

# **Fe-based superconductors (FBS) at high magnetic fields**

Alex Gurevich

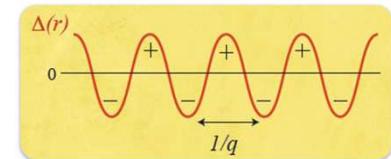
Old Dominion University, Norfolk, VA

Dept. Physics, University of Virginia, Sept. 26, 2011

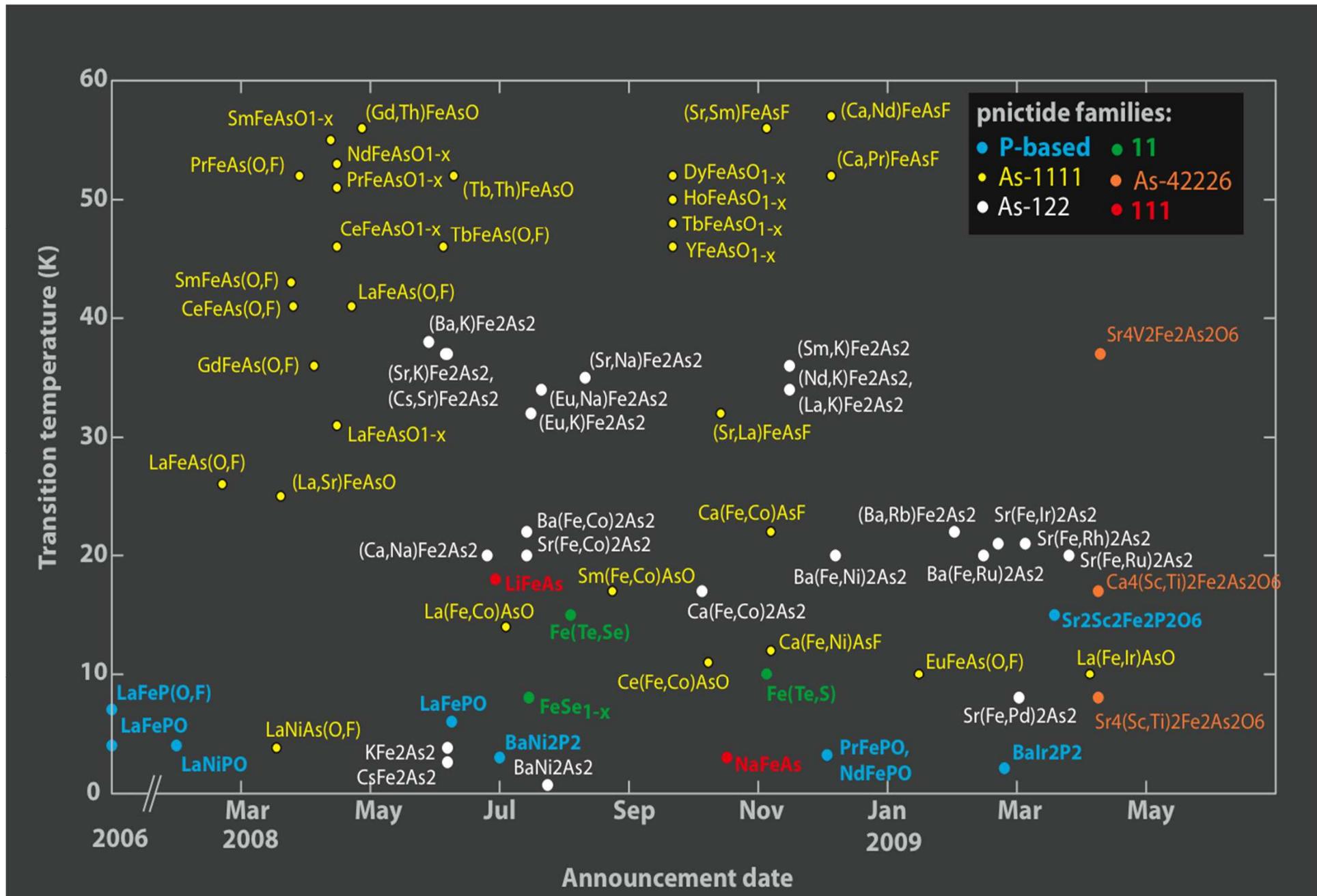
# Outline

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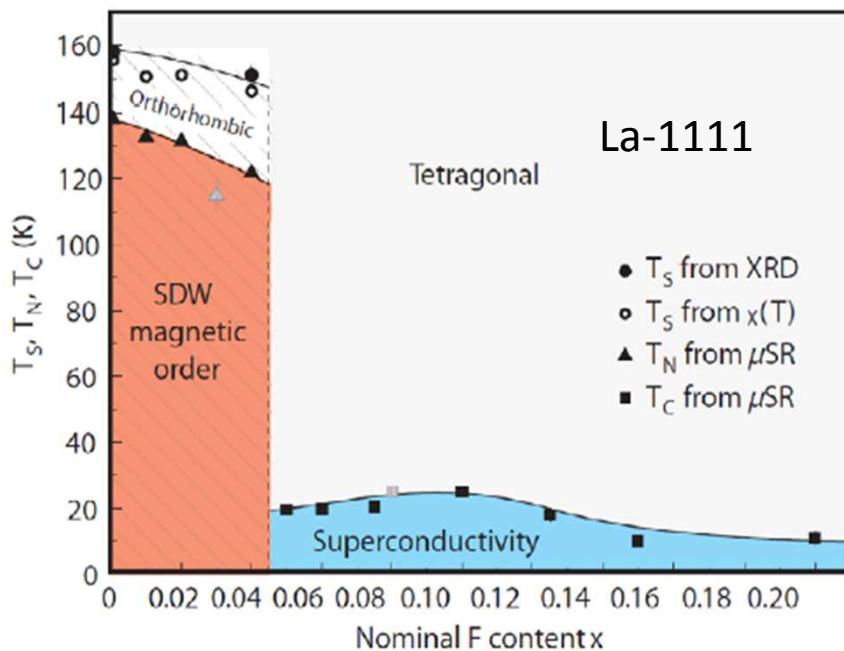
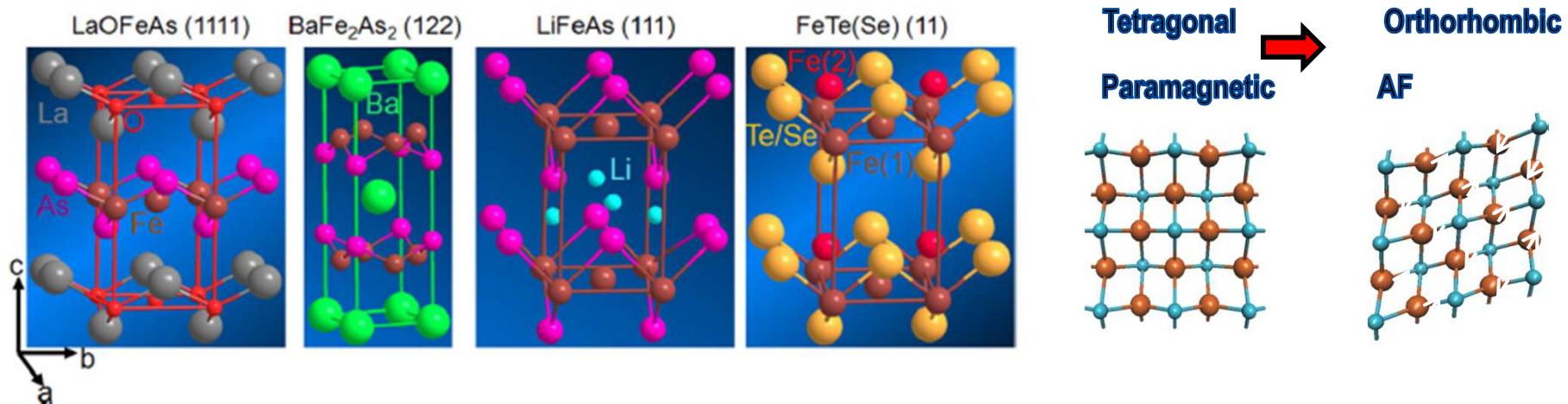
- Fe-based superconductors: unconventional multiband superconductivity mediated by magnetic fluctuations
- High  $T_c$  and huge upper critical magnetic fields. Interplay of orbital and paramagnetic pairbreaking in multiband SCs and their effect on  $H_{c2}(T)$
- Manifestations of the  $s^\pm$  pairing symmetry in the temperature dependence of  $H_{c2}(T)$ .
- Strong Pauli pairbreaking in FBS can lead to FFLO.  
Does the  $s^\pm$  pairing facilitate or inhibit the FFLO instability?
- FFLO in multiband FBS: what happens if one band is FFLO unstable but another one is not?
- Tuning  $H_{c2}(T)$  by doping: FFLO triggered by the Lifshitz transition



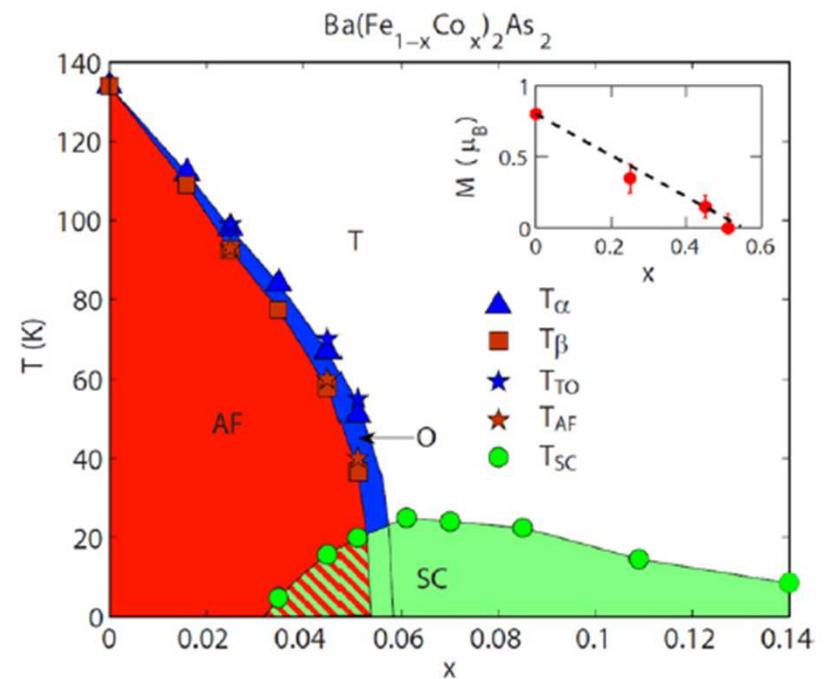
# Diverse family of Fe-based superconductors (FBS)



# Phenomenology of pnictides

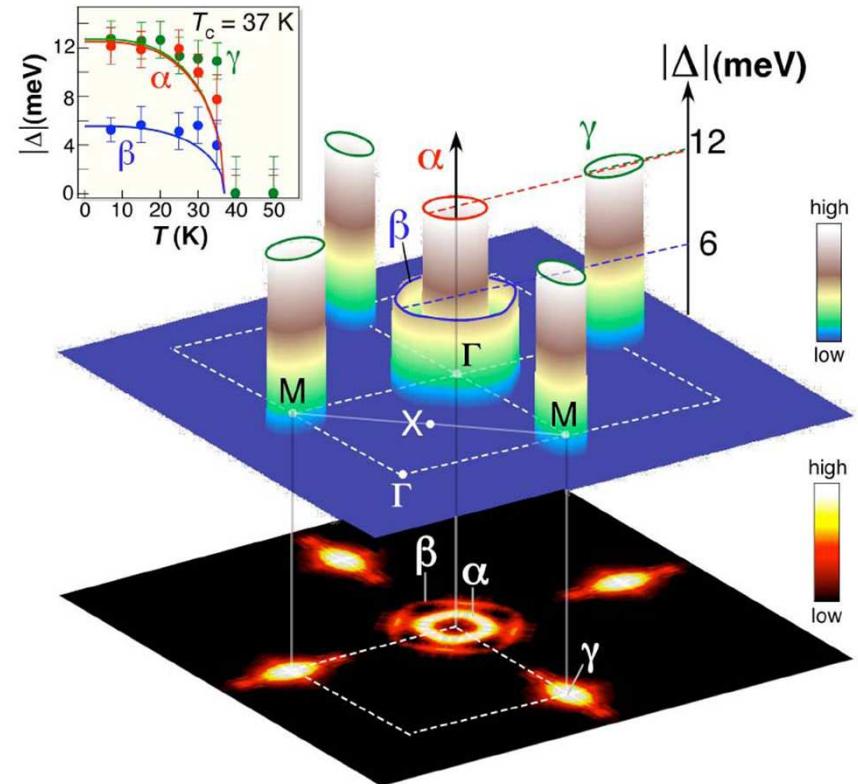
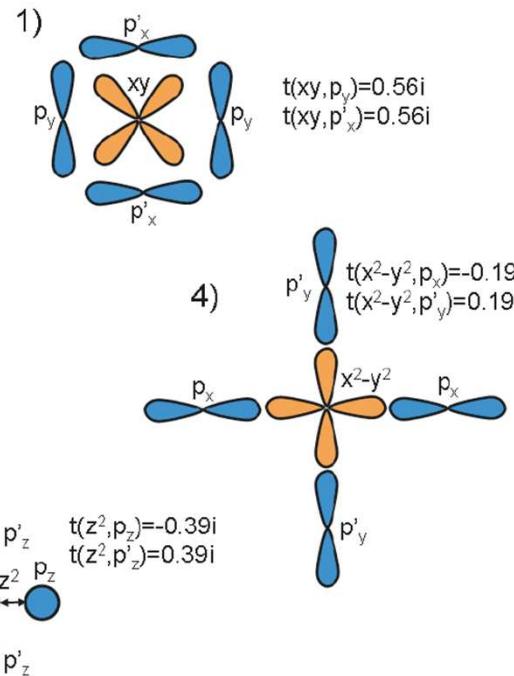
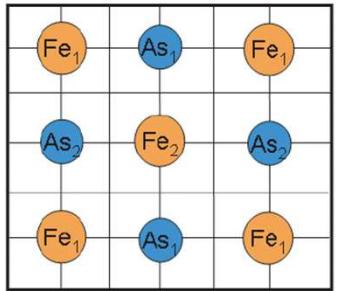


H. Luetkens et al, Nature Mat. 8, 305 (2009)



C. Lester et al, PRB 79, 144523 (2009)

# Multiband superconductivity in oxypnictides



Haule and Kotliar, NJP 025021 (2009)

Five d-orbitals of Fe hybridized with p-orbitals of As

Several disconnected pieces of FS

Multiple superconducting gaps

ARPES and tunneling:  
 $\text{Ba}_{0.6}\text{K}_{0.4}\text{Fe}_2\text{As}_2$

Ding et al, EL 83, 47001 (2008)

# The Matthias rules are violated:

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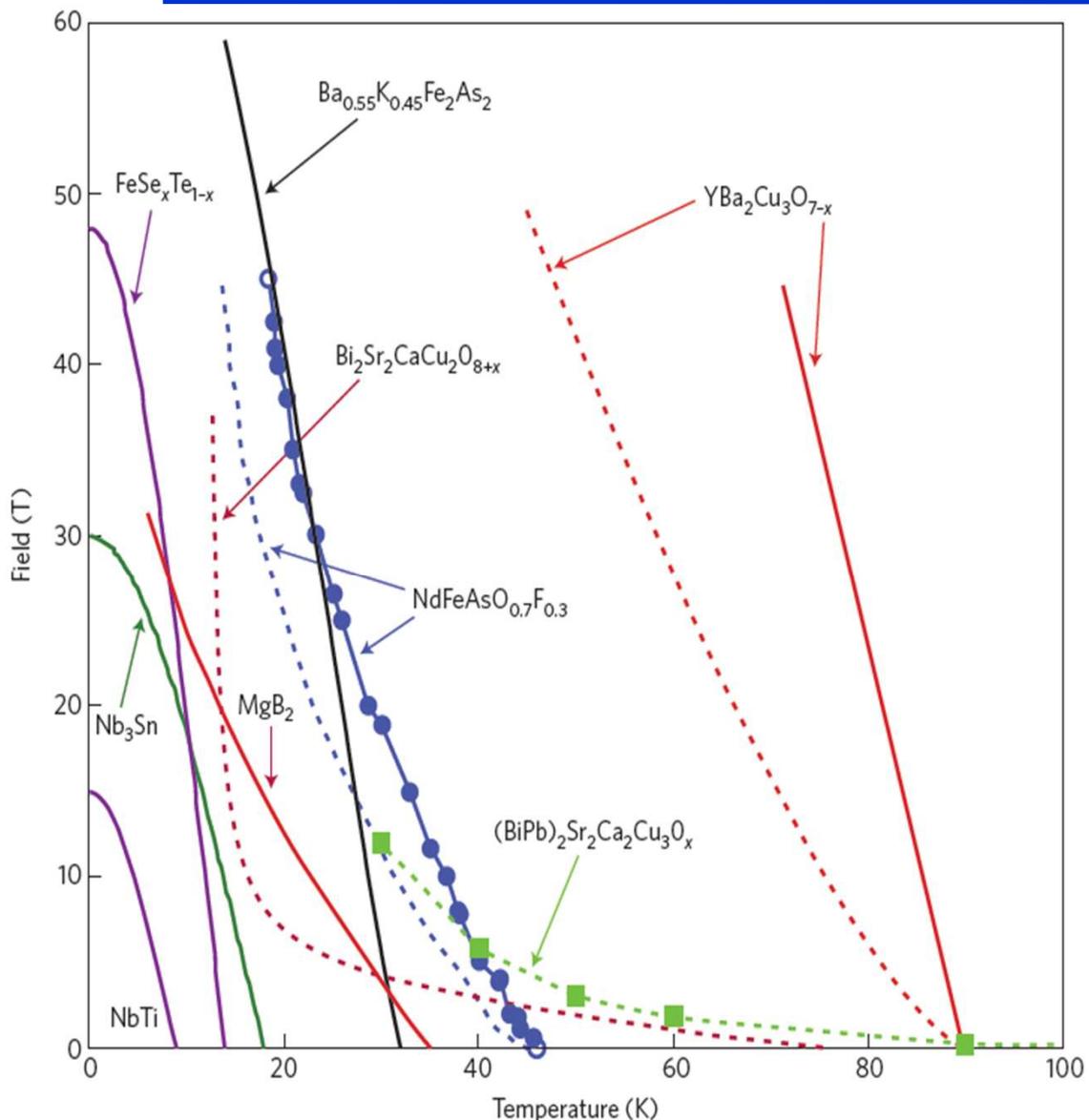
$$T_c = \Omega e^{-1/NV}$$

