Quantum-wire cluster states in the quantum optical frequency comb

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OUTLINE

Background

- **Quantum computer with Qbits and Qmodes**
- **Cluster states definition**
- **Optical Parametric Oscillator (OPO)**

Quantum wire cluster states generation

- **How to get quantum wire from OPO**
- **Experimental Setup**
- **Experimental Progress**

Quantum Computer

Classical Computer

Classical Bit: either 0 or 1

Quantum Computer
Qbit (Qubit): either 0 or 1, or a *superposition of both*

$$|\varphi> = \alpha\,|0> \ +\beta\,|1> \qquad (\,|\alpha|^2+|\beta|^2 = 1)$$

A quantum computer with n qbits can be in an arbitrary superposition of up to 2^n different states simultaneously, while a classical computer can only be in one of these state at any one time.

Quantum Computer

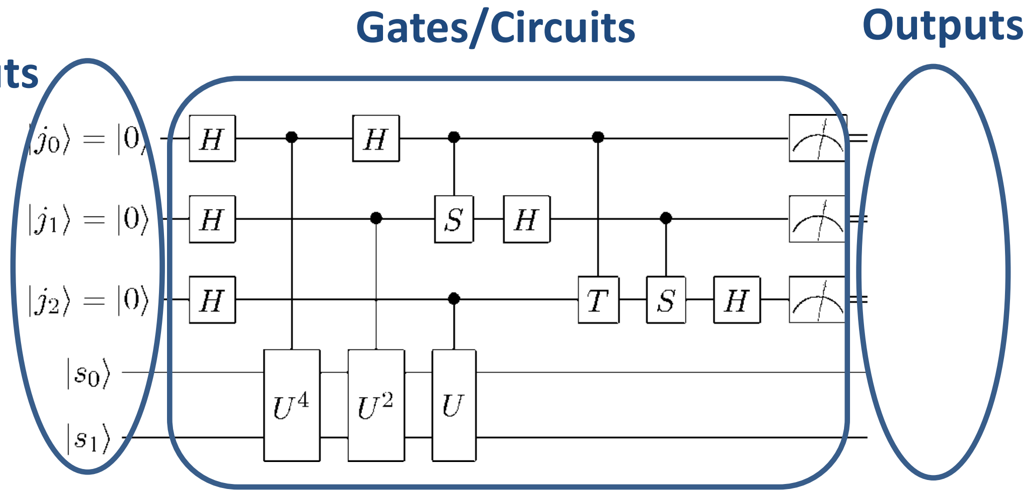
The unit

Qbit	Qmode
Discrete Variable	Continuous Variable
$\alpha\|0> + \beta\|1>$	$\|q>$
$\|+> = (\|0> + \|1>)/\sqrt{2}$ $\|-> = (\|0> - \|1>)/\sqrt{2}$	$\|p>$
$Pauli\ Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ $Z\|0> = \|0>, Z\|1> = -\|1>$ $Z\|+> = \|->, Z\|-> = \|+>$	$e^{i\delta Q}\|q> = e^{i\delta q}\|q>$ $e^{i\delta Q}\|p> = \|p+\delta>$
$Pauli\ X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ $X\|+> = \|+>, X\|-> = -\|->$ $X\|0> = \|1>, X\|1> = \|0>$	$e^{-i\varepsilon P}\|p> = e^{-i\varepsilon p}\|p>$ $e^{-i\varepsilon P}\|q> = \|q+\varepsilon>$
$Hadamard\ H = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$ $H\|0> = \|+>$ $H\|1> = \|->$	Fourier Transform $\|q> = \int e^{-ipq}\|p>dp$ $\|p> = \int e^{iqp}\|q>dq$

→ **Optical Modes (amplitude, phase)**

$$Q = \frac{1}{\sqrt{2}}(a^{+} + a)$$
$$P = \frac{i}{\sqrt{2}}(a^{+} - a)$$

Quantum Computer



Quantum computer models

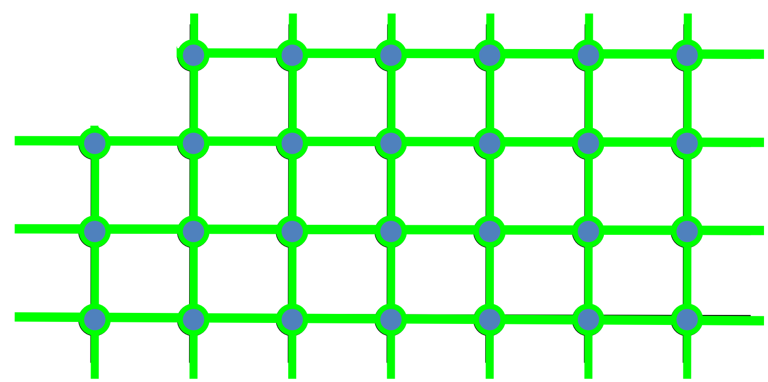
Circuit Model

Measurement-based Model: One way quantum computer

Cluster States

definition

A method of quantum computing that first makes an entangled resource state, a.k.a. Cluster State, and then performs measurements on it.



Different definitions of graph (cluster) states

Std Qbit definition

Briegel and Raussendorf, PRL 2001

Raussendorf and Briegel, PRL 2001

CZ

$|+\rangle_1$ — $|+\rangle_2$

Stabilizers:

$$X_j \prod_{k\in\mathcal{N}(j)} Z_k$$

Std Qmode definition

Zhang and Braunstein, PRA 2006

Gu et al., PRA 2009

$U_{12} = e^{-iQ_1Q_2}$

$|p_1 = 0\rangle_1$ — $|p_2 = 0\rangle_2$

Stabilizers:

$$e^{iP_j} \prod_{k\in\mathcal{N}(j)} e^{-iQ_k}$$

Nullifiers:

$$\begin{pmatrix} P_1 \\ \vdots \\ P_N \end{pmatrix} - \mathbb{A} \begin{pmatrix} Q_1 \\ \vdots \\ Q_N \end{pmatrix} \to \vec{0}$$

Graph adjacency matrix

"H-graph" definition

Menicucci et al., PRA 2007

$H_{12} = a_1^\dagger a_2^\dagger + H.c.$

$|n_1 = 0\rangle_1$ — $|n_2 = 0\rangle_2$

Heisenberg equations:

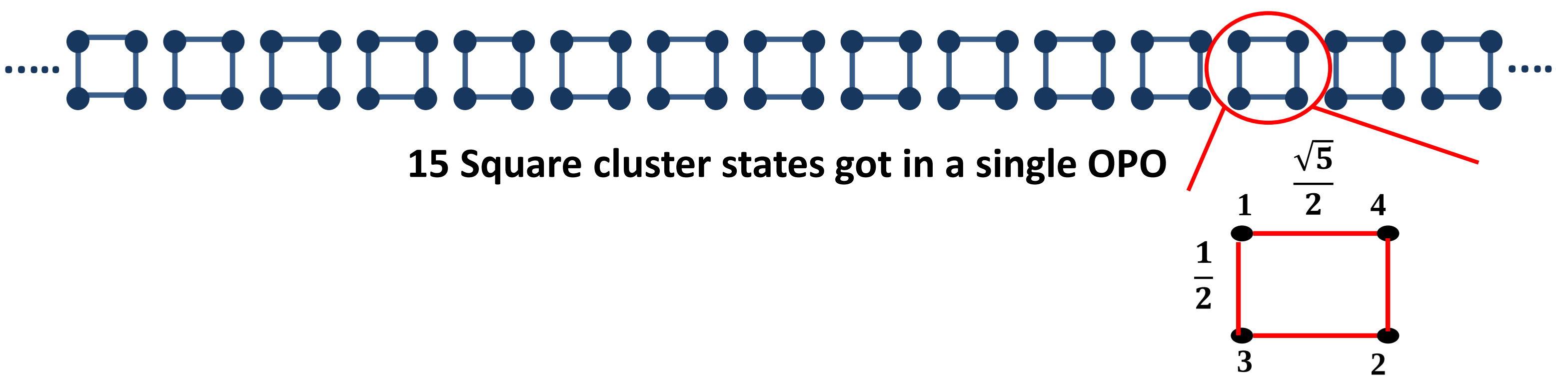
$$\begin{pmatrix} \dot{Q}_1 \\ \vdots \\ \dot{Q}_N \end{pmatrix} - \mathbb{G} \begin{pmatrix} Q_1 \\ \vdots \\ Q_N \end{pmatrix} = \vec{0}$$

H-graph adjacency matrix

Our way of making Cluster State :
VERY SCALABLE in a single OPO (Optical Parametric Oscillator)

One-way quantum computing in the optical frequency comb
N.C. Menicucci, S.T. Flammia, and O. Pfister PRL **101**, 130501 (2008)

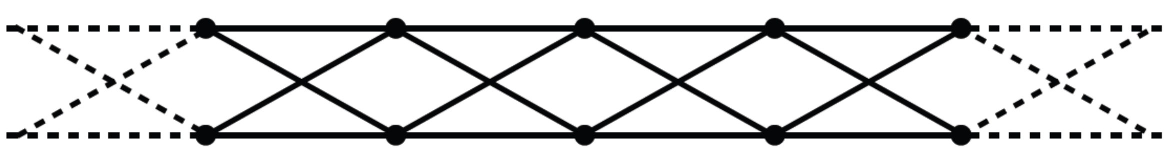
Parallel generation of quadripartite cluster entanglement in the optical frequency comb
M. Pysher, Y. Miwa, R. Shahrokhshahi, R. Bloomer, and O. Pfister PRL **107**, 030505 (2011)



15 Square cluster states got in a single OPO

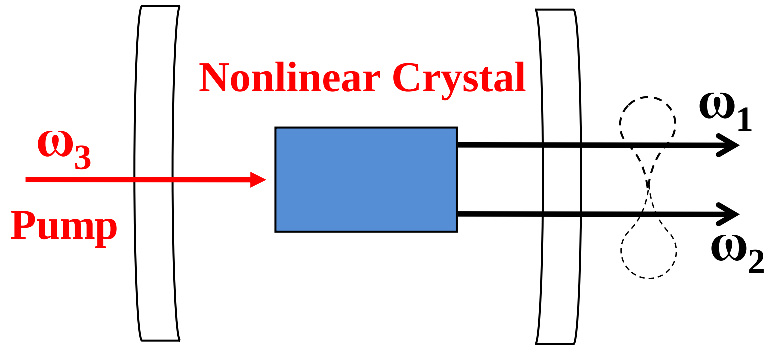
Go further: Making bigger cluster state

Quantum wire Cluster State



Background (OPO)

Optical Parametric Oscillator (OPO)



An OPO converts an input laser wave (called "pump") into two output waves of lower frequency by means of second order nonlinear optical interaction, when two conditions are satisfied:

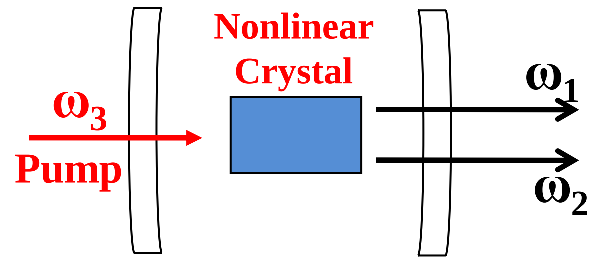
$$\boxed{\begin{gathered}\omega_1 + \omega_2 = \omega_3 \\ \boldsymbol{k}_1 + \boldsymbol{k}_2 = \boldsymbol{k}_3\end{gathered}}$$

Background (EPR)

$$\boxed{H = i\hbar\chi\,(b a_1^+ a_2^+ - b^+ a_1 a_2)}$$

Treat pump classically, $b = \beta e^{i\varphi}, take\ \varphi = 0$

$$H = i\hbar\kappa\,(a_1^+ a_2^+ - a_1 a_2) \qquad \kappa \stackrel{\text{def}}{=} \chi\beta$$



Solving the **Heisenberg Equations** $\boldsymbol{i\hbar\dot{a}_j = [a_j, H]}$ we obtain the following equations:

$$\dot{a}_1 = \kappa a_2^+, \qquad \dot{a}_1^+ = \kappa a_2$$
$$\dot{a}_2 = \kappa a_1^+, \qquad \dot{a}_2^+ = \kappa a_1$$

Solve coupled equations
Initial conditions

$$\boxed{\begin{aligned} a_1(t) &= a_1(0)\cosh(r) + a_2^+(0)\sinh(r) \\ a_2(t) &= a_1^+(0)\sinh(r) + a_2(0)\cosh(r) \end{aligned}} \quad \boldsymbol{(r \stackrel{\text{def}}{=} \kappa t)}$$

