 2D

$$15\hbar\omega_r \sim k_B T \text{ and } \mu \sim \frac{\hbar\omega_z}{10}$$

Determined from
the density profile.

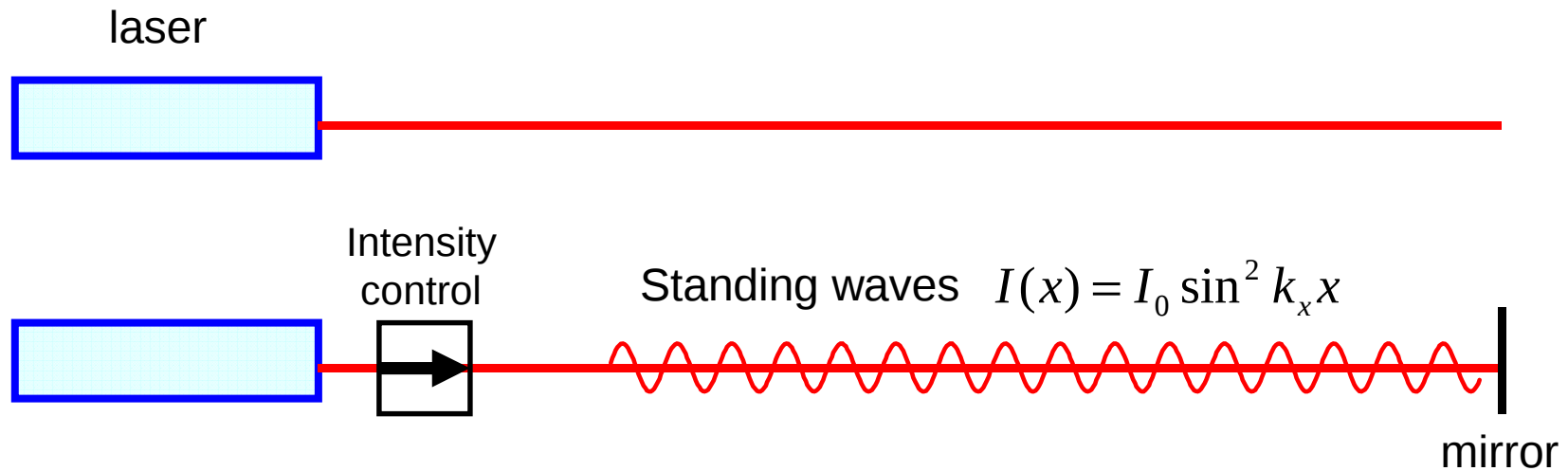
Imaging beam

N.D. Gemelke et al., Nature 460 (2009)

See also: (Harvard) W.S. Bakr et al., Nature 462 (2009)

(MPQ) J.F. Sherson et al., Nature 467 (2010)

What is an optical lattice?



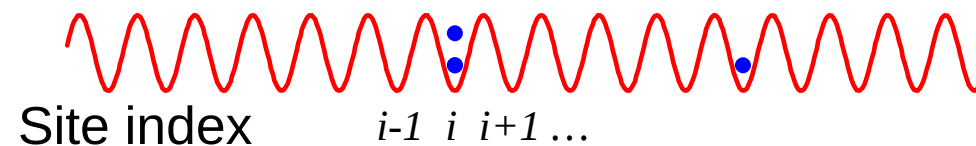
Atomic polarization $P = \epsilon E$

Atom-photon interaction $V = -\int P \cdot dE = -\frac{1}{2} \epsilon E^2 = -\frac{1}{2} \alpha_{AC} I$

α_{AC} : AC polarizability

Optical lattice potential $V(x) = V_0 \sin^2 k_x x$

Optical lattice toolbox



Hopping

$$-t \sum_{\langle i,j \rangle} (\psi_i^\dagger \psi_j + \psi_j^\dagger \psi_i)$$

Controlled by

Lattice potential

Interaction

$$\frac{U}{2} \sum_i n_i (n_i - 1)$$

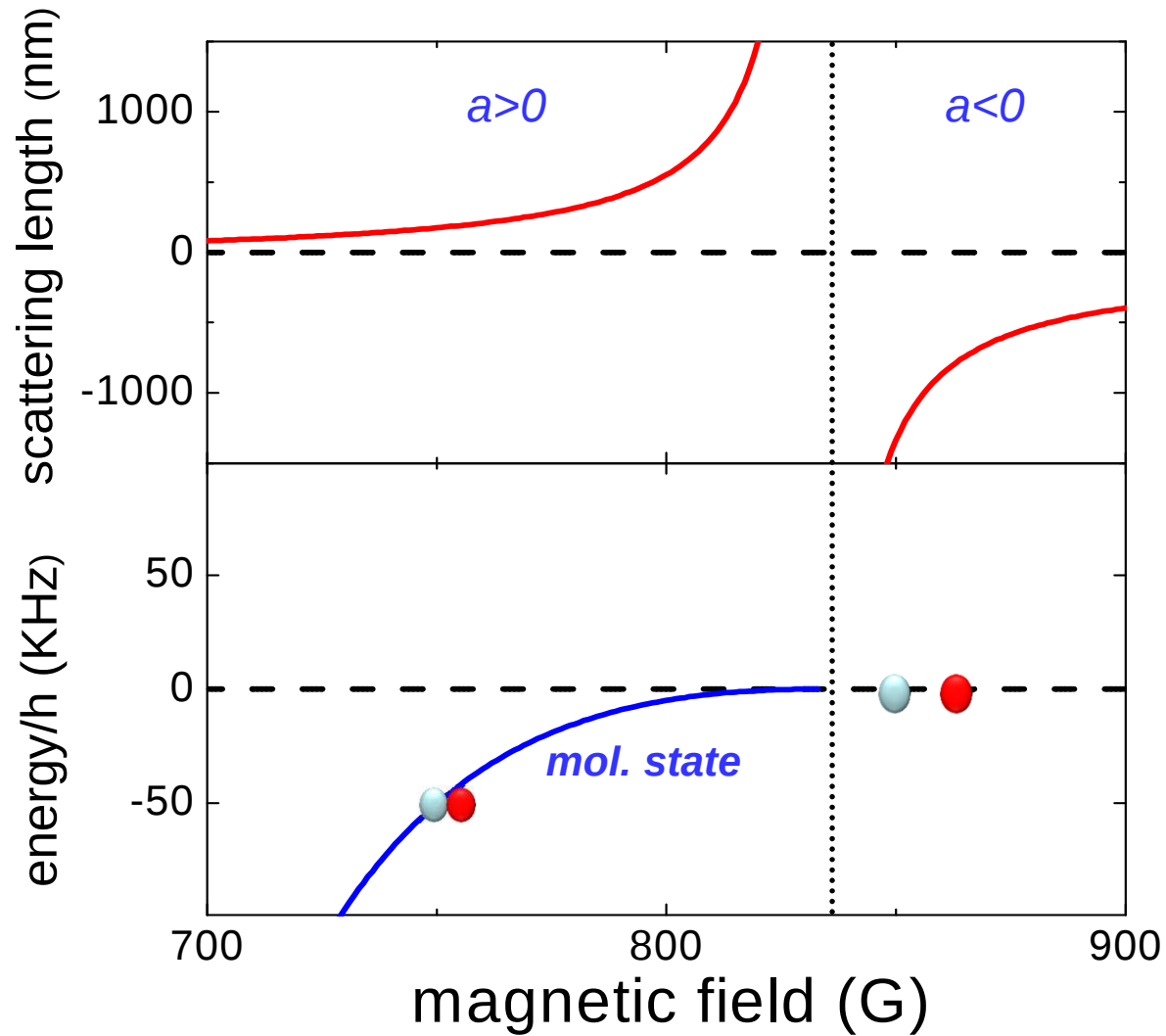
Feshbach tuning

Trapping potential

$$-\sum_i n_i (\mu - V_i)$$

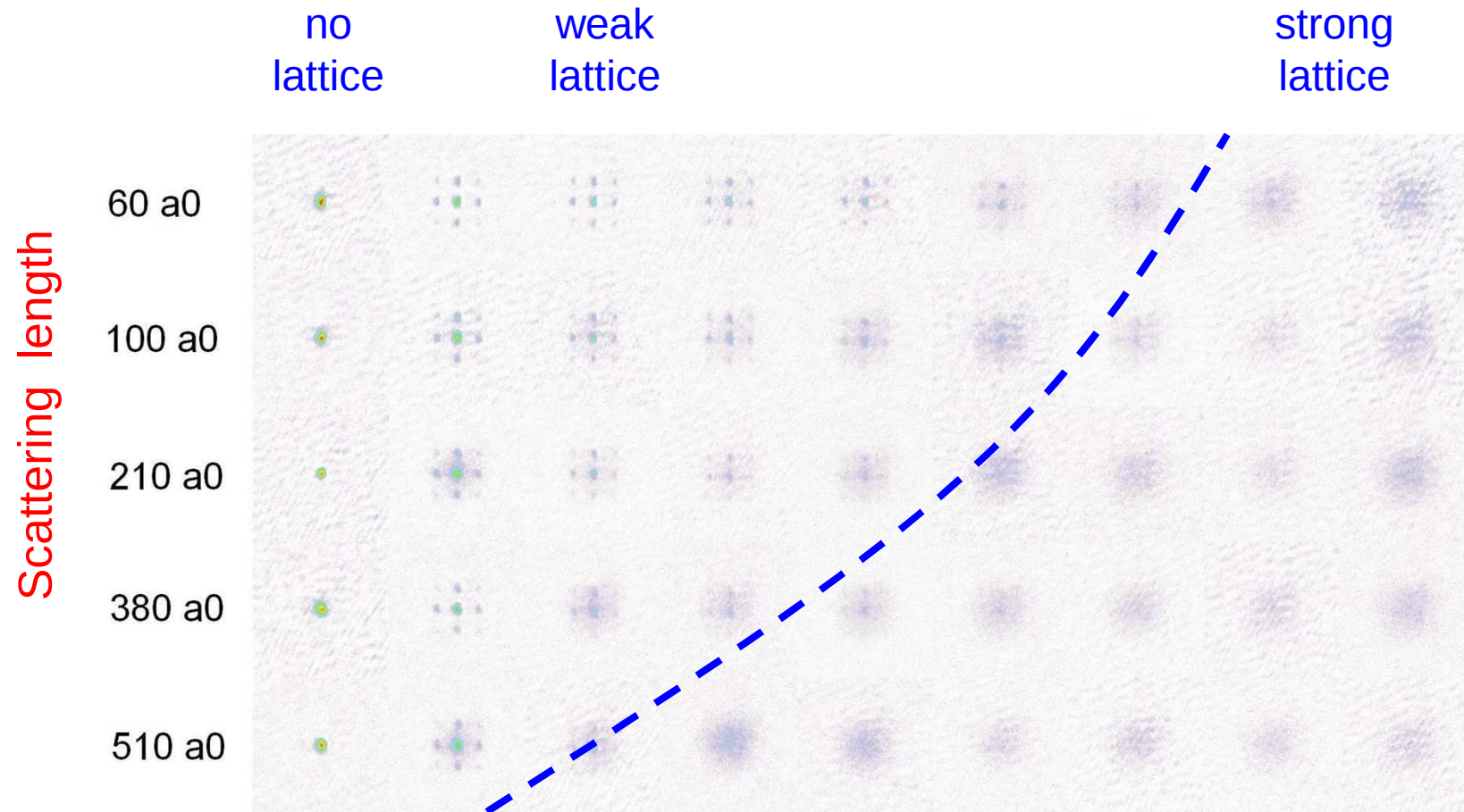
Trap potential

Feshbach resonance in ultracold gases



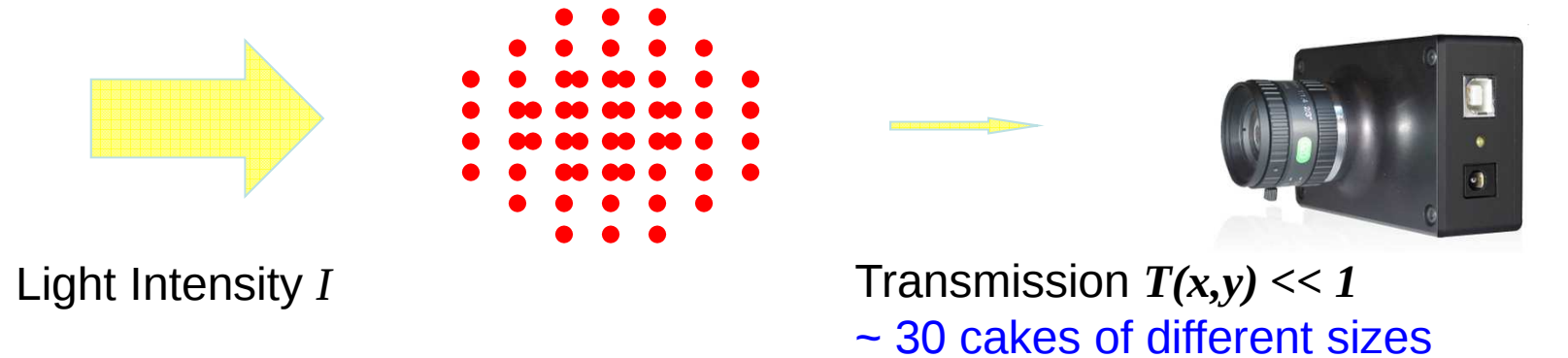
Chin, Grimm, Julienne and Tiesinga, Rev. Mod. Phys. (2010)

SF-MI transition in a single layer of 2D lattice

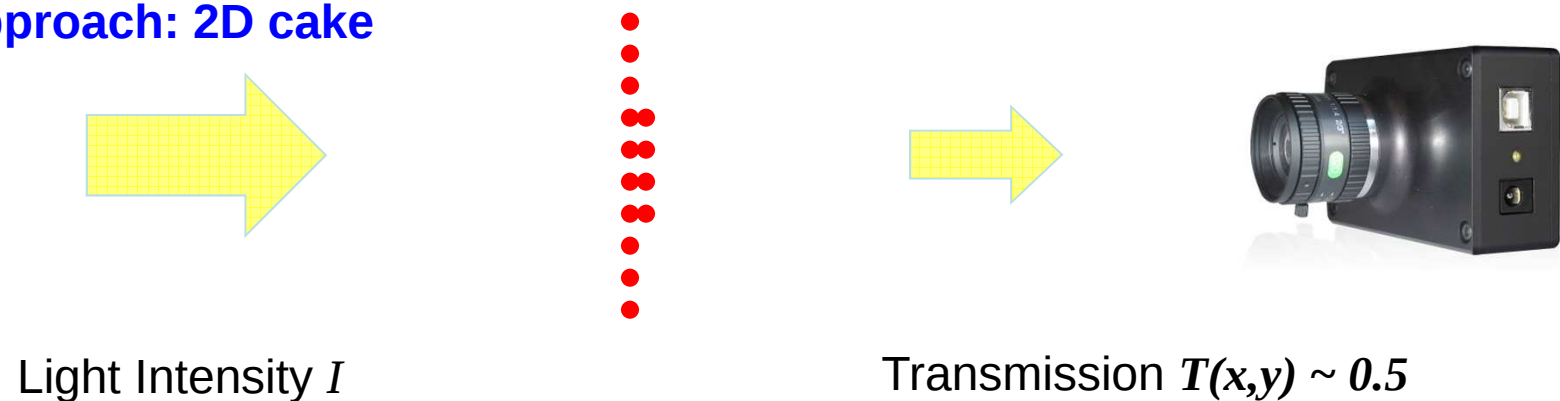


Why is it difficult to see the cake?

