Massless and Massive Electrons: Relativistic Physics in Condensed Matter Systems

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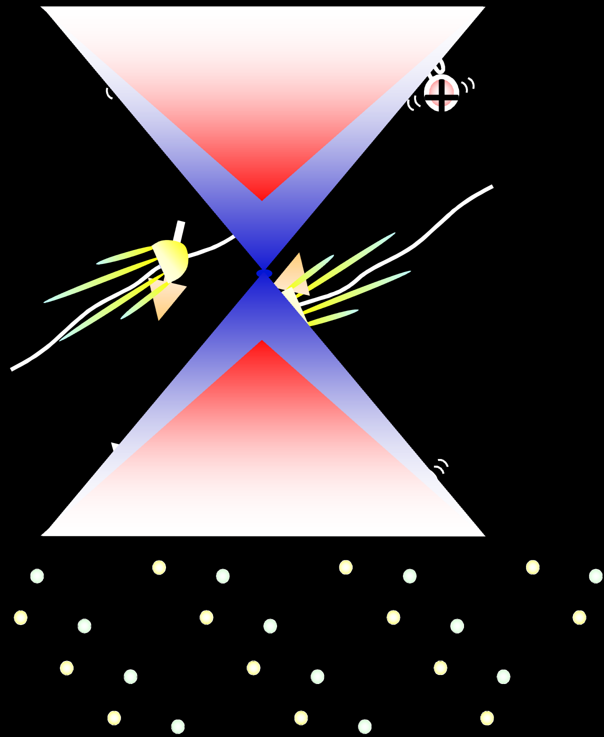
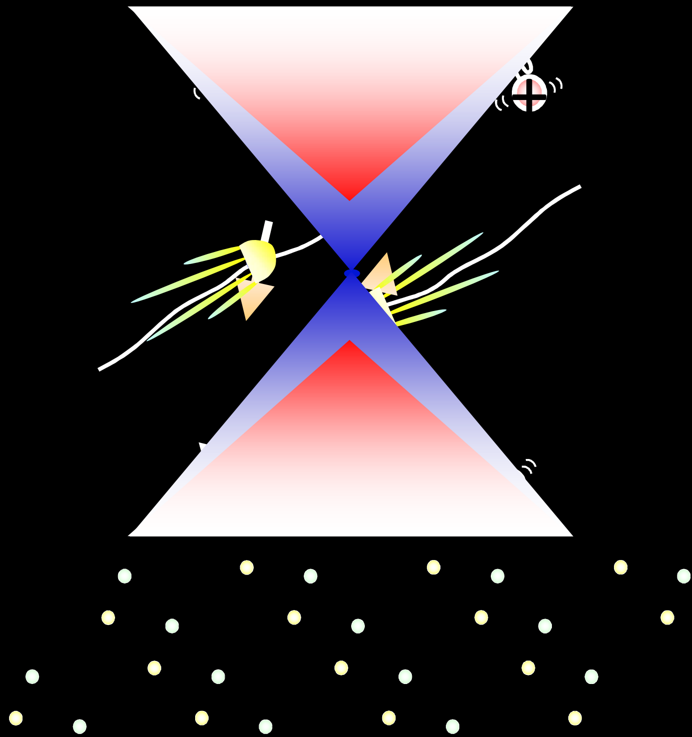


U.S. DEPARTMENT OF ENERGY

Massless Electrons in Condensed Matter Systems



Emergence
(More is different, P.A)

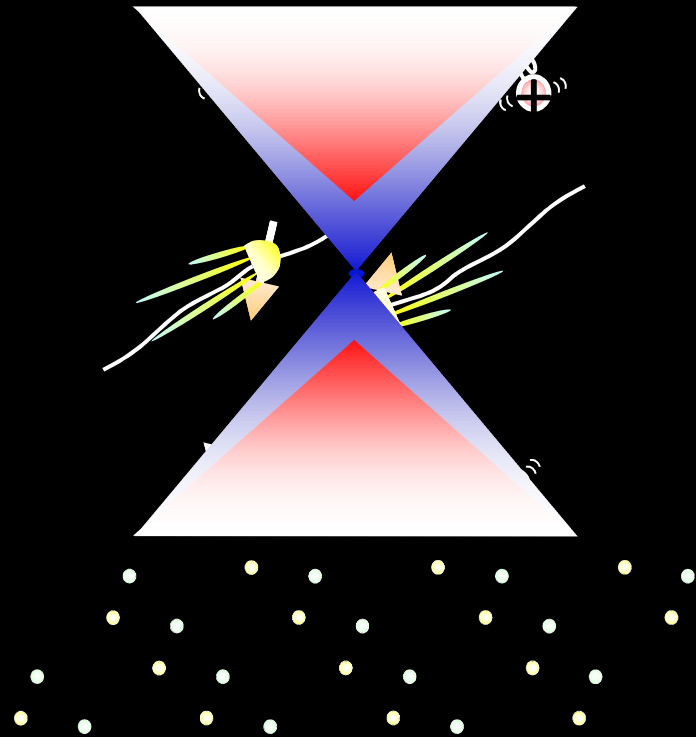
A termite "cathedral" mound produced by a termite colony