Background (EPR)

$$\boxed{\begin{aligned} a_1(t) &= a_1\cosh(r) + a_2^{+}\sinh(r) \\ a_2(t) &= a_1^{+}\sinh(r) + a_2\cosh(r) \end{aligned}}$$

rewrite

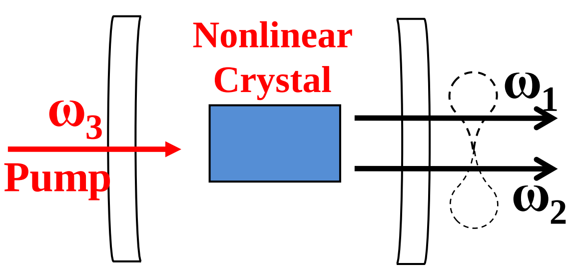
$$a_j = \frac{1}{\sqrt{2}}\left(Q_j + iP_j\right)$$
$$a_j^{+} = \frac{1}{\sqrt{2}}\left(Q_j - iP_j\right)$$

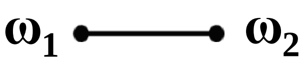
$$\boldsymbol{P_1(t) + P_2(t) = [P_1(0) + P_2(0)]e^{-r} \ \rightarrow 0}$$
$$\boldsymbol{Q_1(t) - Q_2(t) = [Q_1(0) - Q_2(0)]e^{-r} \ \rightarrow 0}$$

Entanglement witnesses for EPR Pair

$(r \stackrel{\text{def}}{=} \kappa t)$ $\ r$ *is squeezing parameter*

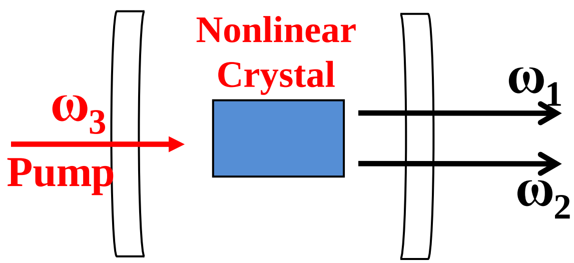
$$\boldsymbol{P_1(t) - P_2(t) = [P_1(0) - P_2(0)]e^{r} \rightarrow inf}$$
$$\boldsymbol{Q_1(t) + Q_2(t) = [Q_1(0) + Q_2(0)]e^{r} \rightarrow inf}$$



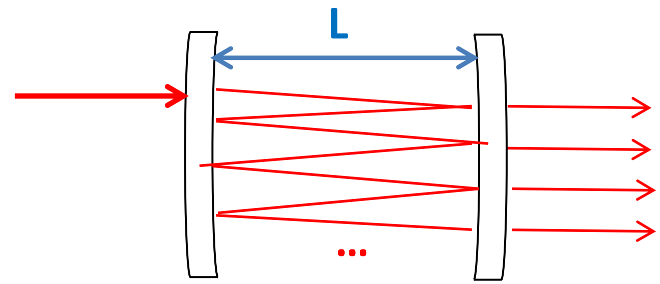


EPR Pair

Background (Optical Frequency Comb)



OPO



Cavity

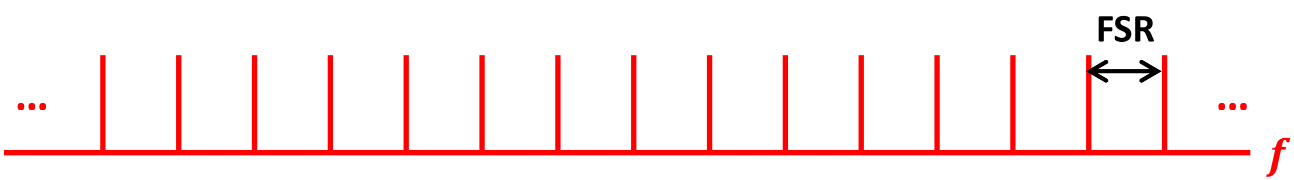
For an optical beam to be "*resonant*" in a cavity, <u>constructive interference</u> is needed, which requires the optical path difference between each transmitted beam is an integer multiple of the wavelength:

$$\boxed{2L = n\lambda}$$

$$\lambda = \frac{c}{f}$$

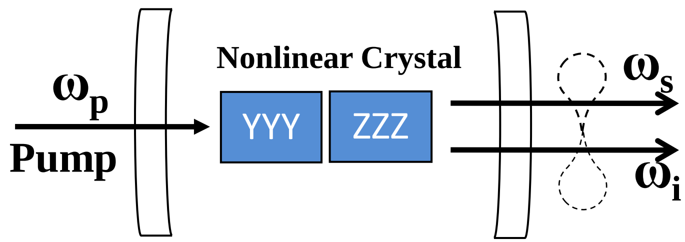
$$f = n\,\frac{c}{2L} = n\,FSR$$

$$\left(\frac{c}{2L} \stackrel{\text{def}}{=} Free\ Spectral\ Range\right)$$

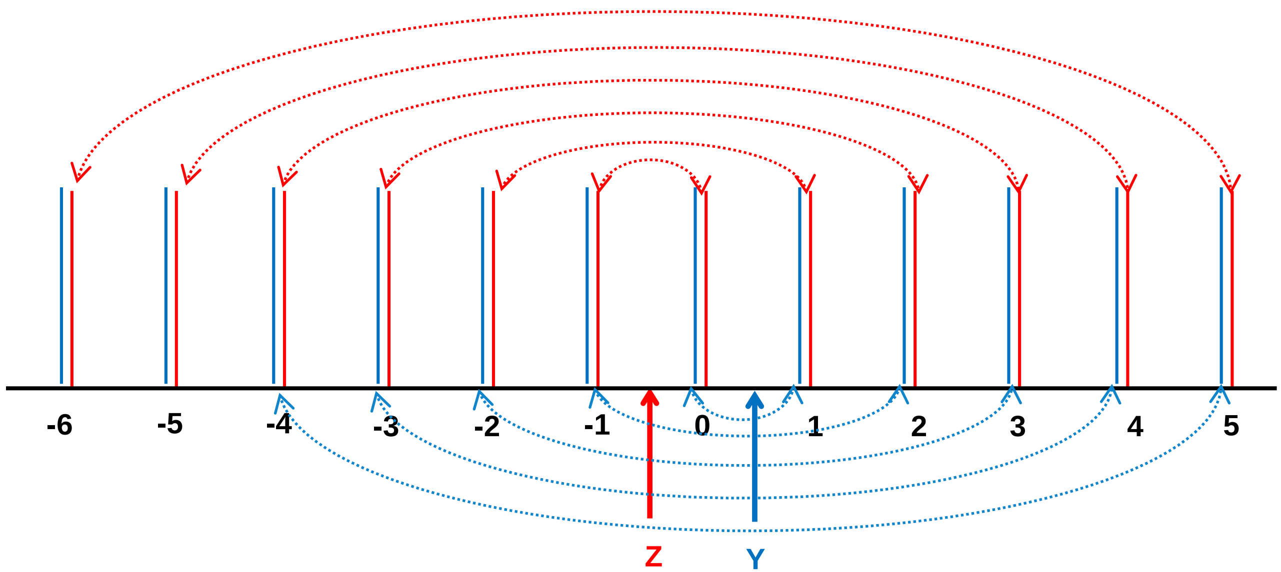


Optical Frequency Comb

How to Generate the Quantum Wire Cluster State

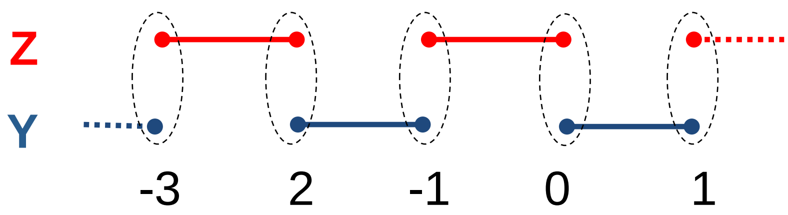


$$\omega_i + \omega_s = \omega_p$$
$$\boldsymbol{k_i} + \boldsymbol{k_s} = \boldsymbol{k_p}$$



Optical Frequency Comb (OFC) of an Optical Parametric Oscillator (OPO)

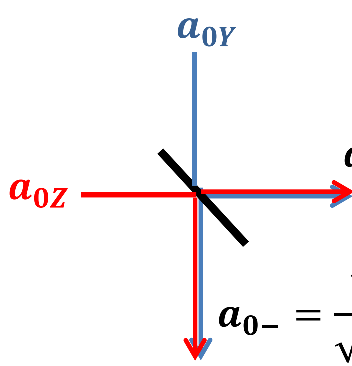
How to Generate the Quantum Wire Cluster State



Original entanglement witnesses:

$$Q_{mZ} - Q_{nZ} \to 0$$
$$P_{mZ} + P_{nZ} \to 0$$

After beam splitter



$$a_{0+} = \frac{1}{\sqrt{2}}(a_{0Y} + a_{0Z})$$

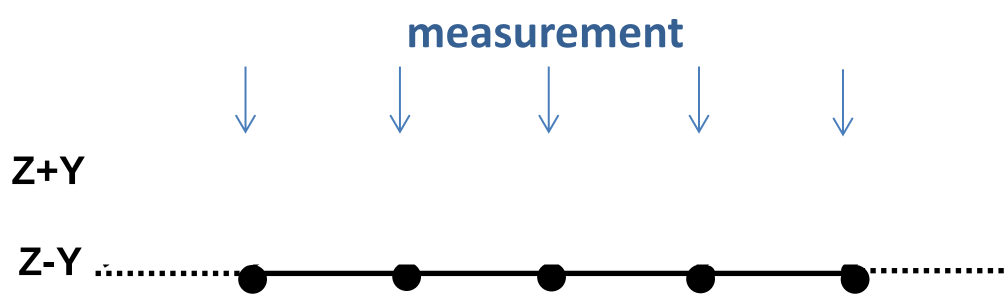
$$a_{0-} = \frac{1}{\sqrt{2}}(a_{0Y} - a_{0Z})$$

$$a_{0Z} = \frac{1}{\sqrt{2}}(a_{0+} - a_{0-})$$
$$a_{0Y} = \frac{1}{\sqrt{2}}(a_{0+} + a_{0-})$$

New entanglement witnesses:

$$(Q_{m+} - Q_{m-}) - (Q_{n+} - Q_{n-}) \to 0$$
$$(P_{m+} - P_{m-}) + (P_{n+} - P_{n-}) \to 0$$

After phase shift and some algebra

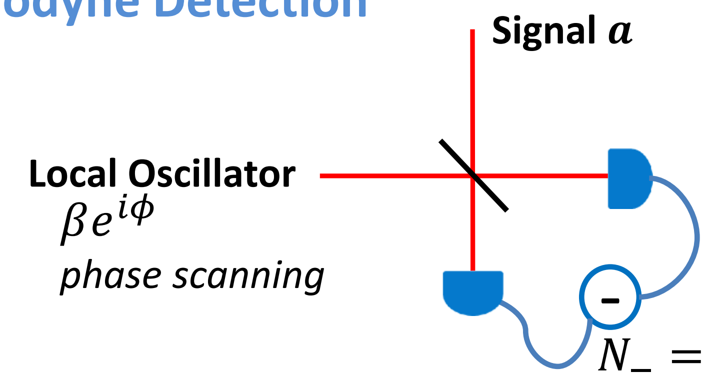


Quantum wire Cluster State!

How to Verify the Quantum Wire Cluster State

To measure its entanglement witnesses

Homodyne Detection



$$N_- = \beta(ae^{-i\phi} + a^+ e^{i\phi}) \begin{cases} \boldsymbol{\phi = 0}: \ \propto \boldsymbol{Q} \\ \boldsymbol{\phi = \frac{\pi}{2}}: \ \propto \boldsymbol{P} \end{cases}$$

If there is no signal -> vacuum input:

$$\boldsymbol{N_- = \beta X_{vac}}$$

(Shot Noise)

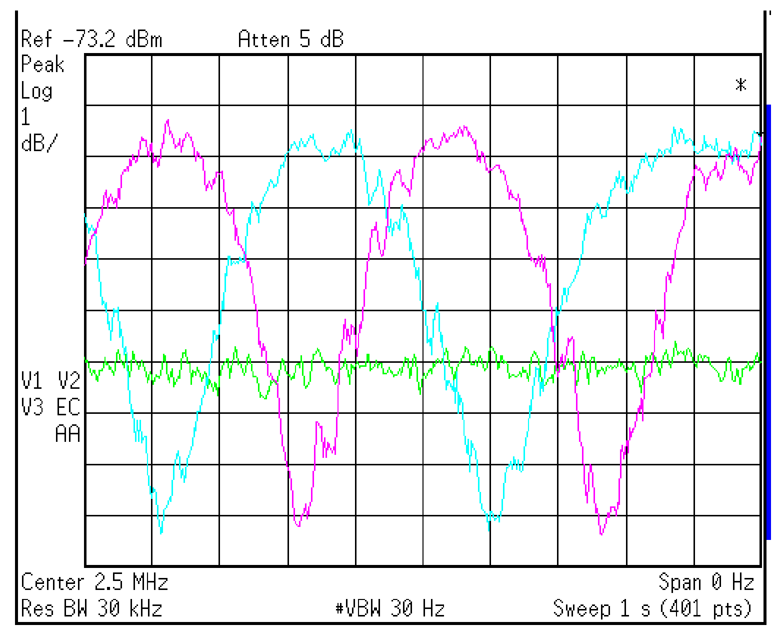
If the Local Oscillator has two frequencies, we could measure:

$$\boxed{\boldsymbol{P_1(t) + P_2(t) = [P_1(0) + P_2(0)]e^{-r} \ \rightarrow 0}}$$

$$\boldsymbol{S = 20\log(e^{-r}) \ \ dB}$$

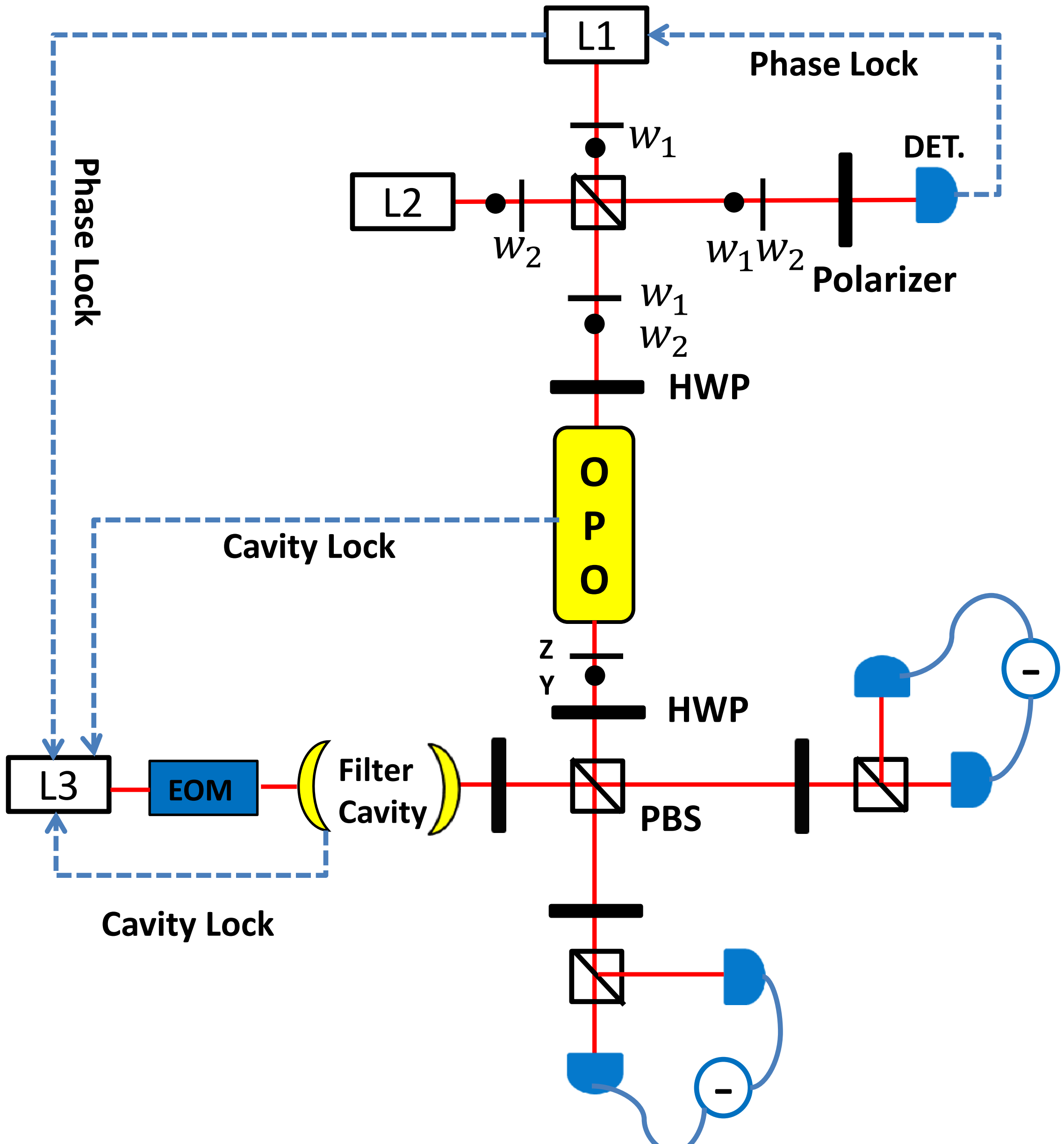
If we use two homdyne detectors together, we could measure:

$$\boldsymbol{(Q_{m+} - Q_{m-}) - (Q_{n+} - Q_{n-}) = e^{-r} \rightarrow 0}$$
$$\boldsymbol{(P_{m+} - P_{m-}) + (P_{n+} - P_{n-}) = e^{-r} \rightarrow 0}$$
$$\boldsymbol{(Q_{m'+} + Q_{m'-}) - (Q_{n'+} + Q_{n'-}) = e^{-r} \rightarrow 0}$$
$$\boldsymbol{(P_{m'+} + P_{m'-}) + (P_{n'+} + P_{n'-}) = e^{-r} \rightarrow 0}$$



ZZZ EPR squeezing measurement

Experimental Setup



Experimental Difficulties

- **Two phase lockings and two cavity lockings have to work at the same time**

- **Squeezing detection super sensitive to any loss or fluctuations (pump shifts, LO not perfectly aligned, air drifts, etc.)**

$$R_{-} = 1 - \eta \xi^2 \zeta \rho \frac{4x}{(1+x)^2 + 4\Omega^2}$$

- **OPO emits back-propagating beams**

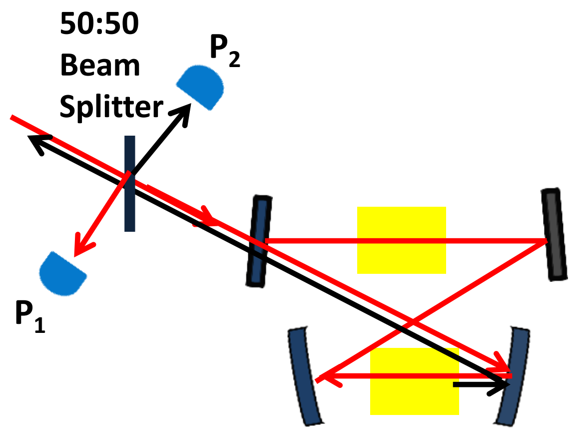
Experimental Progress

Things I have done:

- ✓ **High visibility obtained for Homodyne Detection**
- ✓ **SHG measurement for ZZZ and YYY crystals**
- ✓ **Backward reflection reduction**
- ✓ **Intracavity loss measurement**
- ✓ **Lasing threshold power lowered**
- ✓ **Laser problem fixed with InnoLight**
- ✓ **Phase noise reduction**
- ✓ **>3 dB squeezing obtained for ZZZ EPR pairs**