- High symmetry is good, cubic symmetry is best
- High density of electronic states is good
- Stay away from oxygen
- Stay away from magnetism
- Stay away from insulators
- Stay away from theorists



Bernd Matthias

# Huge $H_{c2}$ in pnictides



- High slopes  $H_{c2}' = 2-100 \text{ T/K}$  at  $T_c$
- $H_{c2}(0)$  for 1111 and 122 FBS, extrapolate to  $> 100\text{T}$
- Short GL coherence lengths

$$\xi_0 = \left[ \frac{\phi_0}{2\pi T_c H'_{c2}} \right]^{1/2} = 1 - 2 \text{ nm}$$

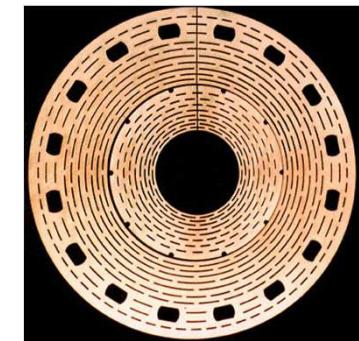
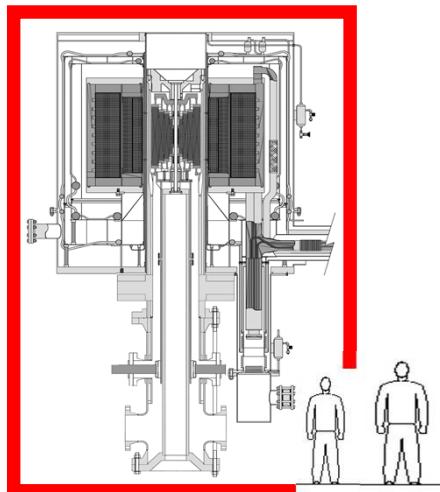
result from high  $T_c$  and low carrier density in semi-metallic FBS

$$\xi_0 = \frac{\hbar v_F}{2\pi T_c}$$

Dirty limit can hardly be reached

# High-field measurements at NHMFL

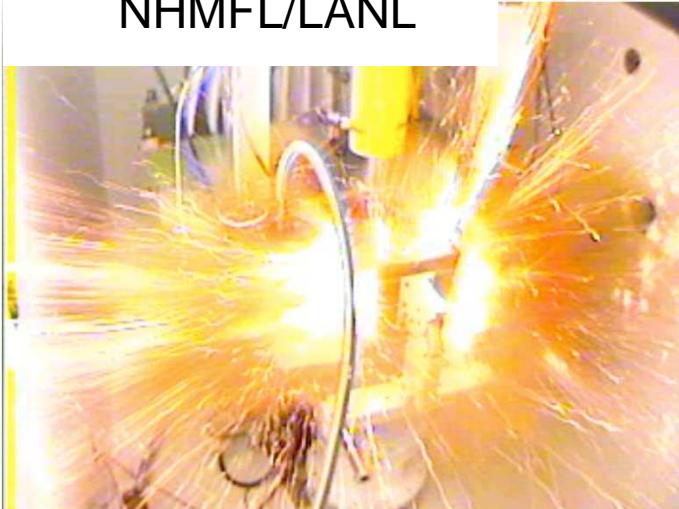
45T Hybrid Magnet  
*Highest DC Magnetic Field*



***"world's highest steady-field  
resistive (35 T) and  
hybrid (45 T) magnets"***

200T, 1 microsecond  
*One pulse per hour*

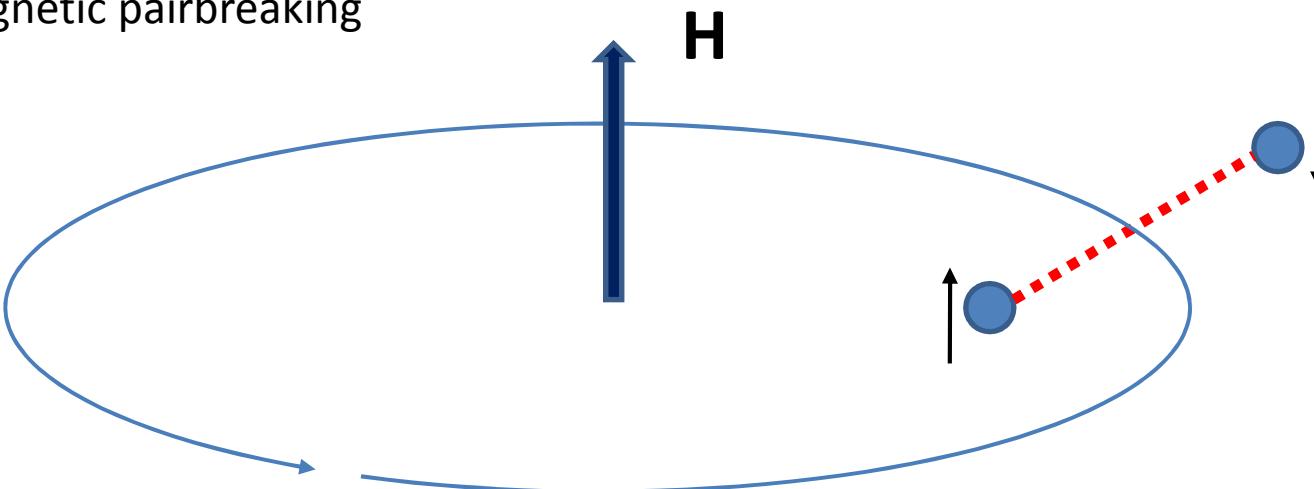
NHMFL/LANL



# Cooper pairs at high magnetic fields

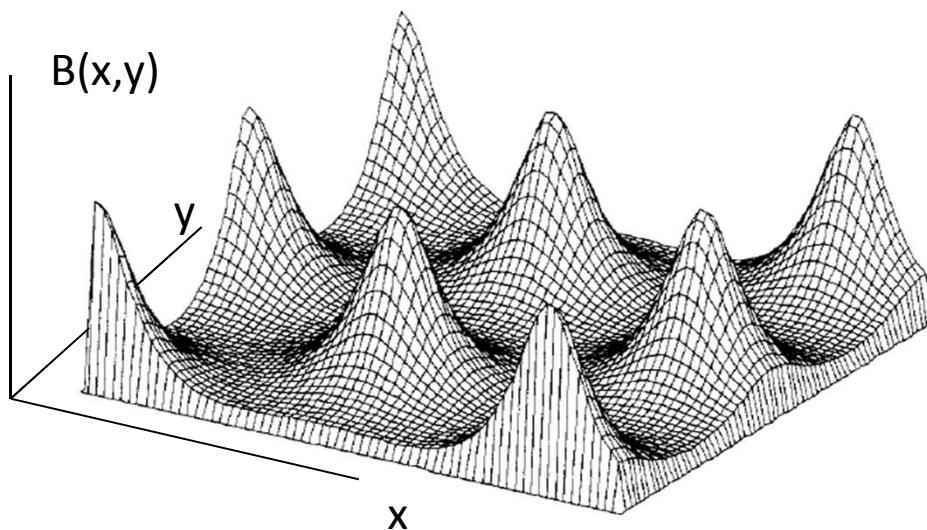
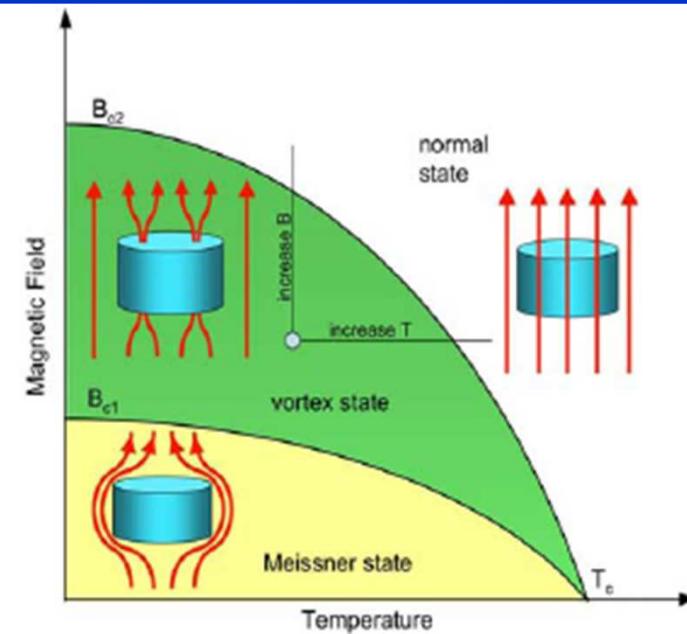
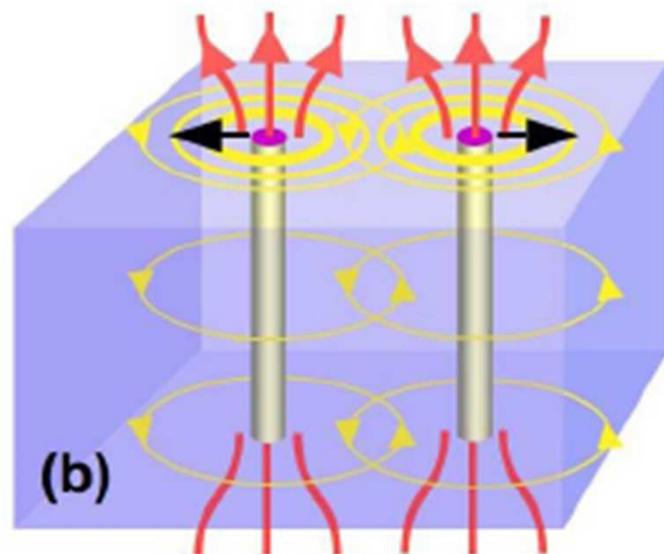
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- Larmor orbital motion of Cooper pairs: Vortex structure and orbital pairbreaking in vortex cores
- Paramagnetic pairbreaking



- Critical orbital velocity  $v_c$  to destroy superconductivity  $\rightarrow$  upper critical field:  $H_{c2}$
- Zeeman energy = binding energy of the Cooper pair  $\rightarrow$  paramagnetic limit:  $\mu_B H_p = \Delta$

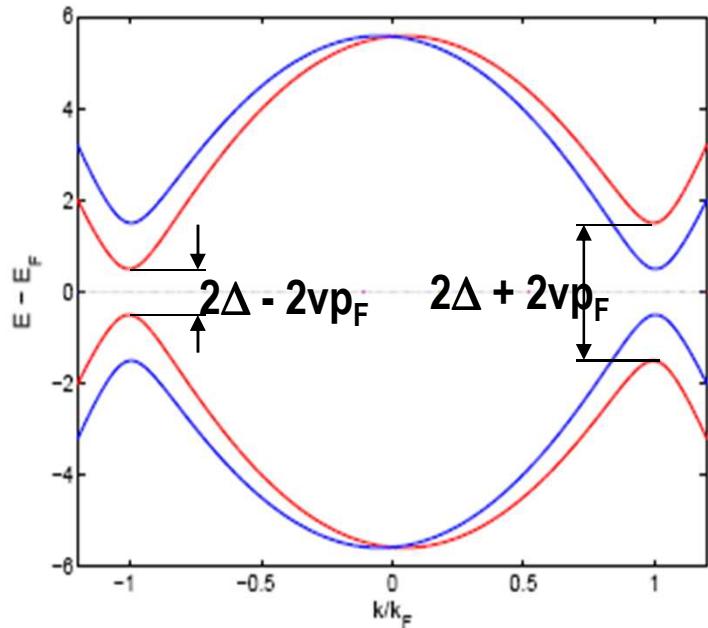
# Type-II superconductors



- Hexagonal lattice of vortex lines, each carrying the flux quantum  $\phi_0$
- Vortex density  $n(B) = \phi_0/B$
- Spacing between vortices:  $a = (\phi_0/B)^{1/2}$

# Paibreaking velocity, vortex core, and coherence length

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Doppler shift of the electron spectrum in the superflow:

$$E(p) = \pm \sqrt{\Delta^2 + (\epsilon_p - E_F)^2} \pm \vec{p}_F \vec{v}_s(t)$$

Gap reduction:  $\Delta(v_s) = \Delta - p_F |v_s|$  →  
the critical velocity

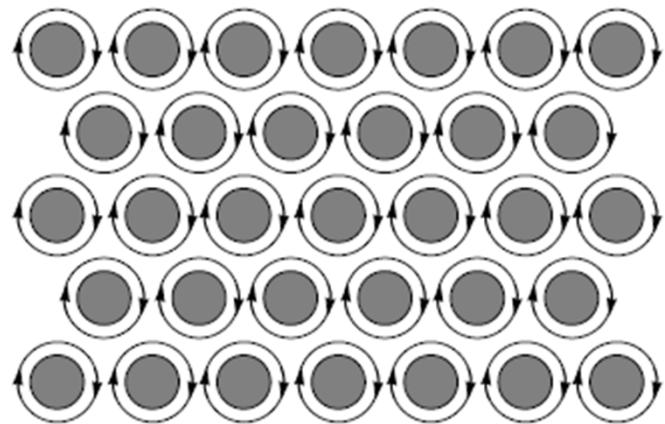
$$v_c = \Delta / p_F$$

Superfluid velocity around the vortex and the coherence length  $\xi$ :

$$v = \frac{\hbar}{m * R},$$

$$\frac{\hbar}{m * \xi} = v_c \quad \Rightarrow \quad \xi = \frac{\hbar v_F}{\Delta}$$

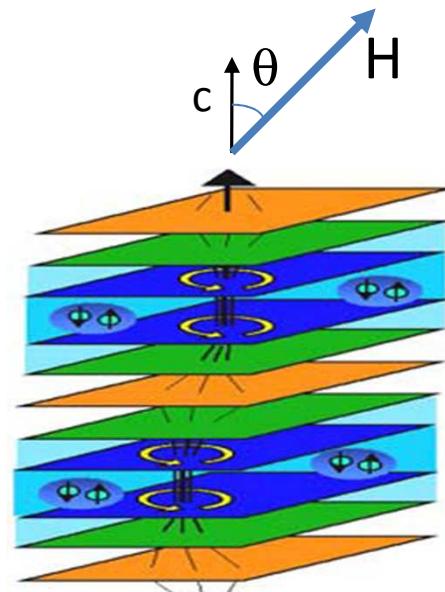
# Upper critical field:



At  $H = H_{c2}$  normal vortex cores overlap:

$$(\phi_0 / H)^{1/2} < \xi \quad \Rightarrow \quad H_{c2} = \frac{\phi_0}{2\pi\xi^2}$$

**Effect of anisotropy:**



$$H_{c2}(\theta) = \frac{H_{c2}}{\sqrt{\cos^2 \theta + \varepsilon \sin^2 \theta}}, \quad \varepsilon = \frac{m_{ab}}{m_c} < 1$$

Enhancement of parallel  $H_{c2}$ :

$$H_{c2}^{\parallel} = H_{c2} \sqrt{\frac{m_c}{m_{ab}}} = \frac{\pi^2 c \sqrt{m_{ab} m_c}}{2e\hbar} \frac{\Delta^2}{E_F}$$

