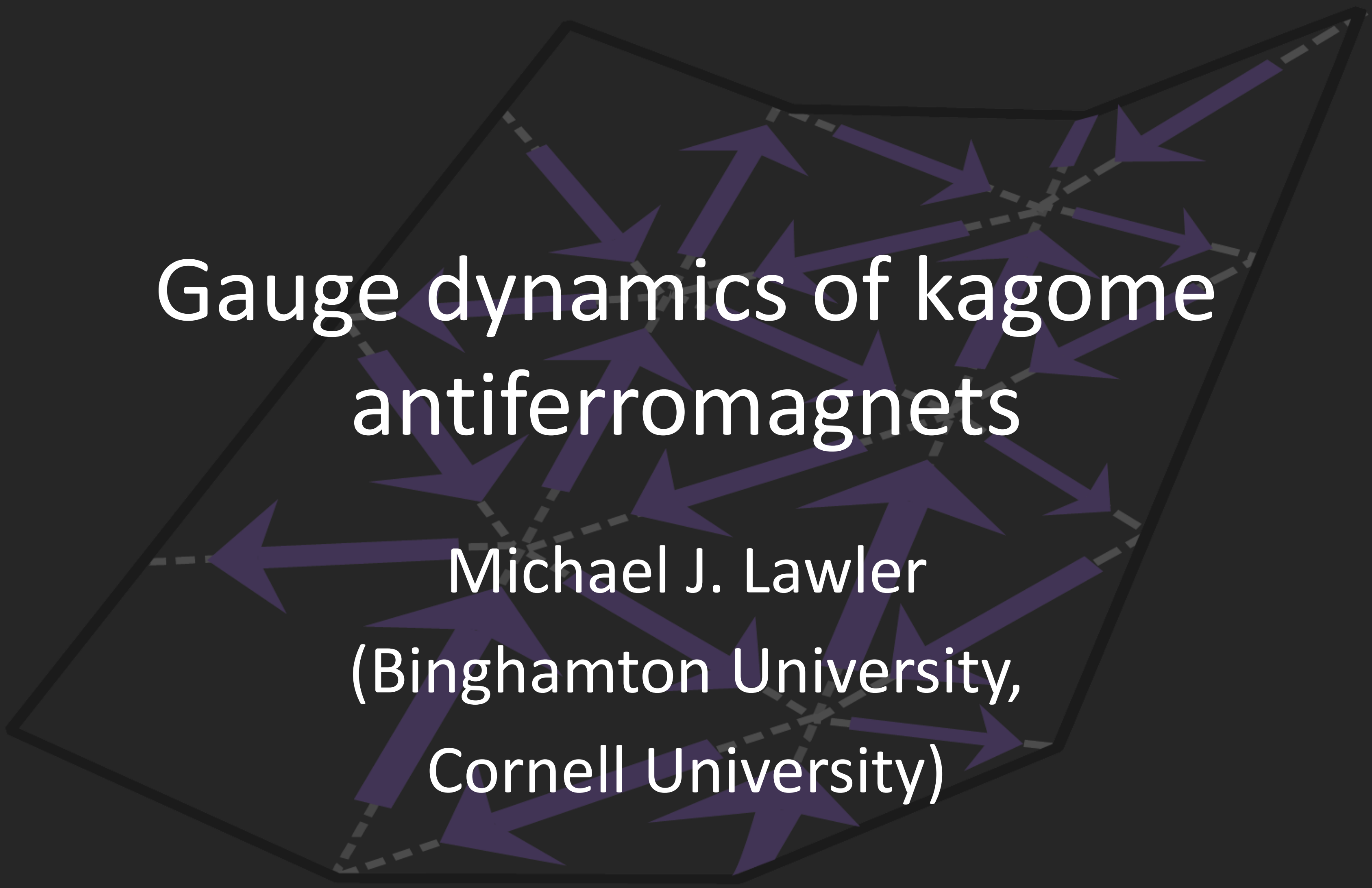

Gauge dynamics of kagome antiferromagnets

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Outline

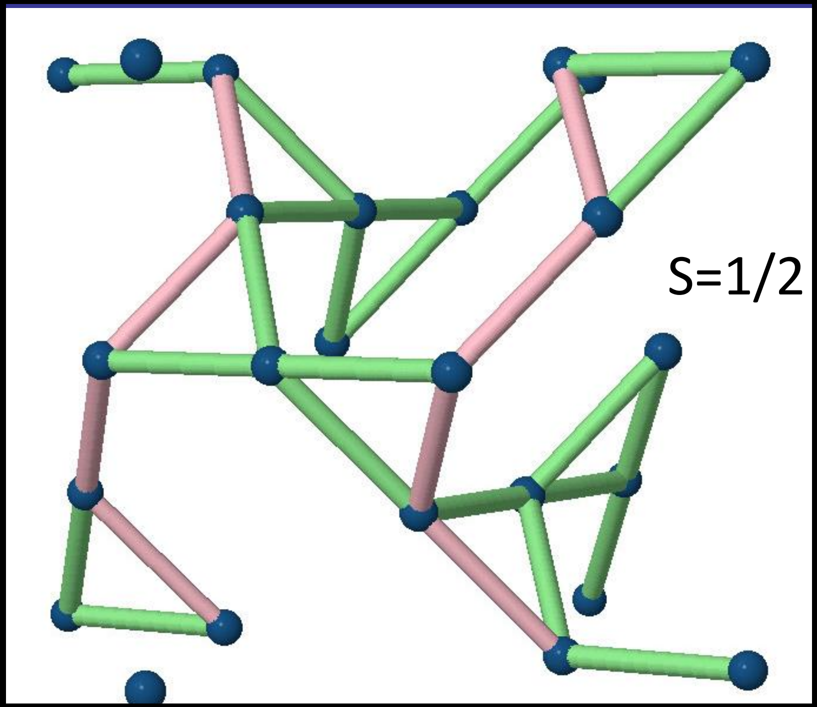
- Introduction to highly frustrated magnets
- Constrained spin models
  - Dirac's generalized Hamiltonian mechanics
  - Degrees of freedom counting
  - Edge states?
- Simulations of spin waves in kagome AFM
- Conclusions

The problem of highly frustrated magnetism

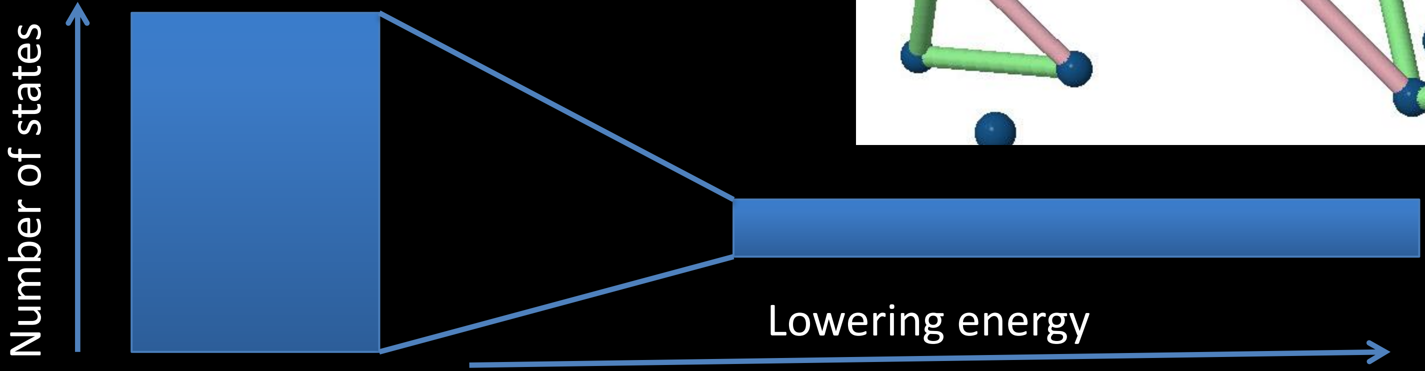
$Na_4Ir_3O_8$, Okamoto et. al. 2007

- Nearest neighbor model just selects a low energy subspace!