Experimental Progress

- Backward Reflection Reduction

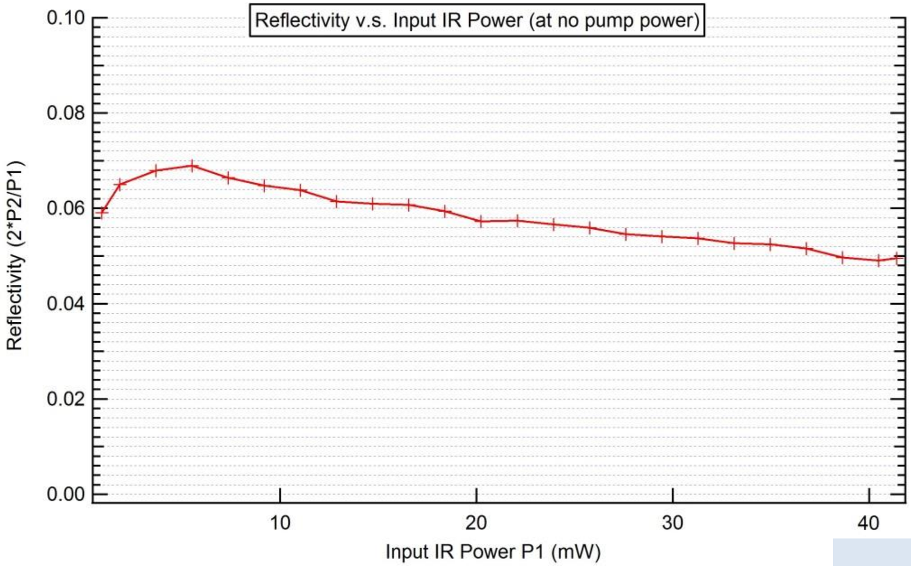


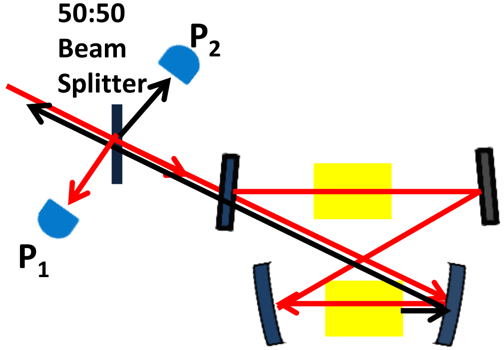
$$P_1 = power\ of\ input\ IR\ to\ OPO$$
$$P_2 = half\ power\ of\ backreflected\ IR$$

$$\boldsymbol{Backward\ Reflectivity = \frac{2P_2}{P_1}}$$

Experimental Progress

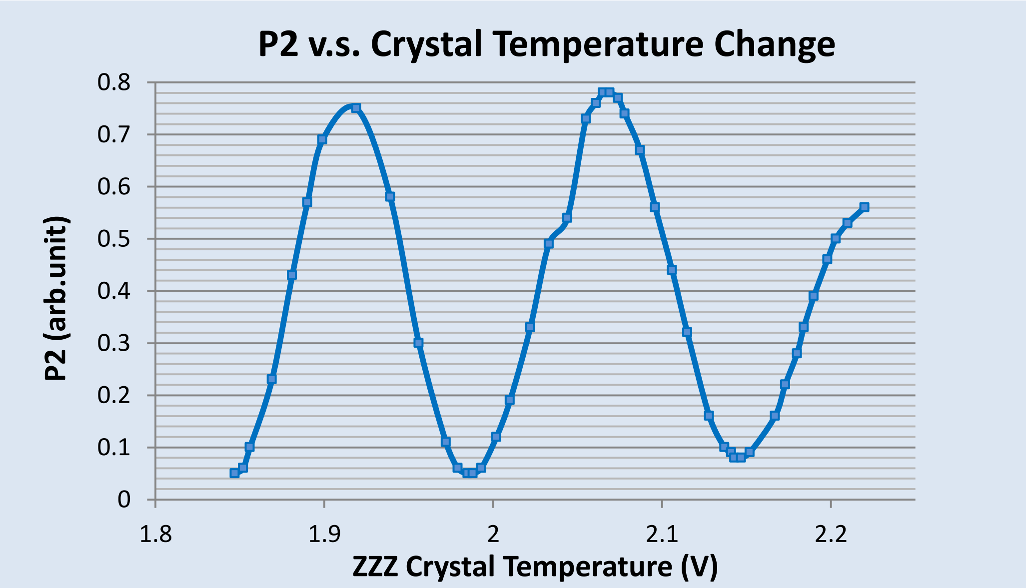
- Backward Reflection Reduction





Guess: Not nonlinear effects, just <u>reflection from the crystal surface</u>?

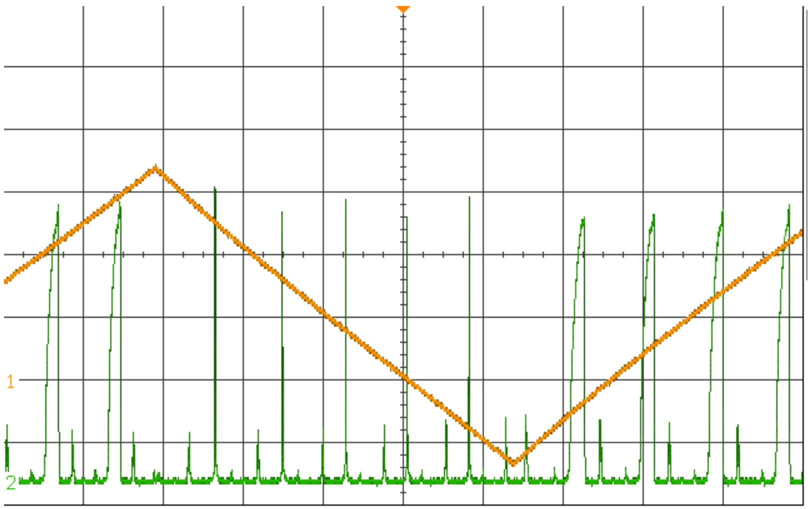
Tilt the crystal, and backward reflection went away!