Mass anisotropy, high  $T_c$  and low  $E_F$  greatly enhance  $H_{c2}$

# Does increasing $H_{c2}$ by disorder work in FBS?

Effect of the elastic mean free path  $\ell$  on the orbitally-limited  
(Werhamer-Helfand-Hohenberg, 1966)

$$H_{c2} = \phi_0 / 2\pi\xi^2$$

Clean limit:  $\ell \gg \xi_0 \Rightarrow \xi = \xi_0$  and

$$H_{c2} \cong \frac{\pi\phi\Delta^2}{2\hbar^2v_F^2} \propto T_c^2$$

Dirty limit :  $\ell \ll \xi_0 \Rightarrow \xi = (\ell\xi_0)^{1/2}$

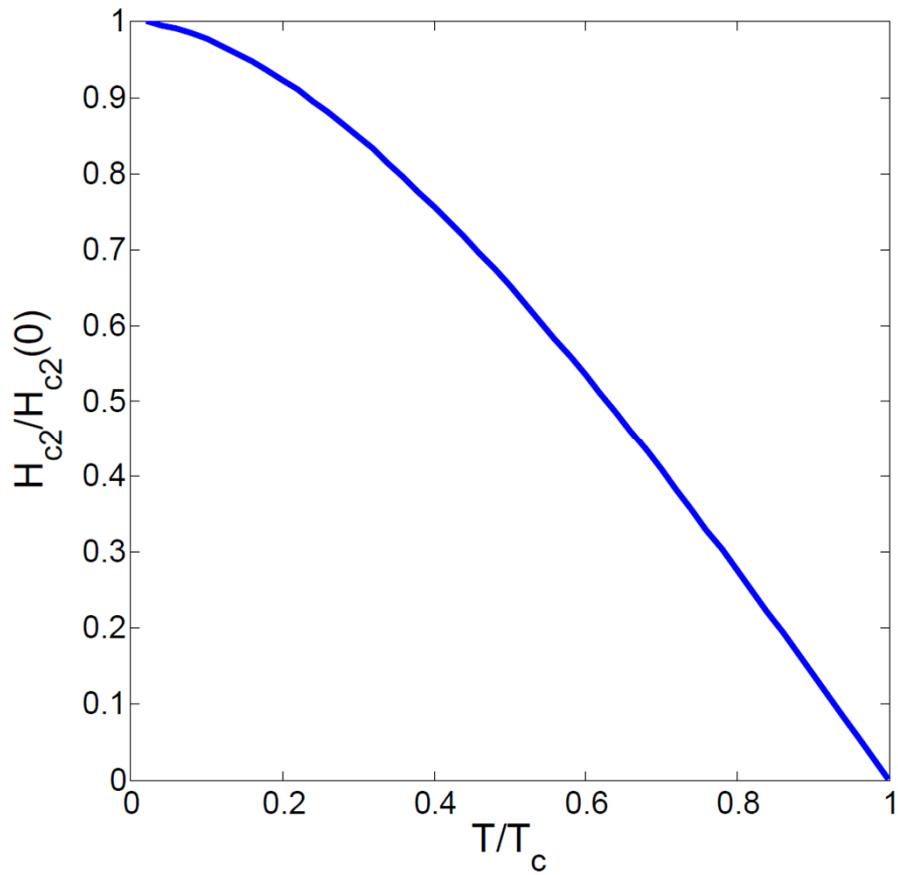
$$H_{c2} = \frac{\phi\Delta}{2\hbar v_F \ell} \propto T_c \rho_n$$

Works in conventional superconductors: 10 –fold increase of  $H_{c2}$  in MgB<sub>2</sub>

Does not work in FBS because  $\ell < \xi_0 \sim 1-2$  nm implies the Joffe-Regel limit and  $\ell k_F < 1$  for which the conventional transport theories fail

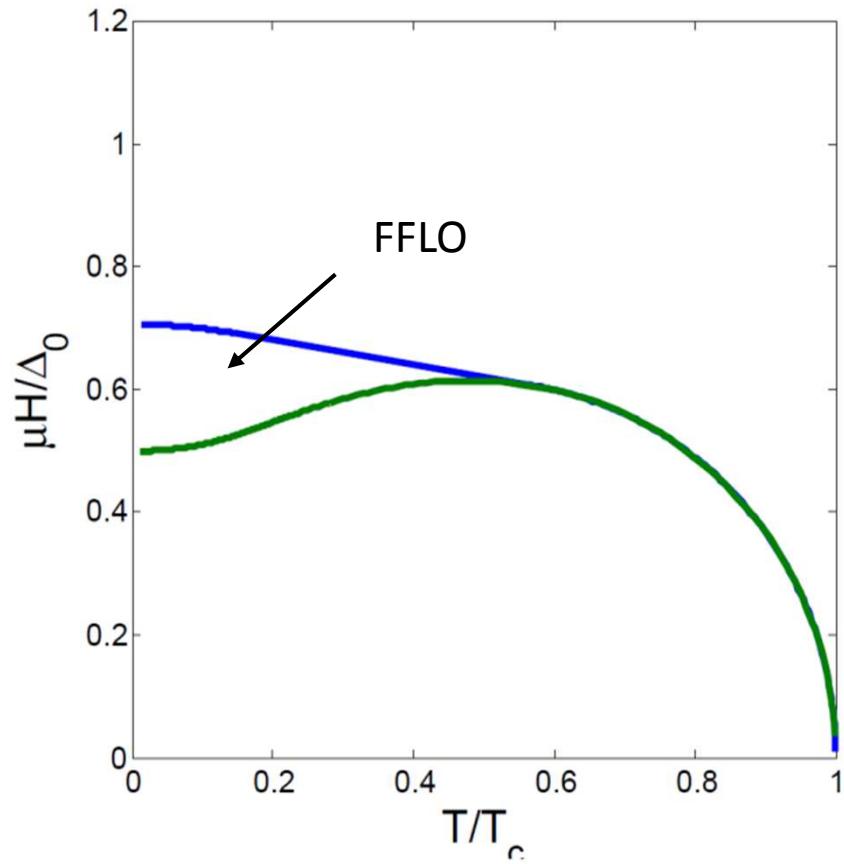
$H_{c2}$  in semi-metallic FBS can be effectively tuned by doping

# Orbital or Pauli-limited $H_{c2}$ ?



Orbitally limited

Werthamer-Helfand-Hohenberg  
1963-1965



Mostly Pauli limited

Sarma, Maki 1963-1964  
Gruenberg and Gunther, 1966

# Pauli pairbreaking

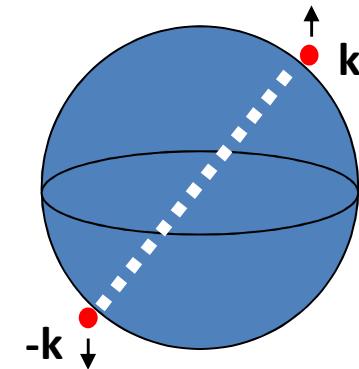
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Chandrasekhar – Klogston limit

$$\frac{\chi_n}{2} H_p^2 = N(0) \frac{\Delta^2}{2}, \quad \chi_n = 2\mu_B^2 N(0)$$

magnetic  
energy

condensation  
energy



$$\mu_B H_p = \Delta / \sqrt{2}$$

First order phase transition

Using BCS

$$\Delta = 1.78k_B T_c$$

yields a useful relation

$$H_p [\text{Tesla}] = 1.84 T_c [\text{Kelvin}]$$

# Relation between orbital and Pauli pairbreaking

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- Maki parameter  $\alpha_M = 2^{1/2} H_{c2}^{\text{orb}} / H_p$  :

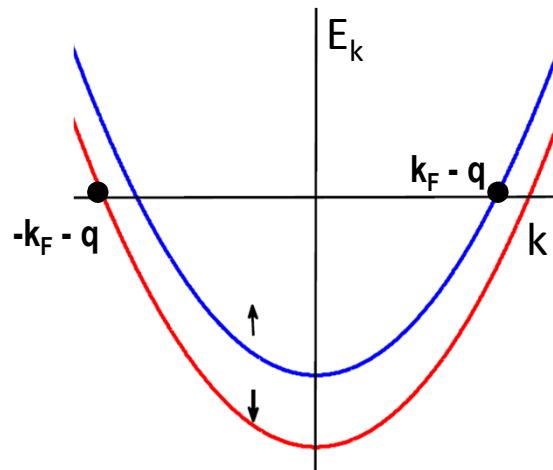
$$\alpha_M = \frac{\pi^2 \Delta}{4 E_F} \frac{m_{ab}}{m_0}, \quad H \perp ab$$
$$\alpha_M = \frac{\pi^2 \Delta}{4 E_F} \frac{\sqrt{m_{ab} m_c}}{m_0}, \quad H \parallel ab$$

- In ordinary metallic BCS superconductors with  $m_{ab} \sim m_0$  and  $\Delta \ll E_F$ , paramagnetic pairbreaking is negligible ,  $\alpha_M \ll 1$

## Pauli-limited superconductors with $\alpha_M > 1$

- Heavy fermions with  $m_{ab}/m_0 \sim 10^3$
- Highly anisotropic materials with  $m_c/m_0 \sim 10^6$  : layered organic SC, high- $T_c$  cuprates (BSCCO), etc for  $H \parallel ab$
- Semi-metallic, strongly correlated FBS with  $E_F < 0.01\text{-}0.1 \text{ eV}$ , and  $m_{ab}/m_0 \sim 10$

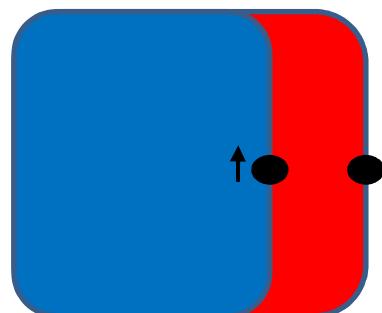
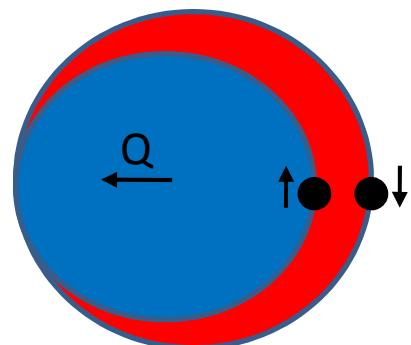
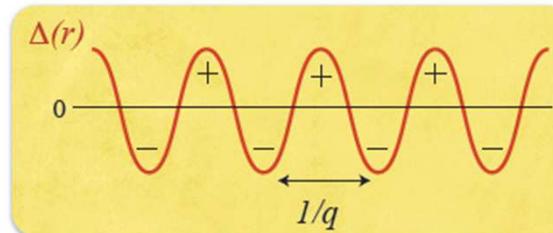
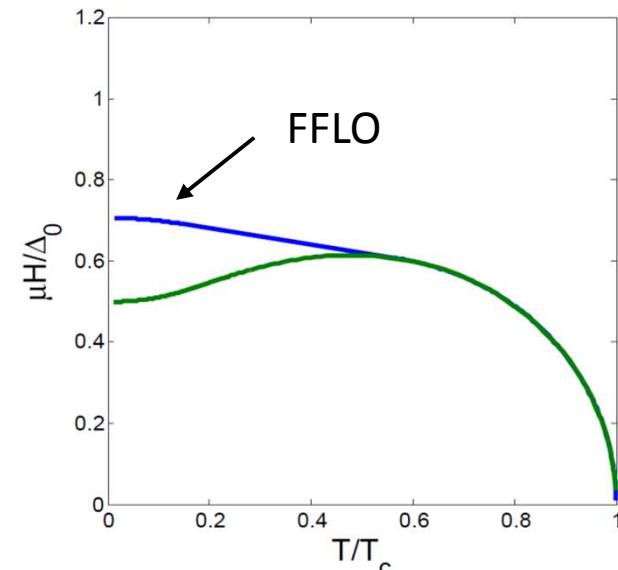
# Orbital and Pauli coupling: FFLO state



Cooper pairing with nonzero momentum  $Q = 2q$ :  
modulation of the order parameter along  $H$

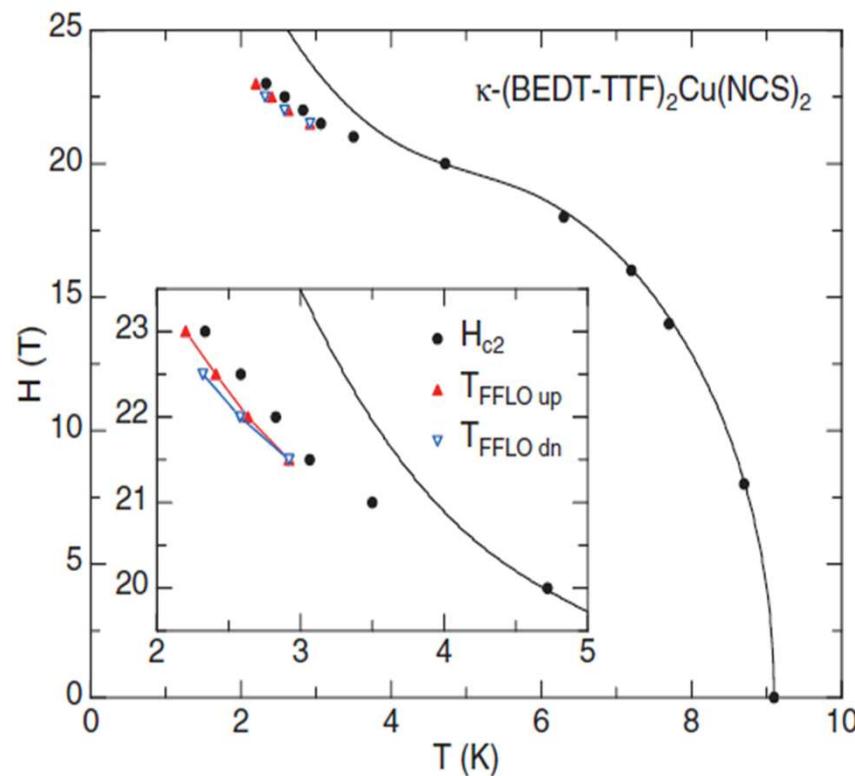
$$\Delta(z) = \Delta_0 \cos(Qz) \quad (\text{Larkin-Ovchinnikov})$$

$$\Delta(z) = \Delta_0 \exp(iQz) \quad (\text{Fulde-Ferrel})$$



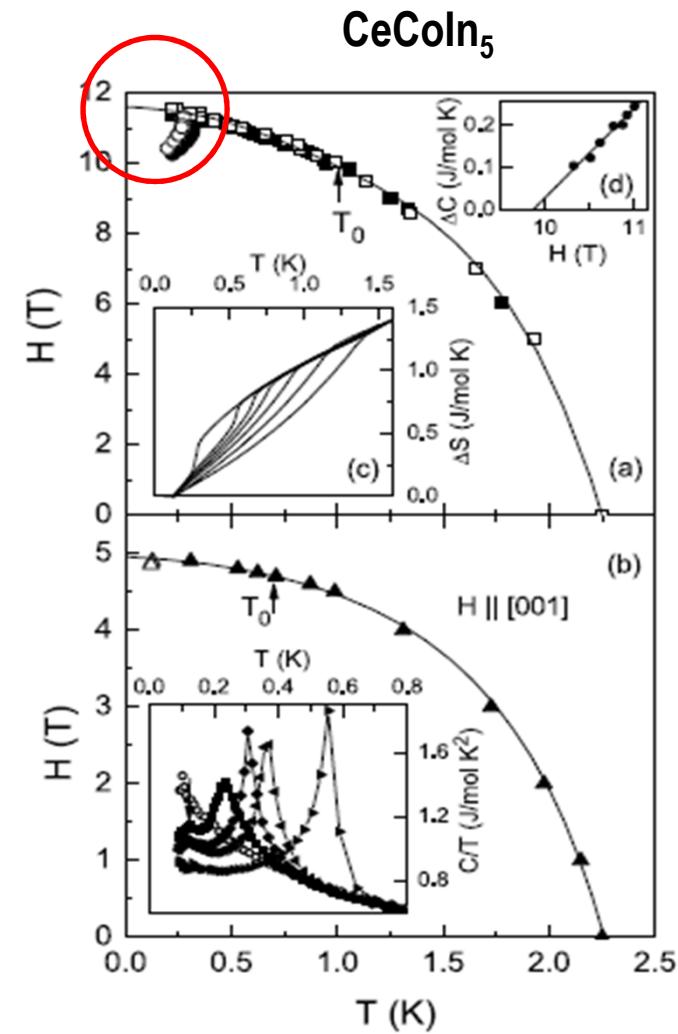
FS nesting facilitates  
the FFLO state

# FFLO in heavy fermions and organics



B. Lortz et al, PRL 99, 187002 (2007)

Layered organic SC



Bianchi et al, PRL 91, 187004 (2003)