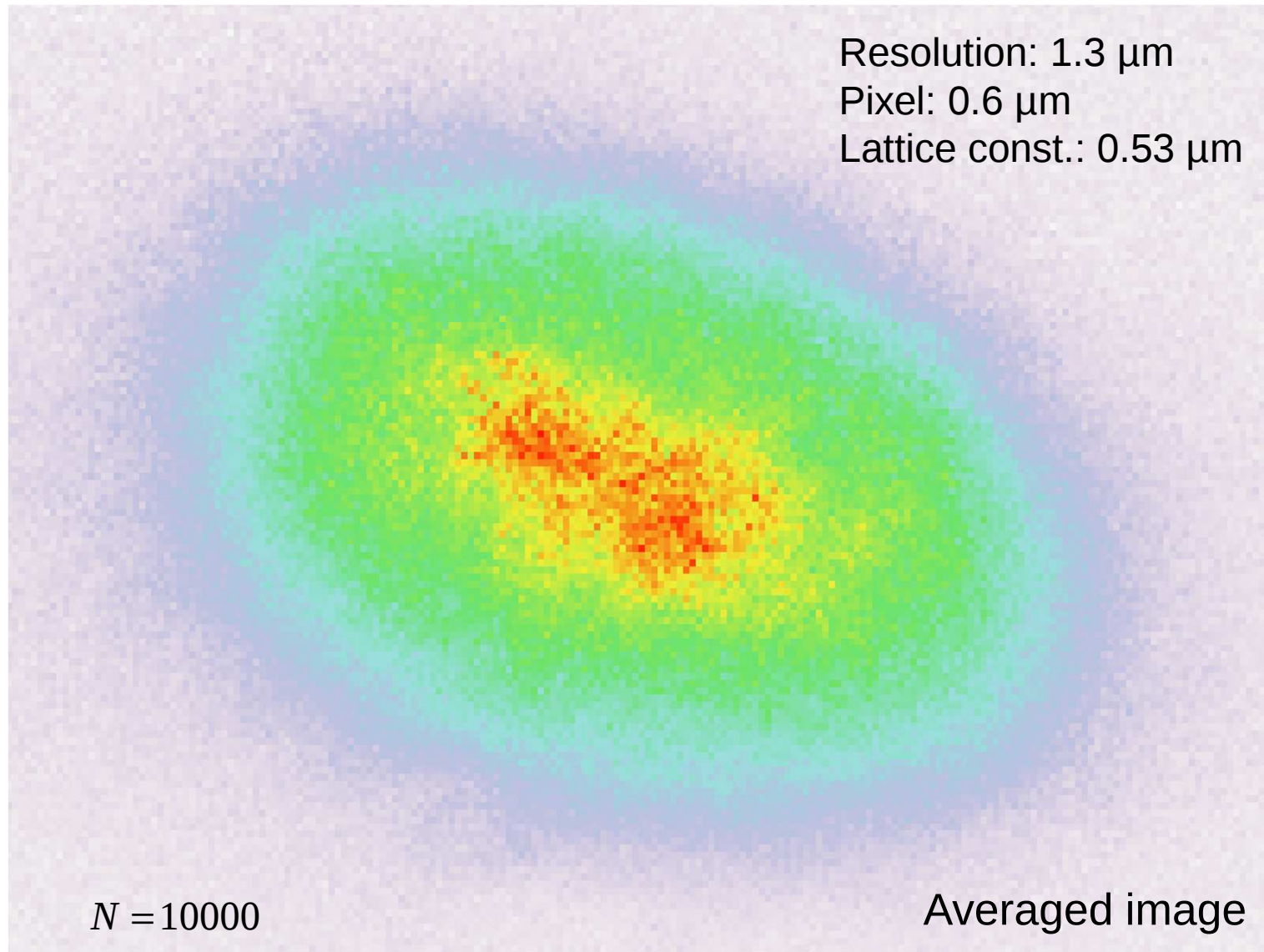
Conventional approach: 3D cake



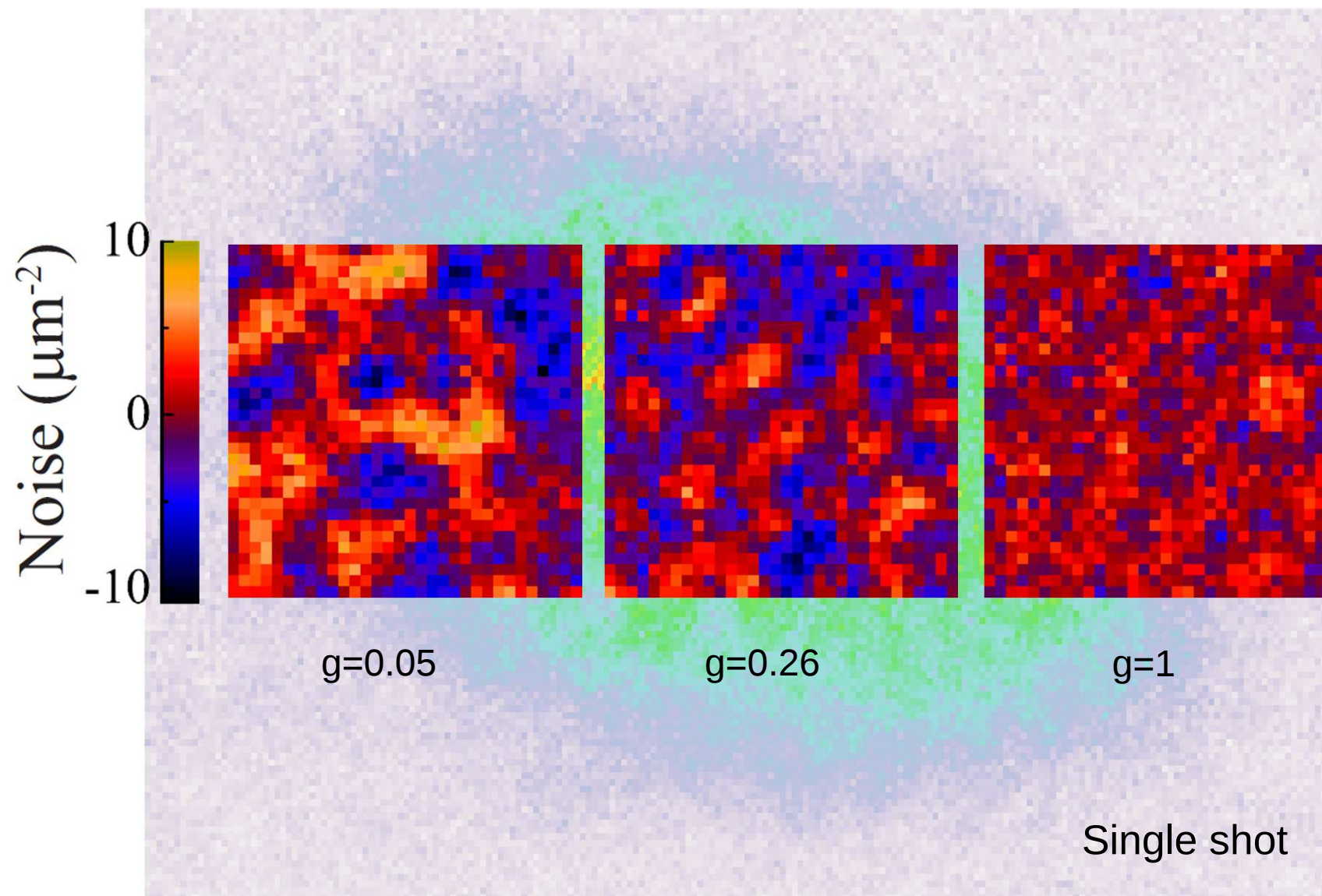
Our approach: 2D cake



A closer look...

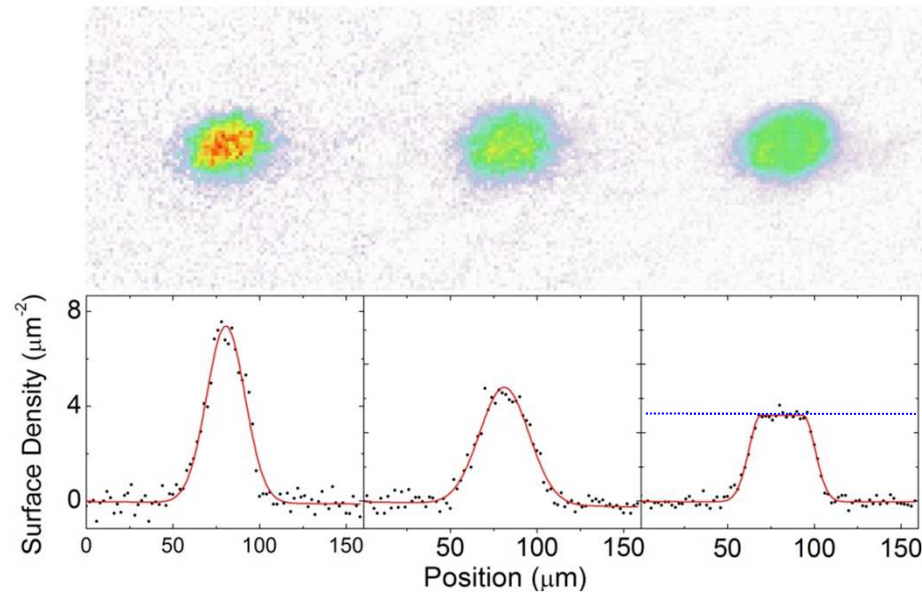


Density fluctuations and correlations



Noise of $A \equiv A - \langle A \rangle$

Observation of the Mott domain (single tier cake)

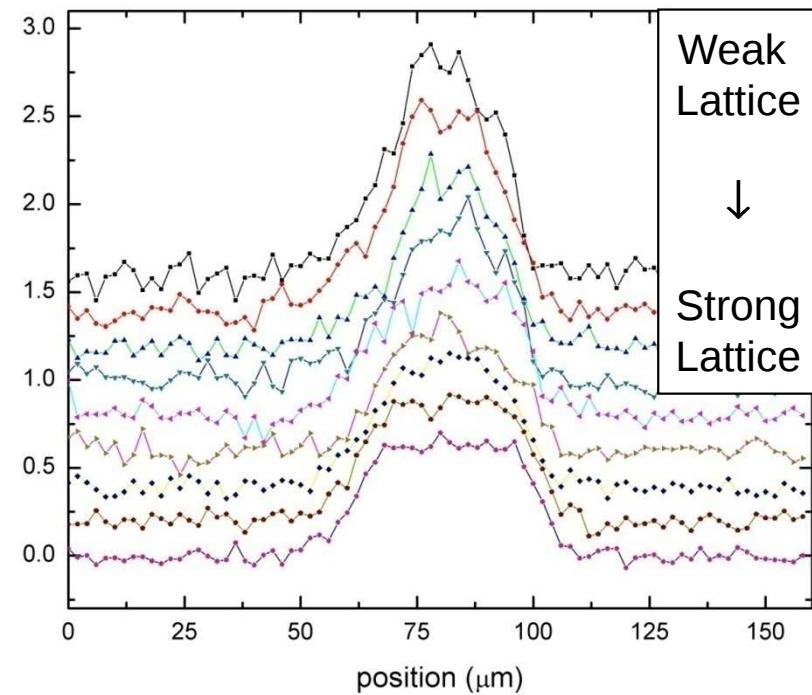


Plateau density:

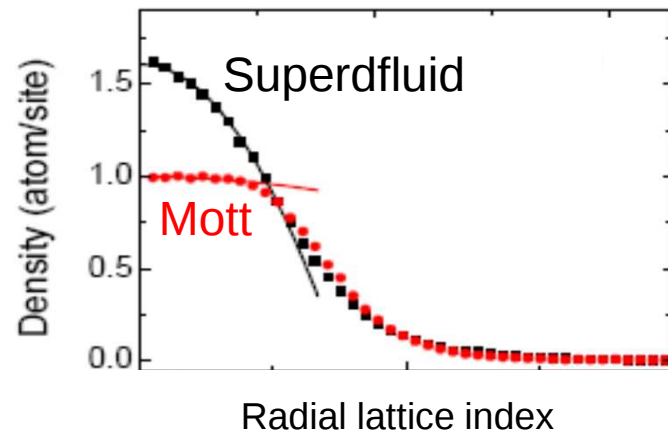
Theory: $1/d^2 = 3.53 / \mu\text{m}^2$

Experiment: $3.5(3) / \mu\text{m}^2$

Emerging of Mott plateau



Compressibility κ

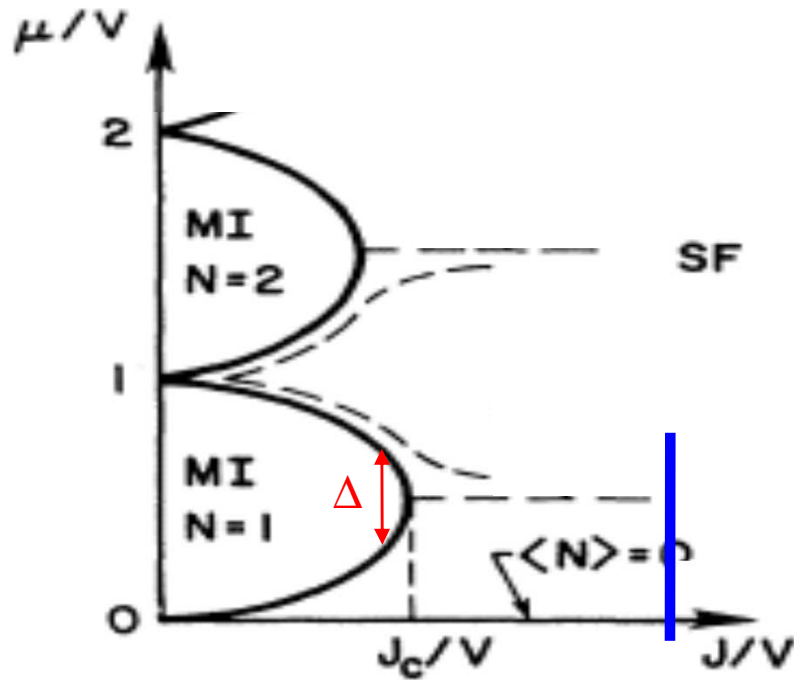


Definition

$$\kappa = \frac{\partial n}{\partial \mu} = \frac{1}{m\omega_r^2} \frac{n'(r)}{r}$$

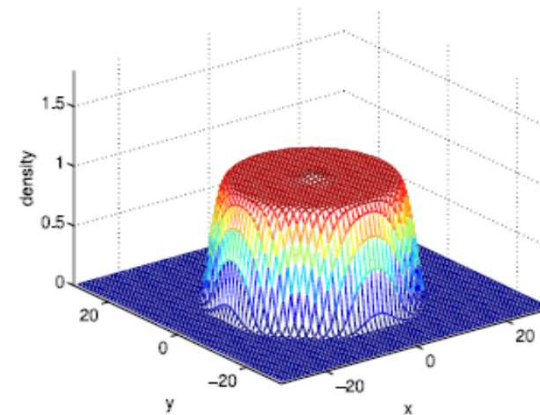
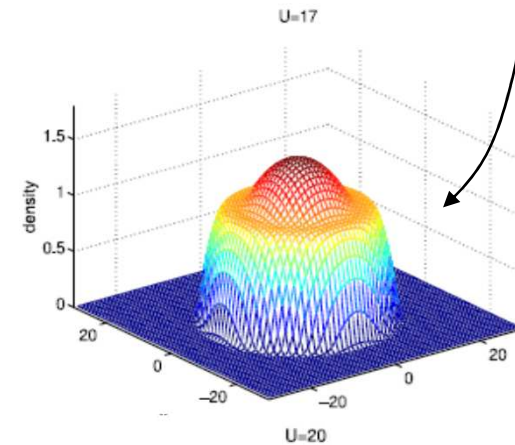
Origin of the cake structure

Phase diagram at $T=0$



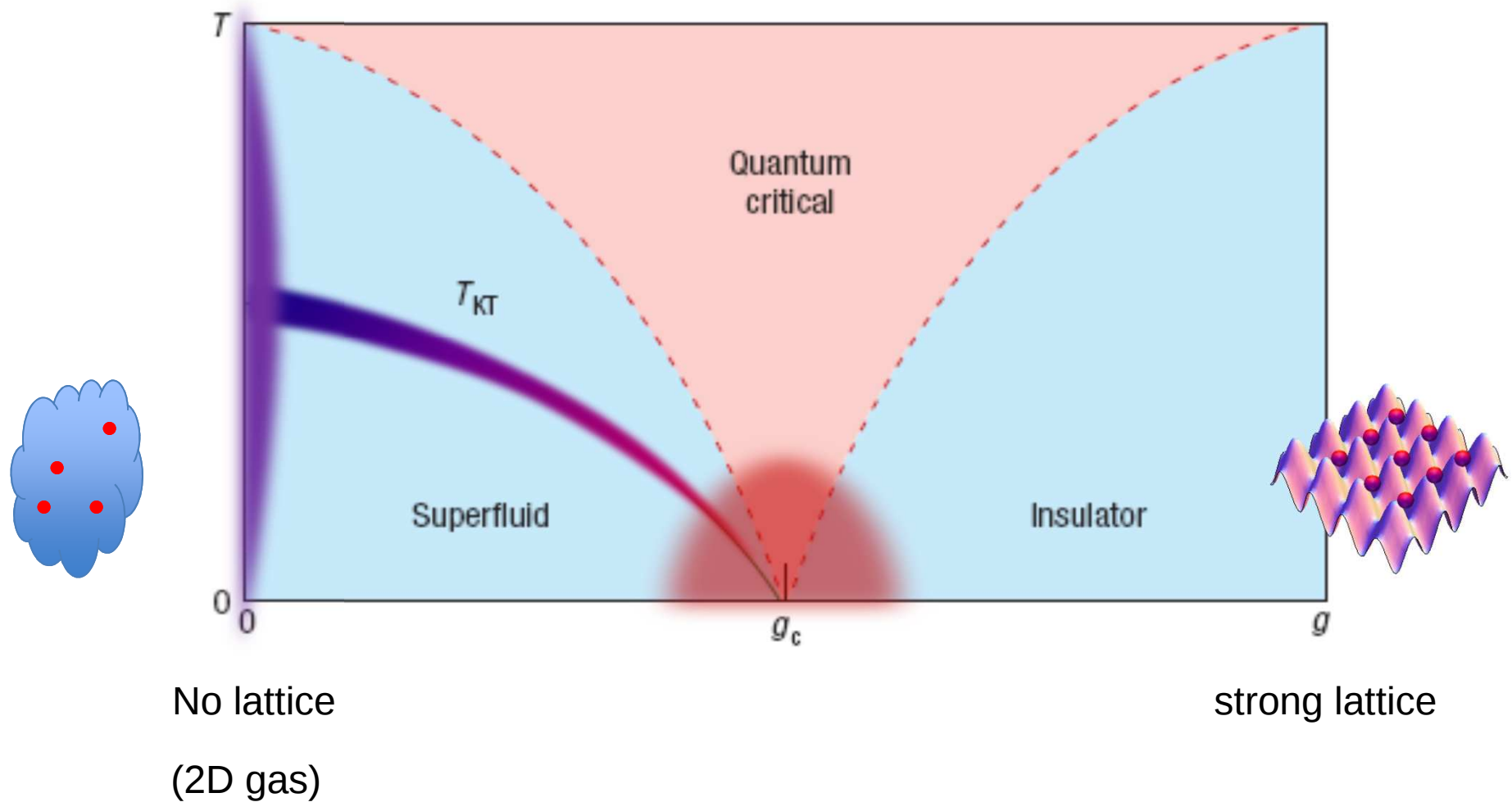
Fisher et al., PRB 40, 546 (1989)

MI plateau appears near $\langle n \rangle = 1$



Quantum Monte Carlo calculation
(R. Scalettar, UC Davis)

Universal scaling regimes in 2D optical lattices



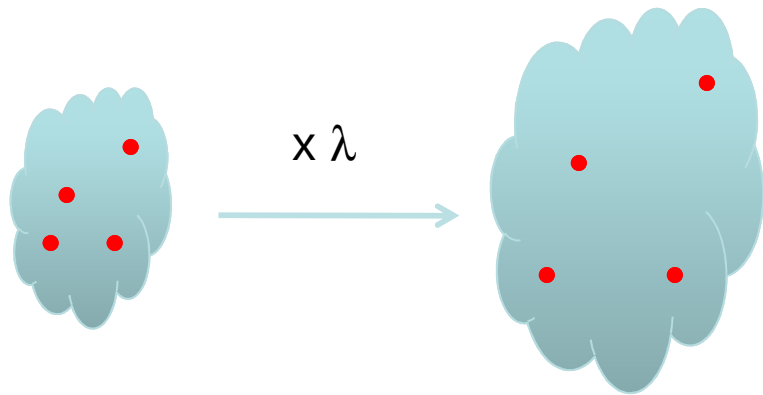
Scaling symmetry



2D gas with constant contact interaction Pitaevskii and Rosch, PRA (1997)

$$H = \sum_i \frac{p_i^2}{2m} + \sum_{ij} g \delta^2(r_{ij})$$

$$H\psi(\lambda x) = \lambda^{-2} H\psi(\lambda x)$$



$$E_k \rightarrow \frac{E_k}{\lambda^2} \quad gn \rightarrow g \frac{n}{\lambda^2}$$

Another example: resonant Fermi gas.