Ripple patterns in a sand dune created by wind or water

The mind may be the ultimate exemplar of emergence. The brain contains several billion neurons that perform a very simple function: relaying electrical messages across synapses to their neighbors. It's a physical action, yet out of their collective firings somehow arises a psychological phenomenon—the conscious mind.

Slime mold: For most of its life, the slime mold exists as thousands of single-celled organisms, invisible to the naked eye as they dine on decaying leaves and wood. But when weather conditions become less than ideal, those cells band together, forming a single entity—"they" become an "it." If you used time-lapse photography, you could actually see the macroorganism crawl along, slow as a starfish.

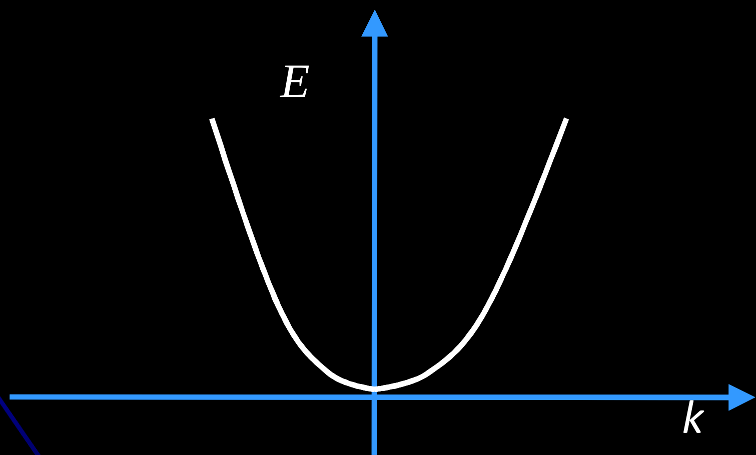
Relativistic Electrons in Condensed Matter Systems



Non-Relativistic Energy and Hamiltonian

$$E = \tfrac{1}{2} m v^2 ; \quad mv = p; \quad v = p/m$$

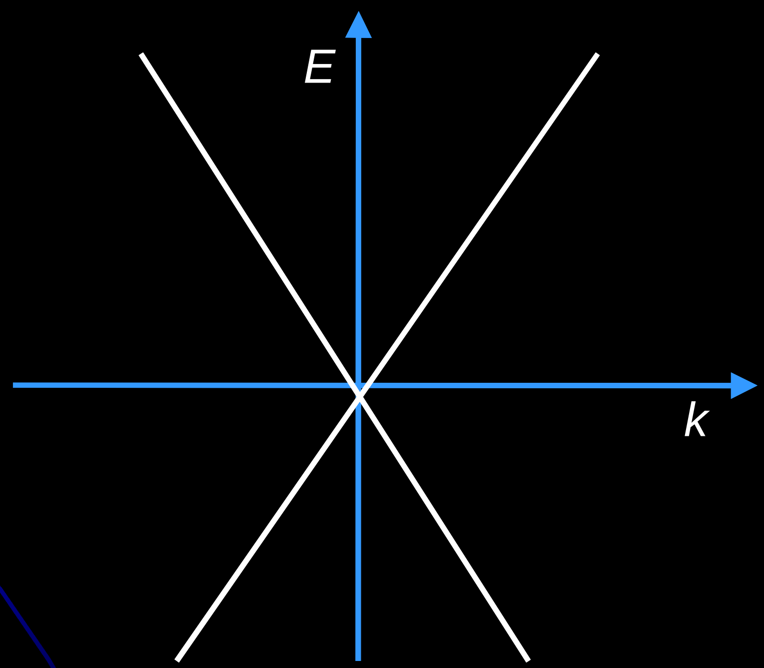
$$E = p^2 / 2m \Rightarrow \hat{H} = \frac{\hat{p}^2}{2m}; \quad p = \hbar k; \quad E = \frac{\hbar^2 k^2}{2m}$$