Heavy fermions

# Theory of anisotropic FFLO (single band)

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Linearized Gor'kov equation in a uniaxial SC with ellipsoidal FS:

$$\Psi(\vec{r}) = \int d^3r' \Psi(\vec{r}') \int D(\vec{k}) \exp\left[i\vec{k}(\vec{r} - \vec{r}') + \frac{i\pi}{\phi_0} \vec{H} \cdot (\vec{r} \times \vec{r}')\right] \frac{d^3k}{(2\pi)^3}$$

$$D(k) = \text{Re} \sum_{\omega>0}^{\Omega} \frac{4\pi\lambda T}{v\sqrt{k_\perp^2 + \epsilon k_z^2}} \tan^{-1} \frac{v\sqrt{k_\perp^2 + \epsilon k_z^2}}{2(\omega + i\mu_B H)}$$

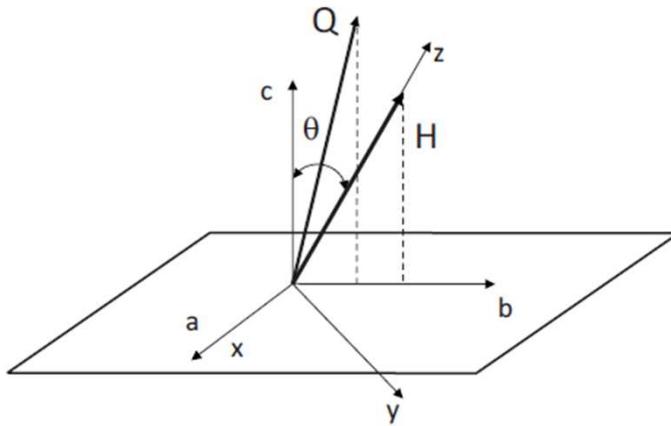
$H_{c2}$  is an eigenvalue of the Schroedinger equation for a particle with  $q = 2e$   
[Werthamer and Helfand, \(1964\)](#)

Tilted first Landau level eigenfunction:

$$\Psi(x, y) = \Delta \exp\left[-\frac{\pi H}{2\phi_0}(c_x x^2 + c_y y^2)\right] \exp[iQ\vec{r}]$$

$c_x$ ,  $c_y$  and the FFLO vector  $\mathbf{Q}$  are determined by the condition that  $H_{c2}$  is maximum

# Exact solution



- FFLO wave vector  $\mathbf{Q}$  is not parallel to  $\mathbf{H}$  unless  $\mathbf{H}$  is along the symmetry axis. The angle  $\gamma$  between  $\mathbf{Q}$  and  $\mathbf{H}$ :

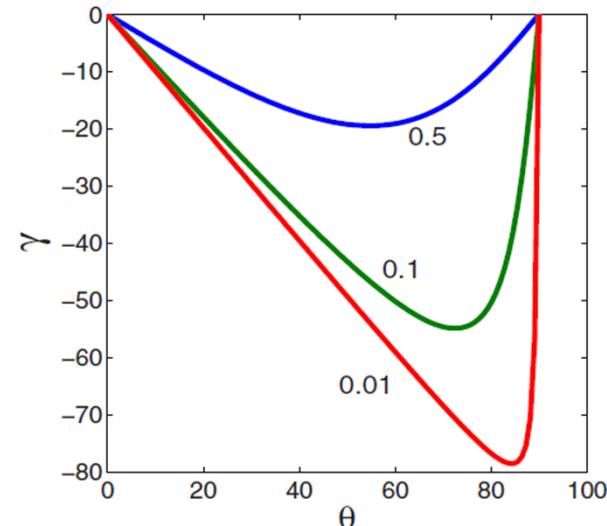
$$\tan \gamma = -\frac{(1-\varepsilon) \sin 2\theta}{2(\cos^2 \theta + \varepsilon \sin^2 \theta)}$$

- Competition between the FFLO kinetic energy  $\varepsilon \mathbf{Q}_z^2$  and the Zeeman energy
- GL angular scaling works for all T  
(Brison et al, Physica C 250, 198 (1995))

- $H_{c2}$  is maximum provided that:  
[AG, PRB 82, 184504 \(2010\)](#)

$$c_x = \mathcal{E}_\theta^{1/2}, \quad c_y = \mathcal{E}_\theta^{-1/2}$$

$$\mathcal{E}_\theta = \cos^2 \theta + \varepsilon \sin^2 \theta$$



$$H_{c2}(\theta) = \frac{H_{c2}(0)}{(\cos^2 \theta + \varepsilon \sin^2 \theta)^{1/2}}$$

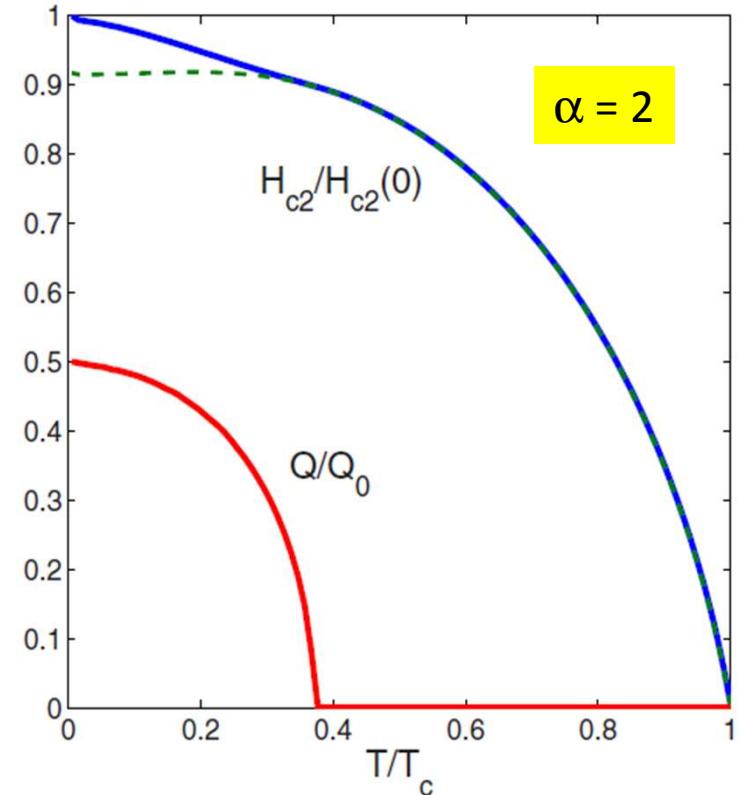
# Equation for $H_{c2}$ and Q

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$$\ln t + U(t, b, q) = 0$$

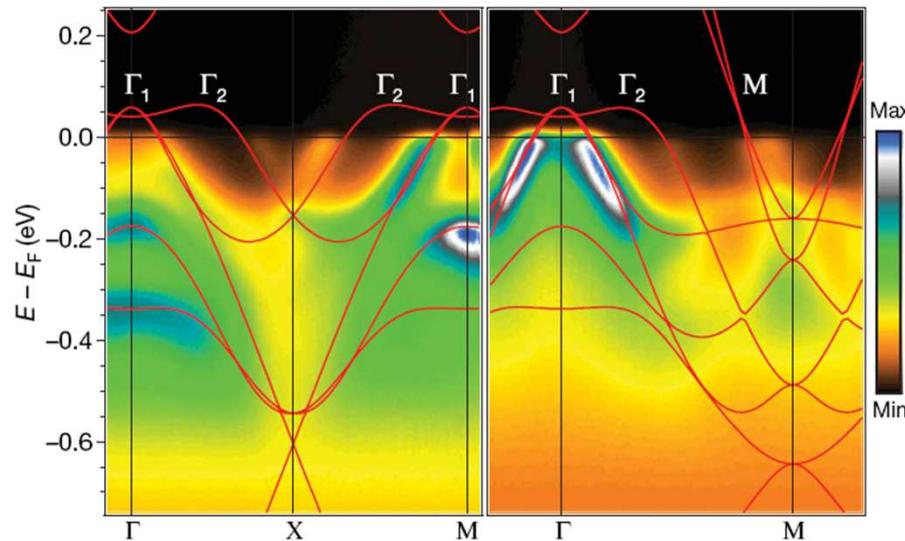
$$U = 2e^{q^2} \operatorname{Re} \sum_{n=0}^{\infty} \int_0^{\infty} du e^{-u^2} \left\{ \frac{u}{n+1/2} - \frac{t}{\sqrt{b}} \tan^{-1} \left[ \frac{u\sqrt{b}}{(n+1/2)t + i\alpha b} \right] \right\},$$

$$b = \frac{\hbar^2 v^2 \epsilon_{\theta}^{1/2} H}{8\pi\phi_0 T_c^2}, \quad \alpha = \frac{4\mu_B \phi_0 T_c}{\hbar^2 v^2 \epsilon_{\theta}^{1/2}}, \quad q^2 = \frac{Q_z^2 \epsilon \phi_0}{2\pi H \epsilon_{\theta}^{3/2}}$$



- FFLO transition for  $\alpha > 1$
- Spontaneous FFLO vector  $Q(T)$  appears at low  $T$
- The FFLO period  $\ell(T) = 2\pi/Q(T)$  diverges at the spinodal:  $T = T_{\text{FFLO}}$
- At zero  $T$ :  $\ell(0) \sim \xi_0$ . First order transition line between two spinodals.

# Electron spectrum from ab-initio calculations and ARPES



**Figure 2 | Comparison between angle-resolved photoemission spectra and LDA band structures along two high-symmetry lines.** ARPES data from LaOFeP (image plots) were recorded using 42.5-eV photons with an energy resolution of 16 meV and an angular resolution of 0.3°. For better

LaFeP(O,F)

- multiple bands crossing the Fermi level
- two hole pockets at  $\Gamma$  and two electron pockets at M

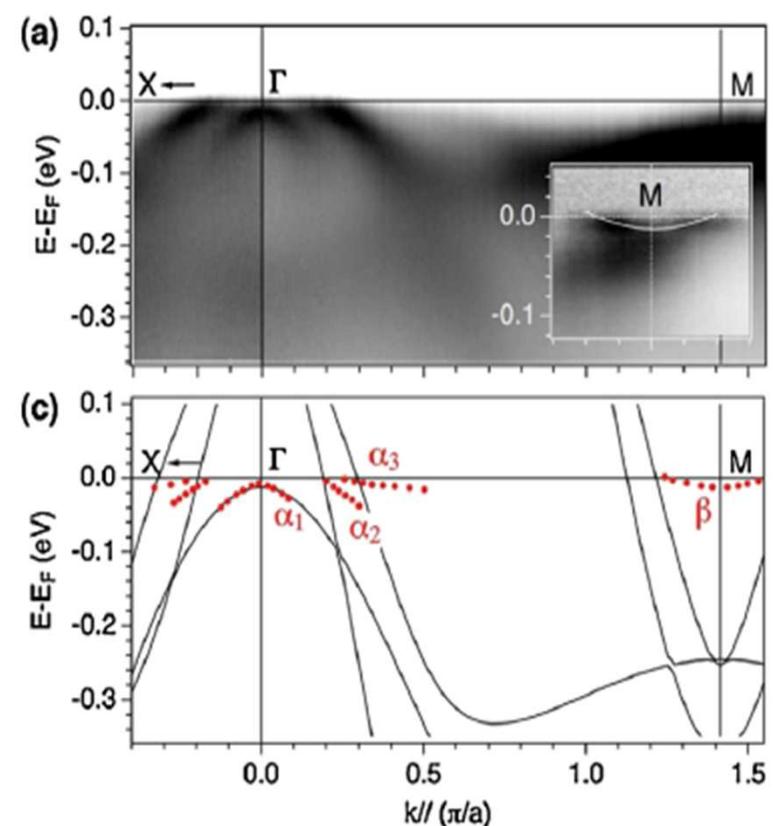
Vol 455 | 4 September 2008 | doi:10.1038/nature07263

nature

LETTERS

## Electronic structure of the iron-based superconductor LaOFeP

D. H. Lu<sup>1</sup>, M. Yi<sup>1</sup>, S.-K. Mo<sup>1,2</sup>, A. S. Erickson<sup>3</sup>, J. Analytis<sup>3</sup>, J.-H. Chu<sup>3</sup>, D. J. Singh<sup>4</sup>, Z. Hussain<sup>2</sup>, T. H. Geballe<sup>3</sup>, I. R. Fisher<sup>3</sup> & Z.-X. Shen<sup>1\*</sup>



FeSe<sub>0.42</sub>Te<sub>0.58</sub>

PRL 104, 097002 (2010)

PHYSICAL REVIEW LETTERS

week ending  
5 MARCH 2010

## Strong Electron Correlations in the Normal State of the Iron-Based FeSe<sub>0.42</sub>Te<sub>0.58</sub> Superconductor Observed by Angle-Resolved Photoemission Spectroscopy

A. Tamai,<sup>1</sup> A. Y. Ganin,<sup>2</sup> E. Rozbicki,<sup>1</sup> J. Bacsa,<sup>2</sup> W. Meevasan,<sup>1</sup> P. D. C. King,<sup>1</sup> M. Caffio,<sup>3</sup> R. Schaub,<sup>3</sup> S. Margadonna,<sup>4</sup> K. Prassides,<sup>5</sup> M. J. Rosseinsky,<sup>2</sup> and F. Baumberger<sup>1</sup>

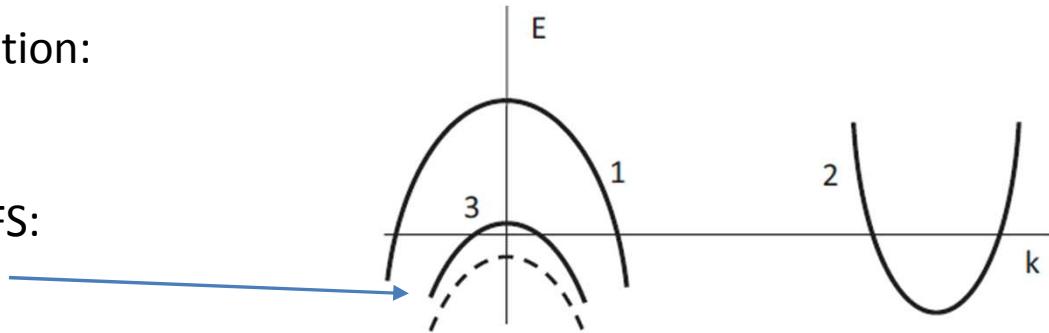
# New features of FBS revealed by ARPES

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- Small Fermi energies:  $E_F \sim 0.02\text{-}0.5 \text{ eV}$

- Large effective mass renormalization:  
 $m^* \sim (2\text{-}16)m_e$

- Several shadow bands near the FS:  
Lifshitz transition upon doping



- Strongly correlated semimetals
- Good candidates for the FFLO state:  $\alpha > 1$

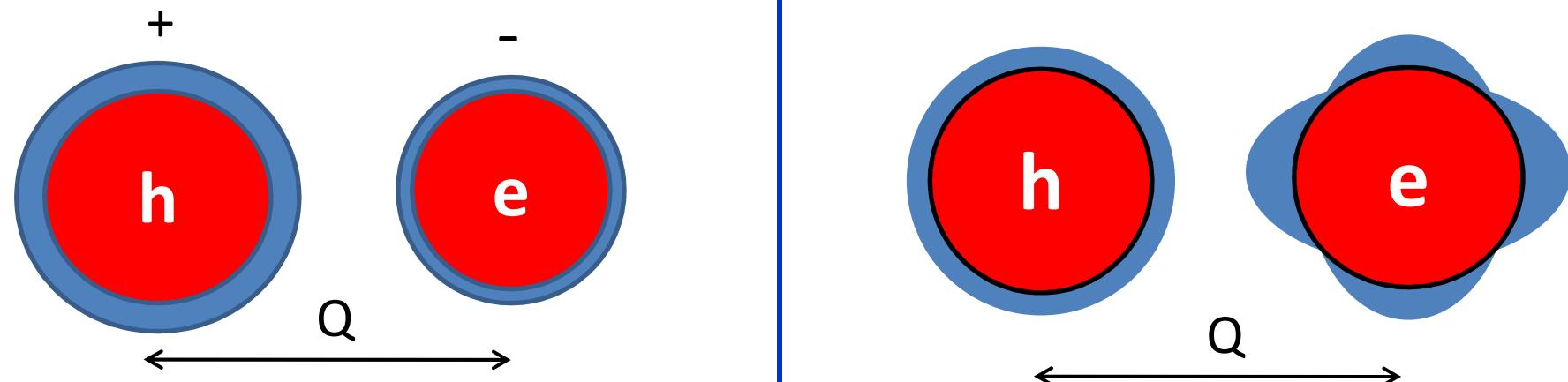
- Example of a Pauli-limited SC:  $\text{FeSe}_{0.5}\text{Te}_{0.5}$  :  
 $T_c = 16\text{K}$ ,  $E_F = 25 \text{ meV}$ ,  $m_{ab} = 10m_e$

$$\alpha = \frac{\pi k_B T_c m_{ab}}{E_F m_e}$$

$\alpha = 1.5$  even for  $H \parallel c$

- In-plane coherence lengths  $\xi \approx 1\text{-}2 \text{ nm}$

# Multiband pairing gap symmetries



- $s^\pm$  pairing: gaps with opposite signs

Mazin, Singh, Johannes, Du, PRL 101, 057003 (2008);  
Kuroki et al, PRL 101, 087004 (2008)

