Superconducting quarks: Condensed Matter in the Heavens

Mark Alford
Washington University in St. Louis

Outline

I Quarks at high density
Confined, quark-gluon plasma, color superconducting

II Color superconductivity
Color-flavor locking (CFL), and beyond

III Compact stars
Transport properties, equation of state

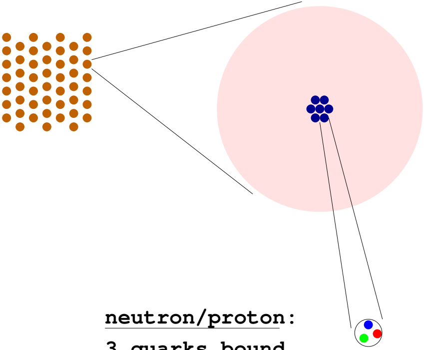
IV Looking to the future

M. Alford, K. Rajagopal, T. Schäfer, A. Schmitt, arXiv:0709.4635 (RMP)
A. Schmitt, arXiv:1001.3294

I. Quarks at high density

Quarks: Building blocks of matter

atom: nucleus & electrons



neutron/proton:

3 quarks bound by color force, Quantum Chromo Dynamics (QCD)

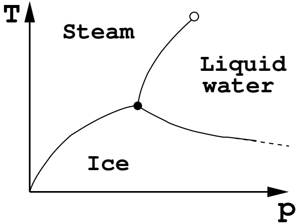
Quarks have color and flavor ("up" or "down")

proton: uud, uud, uud

neutron: udd, udd, udd

Phase Transitions

When you heat up or compress matter, the atoms *reconfigure* themselves. You get phase transitions between solid, liquid, and gas.



At super-high temperatures or densities, when the nuclei are constantly bashed around or remorselessly crushed together, do *quarks* reconfigure themselves?

$$T \sim 150 \text{ MeV} \quad \sim 10^{12} \text{ K}$$
$$\rho \sim 300 \text{ MeV/fm}^3 \sim 10^{17} \text{ kg/m}^3$$

At such a density, a oil super-tanker is 1mm^3 in size.

Where might this occur?

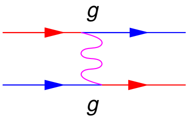
- supernovas, neutron stars;
- Brookhaven (AGS, RHIC); CERN (SPS, LHC)

Interactions between Quarks

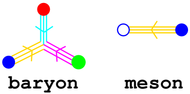
Dominant interaction between quarks is the strong interaction, described by $SU(3)$ "color" non-Abelian gauge theory (QCD).

Properties of QCD

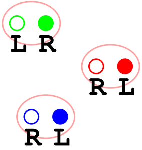
- Short distances, $r \ll 1$ fm, asymptotically free: gauge coupling $g \ll 1$, single gluon exchange dominates, the theory is analytically tractable.



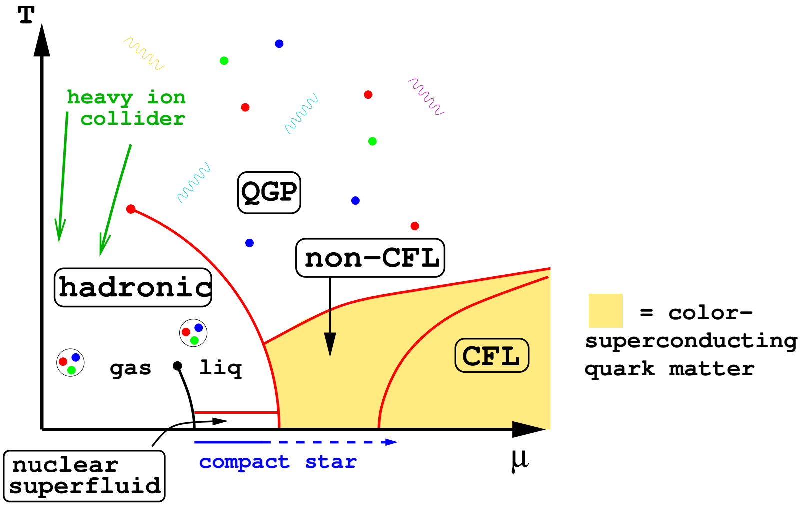
- Long distances $r > 1$ fm, QCD confines: color electric fields form flux tubes, only color-neutral states, baryons and mesons, exist.