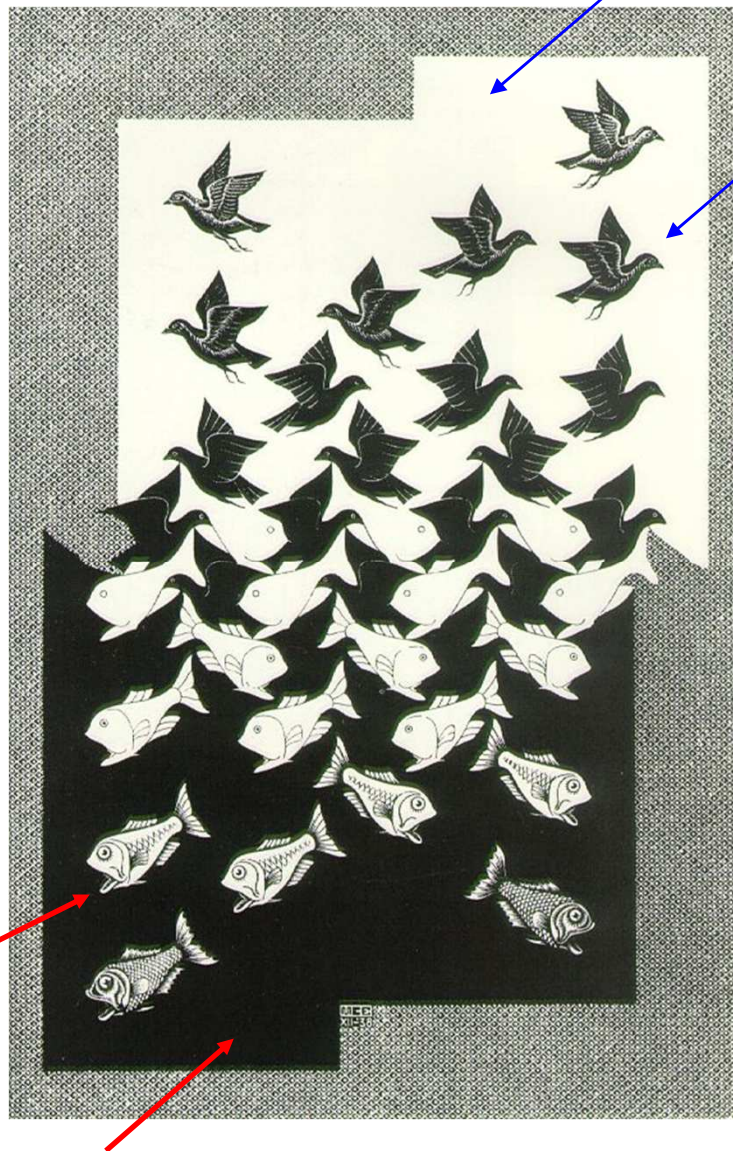
Classical
critical phenomena

Vapor

Droplets in vapor

Bubbles in water

water

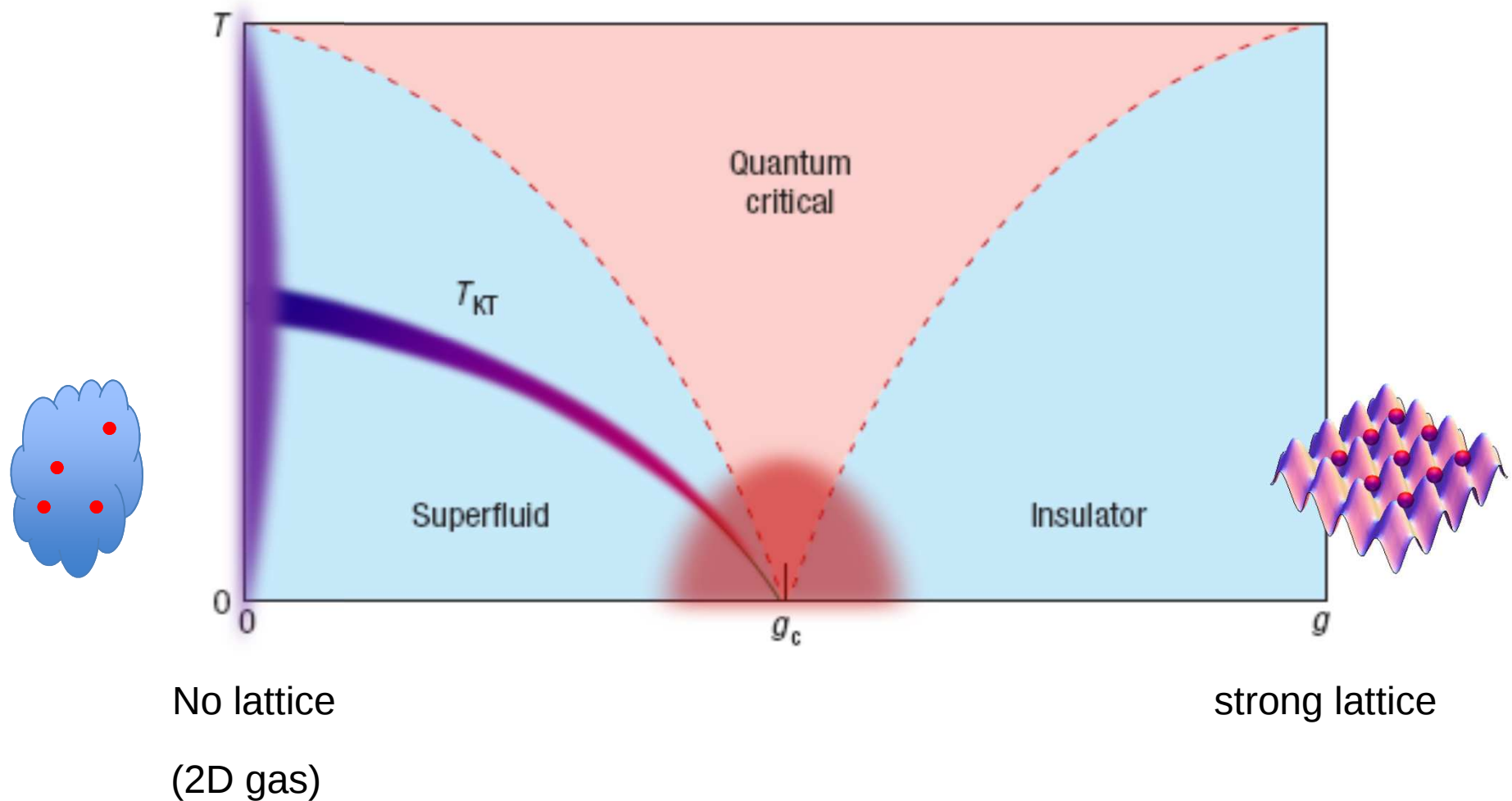


M.C. Escher

YouTube:
Optical Opalescence

Quantum phase transition in optical lattices

$$H = \sum_i \frac{p_i^2}{2m} + \sum_{ij} g \delta^2(r_{ij}) + V(\sin^2 kx + \sin^2 ky)$$



Only the ratio of the length scale matters $\Psi(x) = \lambda^\alpha \Psi(\lambda x)$

Examples:

Equation of state $n(\mu, T) = \lambda_{dB}^{-2} F(\tilde{\mu})$ $\tilde{\mu} = \frac{\mu}{kT}$

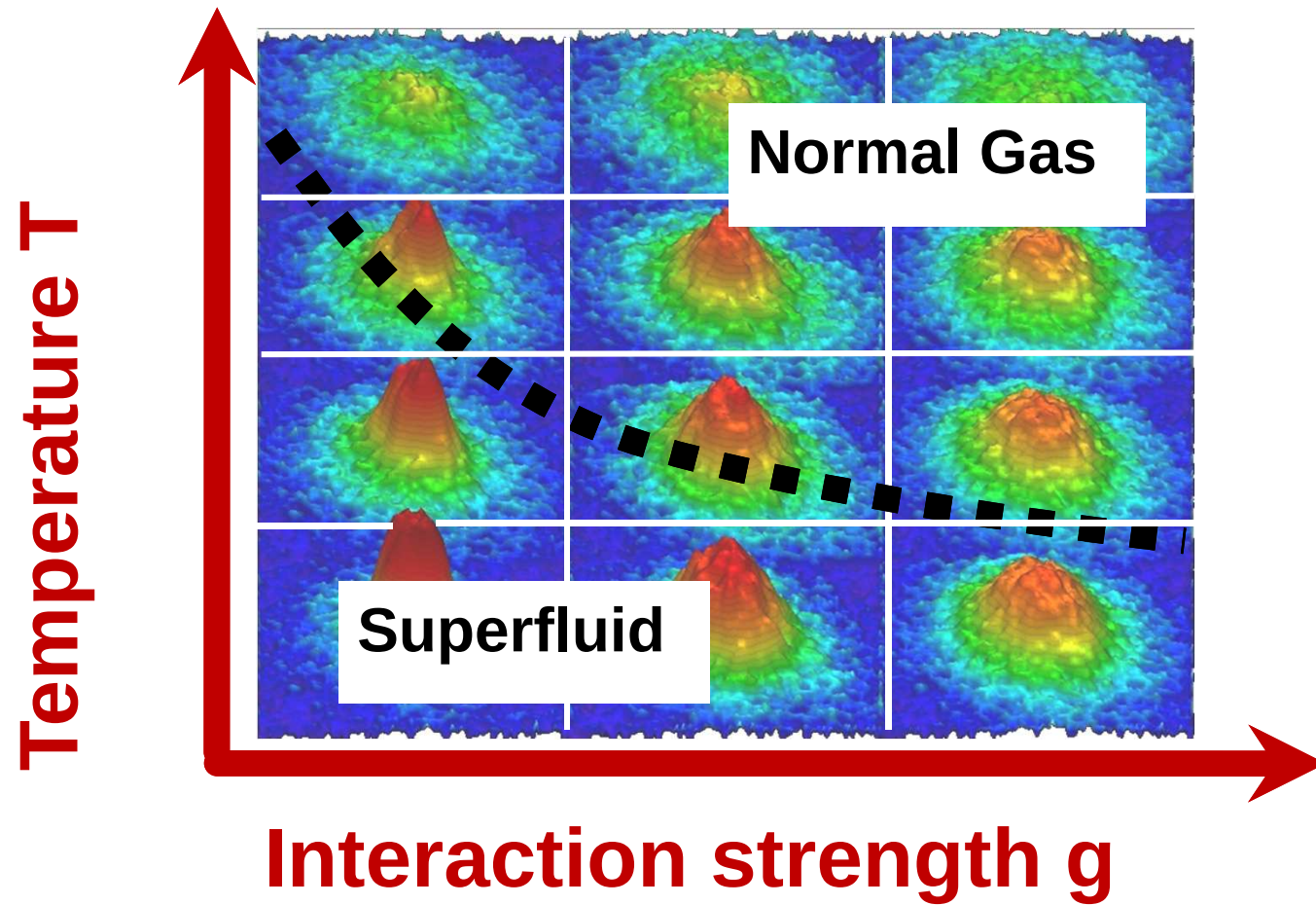
Corollary : Grand potential can be fully determined by EoS.

~~$$d\Omega = -s(\mu, T)dT - n(\mu, T)d\mu$$~~

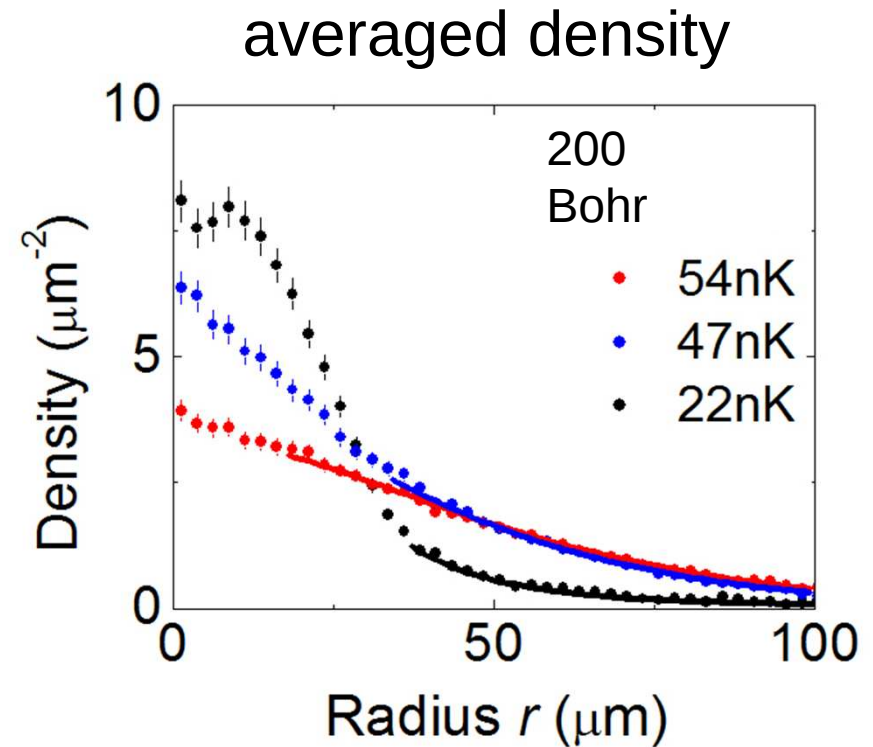
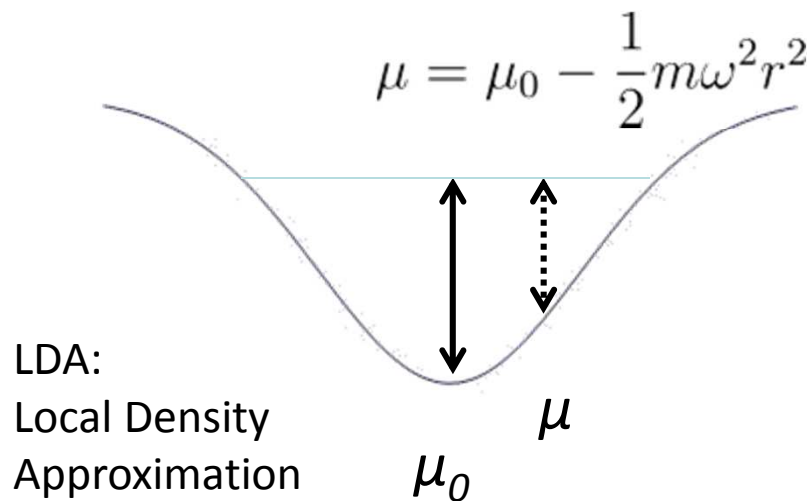
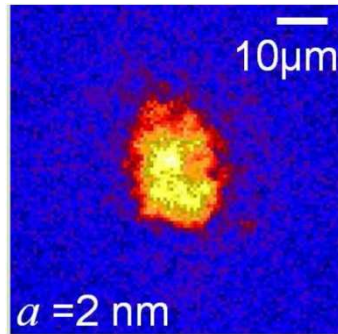
$$d\Omega = \left[-2 \int F(x) dx - F(\tilde{\mu}) \right] T^2 d\tilde{\mu}$$

$$\tilde{n} = n \lambda_{dB}^2$$

Scale invariance and universality in 2D gases



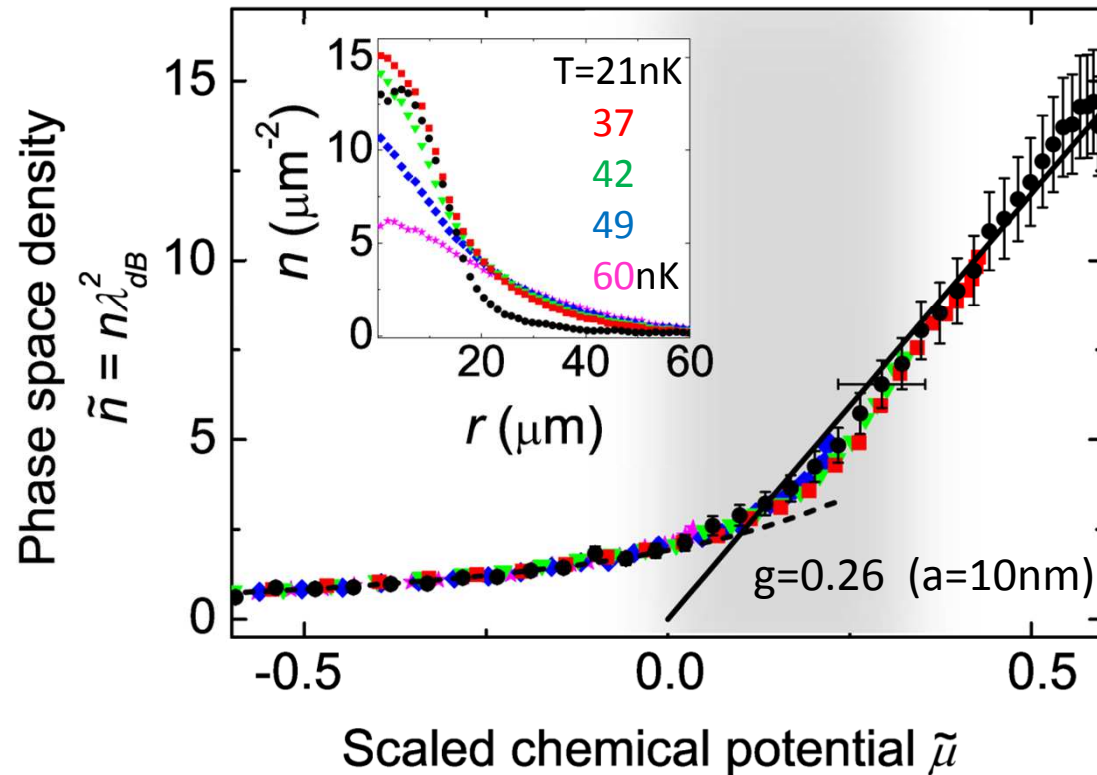
Temperature T and chemical Potential μ



$$n(\mu, T) = -\lambda_{dB}^{-2} \ln[1 - \exp(\mu/k_B T - gn\lambda_{dB}^2/\pi)]$$

Rath et. al., PRA 82, 013609 (2010)