- Combined s and d-wave gaps

Kuroki et al, PRL 101, 087004 (2008);  
Graser, Maier, Hirshfeld, Scalapino, NJP 11, 025016 (2009)

Strong interband repulsion:  $\lambda_{12}\lambda_{21} > \lambda_{11}\lambda_{22}$

Phonons are not sufficient to explain high  $T_c$

Pairing coupling constants

$$\Lambda = \begin{pmatrix} \lambda_{11} & \lambda_{12} \\ \lambda_{21} & \lambda_{22} \end{pmatrix}$$

Impurity scattering rates

$$\Gamma = \begin{pmatrix} \gamma_{11} & \gamma_{12} \\ \gamma_{21} & \gamma_{22} \end{pmatrix}$$

# Critical temperature

Suhl, Mattias, Walker PRL 3, 552 (1959); Moskalenko, FMM 8, 25 (1959):

$$T_{c0} = 1.14\omega_D \exp[-(\lambda_+ - \lambda_0)/2w],$$

$$\lambda_{\pm} = \lambda_{11} \pm \lambda_{22}, \quad \lambda_0 = \sqrt{\lambda_-^2 + 4\lambda_{12}\lambda_{21}},$$
$$w = \lambda_{11}\lambda_{22} - \lambda_{12}\lambda_{21},$$

Pairbreaking interband impurity scattering

$$T_c \cong T_{c0} - \frac{\pi\gamma_{12}}{8} \left( 1 \mp \sqrt{\frac{N_1}{N_2}} \right)^2$$

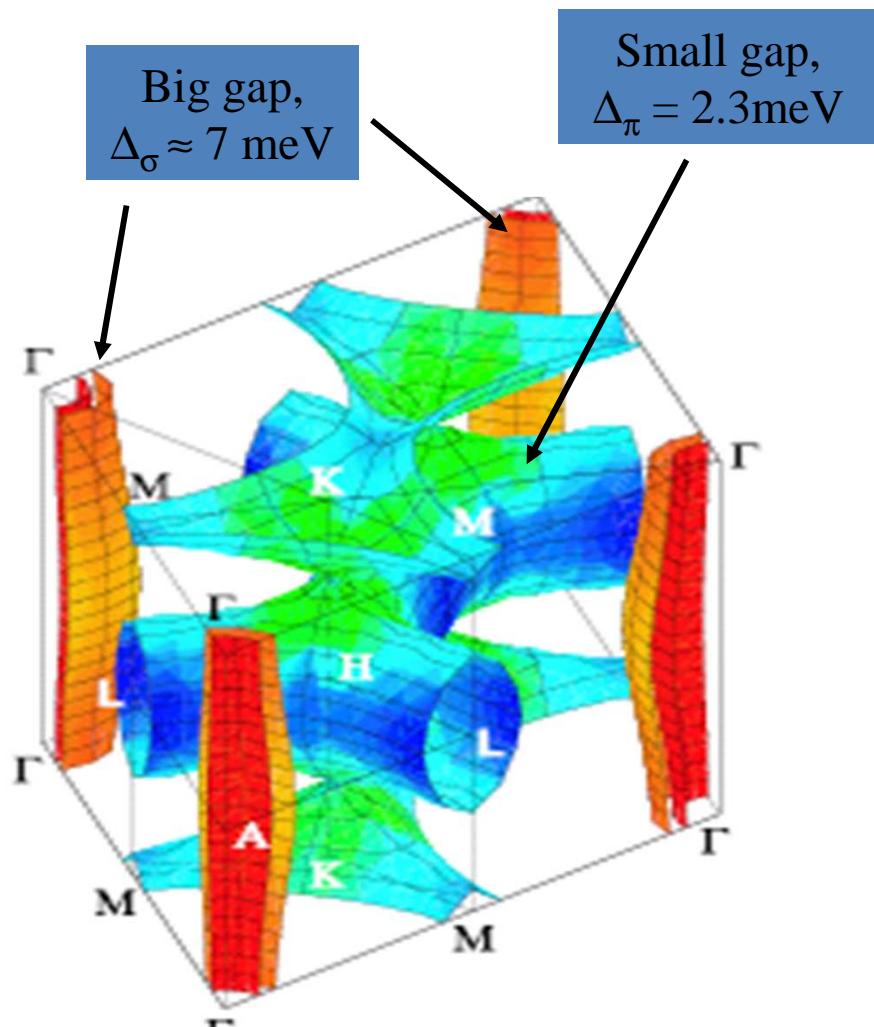
Interband coupling increases  $T_c$

Golubov, Mazin, PRB 55, 15146 (1997);  
AG, PRB 67, 184515 (2003)

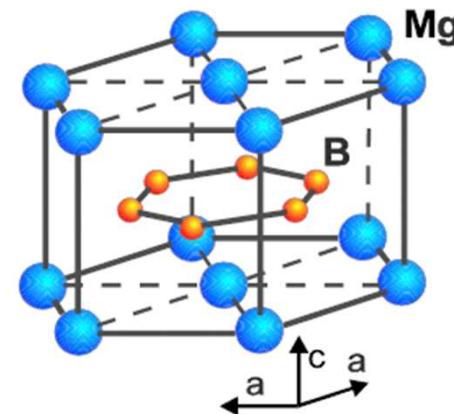
- **Weak interband pairing** in MgB<sub>2</sub>,  $\lambda_{12}\lambda_{21} < \lambda_{11}\lambda_{22}$
- **Strong interband pairing** in pnictides,  $\lambda_{12}\lambda_{21} > \lambda_{11}\lambda_{22}$

$T_c$  suppression by impurities is much weaker for the two-gap s<sup>++</sup>(-) than for s<sup>±</sup> (+)

# Two-gap superconductivity in MgB<sub>2</sub>



Liu, Mazin and Kortus (2002);  
Choi et al, (2002)



- 2D big gap for in-plane  $\sigma$ -orbitals s and 3D small gap for out-of-plane  $\pi$ -orbitals
- Weak interband coupling due to orthogonal  $p_z$  and  $p_{xy}$  orbitals of B

Band structure calculations:

$$\lambda_{11} \approx 0.81, \lambda_{22} \approx 0.3, \lambda_{12} \approx 0.12, \lambda_{21} \approx 0.09$$
$$w = \lambda_{11}\lambda_{22} - \lambda_{12}\lambda_{21} > 0$$

# Multiband superconductivity on repulsion

---

- BCS gap equations for two bands:

$$\Delta_1 = \lambda_{11}\Delta_1 \int \frac{d\varepsilon}{E_1} \tanh\left(\frac{E_1}{2T}\right) + \lambda_{12}\Delta_2 \int \frac{d\varepsilon}{E_2} \tanh\left(\frac{E_2}{2T}\right)$$

where  $E = (\varepsilon^2 + \Delta^2)^{1/2}$

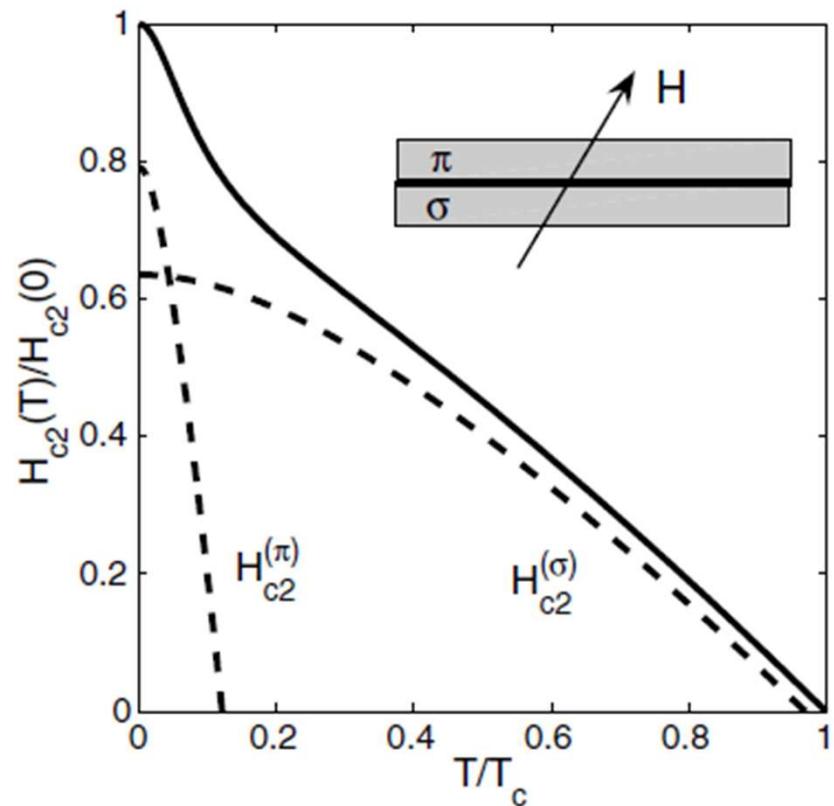
$$\Delta_2 = \lambda_{22}\Delta_2 \int \frac{d\varepsilon}{E_2} \tanh\left(\frac{E_2}{2T}\right) + \lambda_{21}\Delta_1 \int \frac{d\varepsilon}{E_1} \tanh\left(\frac{E_1}{2T}\right)$$

- $s^\pm$  pairing for repulsive interaction  $\lambda_{12} < 0$  and opposite signs of  $\Delta_1$  and  $\Delta_2$
- Pairing glue due to AF spin fluctuations,  $w = \lambda_{11}\lambda_{22} - \lambda_{12}\lambda_{21} < 0$

$$\lambda_{nm} = \frac{JN_n\chi(Q)}{1 - JN_n\chi(Q)}$$

# Convex $H_{c2}(T)$ in two-band models

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- Bilayer toy model of two-band SC
- Interaction of two bands with conventional  $H_{c2}(T)$  can produce unconventional  $H_{c2}(T)$  with upward curvature
- Model independent mechanism

# $H_{c2}$ for two coupled bands (clean limit) $H \parallel c$

---

$$a_1(\ln t + U_1) + a_2(\ln t + U_2) + (\ln t + U_1)(\ln t + U_2) = 0,$$

AG, PRB 82, 184504 (2010)  
Rep. Prog. Phys. 74, 124501 (2011)

$$U_1 = 2e^{q^2} \operatorname{Re} \sum_{n=0}^{\infty} \int_q^{\infty} du e^{-u^2} \left\{ \frac{u}{n+1/2} - \frac{t}{\sqrt{b}} \tan^{-1} \left[ \frac{u\sqrt{b}}{(n+1/2)t + i\alpha b} \right] \right\},$$

$$U_2 = 2e^{sq^2} \operatorname{Re} \sum_{n=0}^{\infty} \int_{q\sqrt{s}}^{\infty} du e^{-u^2} \left\{ \frac{u}{n+1/2} - \frac{t}{\sqrt{b}\eta} \tan^{-1} \left[ \frac{u\sqrt{b}\eta}{(n+1/2)t + i\alpha b} \right] \right\}$$

---

**Band coupling parameters:**

$$a_1 = (\lambda_0 + \lambda_-)/2w, \quad a_2 = (\lambda_0 - \lambda_-)/2w, \quad \lambda_- = \lambda_{11} - \lambda_{22}, \quad \lambda_0 = (\lambda_-^2 + 4\lambda_{12}\lambda_{21})^{1/2}, \quad w = \lambda_{11}\lambda_{22} - \lambda_{12}\lambda_{21}$$

**Band asymmetry parameters:**

$$\eta = \left( \frac{\nu_2}{\nu_1} \right)^2, \quad s = \frac{\mathcal{E}_2}{\mathcal{E}_1}$$

## Limiting cases for $\alpha \ll 1$

---

$$H_{c2}(T) = \frac{24\pi\phi_0 T_c(T_c - T)}{7\zeta(3)\hbar^2(c_+v_1^2 + c_-v_2^2)} \quad T \approx T_c$$

- In the GL region,  $H_{c2}$  is limited by the FS pocket with the largest Fermi velocity. The  $s^{++}$  and  $s^\pm$  scenarios behave similarly

$$H_{c2}(0) = \frac{\pi e^2 \phi_0 T_c^2}{2\gamma \hbar^2 v_1 v_2} \exp(g)$$

$$g = \left[ \frac{\lambda_0^2}{w^2} + \frac{\lambda_-}{w} \ln(\eta) + \frac{\ln^2 \eta}{4} \right]^{\frac{1}{2}} sgn(w) - \frac{\lambda_0}{w}$$

- $H_{c2}(0)$  is limited by the largest Fermi velocity for the  $s^\pm$  pairing but the smallest Fermi velocity for the  $s^{++}$  pairing

# Can $s^\pm$ be distinguished from $s^{++}$ ?

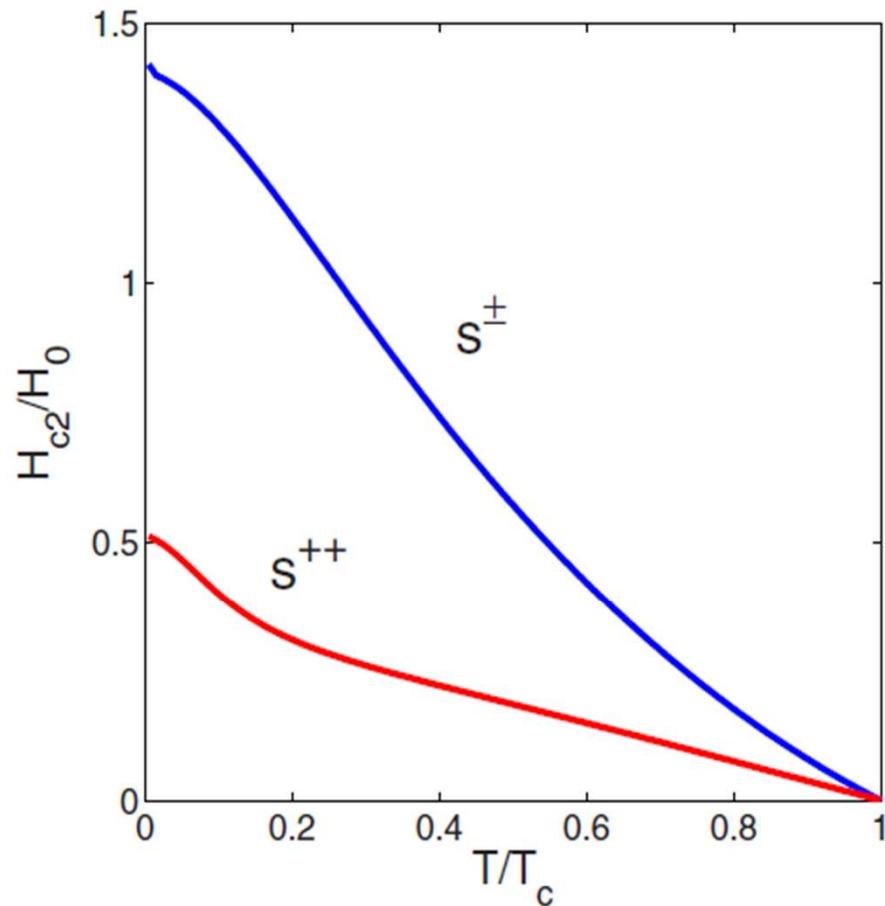


FIG. 5. (Color online) Comparison of  $H_{c2}(T)$  curves for  $s^\pm$  and  $s^{++}$  pairings and  $\alpha=0$ , where  $H_0=8\pi\phi_0k_B^2T_c^2/\hbar^2v_1^2$ . The  $s^\pm$  case was calculated for  $\lambda_{12}\lambda_{21}=0.25$  and  $\eta=0.01$ . The  $s^{++}$  case was calculated for  $\eta=0.01$  and  $\lambda_{lm}$  of MgB<sub>2</sub>:  $\lambda_{11}=0.81$ ,  $\lambda_{22}=0.29$ ,  $\lambda_{12}=0.13$ , and  $\lambda_{21}=0.09$  taken from Ref. 75.