- At low temperature ($T \lesssim 170$ MeV), Chiral (left-right) symmetry is broken: color force can't turn a LH quark to RH, but our vacuum is full of $\bar{q}_L q_R$ pairs



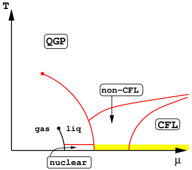
Conjectured QCD phase diagram



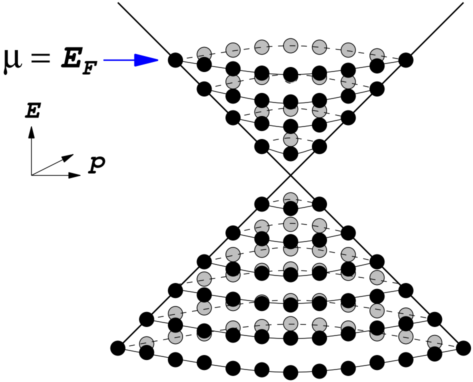
heavy ion collisions: chiral critical point and first-order line

compact stars: color superconducting quark matter core

Color superconductivity



At sufficiently high density and low temperature, there is a Fermi sea of almost free quarks.



$$F = E - \mu N$$

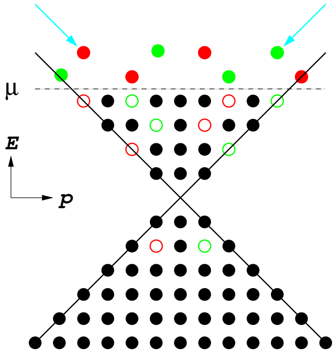
$$\frac{dF}{dN} = 0$$

But quarks have attractive QCD interactions.

Any attractive quark-quark interaction causes pairing instability of the Fermi surface: BCS mechanism of superconductivity.

BCS in quark matter: Ivanenko and Kurdgelaidze, Lett. Nuovo Cim. IIS1 13 (1969).

What is a condensate of Cooper pairs?



$$|\phi_0\rangle = \prod_{\mathbf{p}} \Big(\cos(\theta_{A\mathbf{p}}) + \sin(\theta_{A\mathbf{p}})\, a^\dagger(\mathbf{p}) a^\dagger(-\mathbf{p})\Big)$$

$$\Big(\cos(\theta_{B\mathbf{p}}) + \sin(\theta_{B\mathbf{p}})\, b^\dagger(\mathbf{p}) b^\dagger(-\mathbf{p})\Big) \quad \times \quad |\text{Fermi sea}\rangle$$

$|\phi_0\rangle$, not $|\text{Fermi sea}\rangle$, is the ground state.

Physical consequences of Cooper pairing

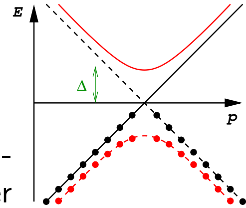
Changes low energy excitations, affecting *transport properties*.

- spontaneous breaking of global symmetries: Goldstone bosons, massless degrees of freedom that dominate low energy behavior.
- spontaneous breaking of local (gauged) symmetries: massive gauge bosons, exclusion of magnetic fields (Meissner effect).
- create a gap in fermion spectrum.

  Adding a fermion of momentum $\vec{p}$ near the Fermi surface disrupts the condensate in that mode:

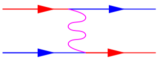
  $$a_p^\dagger(\cos\theta + \sin\theta\, a_p^\dagger a_{-p}^\dagger) = \cos\theta\, a_p^\dagger$$

  This kills that mode's contribution to the binding energy of the condensate, i.e "breaks a Cooper pair", costing energy Δ.

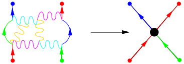


Handling QCD at high density

Lattice: "Sign problem"—negative probabilities
SUSY: Statistics crucial to quark Fermi surface
large N: Large corrections
pert: Applicable far beyond nuclear density.
Neglects confinement and instantons.



NJL: Model, applicable at low density.
Follows from instanton liquid model.



EFT: Effective field theory for lightest degrees of freedom.
"Parameterization of our ignorance": assume a phase, guess coefficients of interaction terms (or match to pert theory), obtain phenomenology.