$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$$



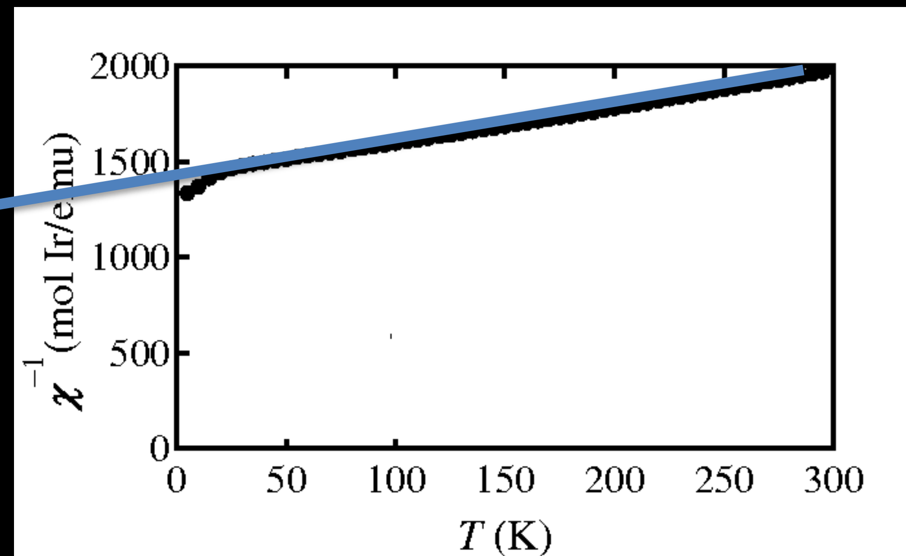
S=1/2



→ defines the "cooperative paramagnet" (Villain, 1979)

Cooperative paramagnets

$Na_4Ir_3O_8$, Okamoto et. al. 2007



Θ_{CW} = -650 K

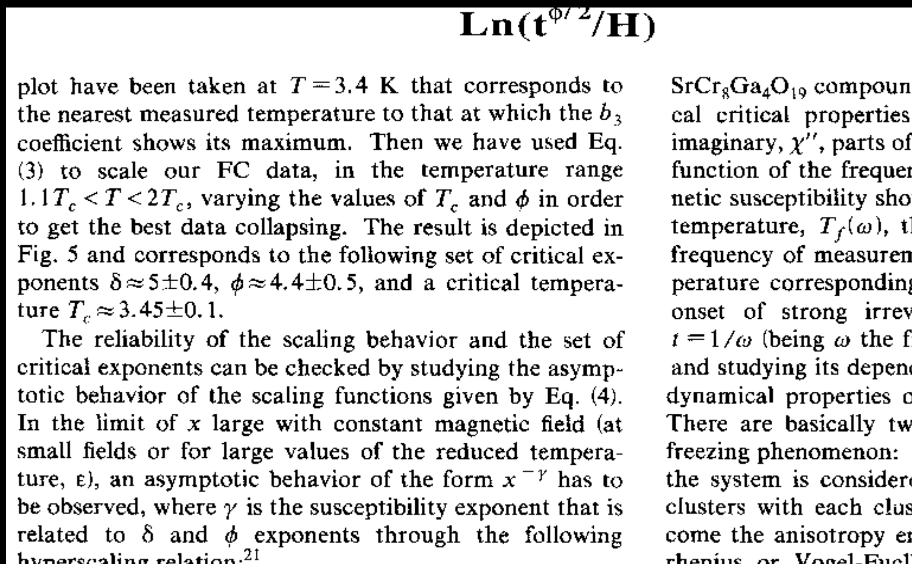
$$\chi(T) = \frac{C}{T - \Theta_{CW}}$$

Frustration parameter:

$$f = \Theta_{CW}/T_c \approx 65$$

→ temperature range of the cooperative paramagnetic

Unusual glassy dynamics?



Ln($t^{\phi/2}$/H)

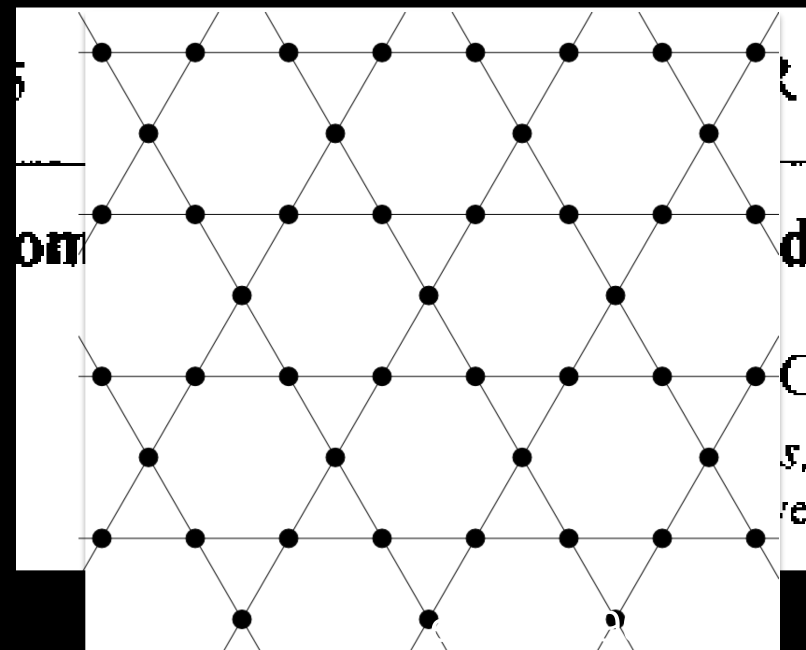
plot have been taken at $T=3.4$ K that corresponds to the nearest measured temperature to that at which the b_3 coefficient shows its maximum. Then we have used Eq. (3) to scale our FC data, in the temperature range $1.1T_c < T < 2T_c$, varying the values of T_c and ϕ in order to get the best data collapsing. The result is depicted in Fig. 5 and corresponds to the following set of critical exponents $\delta \approx 5 \pm 0.4$, $\phi \approx 4.4 \pm 0.5$, and a critical temperature $T_c \approx 3.45 \pm 0.1$.

The reliability of the scaling behavior and the set of critical exponents can be checked by studying the asymptotic behavior of the scaling functions given by Eq. (4). In the limit of x large with constant magnetic field (at small fields or for large values of the reduced temperature, ε), an asymptotic behavior of the form $x^{-\gamma}$ has to be observed, where γ is the susceptibility exponent that is related to δ and ϕ exponents through the following hyperscaling relation:[21]

$SrCr_8Ga_4O_{19}$ compounc
cal critical properties
imaginary, χ'', parts of
function of the frequen
netic susceptibility shov
temperature, $T_f(\omega)$, th
frequency of measurem
perature corresponding
onset of strong irreve
$t=1/\omega$ (being ω the fr
and studying its depend
dynamical properties o
There are basically two
freezing phenomenon:
the system is considere
clusters with each clust
come the anisotropy en
rhenius or Vogel-Fuclh

Martinez et. al. 1994

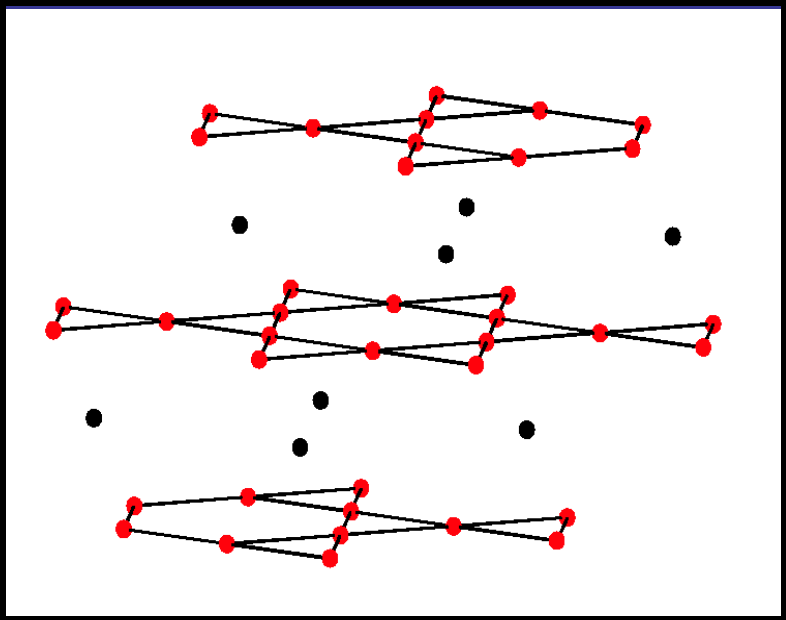
$SrCa_8Ga_4O_{19}$, S=3/2 kagome AFM



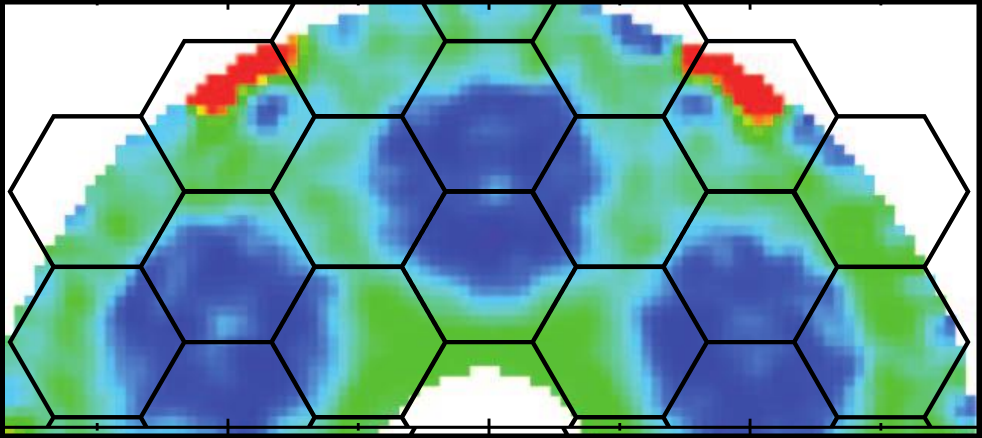
$$\chi_{nl}(H,T) = \chi_0(T) - M(H,T)/H \propto H^{2/\delta}$$

Does not obey hyper-scaling relations

Herbertsmithite: A quantum spin liquid?