Scale Invariance - Density $\tilde{n} = n\lambda_{dB}^2 = F(\frac{\mu}{kT})$

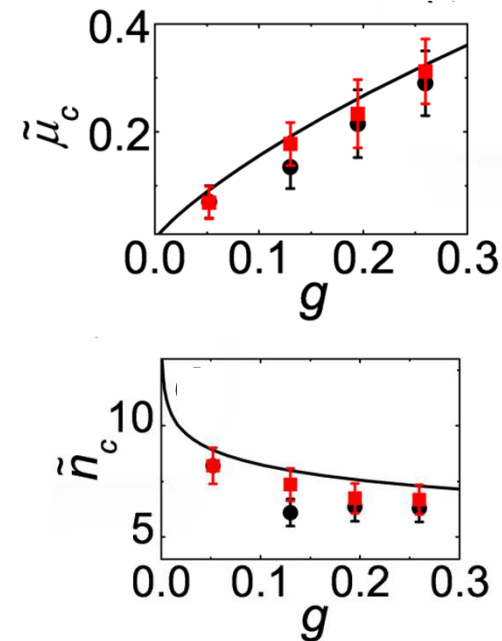
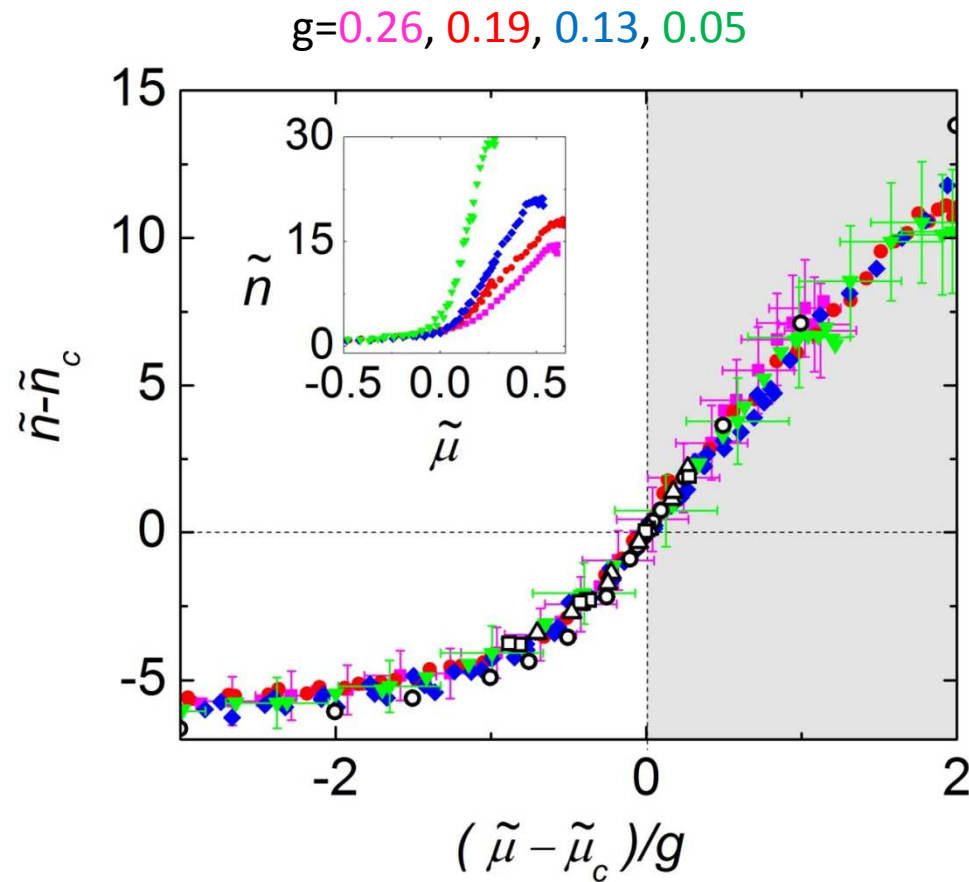


----- $n(\mu, T) = -\lambda_{dB}^{-2} \ln[1 - \exp(\mu/k_B T - gn\lambda_{dB}^2/\pi)]$

———— $\tilde{n} = 2\pi\tilde{\mu}/g$ (Thomas Fermi approximation)

Universality – Density

$$\tilde{n} - \tilde{n}_c = H\left(\frac{\mu - \mu_c}{kTg}\right)$$



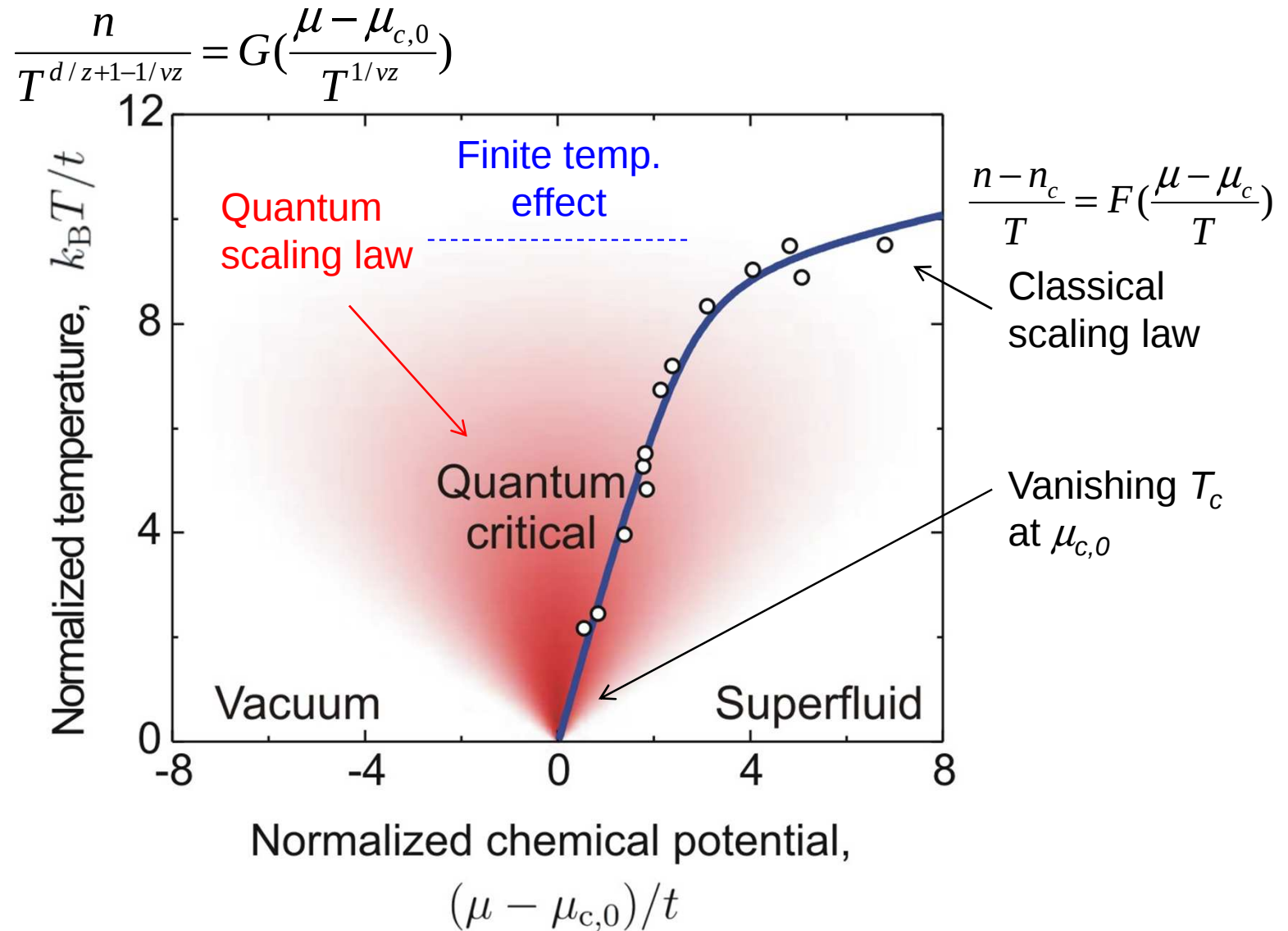
$$\tilde{n}_c = \ln(\xi/g) \quad \xi = 380$$

$$\tilde{\mu}_c = (g/\pi) \ln(\xi_\mu/g) \quad \xi_\mu = 13.2$$

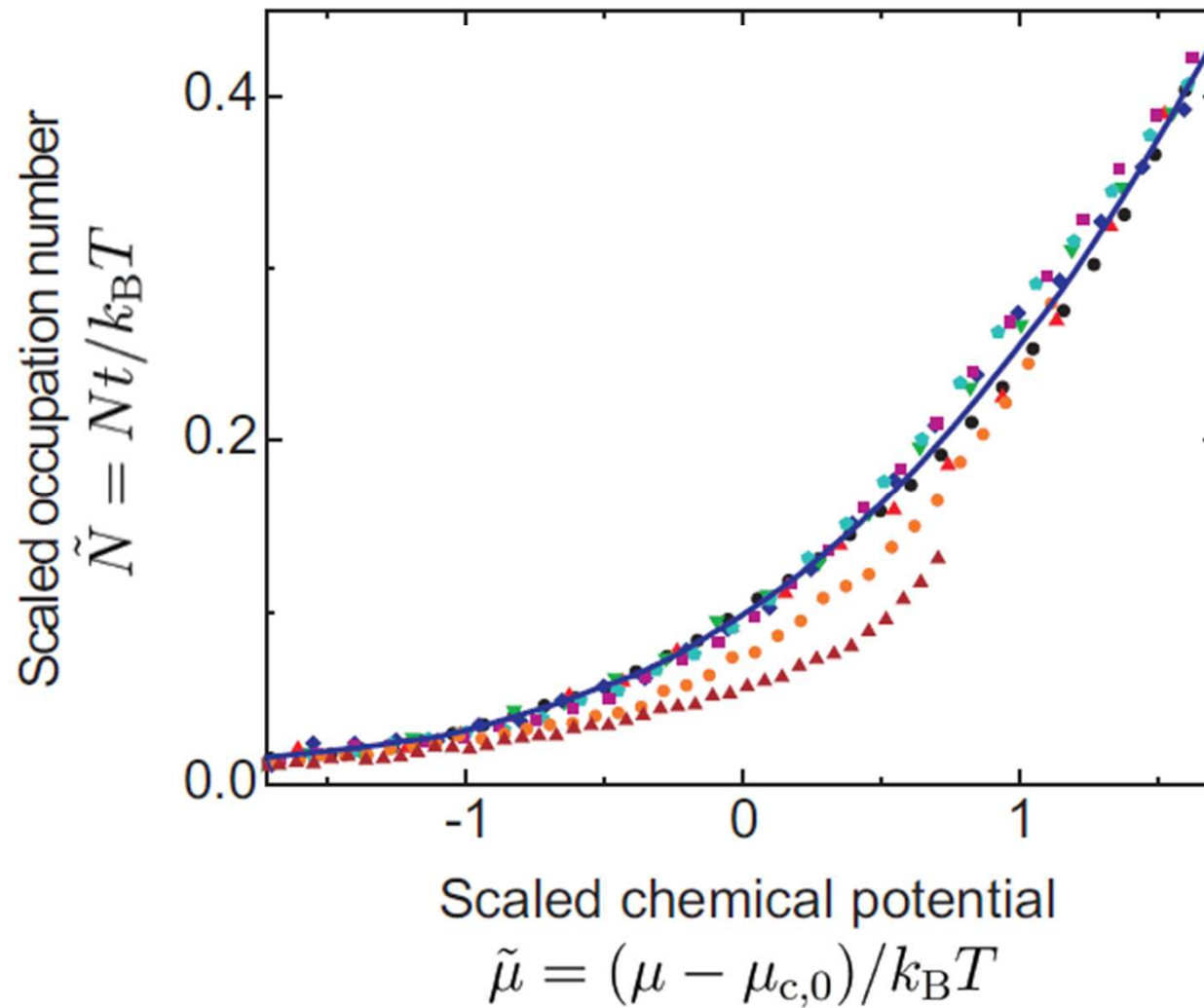
- $\triangle \square$ Holzmam et. al., PRA 81, 043622 (2010)
- $\circ$ Prokof'ev et. al., PRA 66, 043608 (2002)

Prokof'ev et. al., PRL 87, 270402 (2001)