High-density QCD calculations

Guess a color-flavor-spin pairing pattern P; to obtain gap Δ_P, minimize free energy Ω with respect to Δ_P and impose color and electric neutrality

$$\frac{\partial\Omega}{\partial\Delta_P} = 0 \qquad \frac{\partial\Omega}{\partial\mu_i} = 0$$

The pattern with the lowest $\Omega(\Delta_P)$ wins!

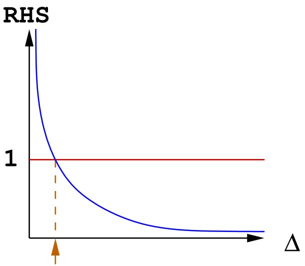
1. Weak-coupling methods. First-principles calculations direct from QCD Lagrangian, valid in the asymptotic regime, currently $\mu \gtrsim 10^6$ MeV.
2. Nambu–Jona-Lasinio models, ie quarks with four-fermion coupling based on instanton vertex, single gluon exchange, etc. This is a semi-quantitative guide to physics in the compact star regime $\mu \sim 400$ MeV, not a systematic approximation to QCD.

NJL gives $\Delta \sim 10-100$ MeV at $\mu \sim 400$ MeV.

Gap equation in a simple NJL model

Minimize free energy wrt Δ:

$$1 = \frac{8K}{\pi^2}\int_0^{\Lambda} p^2 dp \left\{ \frac{1}{\sqrt{\Delta^2 + (p-\mu)^2}} \right\}$$



Note BCS divergence as $\Delta \to 0$: there is *always* a solution, for any interaction strength K and chemical potential μ.
Roughly,

$$1 \sim K\mu^2 \ln(\Lambda/\Delta)$$
$$\Rightarrow \Delta \sim \Lambda \exp\left(-\frac{1}{K\mu^2}\right)$$

Superconducting gap is non-perturbative.

Color superconducting phases

Quark Cooper pair: $\langle q^{\alpha}_{ia} q^{\beta}_{jb} \rangle$

color $\alpha, \beta = r, g, b$
flavor $i, j = u, d, s$
spin $a, b = \uparrow, \downarrow$

There is a 9×9 matrix of possible BCS pairing patterns!

The attractive channel is:
color antisymmetric
spin antisymmetric
$\Rightarrow$ flavor antisymmetric

So pairing between *different flavors* is favored.

Let's start with the most symmetric case, where all three flavors are massless.

Color supercond. in 3 flavor quark matter: Color-flavor locking (CFL)

Equal number of colors and flavors gives a special pairing pattern
(Alford, Rajagopal, Wilczek, hep-ph/9804403)

$$\langle q_i^\alpha q_j^\beta \rangle \sim \delta_i^\alpha \delta_j^\beta - \delta_j^\alpha \delta_i^\beta = \epsilon^{\alpha\beta n}\epsilon_{ijn}$$

color α, β
flavor i, j

This is invariant under equal and opposite rotations of color and (vector) flavor

$$SU(3)_{\text{color}} \times \underbrace{SU(3)_L \times SU(3)_R}_{\supset\, U(1)_Q} \times U(1)_B \to \underbrace{SU(3)_{C+L+R}}_{\supset\, U(1)_{\tilde{Q}}} \times \mathbb{Z}_2$$

- Breaks chiral symmetry, but *not* by a $\langle \bar{q} q \rangle$ condensate.
- There need be no phase transition between the low and high density phases: ("quark-hadron continuity")
- Unbroken "rotated" electromagnetism, $\tilde{Q}$, photon-gluon mixture.

Color-flavor-locked (“CFL”) quark pairing

$\tilde{Q}$	0	0	0	−1	+1	−1	+1	0	0
	u	d	s	d	u	s	u	s	d
u		Δ	Δ						
d	Δ		Δ						
s	Δ	Δ							
d					$-\Delta$				
u				$-\Delta$					
s							$-\Delta$		
u						$-\Delta$			
s									$-\Delta$
d								$-\Delta$	

III. Quark matter in compact stars

Where in the universe is color-superconducting quark matter most likely to exist? In compact stars.

A quick history of a compact star.

A star of mass $M \gtrsim 10 M_\odot$ burns Hydrogen by fusion, ending up with an Iron core. Core grows to Chandrasekhar mass, collapses $\Rightarrow$ supernova. Remnant is a compact star:

mass	radius	density	initial temp
$\sim 1.4 M_\odot$	$\mathcal{O}(10\ \text{km})$	$\gtrsim \rho_{\text{nuclear}}$	~ 30 MeV

The star cools by neutrino emission for the first million years.