Relativistic Energy and `Hamiltonian'

$$E \neq p^2/2m\ ; \qquad E^2 = p^2c^2 + m^2c^4; \quad let\ m = 0$$

$$E = \pm pc \Rightarrow \hat{H} = \pm \hat{p}c; \quad E = \pm \hbar kc$$



Dirac Hamiltonian

$$\boxed{\hat{H}^2 \approx \hat{p}^2 c^2}$$

$$\boxed{\hat{H} = c\boldsymbol{\sigma}.p + mc^2\sigma_z}$$

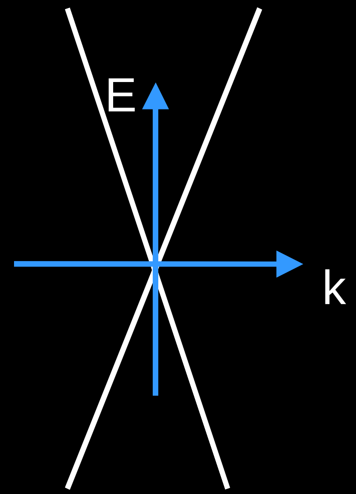
(in 2D)

Consequences of Dirac Hamiltonian

$$\hat{H} = c\boldsymbol{\sigma}.p$$

Massless Dirac Particles

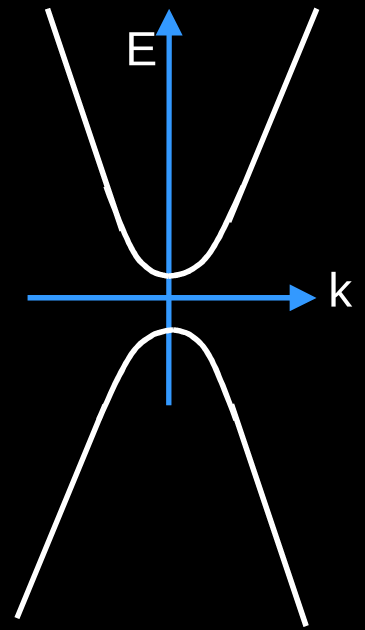
$$E(k) = \pm\sqrt{\hbar^2 c^2 k^2}$$



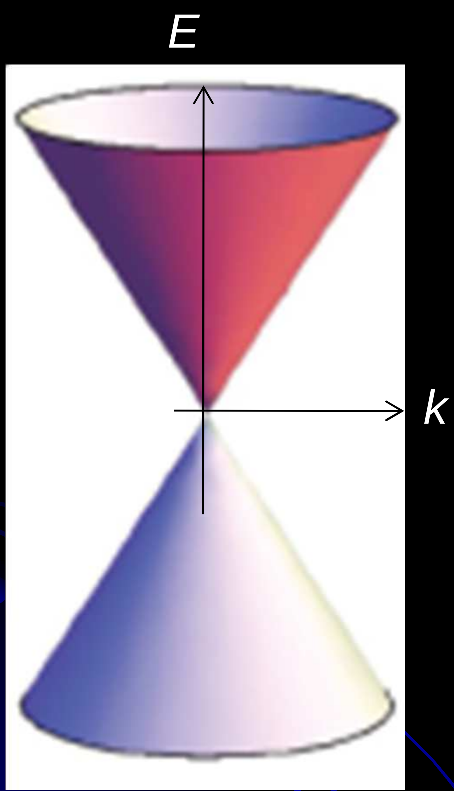
$$\hat{H} = c\boldsymbol{\sigma}.p + mc^2\sigma_z$$

Massive Dirac Particles

$$E(k) = \pm\sqrt{\hbar^2 c^2 k^2 + (mc^2)^2}$$



Dirac Electrons in 2D



$$\hat{H} = v_F \sigma . p + m v_F^2 \sigma_Z$$

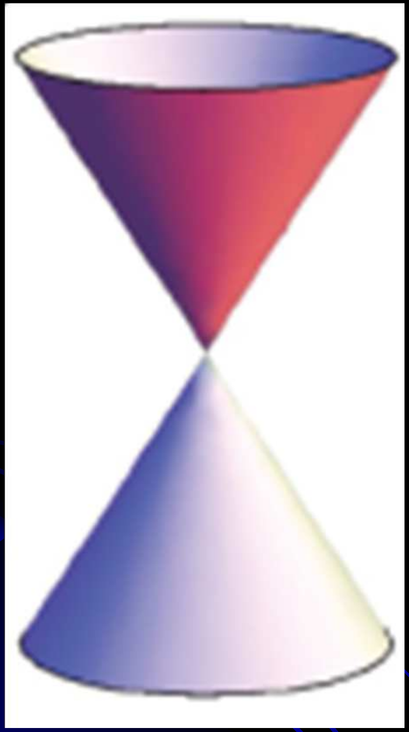
Graphene (2004)