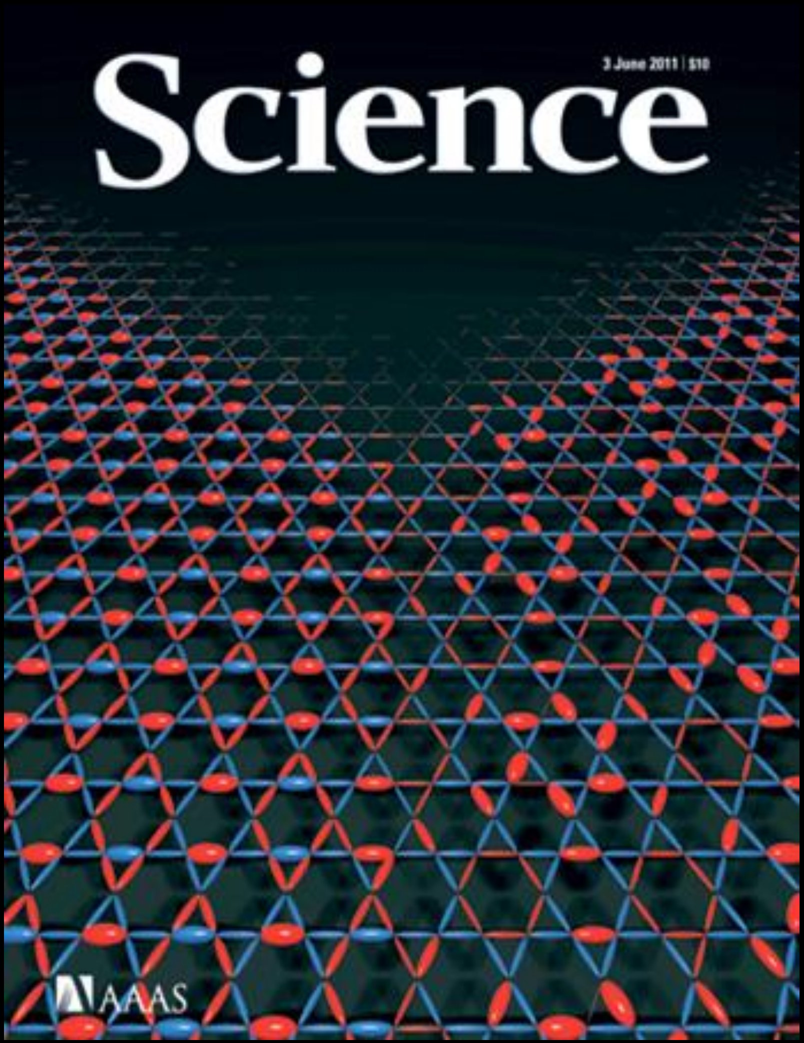
Han et. al., 2012



Neutron scattering at 0.75 meV

- No magnetic order down to 50 mK
- Continuum of spin excitations at low energies

DMRG Ground State



Yan et. al., Science, 2011
Depenbrock et. al., 2012
Jiang et. al., 2012

Strong numerical evidence that the spin ½ ground state is a "Z2 spin liquid"

Definition of quantum spin liquid

- Experimental definition:
  - No sign of magnetic ordering
  - No sign of “freezing” or glassy behavior
  - Odd number of half-odd-integer spins in unit cell
- Theoretical definition:
  - A state with long range entanglement between the spins

Why does frustration produce a quantum spin liquid phase?

Constrained spin models

Lawler, 2013

On the kagome lattice, we can write

$$H_{nn} = \frac{J}{2}\sum_{\langle ijk\rangle}\left(\vec{S}_i + \vec{S}_j + \vec{S}_k\right)^2 + const$$

So the low energy subspace of states obeys

$$\vec{\phi}_{ijk} \equiv \vec{S}_i + \vec{S}_j + \vec{S}_k = 0$$

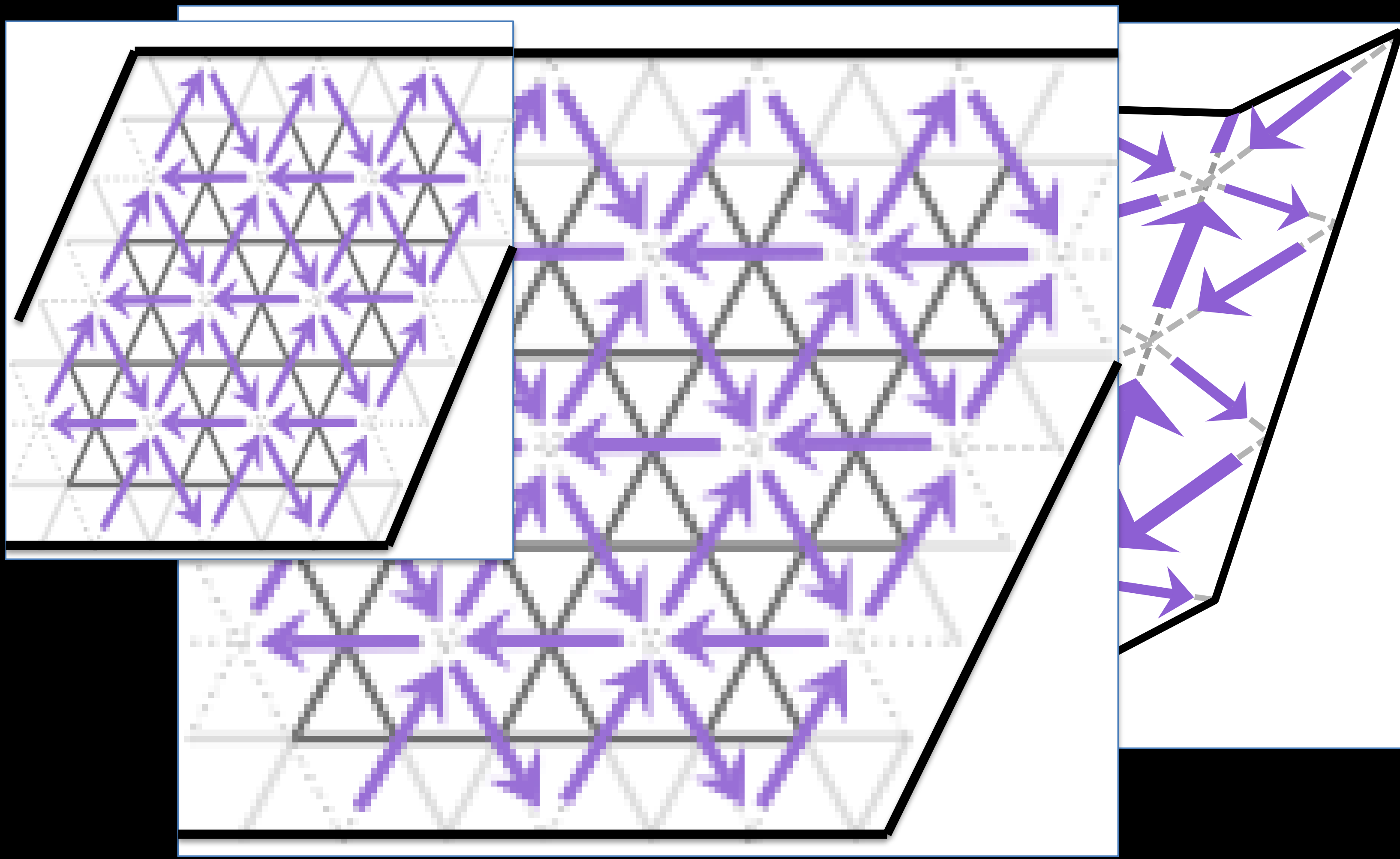
Lets then focus on the simpler model

$$H = H_{eff} - \sum_{\langle ijk\rangle} \vec{h}_{ijk}\cdot\vec{\phi}_{ijk}$$

Lagrange multipliers

Spin origami

Shender et. al, 1993



Constrained spin model is that of a fluctuating membrane!