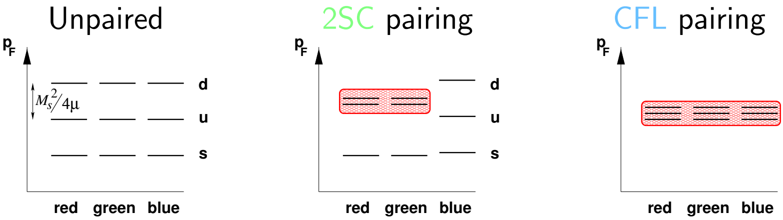
The real world: M_s and neutrality

In the real world there are three complications to the simple account given so far.

1. Strange quark mass is not infinite nor zero, but intermediate. It depends on density, and ranges between about 500 MeV in the vacuum and about 100 MeV at high density.
2. Neutrality requirement. Bulk quark matter must be neutral with respect to all gauge charges: color and electromagnetism.
3. Weak interaction equilibration. In a compact star there is time for weak interactions to proceed: neutrinos escape and flavor is not conserved.

So quark matter in a compact star might be CFL, or something else: gapless CFL; kaon-condensed CFL, 2SC, 1SC, crystalline,…

Cooper pairing vs. the strange quark mass



CFL: Color-flavor-locked phase, favored at the highest densities.

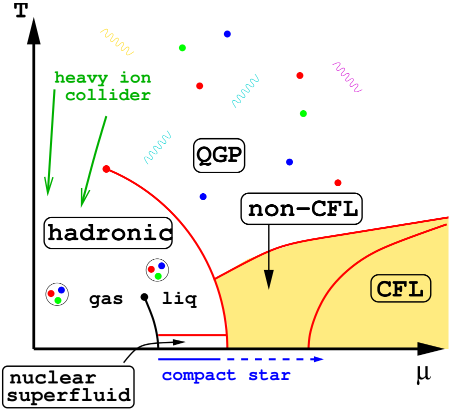
$$\langle q_i^\alpha q_j^\beta \rangle \sim \delta_i^\alpha \delta_j^\beta - \delta_j^\alpha \delta_i^\beta = \epsilon^{\alpha\beta N} \epsilon_{ijN}$$

2SC: Two-flavor pairing phase. May occur at intermediate densities.

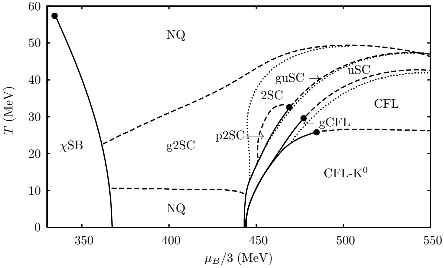
$$\langle q_i^\alpha q_j^\beta \rangle \sim \epsilon^{\alpha\beta 3} \epsilon_{ij3} \sim (rg - gr)(ud - du)$$

or: Exotic non-BCS pairing: LOFF (crystalline phase), *p*-wave meson condensates, single-flavor pairing (color-spin locking, ∼liq ^{3}He-B).

Phases of quark matter, again



NJL model, uniform phases only

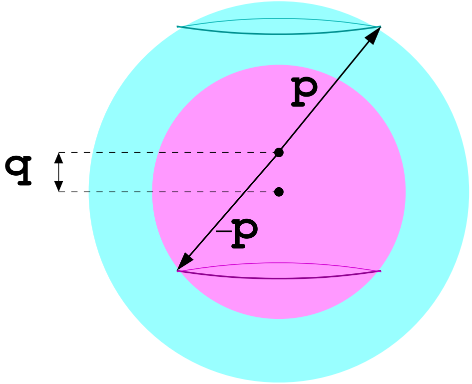


Warringa, hep-ph/0606063

But there are also non-uniform phases, such as the crystalline ("LOFF"/"FFLO") phase. (Alford, Bowers, Rajagopal, hep-ph/0008208)

Crystalline (LOFF) superconductivity

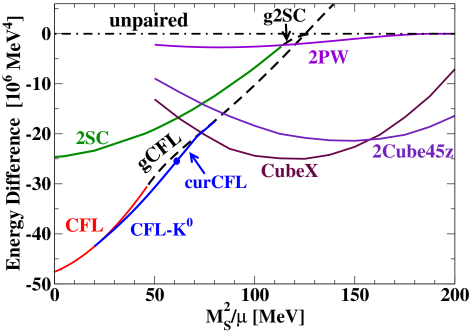
When the Fermi momenta are such that one flavor of quark is just barely excluded from pairing with another, it may be favorable to make pairs with a net momentum, so each flavor can be close to its Fermi surface.