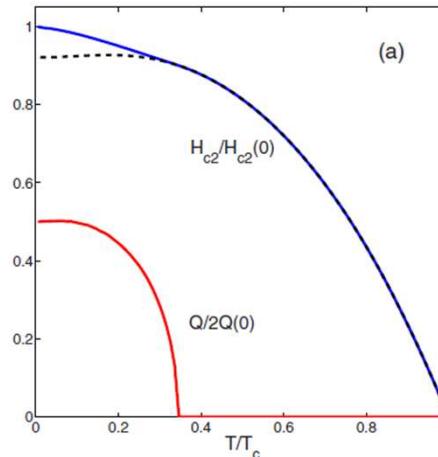
No paramagnetic effects,  $\alpha \ll 1$ :

Strong band asymmetry causes upward curvature in  $H_{c2}(T)$

Conventional  $s^{++}$  for MgB<sub>2</sub> :  $w > 0$ : convex  $H_{c2}(T)$  at **low T**

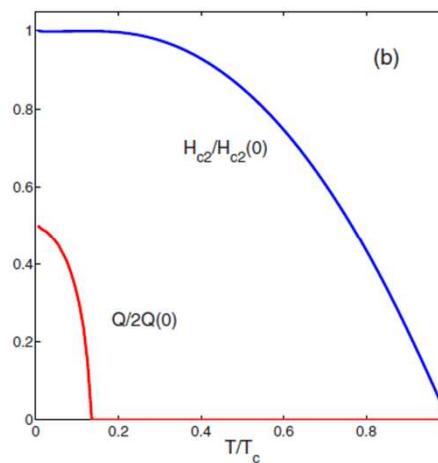
Unconventional  $s^\pm$  for pnictides :  $w < 0$ : convex  $H_{c2}(T)$  at **intermediate T**

$s^\pm$  increases orbitally-limited  $H_{c2}$



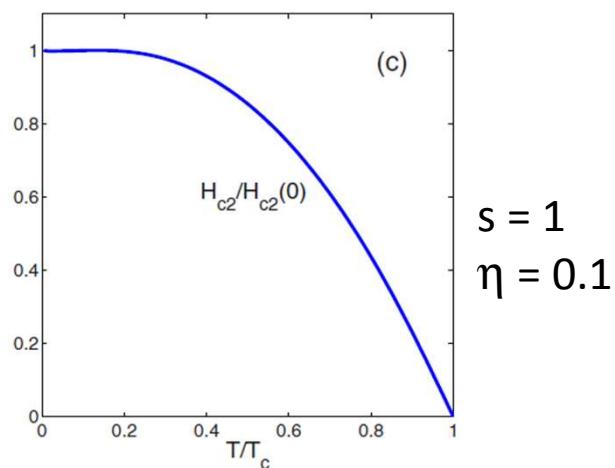
## Mass anisotropy facilitates FFLO

$s = 0.01$



$s = 0.5$

$$\eta = \left( \frac{v_2}{v_1} \right)^2, \quad s = \frac{\epsilon_2}{\epsilon_1}$$

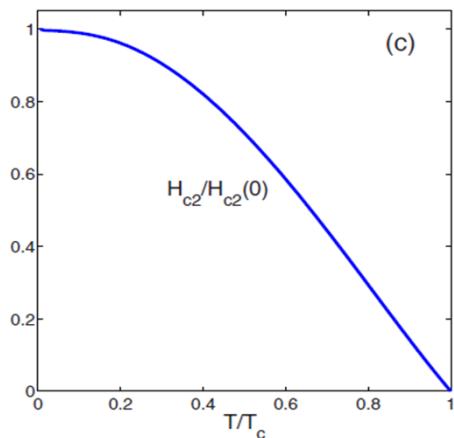
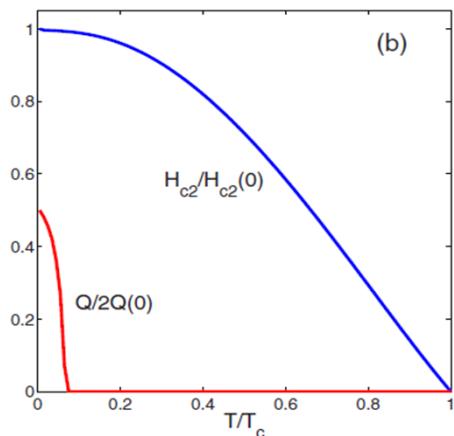
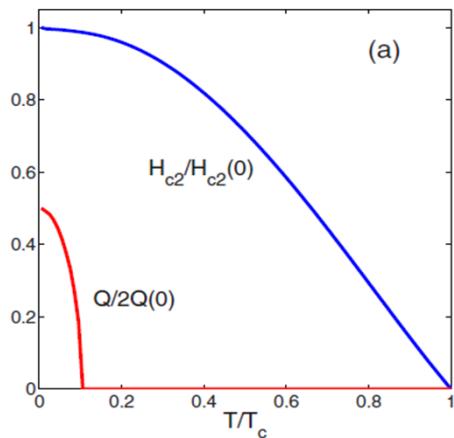


$s = 1$   
 $\eta = 0.1$

- If the passive band with  $\alpha < 1$  has large mass anisotropy, the active band with  $\alpha > 1$  can enforce the global FFLO state
- Reduction of the FFLO kinetic energy  $\epsilon_2 Q_z^2$  in the passive band 2 with  $\alpha_2 < 1$ .

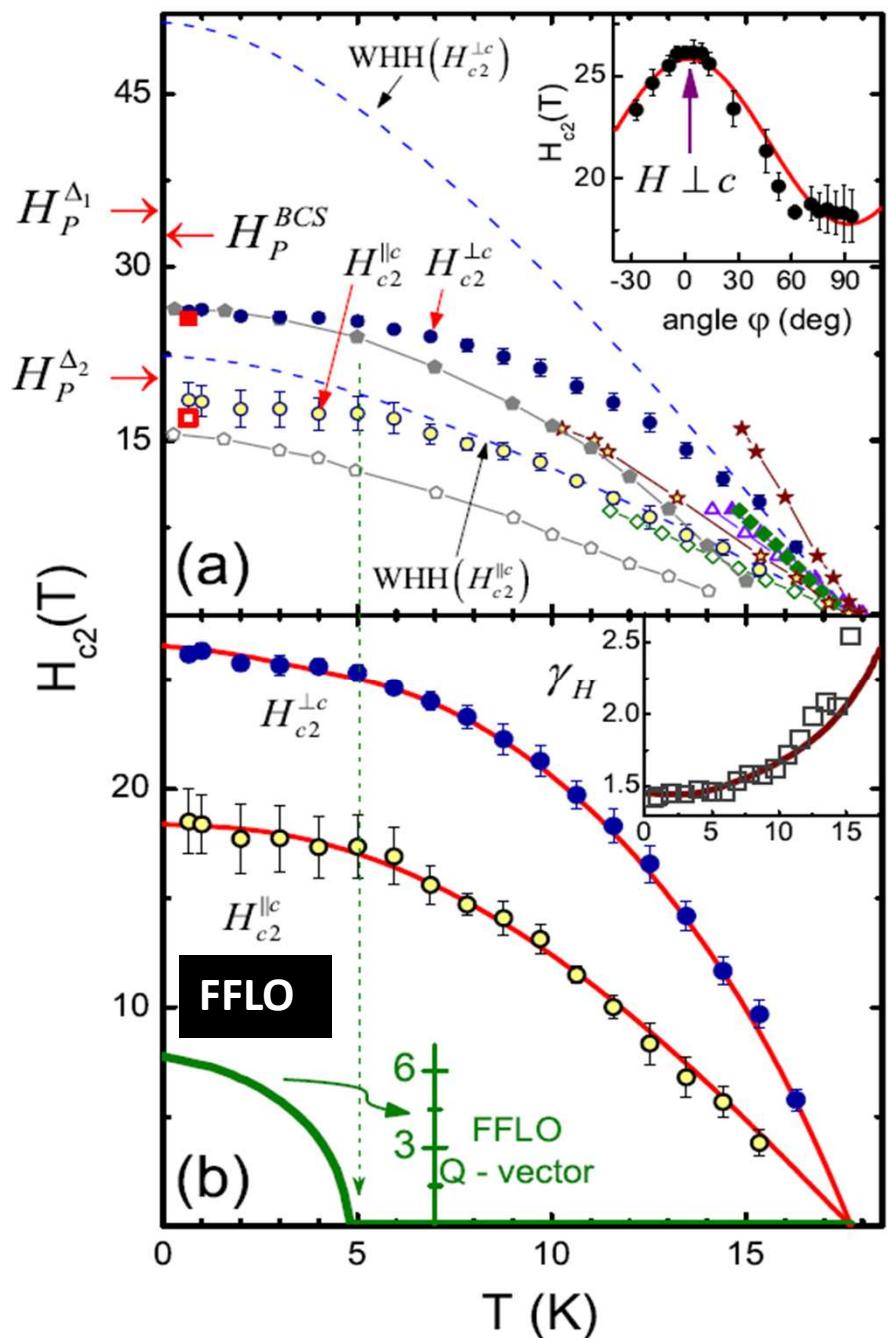
## Band competition: hidden FFLO

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$$\alpha_1 = \frac{2\pi k_B T_c}{v_1^2 m_e}, \quad \alpha_2 = \frac{2\pi k_B T_c}{v_2^2 m_e}$$

- Due to the significant differences in the band parameters, one band can be FFLO unstable ( $\alpha_1 > \alpha_c$ ) but another one is not ( $\alpha_2 < \alpha_c$ ).
- Passive band reduces manifestations of the FFLO in the WHH-like shape of  $H_{c2}(T)$ , but FFLO is still there
- “Hidden” FFLO: no apparent signs in  $H_{c2}(T)$  but can be revealed as the first order PT by magnetic torque and specific heat or NMR



## Experiment-I: LiFeAs

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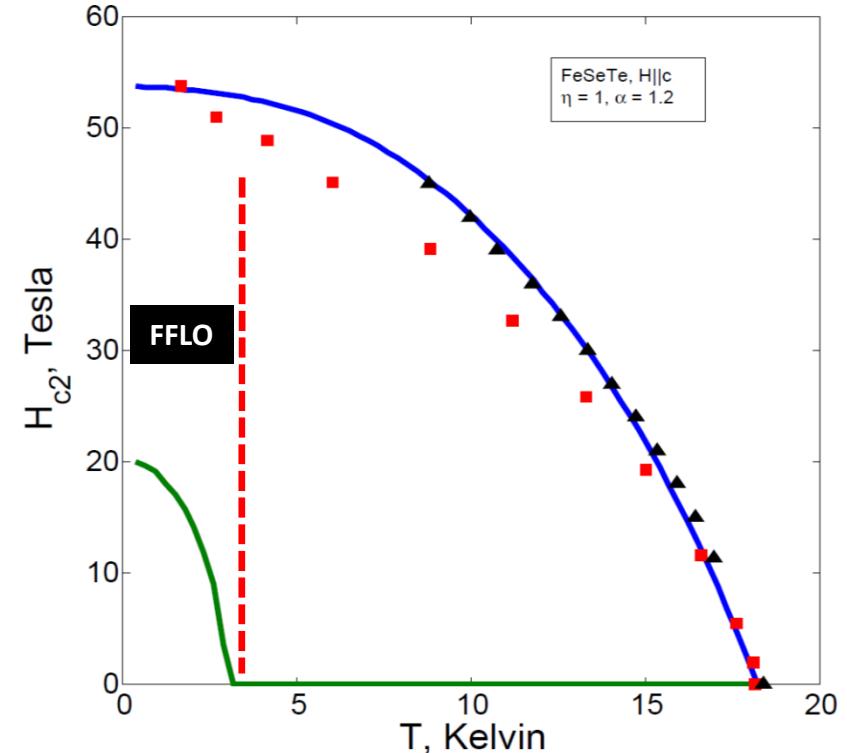
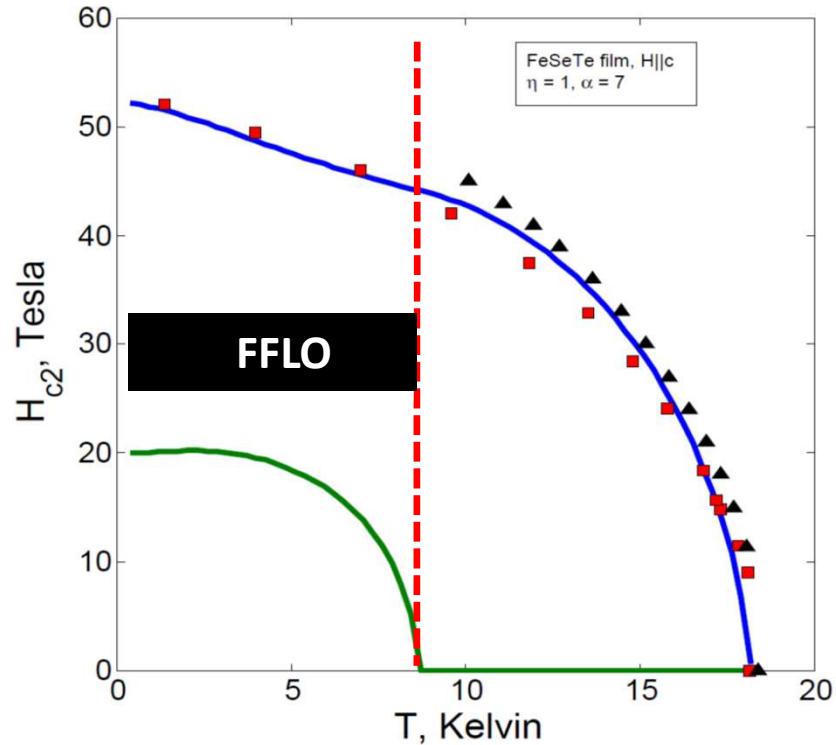
- Undoped composition corresponds to the maximum  $T_c$
- No suppression of FFLO by doping - induced disorder
- Good candidate to search for FFLO, mean free path  $>> \xi$

K. Cho, H. Kim, M. A. Tanatar, Y. J. Song, Y. S. Kwon, W. A. Coniglio, C. C. Agosta, AG, R. Prozorov, PRB, 83, 060502(R) (2011)

- Small jump in magnetic torque develops below 8K

N. Kurita et al. J. Phys. Soc. Jpn. 80, 013706 (2011)

## Experiment-II: FeSe<sub>0.5</sub>Te<sub>0.5</sub> films



- 100-400 nm thick FeSeTe films,  $T_c = 16.2$  K. Huge slopes  $H_{c2}' > 100$  T/K for  $H \parallel ab$

# FFLO triggered by the Lifshitz transition

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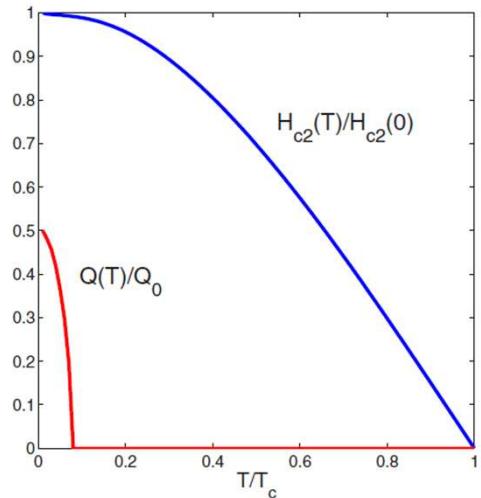


FIG. 9. (Color online)  $H_{c2}(T)$  and  $Q(T)$  calculated from Eq. (37) for  $\alpha=0.3$ ,  $\lambda=0.5$ ,  $g=0.2$ ,  $s=50$ ,  $\eta=0.04$ , and  $Q_0=2Q(0)$ .

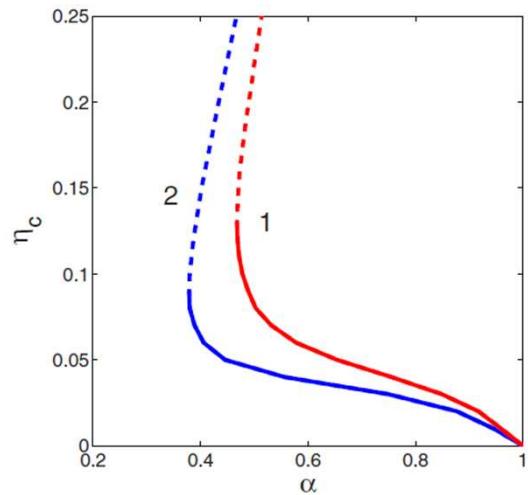
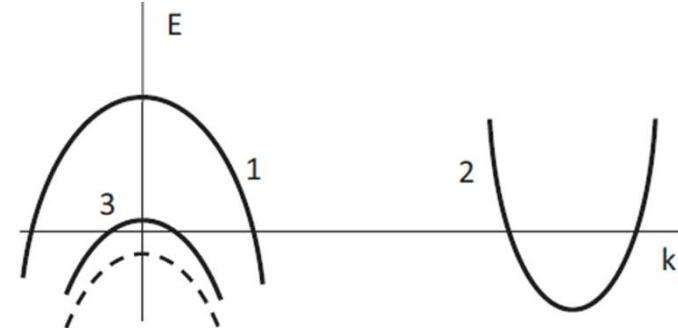


FIG. 10. (Color online) The critical value  $\eta_c$  calculated from Eqs. (41) for  $m_3=2m_1$  and  $s=25$  (1) and  $s=10$  (2). The dashed curves show unstable branches of  $\eta_c(\alpha)$ .