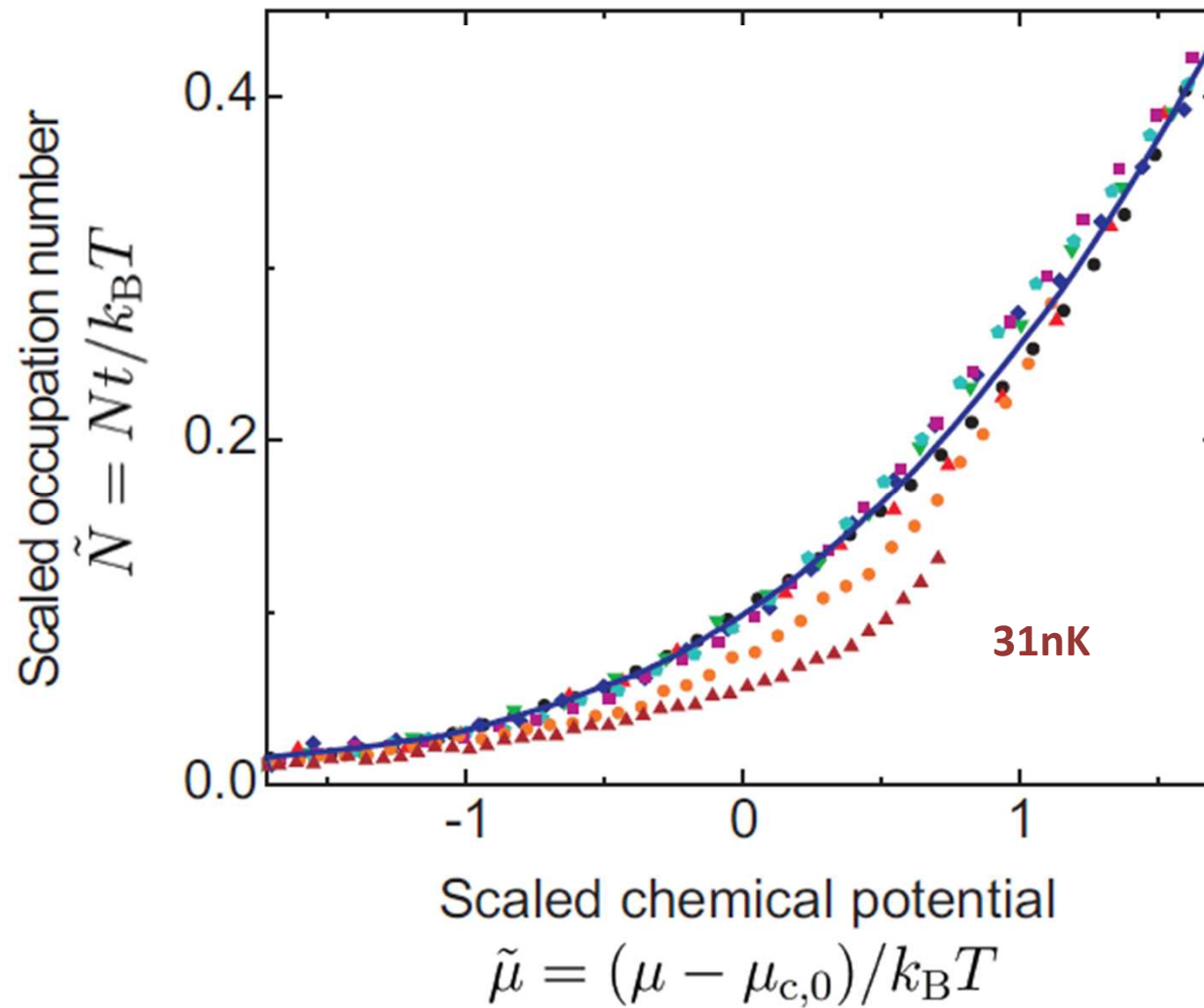
Quantum phase transition in 2D lattice at $V=7 E_R$



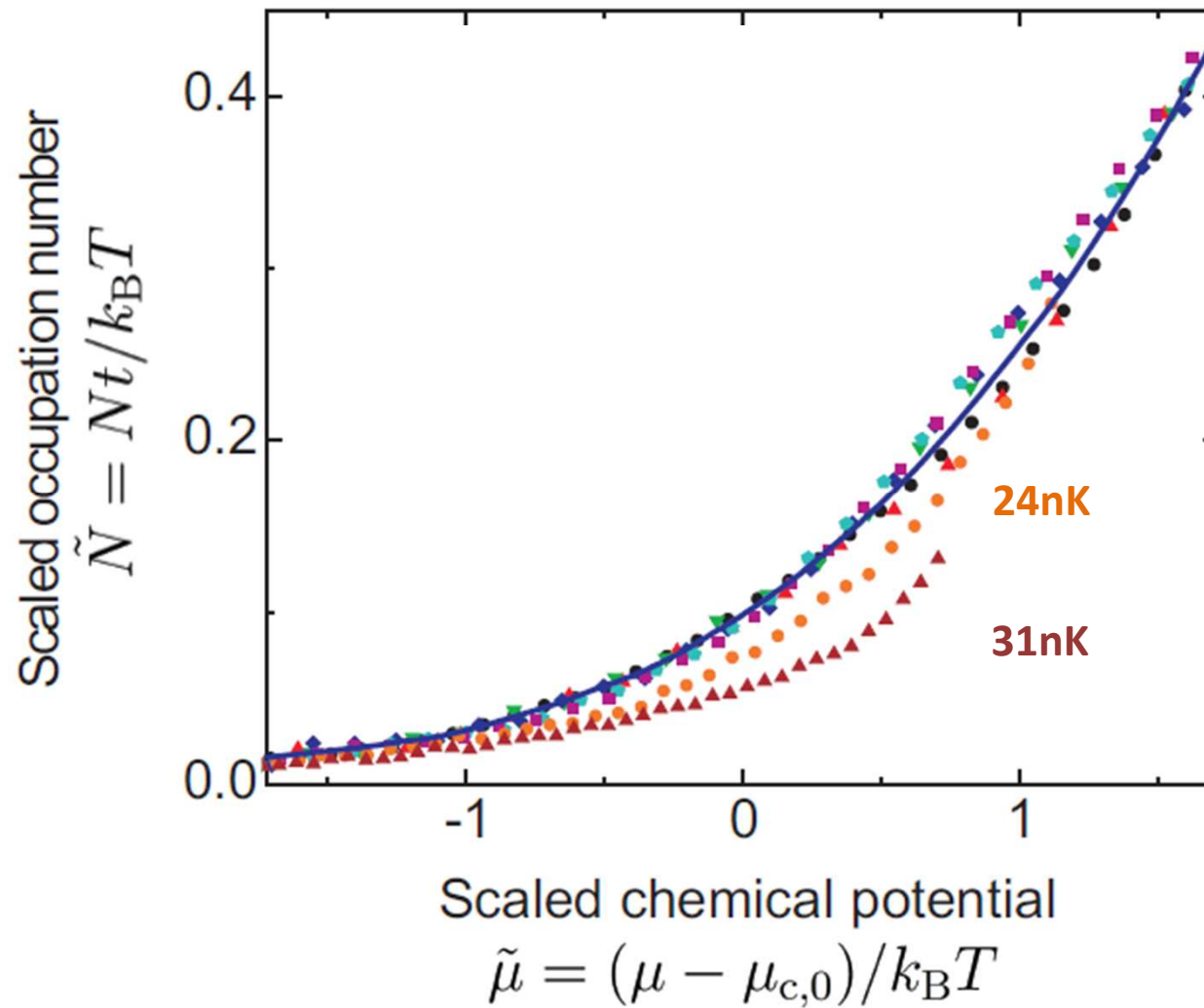
1. Universal scaling.



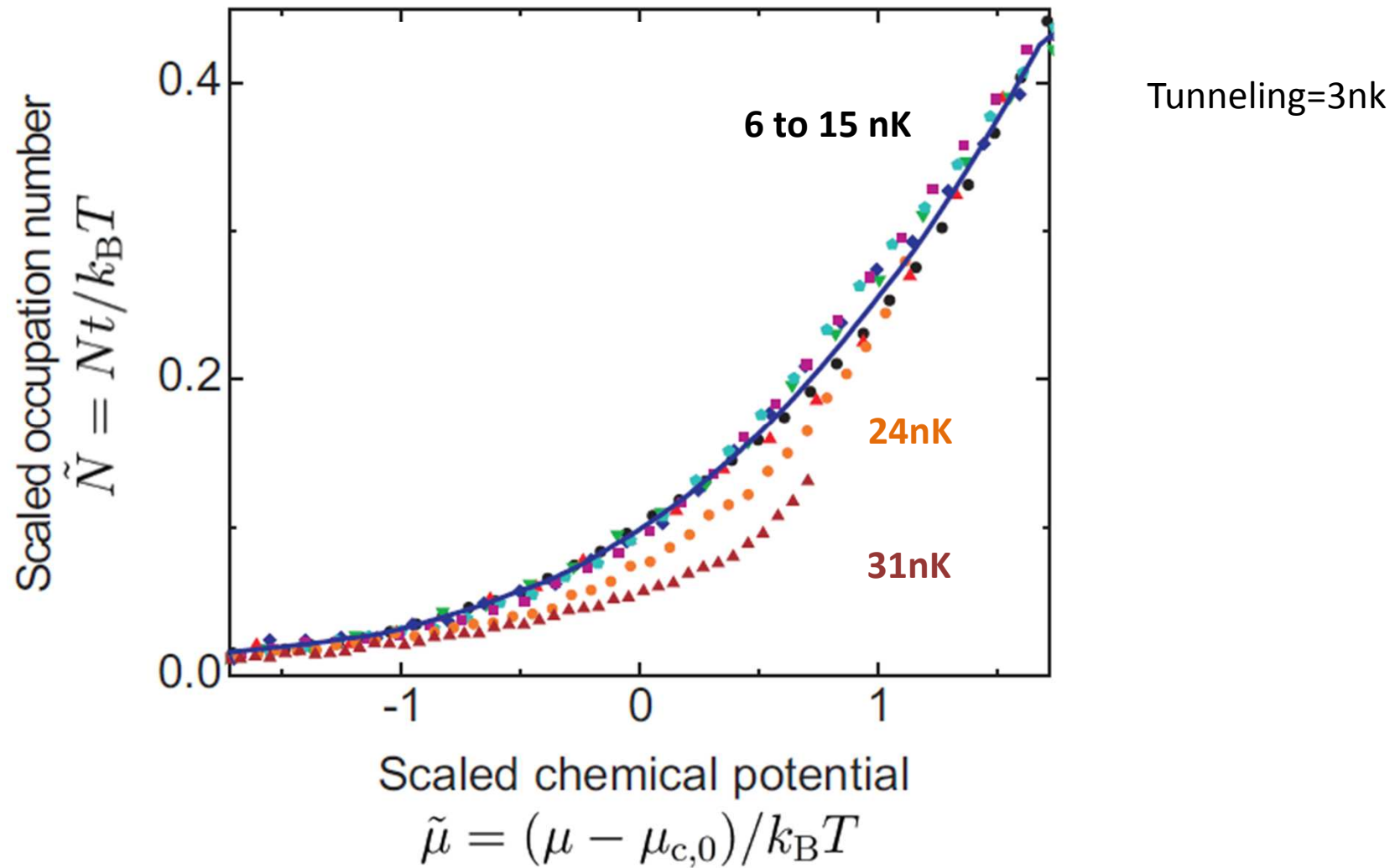
1. Universal scaling.



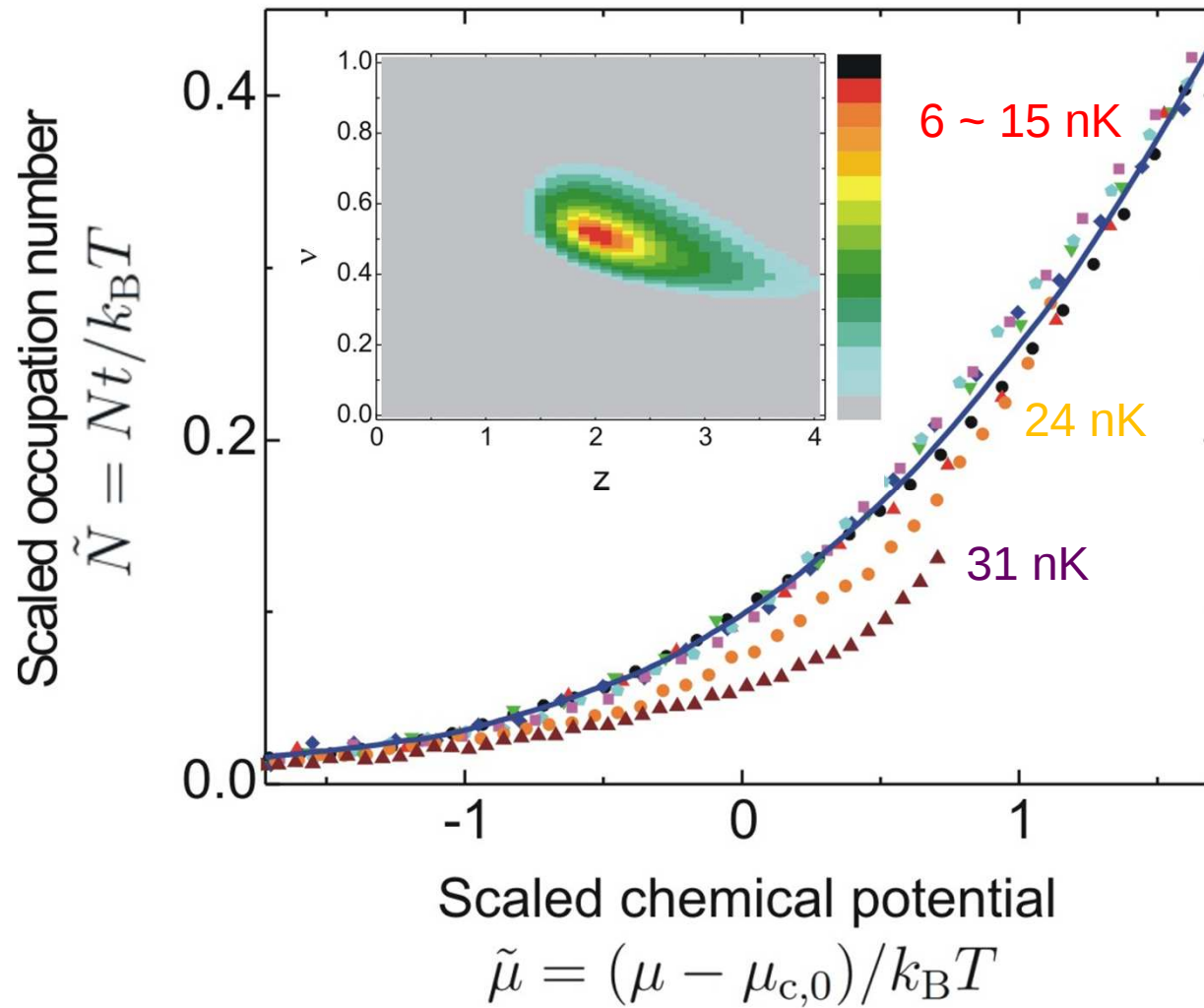
1. Universal scaling.