Constrained Hamiltonian Mechanics

Dirac, 1950,1958

Follow Dirac, and fix the Lagrange multipliers h_n by

$$\frac{d}{dt}\phi_m = \{\phi_m, H_{eff}\} - \sum_n \{\phi_m, \phi_n\} h_n = 0$$

This is a linear algebra problem! If

$$\det C_{mn} \equiv \det\{\phi_m, \phi_n\} \neq 0$$

We can invert and solve for h_n.

Otherwise, some combinations of h_n remain arbitrary!

Gauge dynamics

- A “gauge theory” in mechanics is one with multiple solutions to its equations of motion.
- Example: Maxwell electrodynamics
  - There are many solutions to the scalar and vector potential
  - The electric and magnetic fields evolve the same way for each solution

The single triangle model

This model has constraints

$$\vec{\phi} = \vec{S}_1 + \vec{S}_2 + \vec{S}_3 = 0$$

and Hamiltonian

$$H = H_{eff} - \vec{h} \cdot \vec{\phi}$$

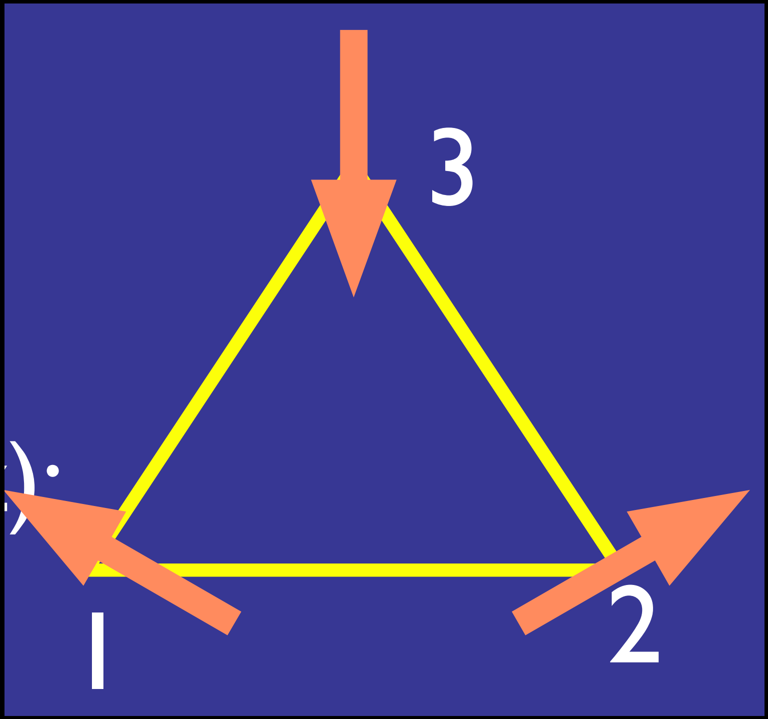
The constraints obey

$$\{\phi_x, \phi_y\} = \phi_z = 0$$

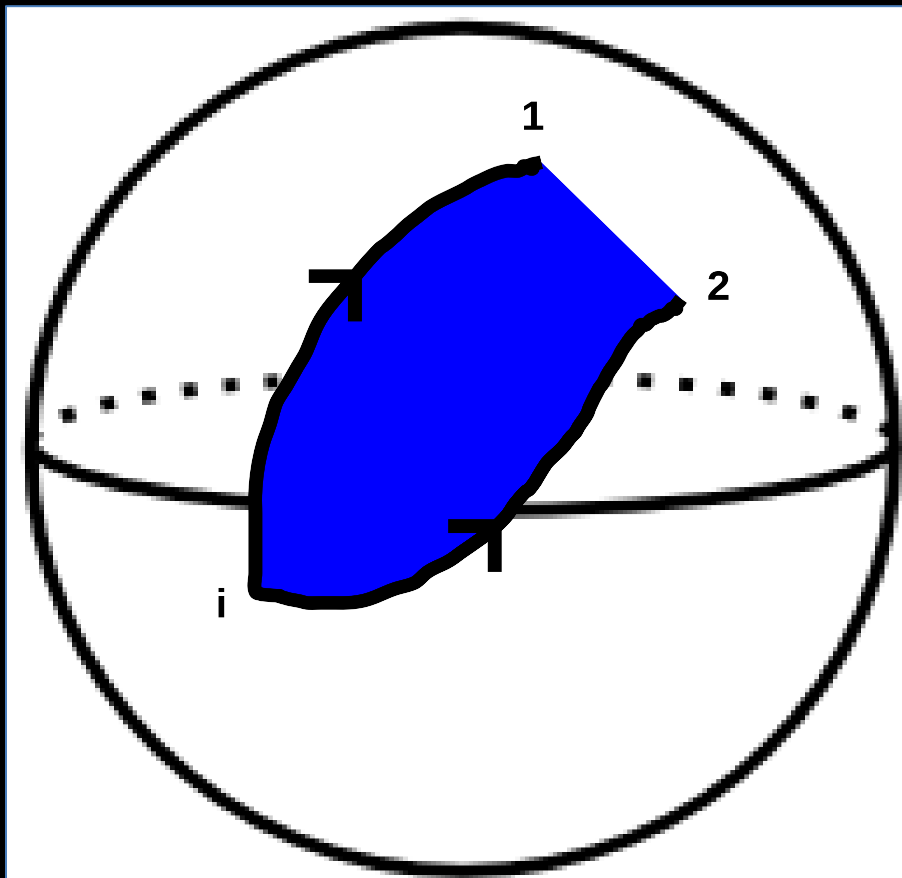
So

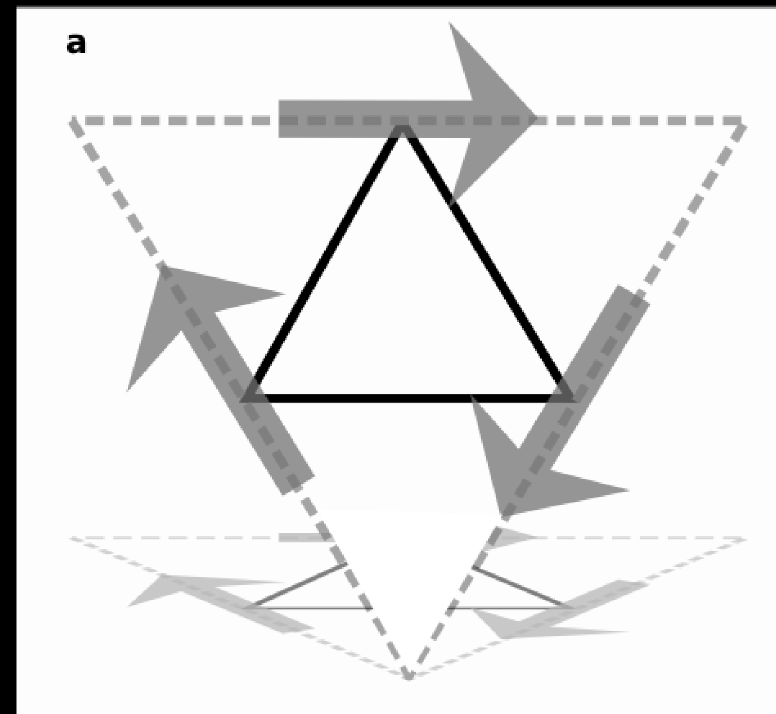
$$\frac{d\phi_a}{dt} = \{\phi_a, H_{eff}\} - h_b\{\phi_a, \phi_b\} = \{\phi_a, H_{eff}\} = 0$$

h_x, *h_y* and *h_z* are arbitrary!