- $H_{c2}$  equation in effective 2-band form:

$$\lambda(\ln t + \tilde{U}_1)(\ln t + U_2) = 2\ln t + \tilde{U}_1 + U_2,$$

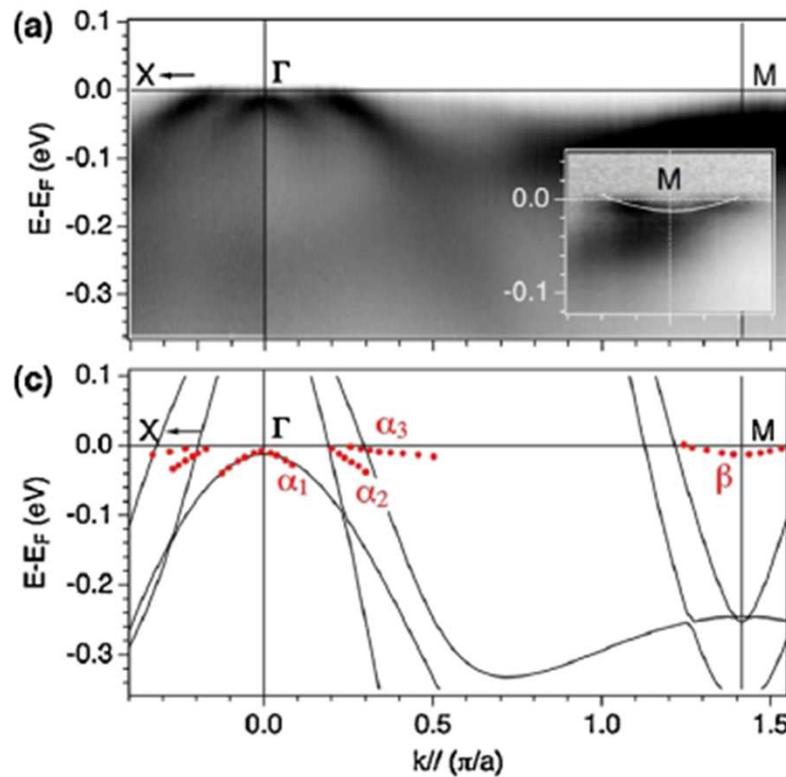
$$\tilde{U}_1 = \frac{U_1 + gU_3}{1+g}, \quad g = \frac{\lambda_{23}\lambda_{32}}{\lambda_{12}\lambda_{21}} = \frac{m_3^2 V_{23}^2}{m_1^2 V_{12}^2} \sqrt{\frac{\eta}{s}},$$

$$\lambda = (\lambda_{12}\lambda_{21} + \lambda_{23}\lambda_{32})^{1/2}$$

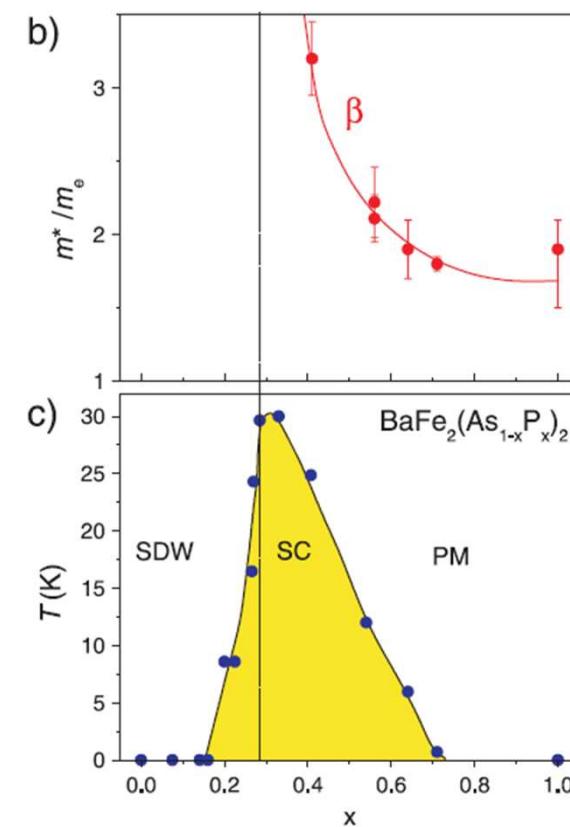
- reduction of the FFLO instability threshold

# Enhancement factors

$$g = \frac{\lambda_{23}\lambda_{32}}{\lambda_{12}\lambda_{21}} = \frac{m_3^2 V_{23}^2}{m_1^2 V_{12}^2} \sqrt{\frac{\eta}{s}},$$



Small Fermi energies in FeSeTe  
Tamai et al, PRL 104, 097002 (2010)

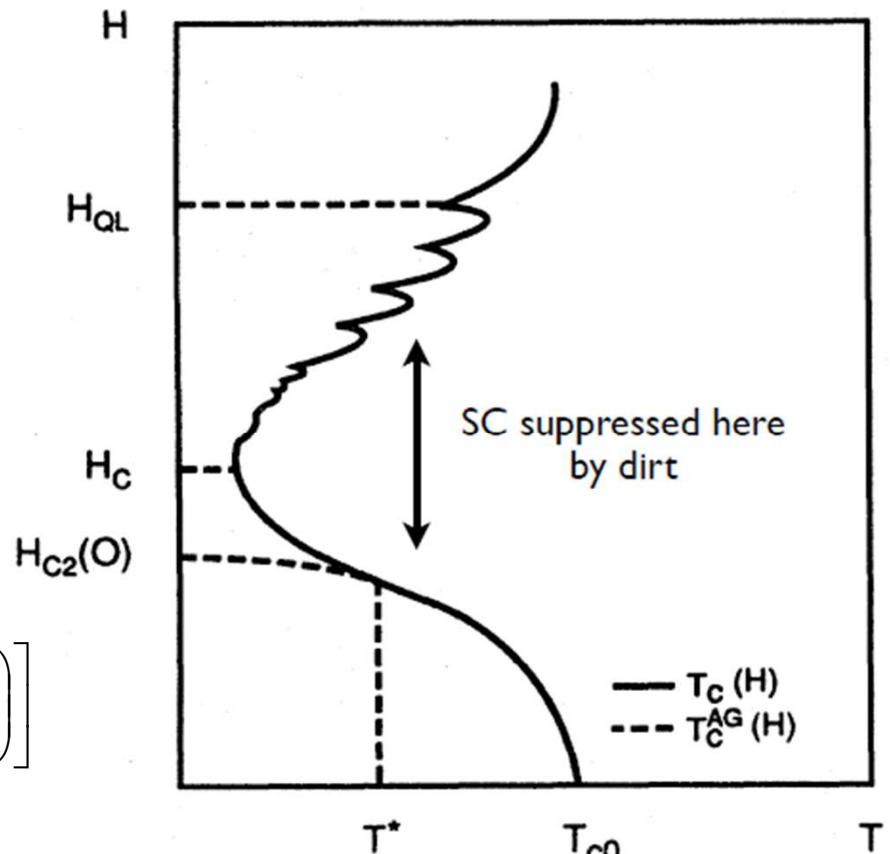


Mass enhancement of a shrinking FS  
Pocket  $\text{BaFe}_2(\text{As}_x\text{P}_{1-x})_2$  revealed by  
dHvA oscillations  
Shishido et al, PRL 104, 057008 (2010)

# Quantum oscillations in $H_{c2}(T)$

- First LL quasiclassic solution works for  $\alpha < 7$ .
- For higher  $\alpha$  (small  $E_F$  of the emerging FS pocket), quantum oscillations due to higher LLs become important

$$T_c(H) = T_{c0}(H) \left[ 1 - \sqrt{\frac{2\omega_c}{E_F}} \exp\left(-\frac{2\pi^2 T_{c0}}{\omega_c}\right) \sin\left(\frac{2\pi E_F}{\omega_c} + \frac{\pi}{4}\right) \right]$$

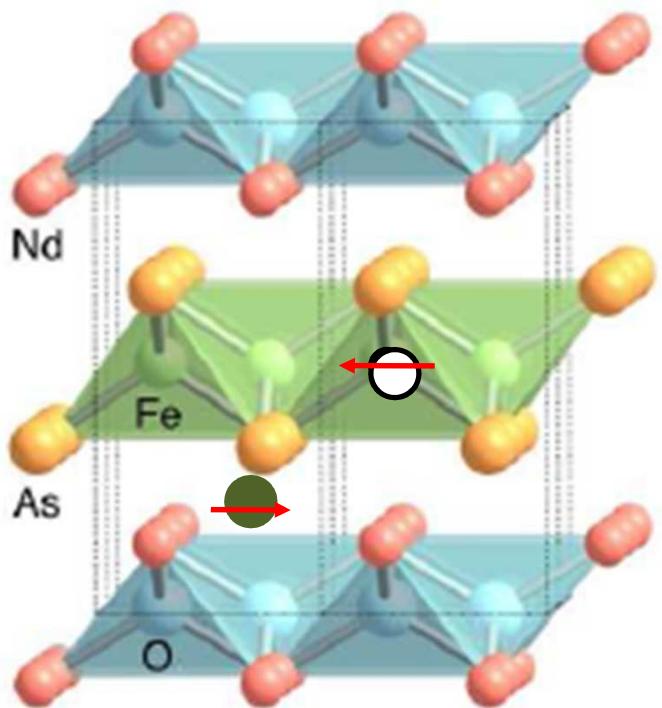


Rajagopal and Vasudevan, Phys. Lett. 23, 539 (1966)

For a single band, see, e.g.  
Rasolt and Tesanovic, RMP, 64, 709 (1992)

For  $E_F = 3$  meV, the field range  $H \sim 30T$  is accessible at low temperatures

# Magnetic defects caused by $\alpha$ particle irradiation



- A “clean” way of introducing disorder without doping and segregation of impurity phases
- $\alpha$  particles mostly interact with Nd, As and Fe.
- Displacement of Fe produces the Frenkel radiation defects
- Partial restoration of magnetic moment of Fe<sup>2+</sup> ion
- Irradiation defects cause both nonmagnetic and magnetic scattering

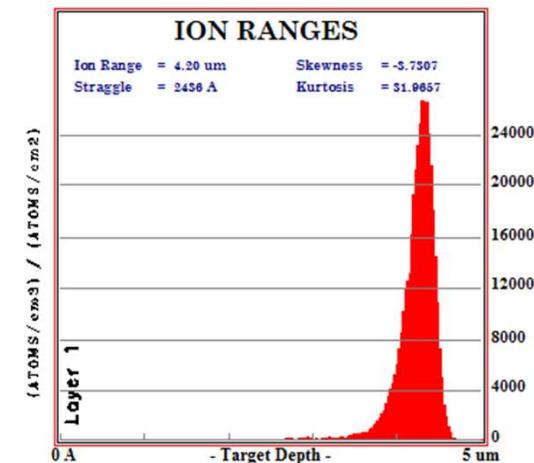
Lee, Prado and Pickett, PRB 78, 174502 (2008)

Kemper et al, PRB 80, 104511 (2009)

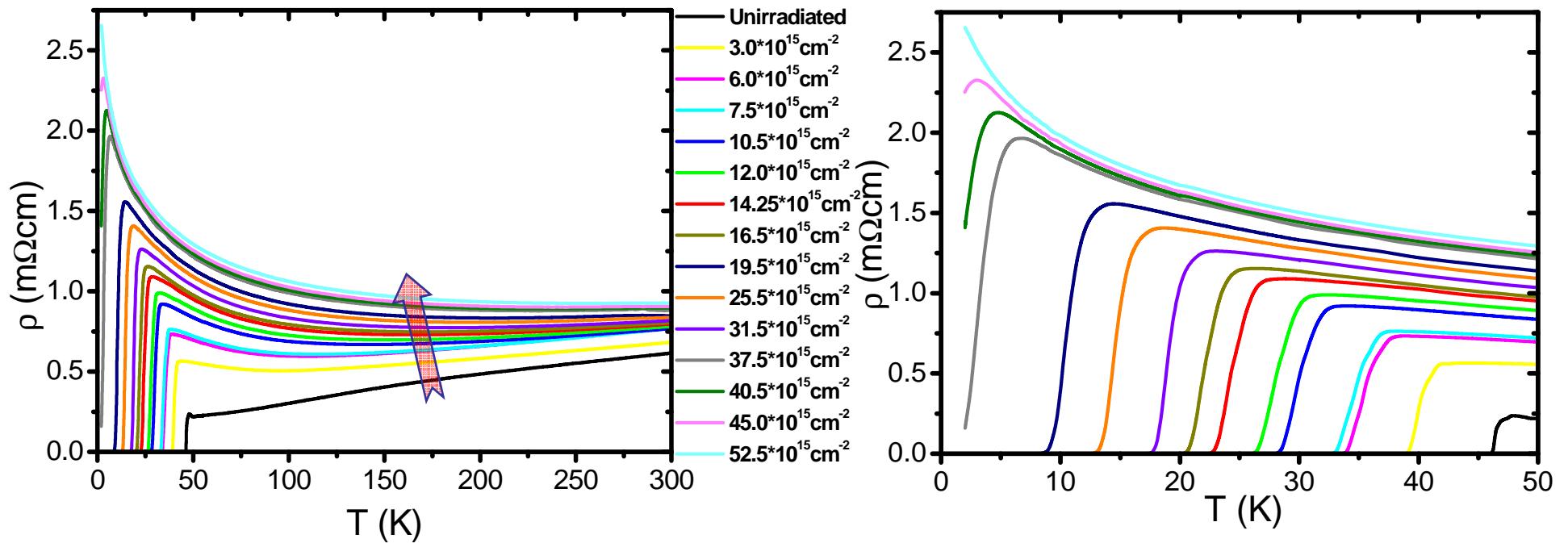
2 MeV  ${}^4\text{He}^{2+}$  ion beam

Ion range in NdFeAsO<sub>0.7</sub>F<sub>0.3</sub> ~ 4.2  $\mu\text{m}$

Uniform damage across the 1  $\mu\text{m}$  thick crystal

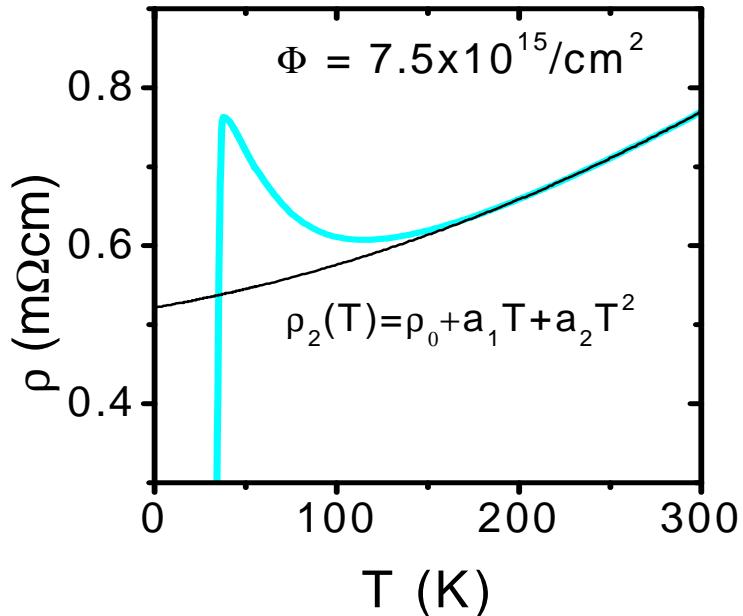


# Effect of irradiation on $\rho(T)$ of a Nd-1111 single crystal



- $T_c$  gradually decreases and the resistivity increases after each irradiation dose with a significant upturn developing at low  $T$
- $T_c$  vanishes at a rather high dose =  $5.25 \times 10^{16} \text{ cm}^{-2}$ .