Conventional Z_2 3D Topological Insulators (2007/2008)

Topological Crystalline Insulators (2012)

Dirac Electrons in 2D

$$\hat{H} = v_F \sigma . p + m v_F^2 \sigma_Z$$



Graphene (2004)

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Topological Crystalline Insulators (2012)

Outline

- The System: Topological Insulators
  - Unique properties of Conventional Z2 Topological Insulators
  - New Topological Material: Topological Crystalline Insulator
- The Technique: Scanning Tunneling Microscopy
  - Interference Patterns
  - Landau Level Spectroscopy
- The Experiment and Results: Breaking Mirror Symmetry to impart Mass to Massless Dirac Fermions
- Outstanding Questions and the Future

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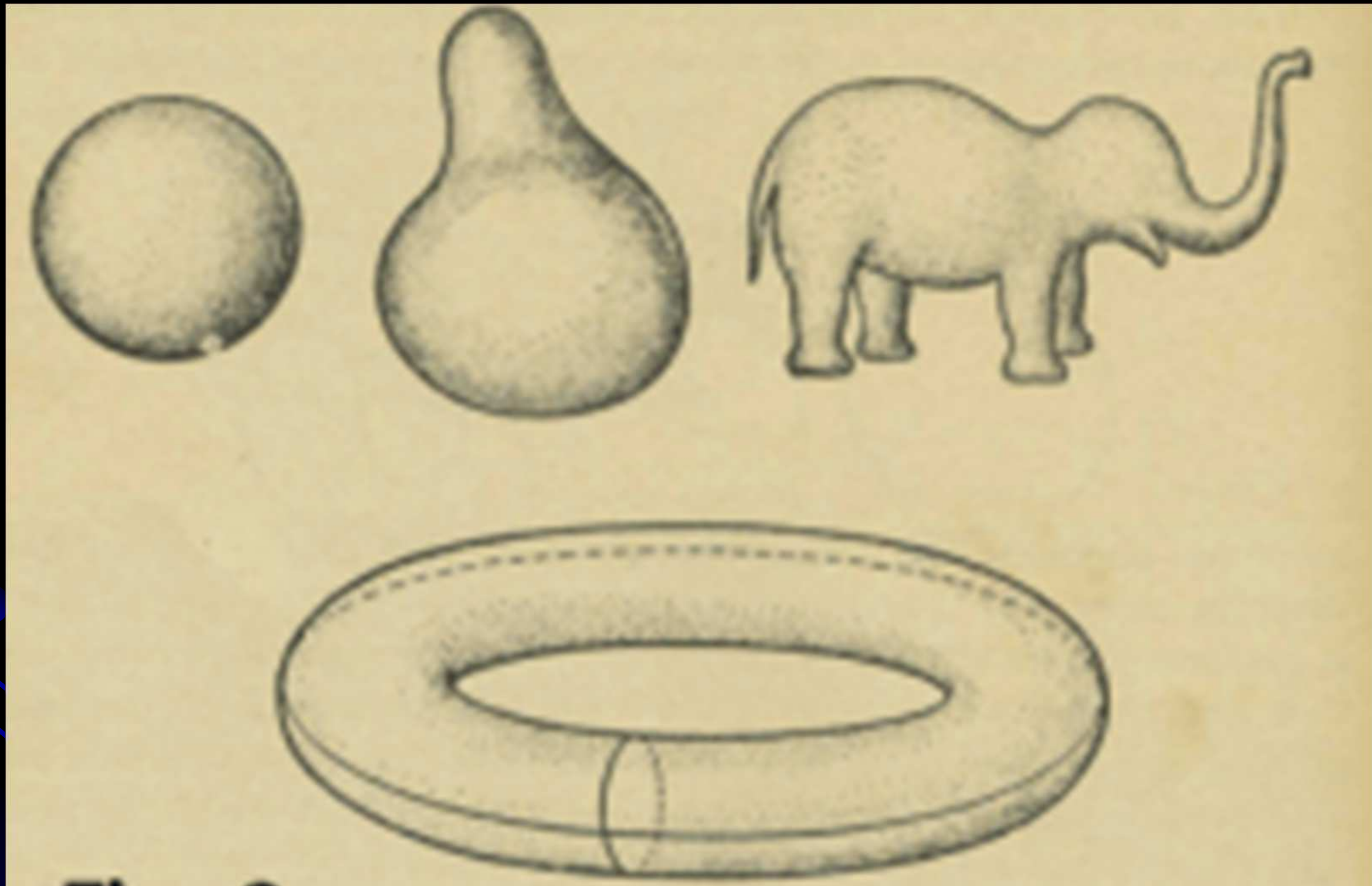
Conventional Topological Insulators