Map all solutions





Spin origami construction

Physical observables evolve the same way
independent of the choice of the arbitrary functions

Degrees of freedom counting

- How many physical observables are there?
  - Dirac discovered

$$N_{canonical} = D - M - N_L$$

  where

    - D: the number of unconstrained coordinates
    - M: the number of constraint functions ϕ_m
    - N_L: the number of arbitrary Lagrange multipliers

Two polarizations of light

- Consider electricity and magnetism
  - D = 8

$$\phi, \quad \vec{A}, \quad \pi_0 = \frac{\delta L}{\delta \dot{\phi}}, \quad \pi_a = E_a = \frac{\delta L}{\delta \dot{A}_a}$$

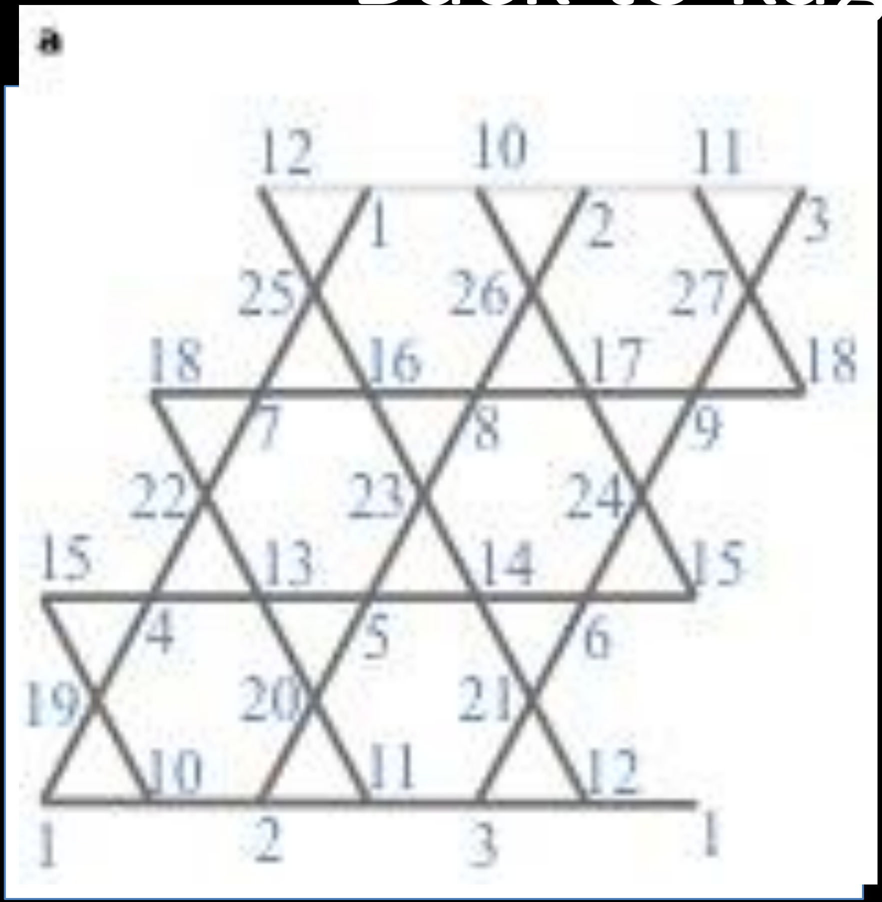
  - M = 2

$$\pi_0 = 0, \quad \nabla \cdot \vec{E} - \rho = 0$$

  - N_L = 2 (the above two constraints commute)

So $N_{canonical}$ = 8 – 2 – 2 = 4 → two polarizations of light!

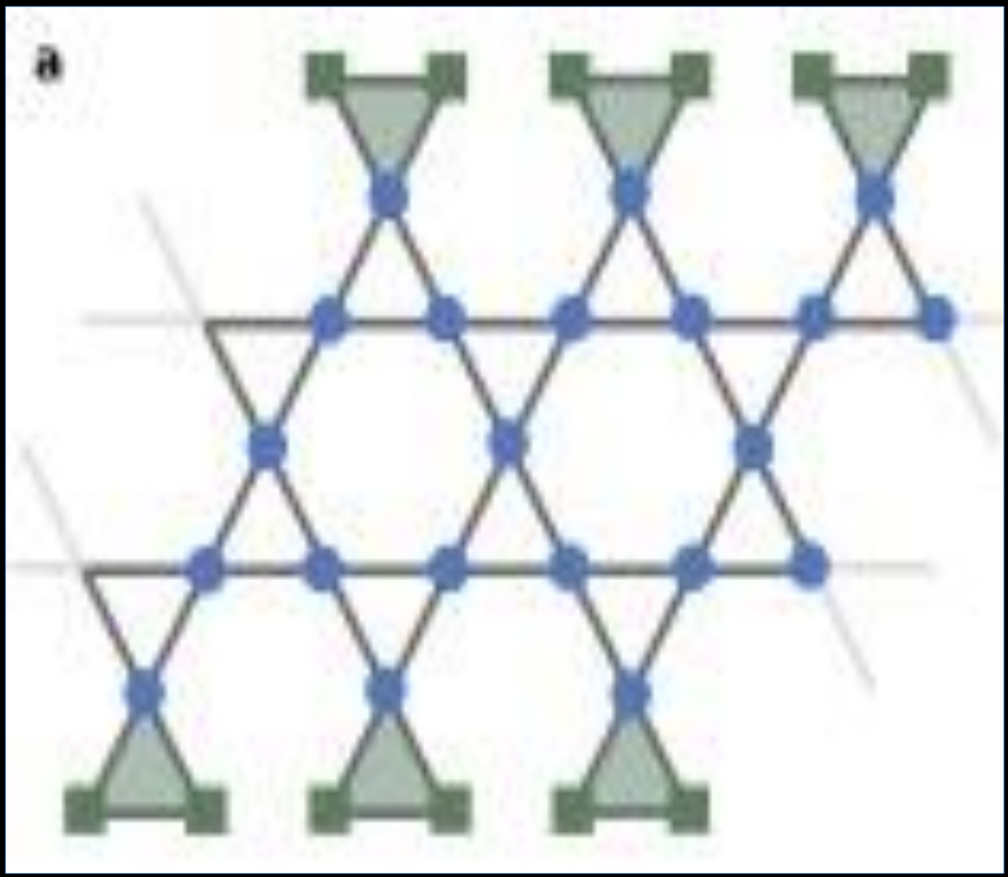
Back to kagome: pbc's



For every spin configuration that satisfies the constraints:

$N_{canonical} = 0!$

Edge states?



Open boundary conditions

$N_{canonical}$ = number of dangling triangles

But a local mechanical object requires a position *and* a momentum coordinate!

Chern-Simon’s electrodynamics

- Similar to “doubled” Chern-Simon’s electrodynamics in 2 spatial dimensions

$$\vec{E} = 0, B = 0$$

- Changes only the statistics of particles
- Quantum model has long range entanglement
- Proposed to govern Z_2 spin liquids (Xu and Sachdev, 2009)

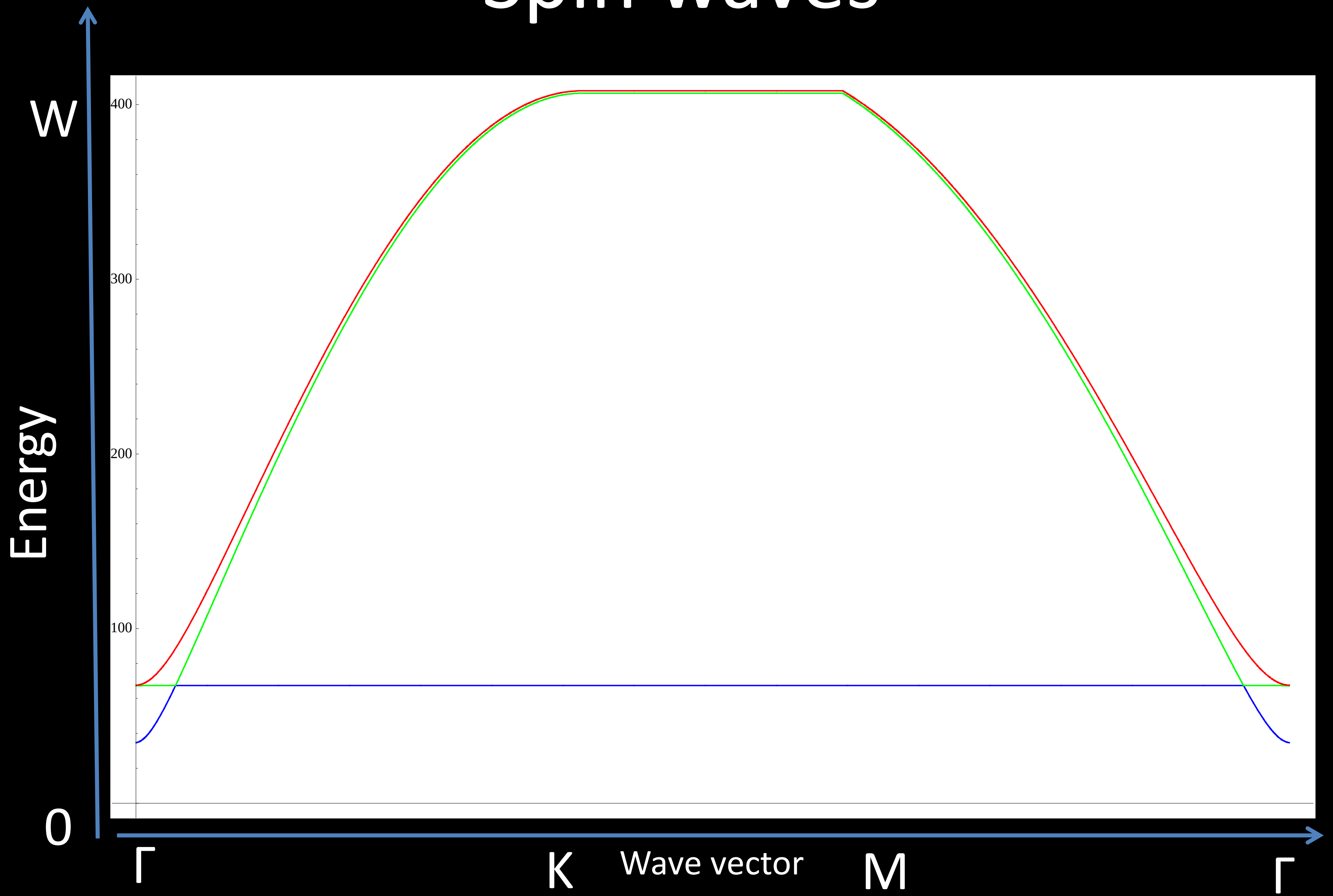
Ordinary Kagome antiferromagnets

Now consider an ordinary kagome antiferromagnet with Hamiltonian

$$H = \sum_{\langle ij \rangle} \left[J \vec{S}_i \cdot \vec{S}_j + \vec{D}_{ij} \cdot \vec{S}_i \times \vec{S}_j \right]$$

How is the discovered gauge dynamics important here?