1. Universal scaling.



2. Critical exponents z and ν : $\frac{n - n_c}{T^{d/z+1-1/\nu z}} = h(\frac{\mu - \mu_c}{T^{1/\nu z}})$



Theory:

$$z=2$$

$$\nu=1/2$$

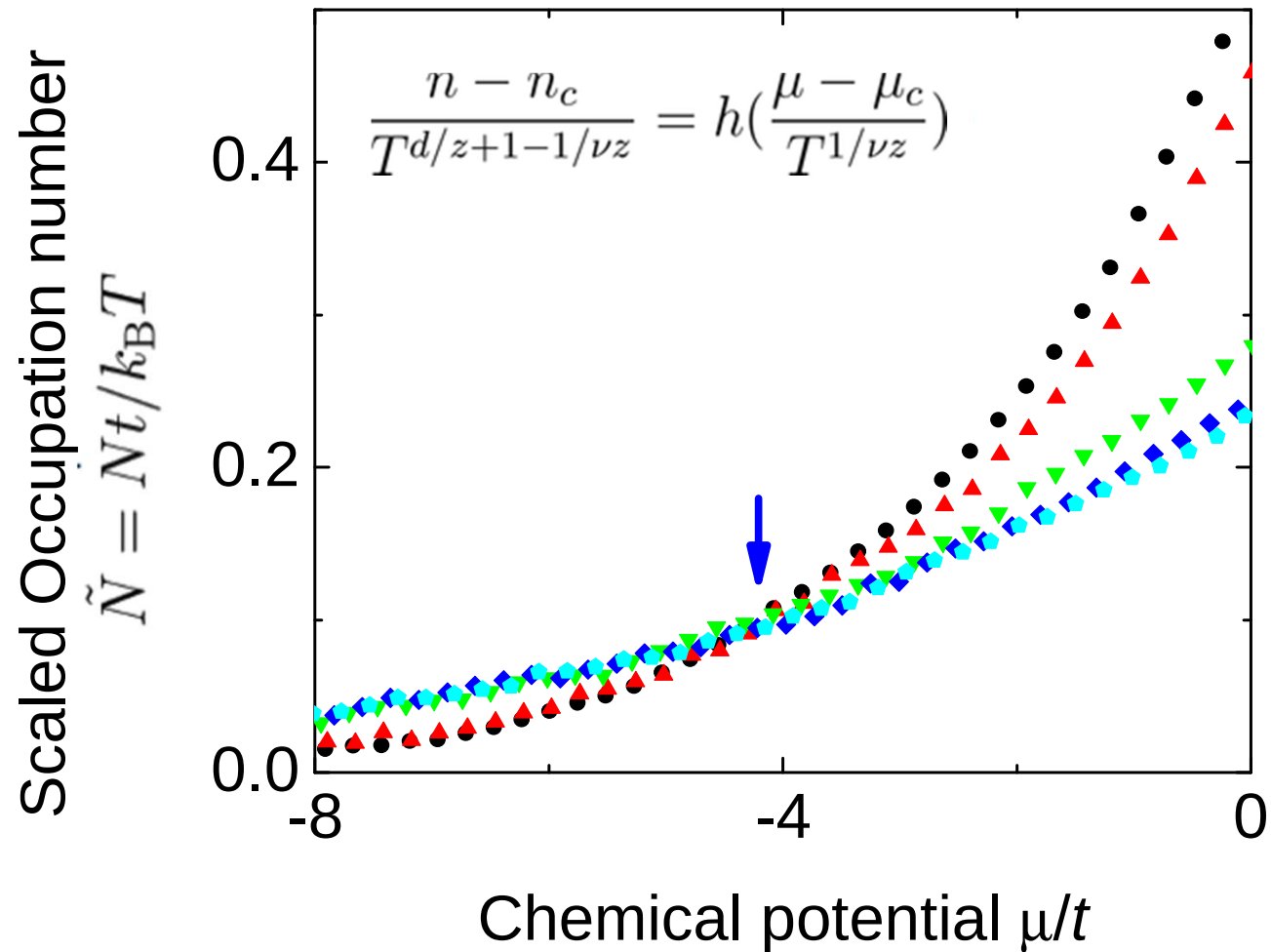
Experiment:

$$z=2.0 \text{ (3)}$$

$$\nu=0.53 \text{ (5)}$$

3. Quantum Critical Point

$$\mu_{c,0} = -4.2(6)t \quad (\text{theory: } -4t)$$



Conclusion

Scale invariance and universality in 2D

In situ imaging: Cake structure

Nature (2008)

Scale invariance and universality

Nature (2010)

Quantum criticality: QPT in optical lattices

...

New experiments:

Quantum fluctuations and correlations

NJP (2011)

Quantum critical transport

PRL (2010), NJP (2011)

Cold atom experiments at Univ. of Chicago

Group members



Graduate
Chen-Lung
Hung



Graduate
Xibo Zhang

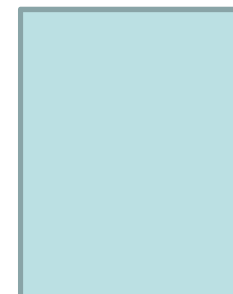


New Student
Harry Ha

Postdocs



Dr. Shih-Kuang Tung



Dr. Colin Parker

Theory Collaborators

2D gas:

N. Prokofev, B. Svistunov

Criticality:

Q. Zhou, D.W. Wang, T.-L. Ho,
K. Hazzard, N. Trevidi

Cold molecules:

P. Julienne, E. Tiesinga

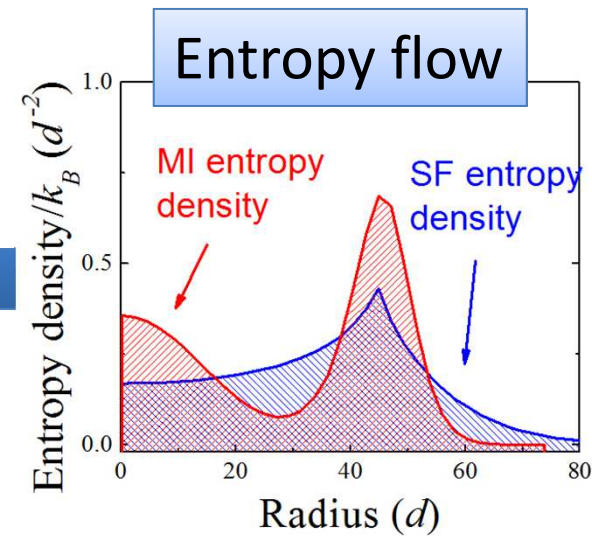
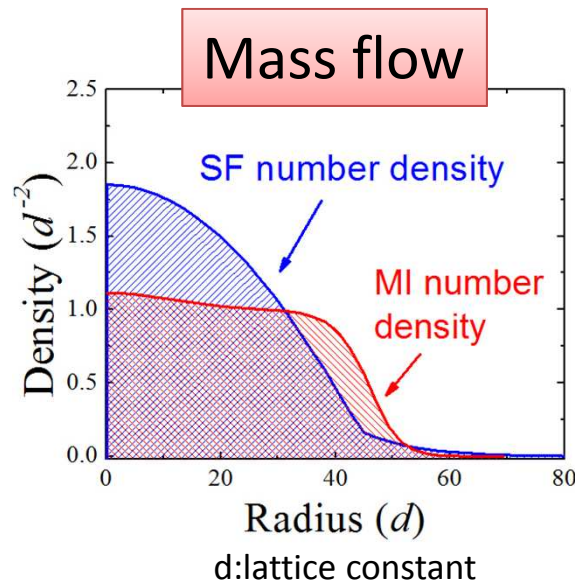


Prof. Nathan Gemelke
(Penn State Univ.)



Dr. Kathy-Anne Soderberg
(Booz allen Hamilton)

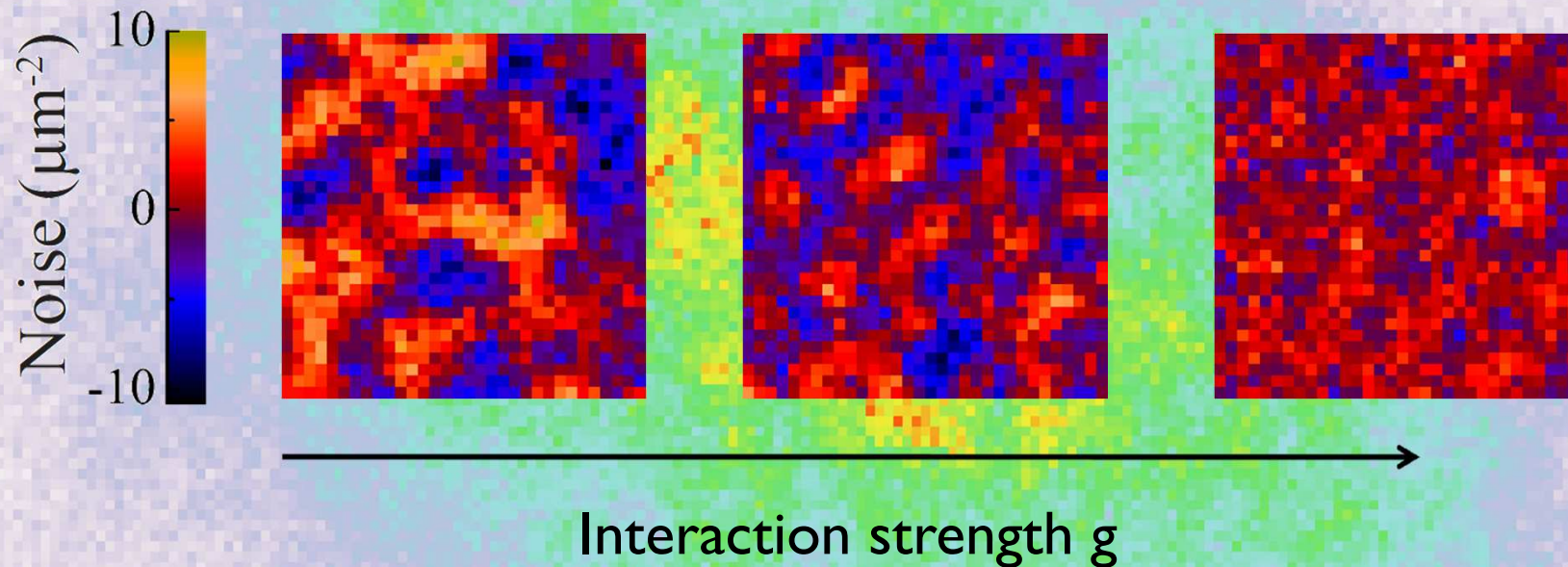
Dynamics across the SF-MI transition



Near-equilibrium
dynamics

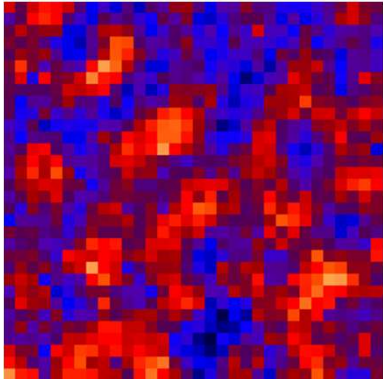
$$\begin{pmatrix} J_m \\ J_s \end{pmatrix} = - \begin{bmatrix} A_{00} & A_{01} \\ A_{10} & A_{11} \end{bmatrix} \begin{pmatrix} \nabla \mu \\ \nabla T \end{pmatrix}$$

Extracting density-density correlations from in situ images of 2D quantum gases



Discrete Fourier Transform (FFT)

→ Density noise power spectrum

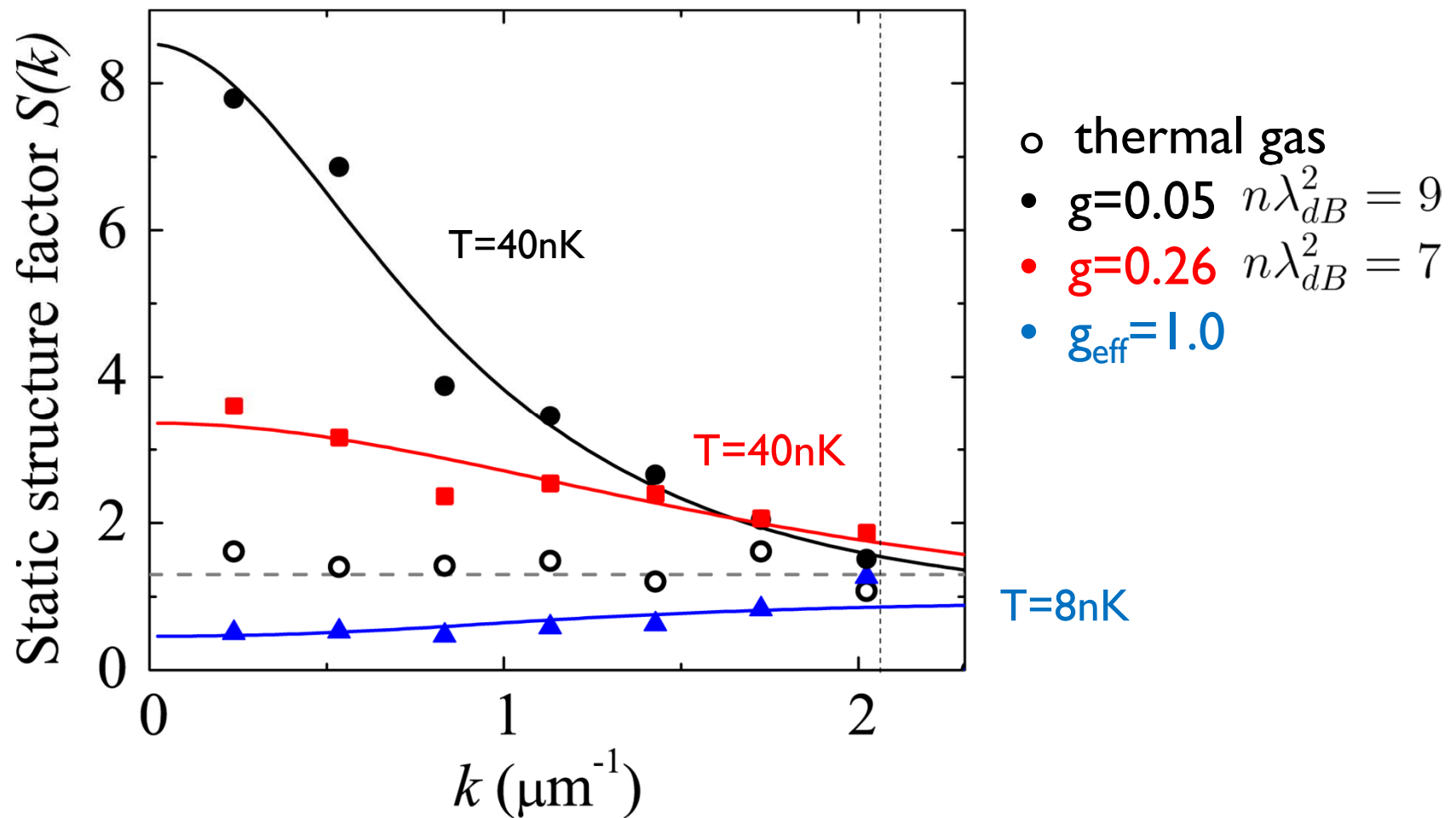
$$|\text{FFT}(\text{img})|^2$$


→ Static structure factor $S(k)$
after removing imaging systematics

$$S(k) = \frac{1}{n} \int \langle \delta n(0) \delta n(r) \rangle e^{-i\mathbf{k} \cdot \mathbf{r}} d^2r = \frac{\langle |\delta n(k)|^2 \rangle}{N}$$