# Logarithmic resistivity



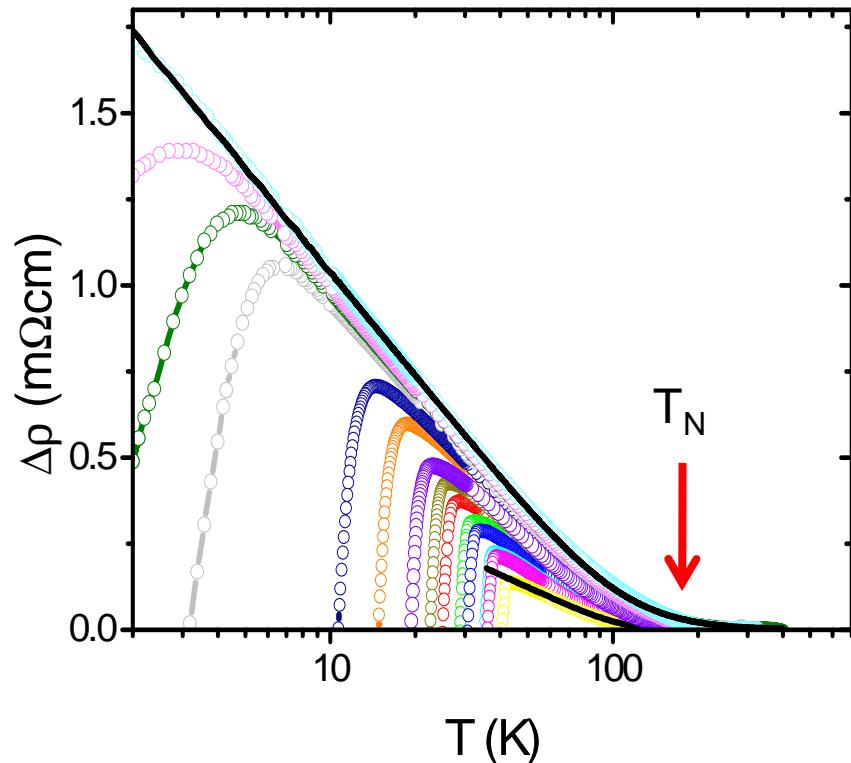
$$\rho_2(T) = \rho_0 + a_1 T + a_2 T^2$$

$$\Delta\rho(T) = \rho(T) - \rho_2(T)$$

- Logarithmic temperature dependence

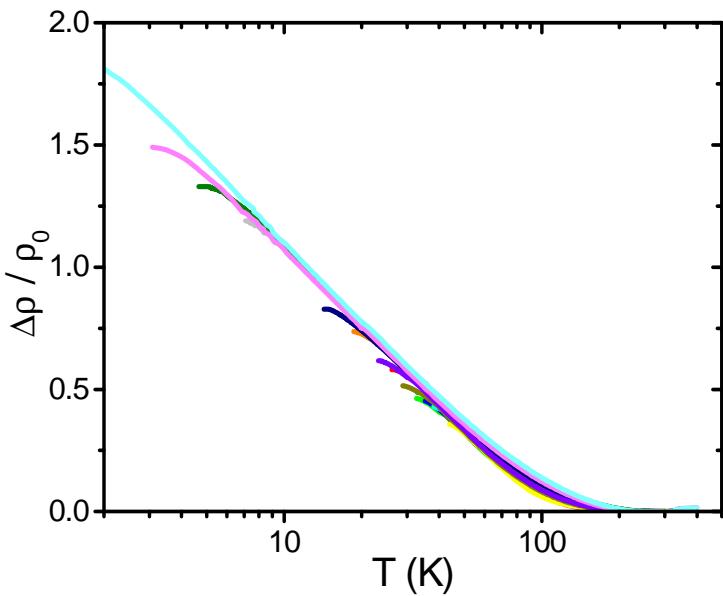
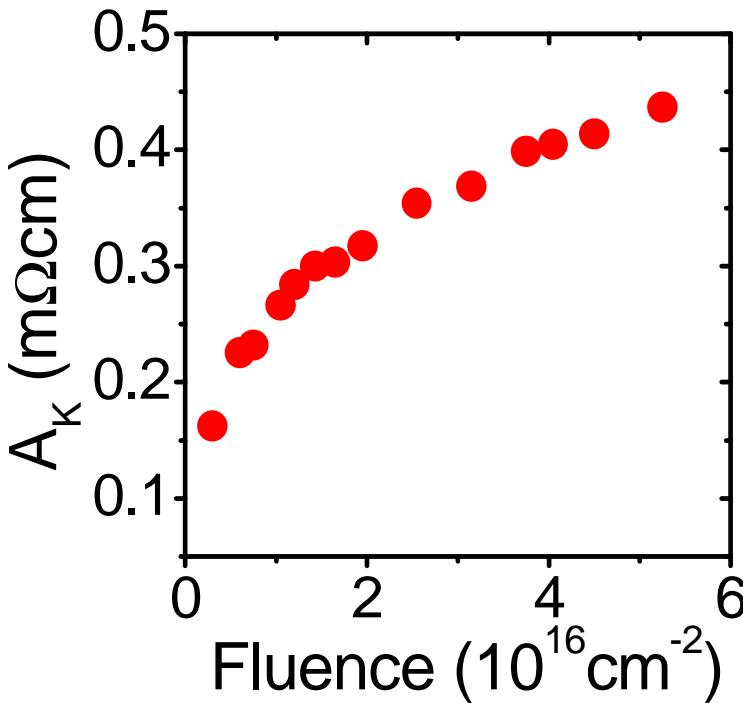
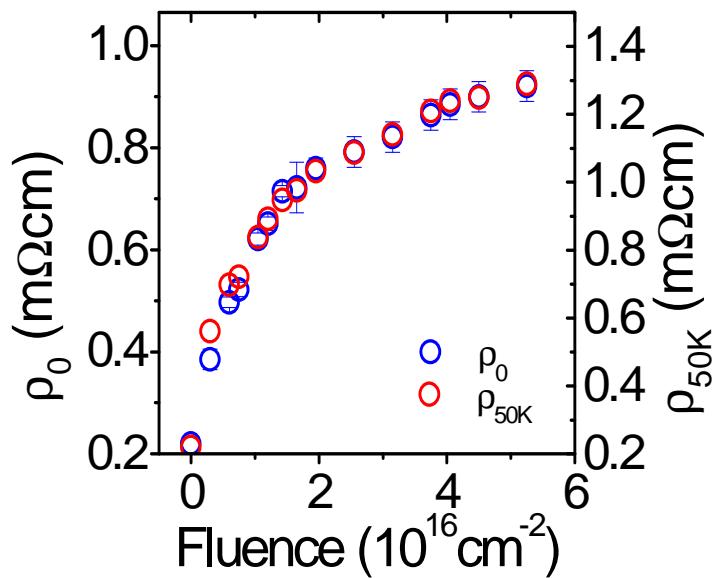
$$\Delta\rho(T) = A_K \ln(T_0 / T)$$

- $A_K$  increases with fluence
- $T_0 \approx 110\text{-}120 \text{ K}$  is independent of fluence



- Kondo scattering induced by irradiation?
- No sign of saturation at low T.  
Kondo temperature  $T_K < 2\text{K}$

## Dependence of fluence



$\rho_0$  and  $A_K$  quantify non-magnetic and magnetic scattering

Both  $\Delta\rho$  (T) and  $\rho_0$  have the same dependence on the concentration of the irradiation defects

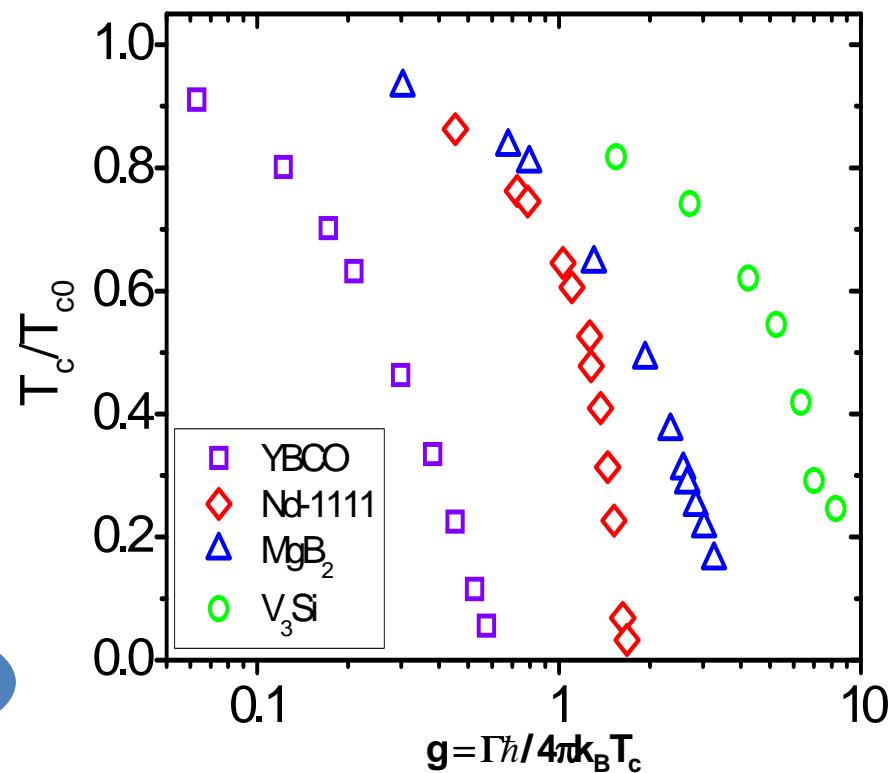
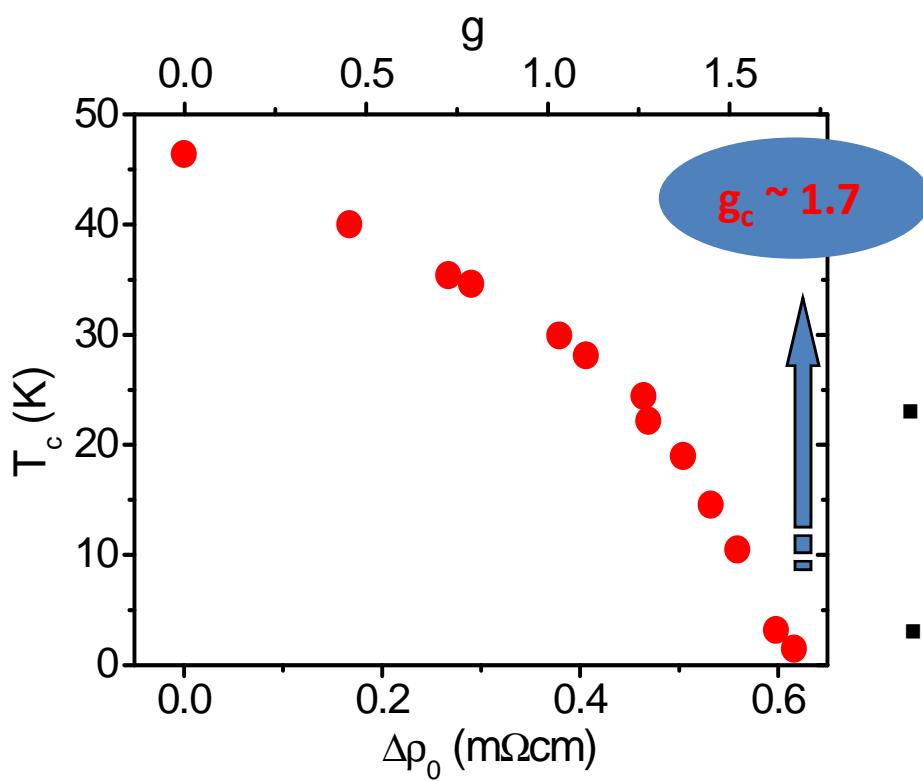
# Suppression of $T_c$ by irradiation defects

Non-magnetic scattering rate  $\Gamma$

$$\Gamma = \Delta\rho_0 / \mu_0\lambda_0^2$$

Pairbreaking interband scattering rate  $g$

$$g = \Gamma\hbar / 4\pi k_B T_{c0}$$



- Nd-1111 looks more like the s-wave MgB<sub>2</sub> and V<sub>3</sub>Si rather than the d-wave cuprates
- High density of irradiation defects: mean free path  $l = v_F/\Gamma \sim 2.4$  nm

# Effect of scattering on $T_c$

Equal gaps:  $\Delta_1 = -\Delta_2$ :

## Pairbreaking:

- Nonmagnetic inter-band scattering
- Magnetic intra-band scattering

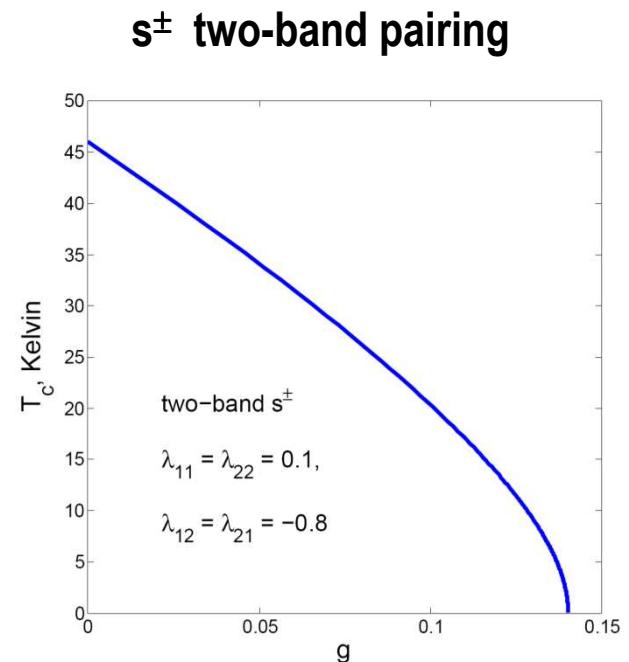
## Non-pairbreaking:

- Intra-band nonmagnetic scattering
- Inter-band magnetic scattering

Reduction of nonmagnetic inter-band scattering for strong impurity potentials

$$\frac{1}{2\tau} \rightarrow \frac{\pi c N V^2}{1 + (\pi N V)^2}$$

G. Priosti and P. Muzikar, PRB 54, 3489 (1996);  
M. Kulic and O. Dolgov, PRB 60, 13062 (1999).



$$g_c = 1/4\pi T_c \tau \approx 0.15$$

Irradiation experiment gives  
 $g_c \approx 1.7$

# Magnetic Kondo scattering

- Strong intra-band Abrikosov-Gorkov magnetic pairbreaking

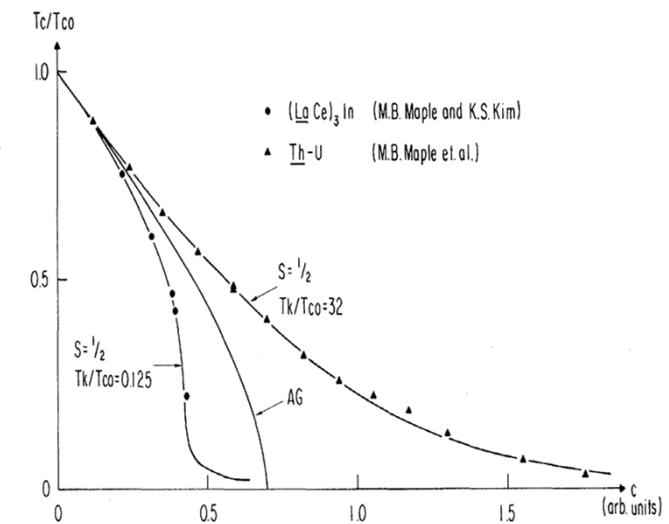
$$\ln \frac{T_{c0}}{T_c} = \psi\left(\frac{1}{2} + \frac{1}{4\pi\tau_s T_c}\right) - \psi\left(\frac{1}{2}\right)$$

- Effect of Kondo scattering on BCS superconductivity:

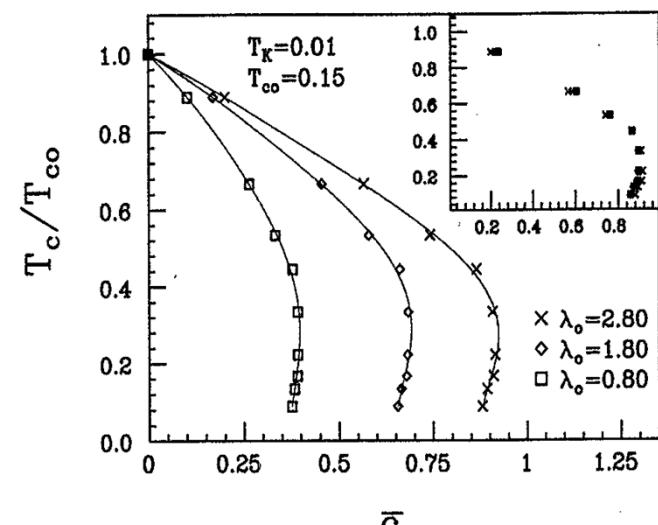
E. Muller-Hartmann, J. Zittartz, PRL 26, 428 (1970); P. Schlottmann, SSC 21, 663 (1973);  
 T. Matsuura, S. Ichinose , Y. Nagaoka, Prog. Theor. Phys. 57, 713 (1977);  
 M. Jarrell, PRB 41, 4815 (R) (1990).

- $T_K > T_{c0}$ : Kondo screening reduces pairbreaking as compared to the AG theory
- $T_K < T_{c0}$ : Kondo scattering enhances pairbreaking as compared to the AG theory. Multi-valued dependence of  $T_c$  on the impurity concentration

For Nd-1111 crystals,  $T_K < 2K$  and  $T_c = 46K$ : any multiband BCS superconductivity would be suppressed irrespective of pairing symmetry



E. Muller-Hartmann, J. Zittartz, PRL 26, 428 (1970)



M. Jarrell, PRB 41, 4815(R) (1990).

# Conclusions

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- Anomalous temperature dependencies of  $H_{c2}(T)$  reflect the effects of multiband pairing and strong Pauli limiting in FBS
- $s^\pm$  pairing, low carrier density and high  $T_c$  enhance the orbitally-limited  $H_{c2}(T)$
- Strong Pauli pairbreaking in FBS can lead to FFLO.
- Hidden FFLO in multiband FBS: The WHH shape of  $H_{c2}(T)$  does not mean the absence of FFLO . Torque, NMR and specific heat experiments at high fields are needed.
- Possibility of tuning  $H_{c2}(T)$  by doping but not disorder: FFLO triggered by the Lifshitz transition
- Unusual behavior of Kondo impurities in pnictides: strong enhancement of resistivity but weak suppression of  $T_c$  even for the mfp  $\approx \xi$
- Magnetic impurity scattering is to be intertwined with pairing AF interaction
- Grossly enhanced paramagnetic limit: evidence for the AF enhancement of  $H_p$