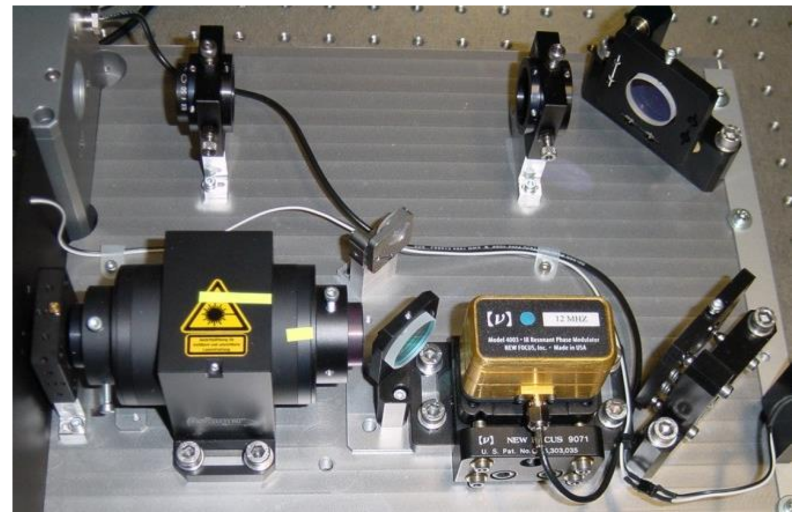


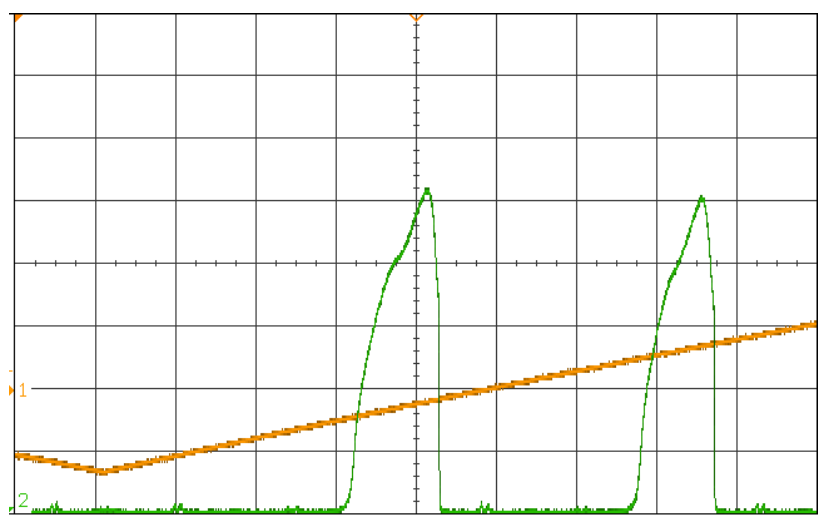
Experimental Progress

- Improve the laser alignment

Problem: Laser flashing/blinking, mode not stable



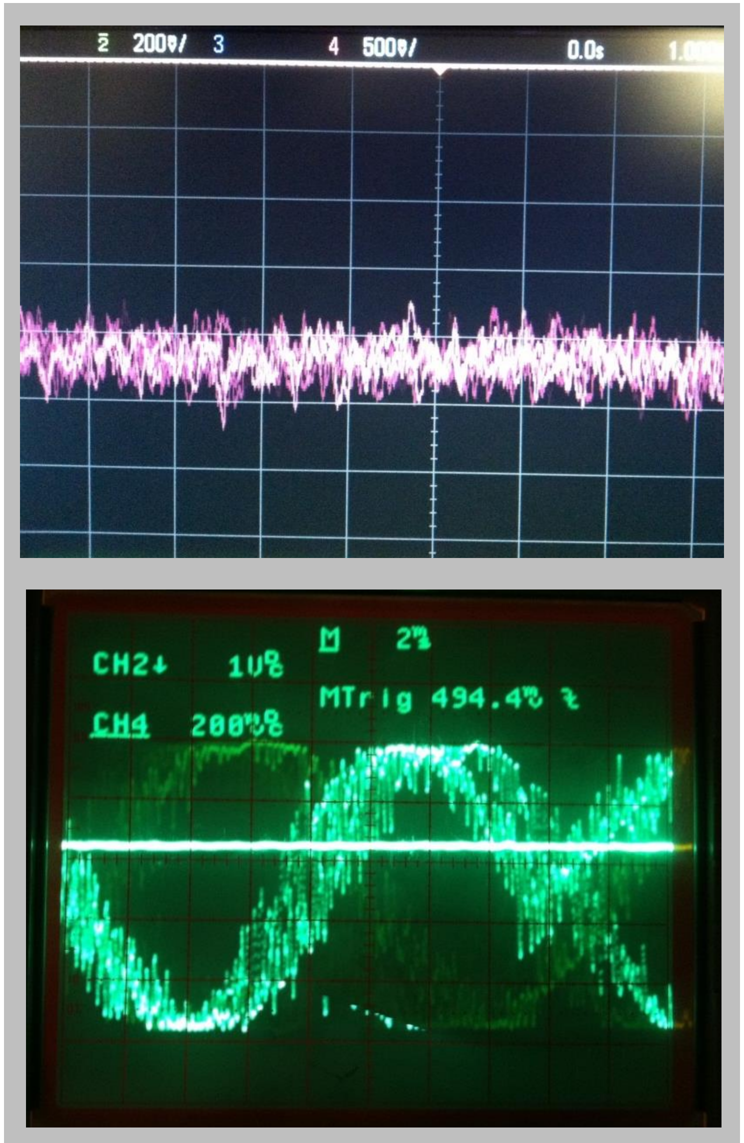




Experimental Progress

- Decrease the phase fluctuation

Problem: phase lock noise was big

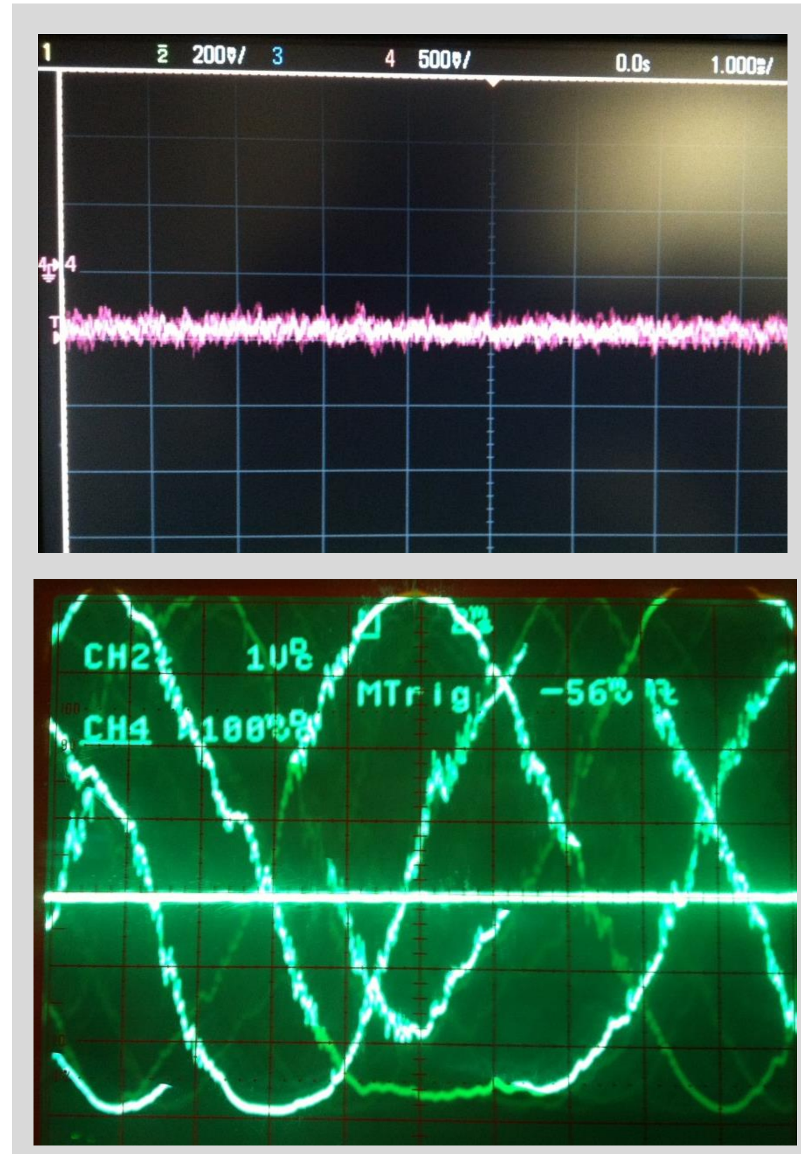


Phase lock error signal

and

inter-ference visi-bility

before v.s. after



Future work

- **Confirm the obtain of the Quantum-Wire cluster state structure**
- **Increase the measurement range to find “how long” the wire is**
- **Add in another OPO to make a Square Grid cluster state (Pei Wang)**

Thank you!

Cluster States

The definition of the standard **Continuous Variable Cluster State** is:

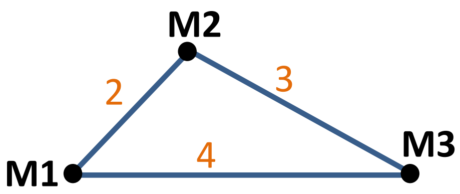
$$\boxed{\boldsymbol{p} - A\boldsymbol{q} \to \boldsymbol{0}}$$

where ***p*** and ***q*** are column vectors of quadratures :

$$P_j = \frac{i}{\sqrt{2}}(a_j^+ - a_j)$$

$$Q_j = \frac{1}{\sqrt{2}}(a_j^+ + a_j)$$

matrix A is the adjacent matrix for the cluster state's graph.



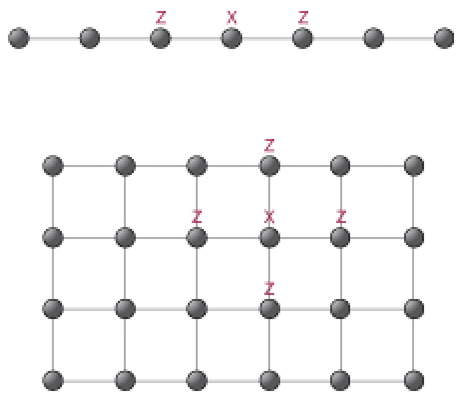
A graph

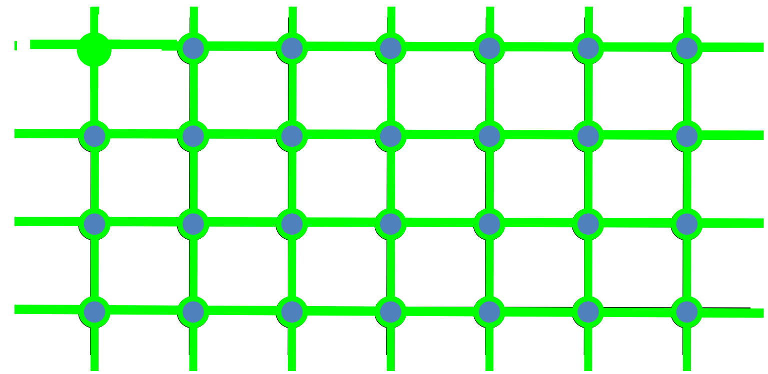