k : spatial frequency

Static structure factor



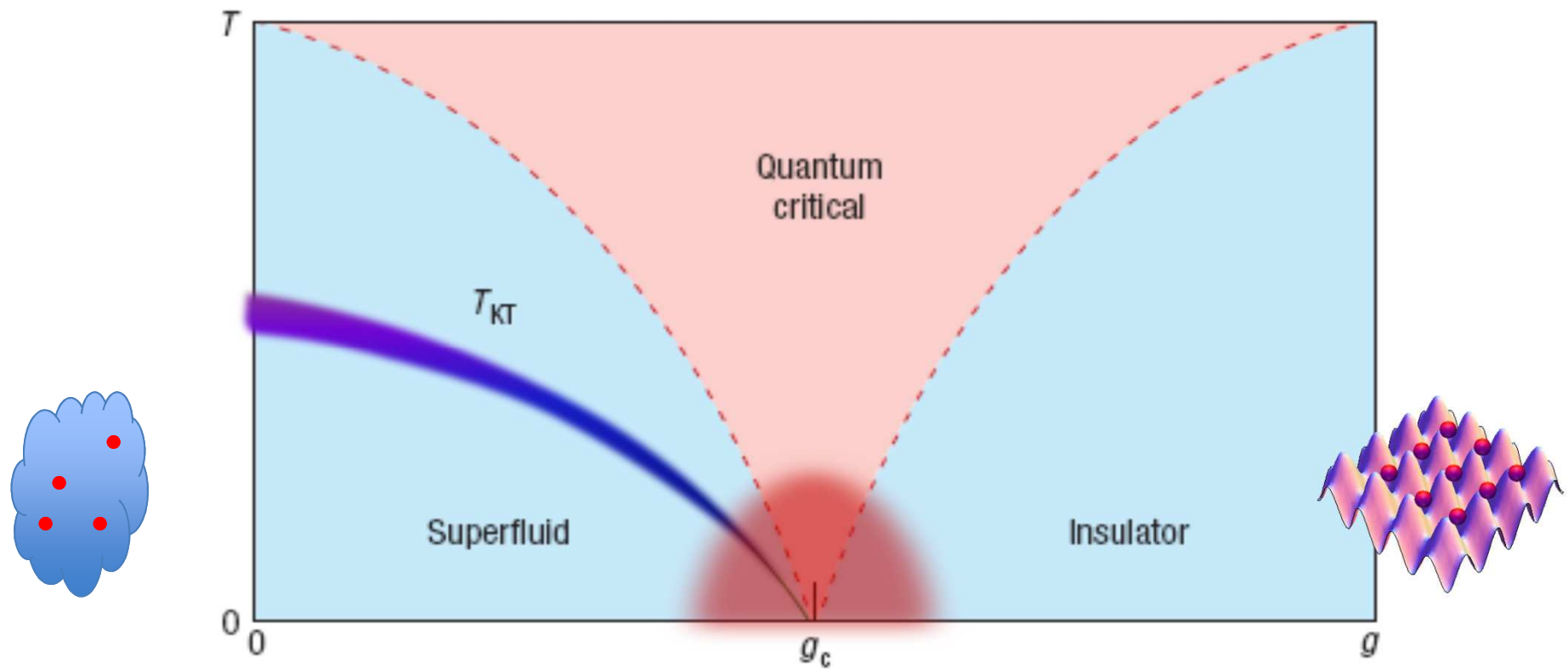
Universal scale behaviors regarding scale
invariance and dilute Bose gas universality in 2D
Critical fluctuations and correlations

Classical superfluid phase transition at finite temperatures



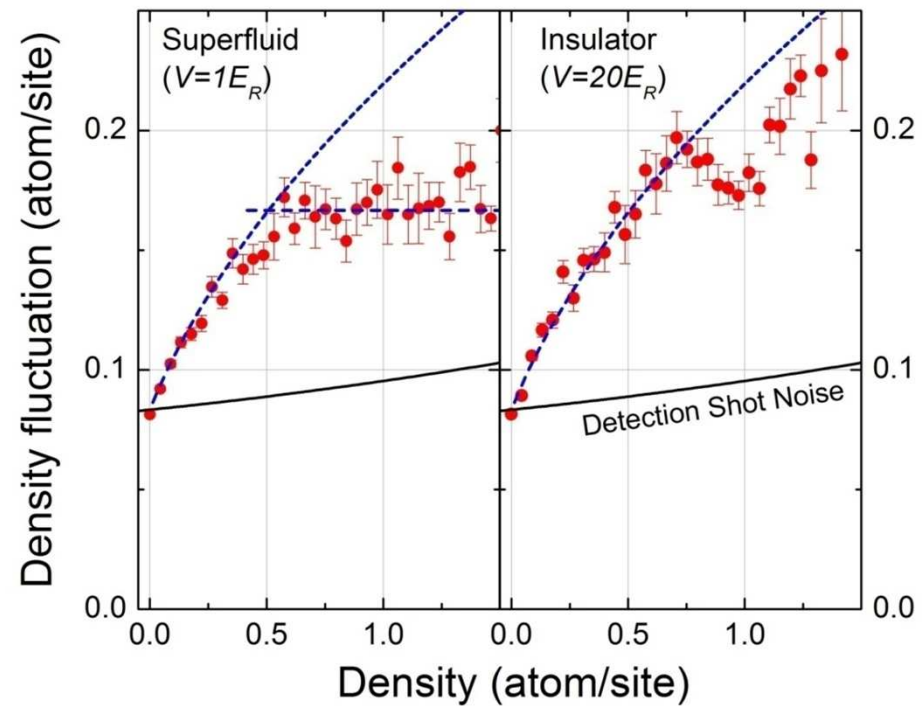
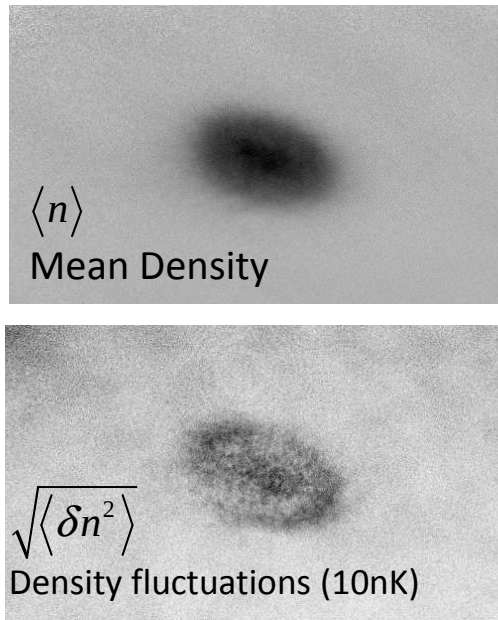
Quantum phase transitions near
absolute zero temperature

Phase transition and quantum phase transition



Phase diagram and AdS4-CFT3 duality: Sachdev, Nature physics (2007)

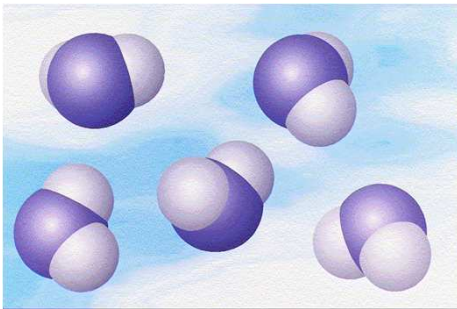
Reduced density fluctuations in a Mott Insulator



Fluctuation-dissipation theorem

fluctuation $\sum_j \langle \Delta n_i \Delta n_j \rangle = k_B T \kappa_i$ compressibility

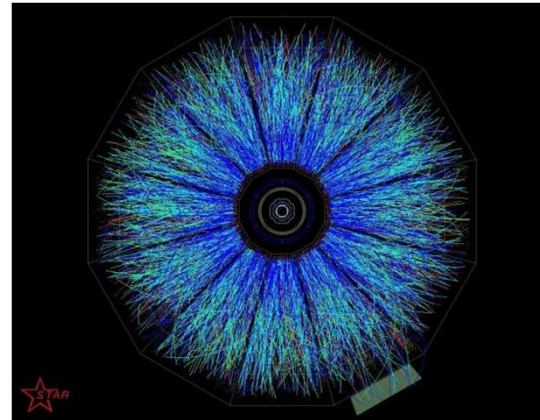
Quantum Simulation



Atoms and molecules

*Other applications:
quantum chemistry,
quantum information,
precision metrology*

Nuclear physics

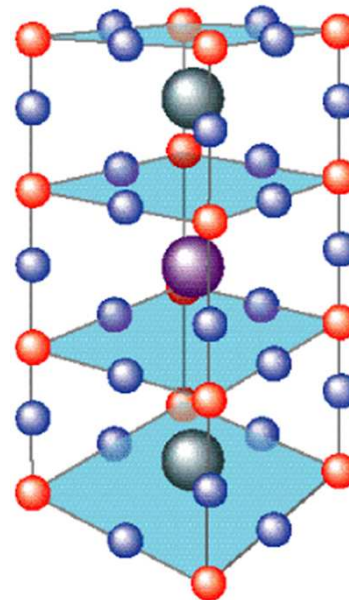


[Efimov physics](#)

[Quark-gluon plasma](#)

...

Condensed matter physics



[BEC-BCS crossover](#)

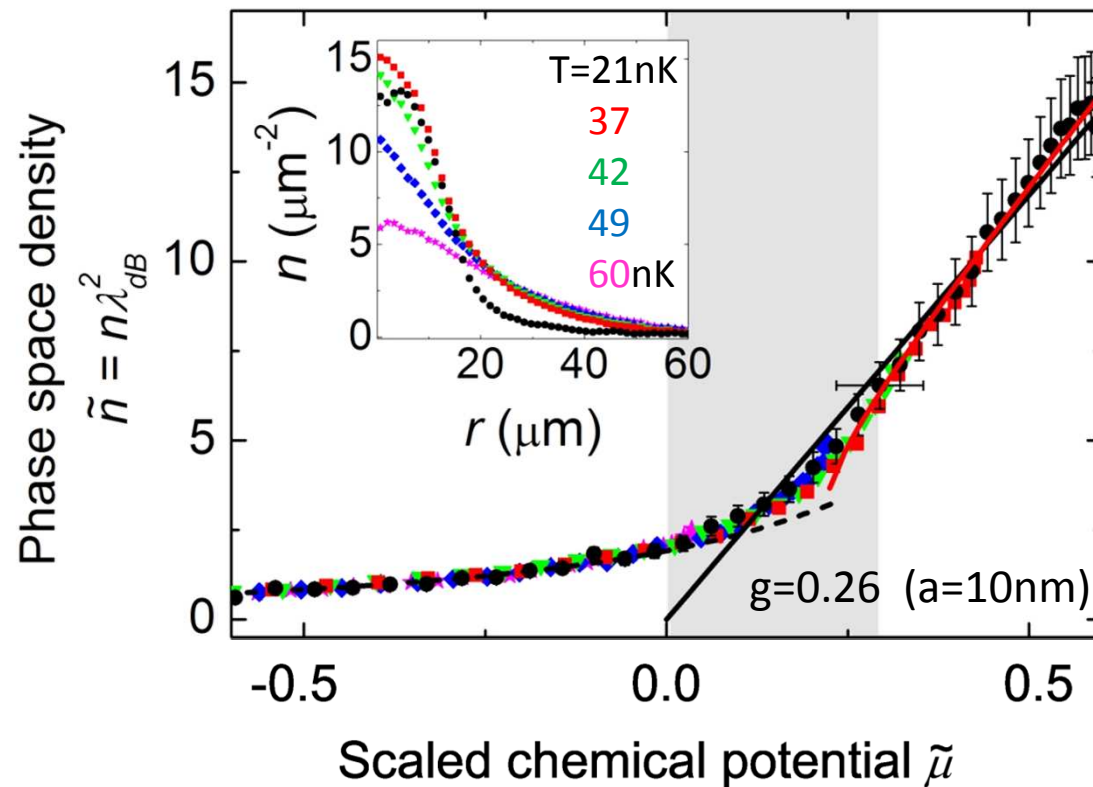
[Quantum magnetism](#)

...

Illustration of scaling symmetry



Scale Invariance - Density $\tilde{n} = n\lambda_{dB}^2 = F(\frac{\mu}{kT})$



----- $n(\mu, T) = -\lambda_{dB}^{-2} \ln[1 - \exp(\mu/k_B T - gn\lambda_{dB}^2/\pi)]$

— $\tilde{n} = 2\pi\tilde{\mu}/g + \ln(\frac{2\tilde{n}g}{\pi} - 2\tilde{\mu})$ Prokof'ev and Svistunov, PRA 66, 043608 (2002)