Every quark pair in the condensate has the same nonzero total momentum $2\mathbf{q}$ (single plane wave LOFF).

Free energy comparison of phases

Assuming $\Delta_{\rm CFL} = 25$ MeV.



CFL-K^0	K^0 condensate
curCFL	K^0 cond current
2PW	LOFF, 2-plane-wave
CubeX	LOFF crystal, G-L approx
2Cube45z	LOFF crystal, G-L approx

(Alford, Rajagopal, Schäfer, Schmitt, `arXiv:0709.4635`)

Curves for CubeX and 2Cube45z use G-L approx far from its area of validity: favored phase at $M_s^2 \sim 4\mu\Delta$ remains uncertain.

Signatures of color superconductivity in compact stars

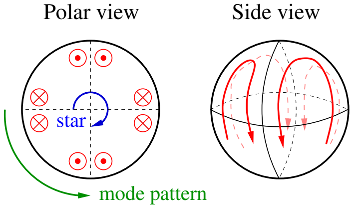
Pairing energy — affects Equation of state. Hard to detect.
(Alford, Braby, Paris, Reddy, nucl-th/0411016)

Gaps in quark spectra and Goldstone bosons — affect Transport properties:
emissivity, heat capacity, viscosity (shear, bulk), conductivity (electrical, thermal)...

1. Cooling by neutrino emission, neutrino pulse at birth
2. Glitches and crystalline ("LOFF") pairing
3. Gravitational waves: r-mode instability, shear and bulk viscosity

r-modes: gravitational spin-down of compact stars

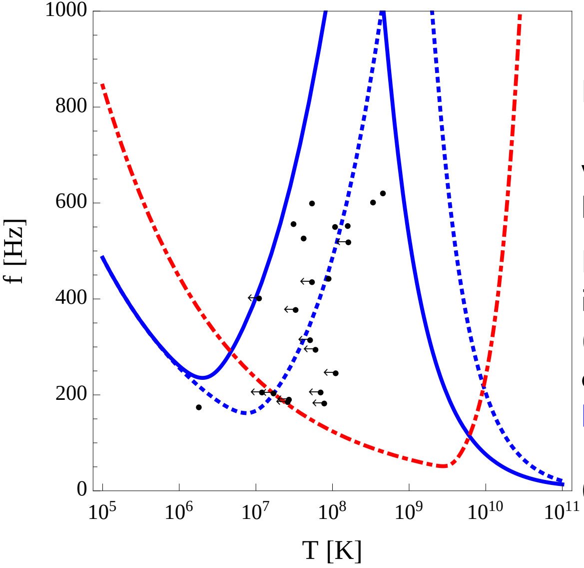
An r-mode is a quadrupole flow that emits gravitational radiation. It becomes unstable (i.e. arises spontaneously) when a star spins fast enough, and if the shear and bulk viscosity are low enough.



Andersson `gr-qc/9706075`

Friedman and Morsink `gr-qc/9706073`

Constraints from r-modes



Regions above curves are "forbidden" because viscosity is too low to hold back the *r*-modes.

Data for accreting pulsars in binary systems (LMXBs) vs instability curves for nuclear and hybrid stars.

(Schwenzer `arXiv:1212.5242`)

IV. Looking to the future

- Neutron-star phenomenology of color superconducting quark matter:
  - mass-radius and equation of state
  - analysis of r-mode spindown vs data
  - elimination/evaluation of other r-mode damping mechanisms
  - neutrino emissivity and cooling
  - structure: nuclear-quark interface (gravitational waves?)
  - color supercond. crystalline phase (glitches) (gravitational waves?)
  - CFL: vortices but no flux tubes; stability of vortices. . .
- More general questions:
  - instability of gapless phases; better treatment of LOFF
  - better weak-coupling calculations
  - role of large magnetic fields
  - solve the sign problem and do lattice QCD at high density.