$$\boldsymbol{A} = \begin{pmatrix} \mathbf{0} & \mathbf{2} & \mathbf{4} \\ \mathbf{2} & \mathbf{0} & \mathbf{3} \\ \mathbf{4} & \mathbf{3} & \mathbf{0} \end{pmatrix}$$

$$\begin{pmatrix} p_1 \\ p_2 \\ p_3 \end{pmatrix} - \begin{pmatrix} 0 & 2 & 4 \\ 2 & 0 & 3 \\ 4 & 3 & 0 \end{pmatrix} \begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix} \to 0$$

Cluster States

What is cluster state?





Why important?

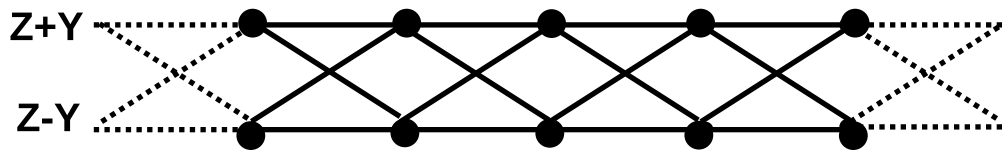
Measurement-based Model (One-way quantum computer):

A method of quantum computing that first makes an entangled resource state, a.k.a. Cluster State, and then performs measurements on it.

It's called "one-way" because the resource state is destroyed by the measurement.

How to Verify the Quantum Wire Cluster State

The way to verify the obtain of the quantum wire cluster state is to measure its entanglement witnesses



$$
\begin{aligned}
&(Q_{m+} - Q_{m-}) - (Q_{n+} - Q_{n-}) \to 0 \\
&(P_{m+} - P_{m-}) + (P_{n+} - P_{n-}) \to 0 \\
&(Q_{m'+} + Q_{m'-}) - (Q_{n'+} + Q_{n'-}) \to 0 \\
&(P_{m'+} + P_{m'-}) + (P_{n'+} + P_{n'-}) \to 0
\end{aligned}
$$

Use Homodyne Detection to measure nullifiers →