Spin waves



What do the eigenmodes corresponding to gauge modes look like?

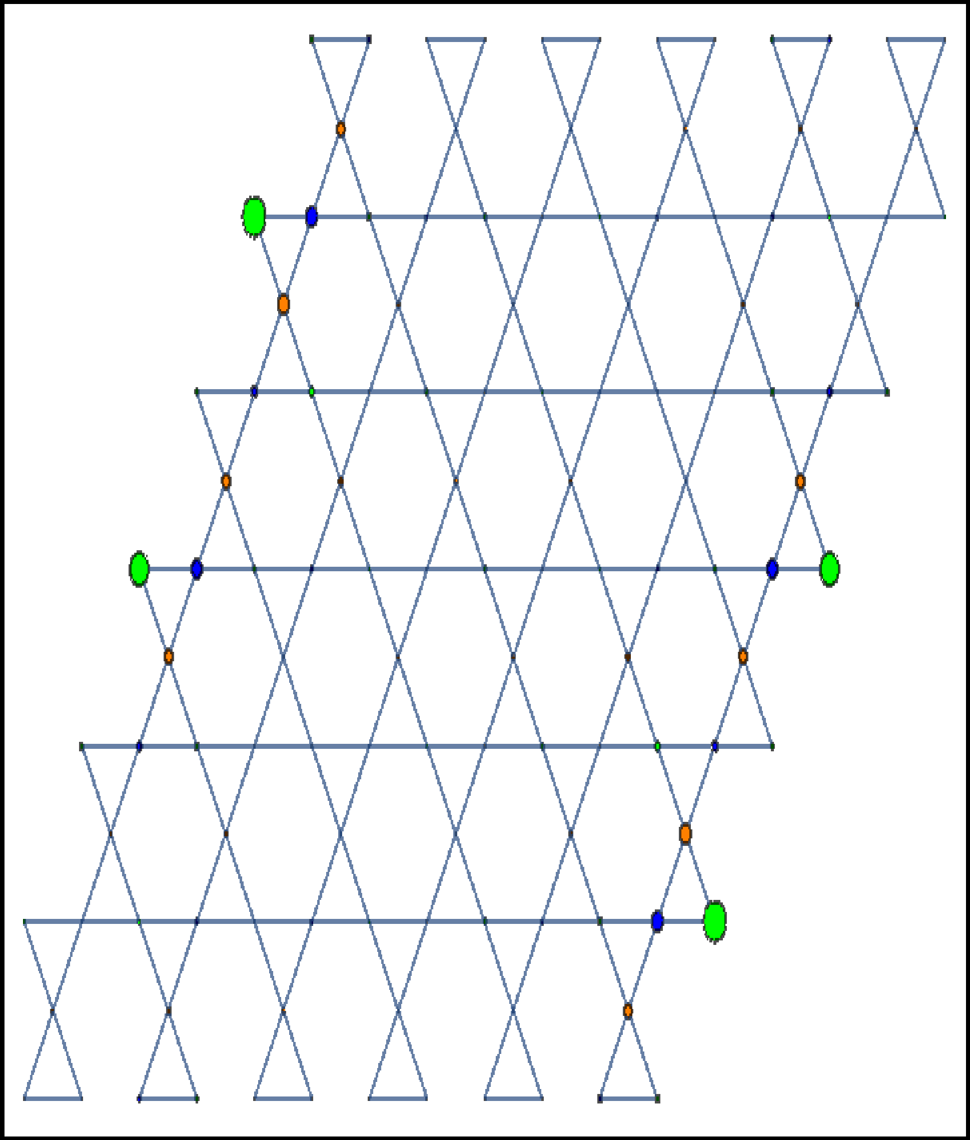
Motion of spins along the side edges

Blue Dots: Spin A

Green Dots: Spin B

Orange Dots : Spin C

Size of dots is proportional to motion of the spins



What do the eigenmodes corresponding to canonical modes look like?

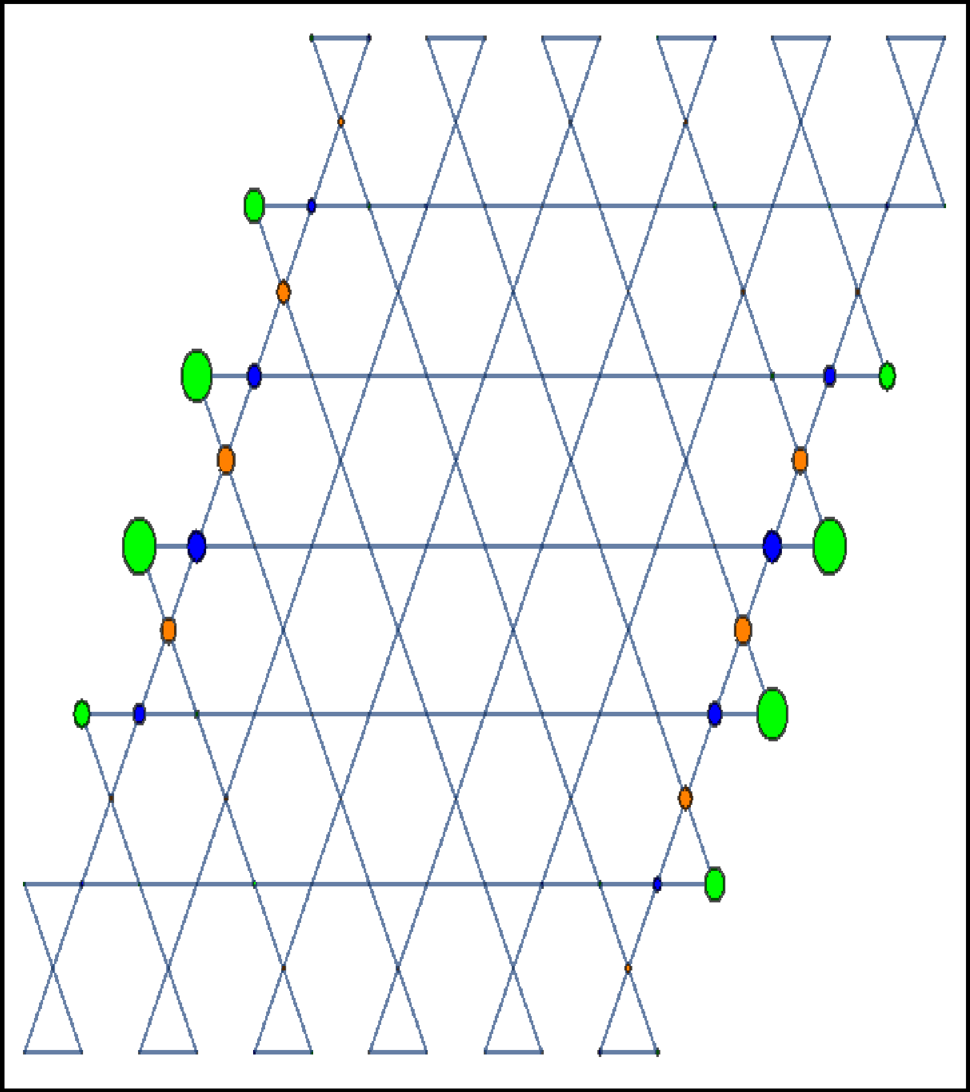
Motion along side edges – Proposed canonical edge states

Blue Dots: Spin A

Green Dots: Spin B

Orange Dots : Spin C

Size of dots is proportional to motion of the spins



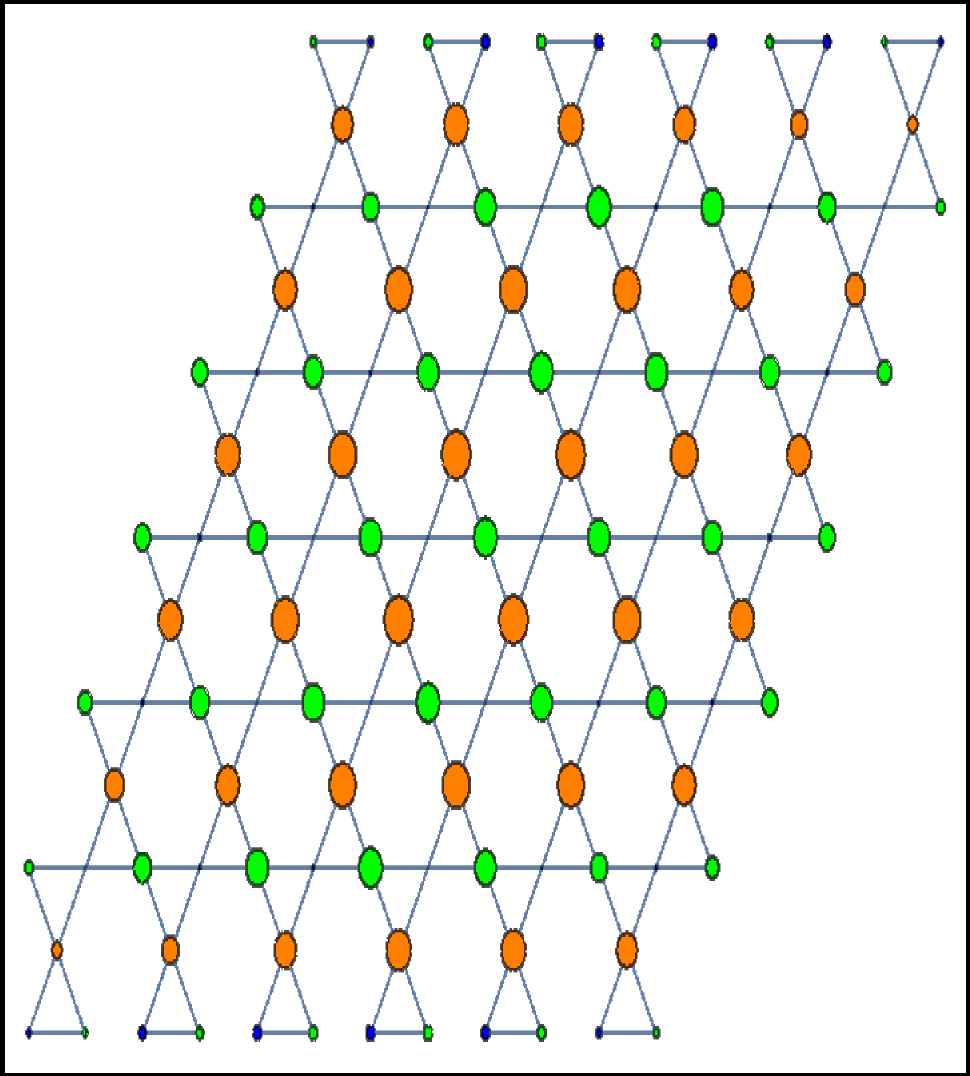
Motion of bulk - Folding

Blue Dots: Spin A

Green Dots: Spin B

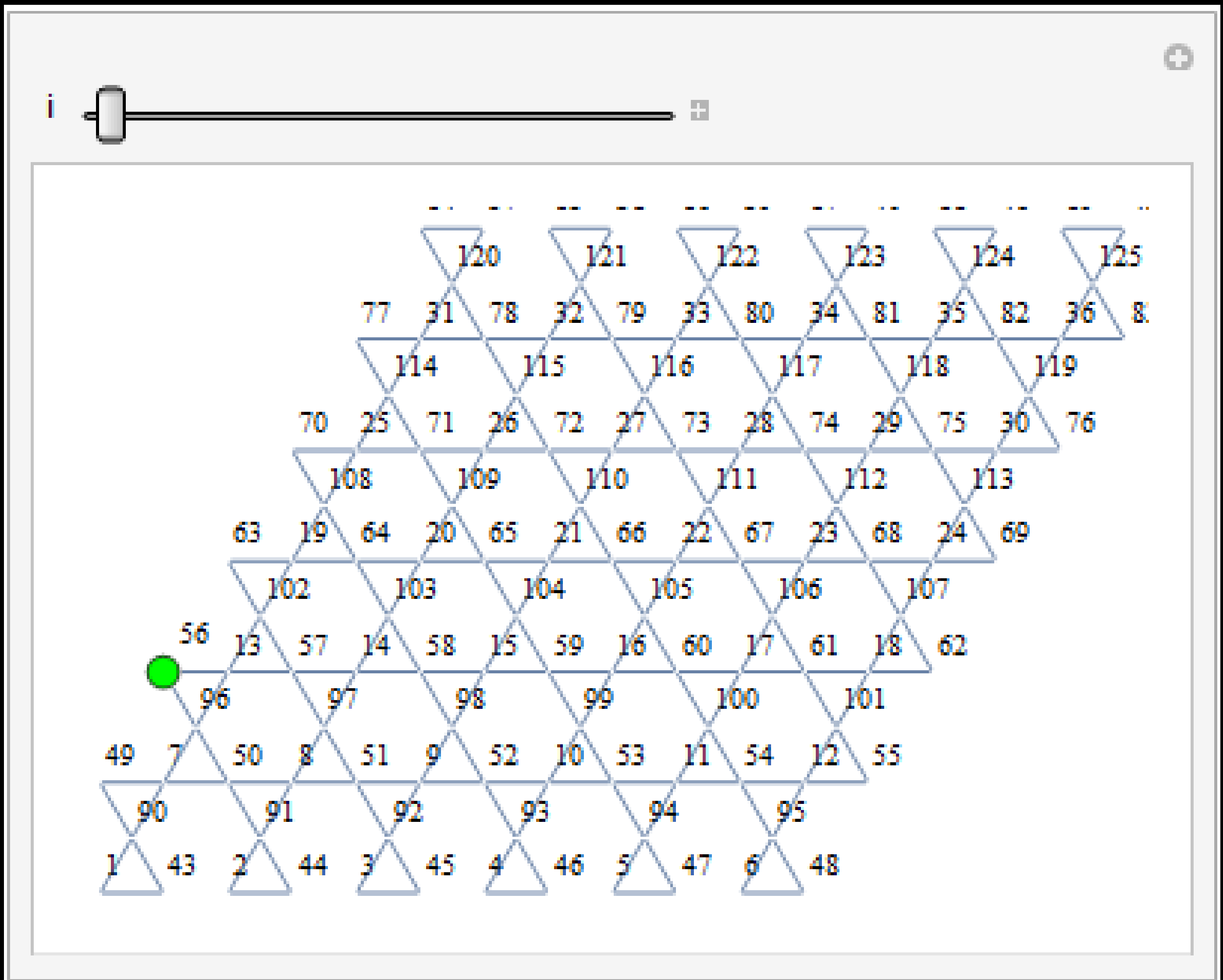
Orange Dots : Spin C

Size of dots is proportional to motion of the spins

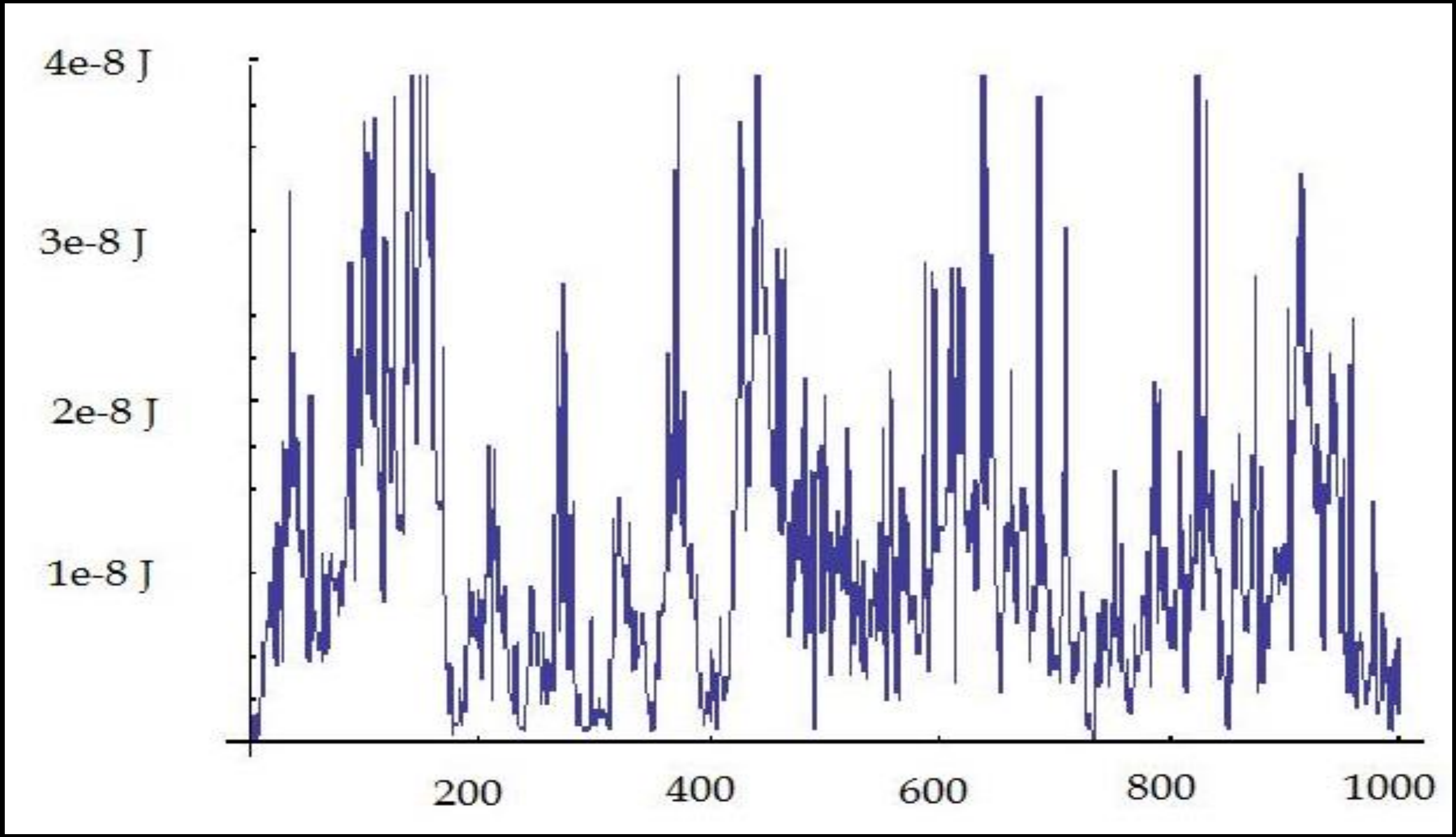


Just a global spin rotation mode!

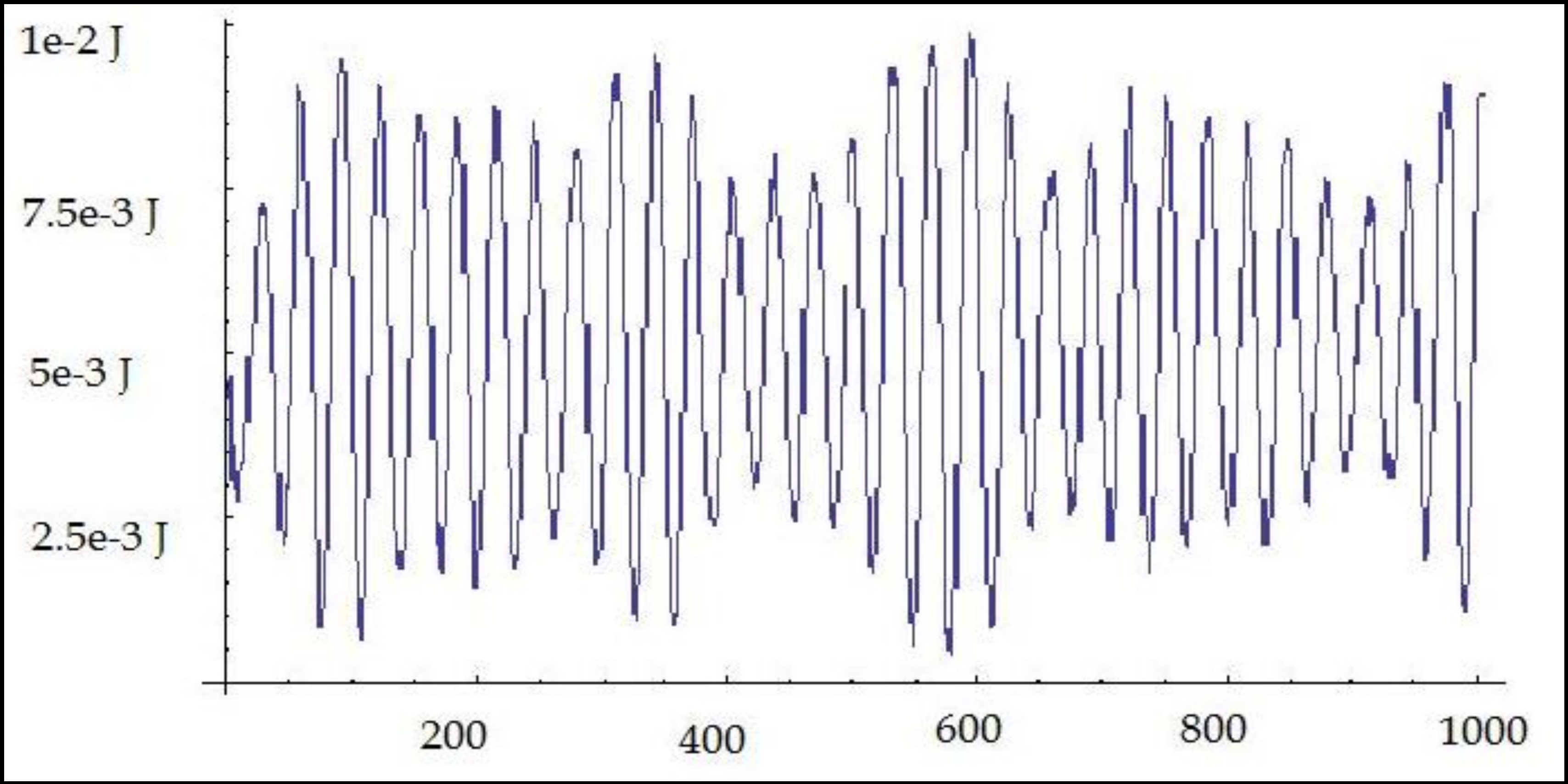
Simulation of edge excitations



Energy in the “gauge” modes

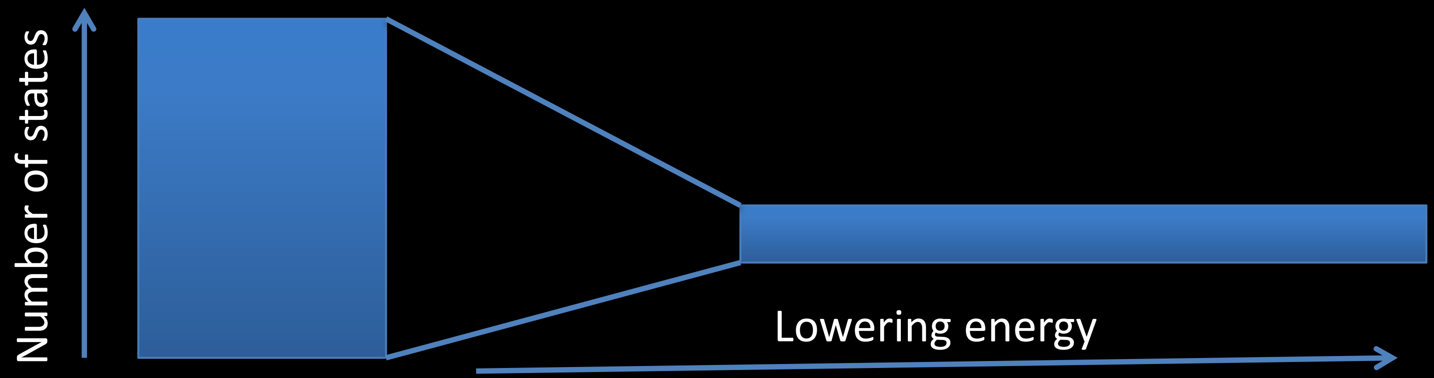


Energy in the “canonical” modes



Conclusions

- Spins constrained to classical ground states of HFMs obeys a kind of electrodynamics.



- Conjecture: frustration is important for the formation of a quantums spin liquid phase.

Strongly correlated metals

- Some strongly correlated metals are also gauge theories.
- Examples:
  - Double occupancy constraint implies

$$\hat{G}_i|phys\rangle = \hat{n}_{i\uparrow}\hat{n}_{i\downarrow}|phys\rangle = 0, [\hat{G}_i, \hat{G}_j] = 0$$

  - No nearest neighbor constraint of spinless fermions

$$\hat{G}_{ij}|phys\rangle = \hat{n}_i\hat{n}_j|phys\rangle = 0, [\hat{G}_{ij}, \hat{G}_{kl}] = 0$$