A new phase of matter that is characterized by a <u>topological invariant</u> associated with the bands rather than a broken symmetry

Example of Topological Invariant: Quantized conductance in integer quantum Hall effect

Spot the ~~difference~~ similarity



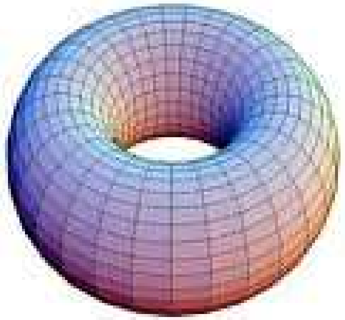
Topological Invariant for geometric shapes

Gaussian curvature   Geodesic curvature

$$\chi = \frac{1}{2\pi}\left[\int_M K\,dA + \int_{\partial M} k_g\,ds\right]$$

χ=2

χ=0

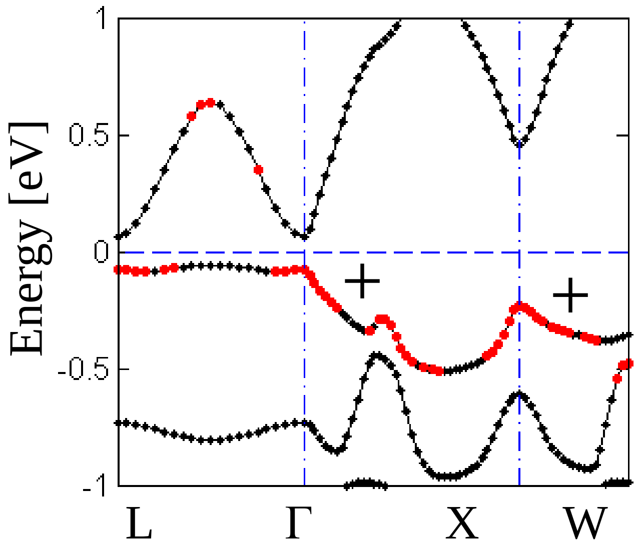


χ=-2

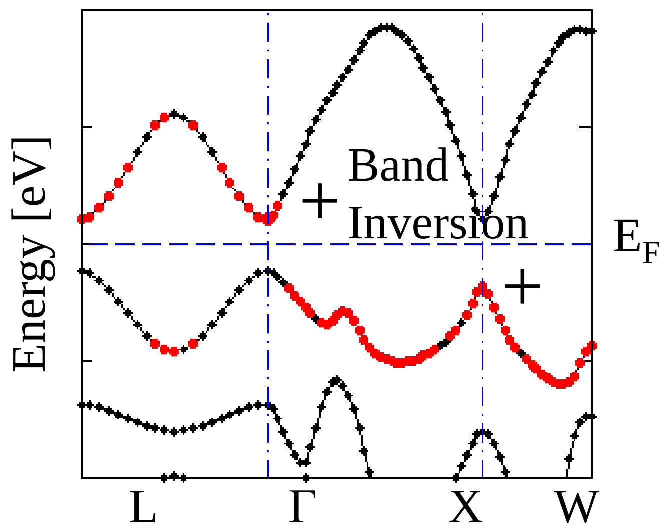
Courtesy Hsin Lin

Z_2 Topological Invariant

NORMAL INSULATOR



INVERTED BAND INSULATOR



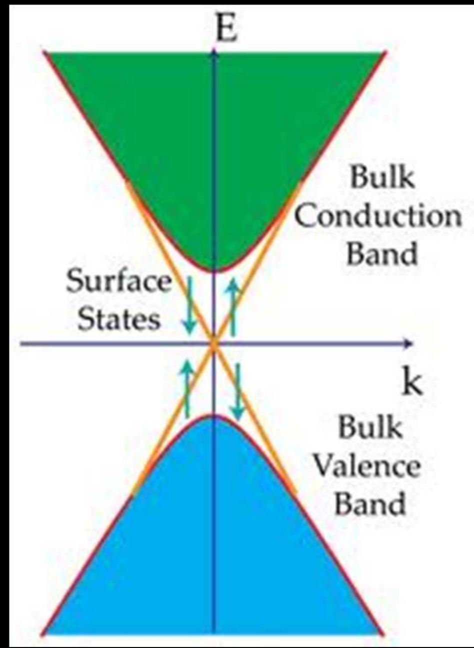
Z_2= Even → Z_2=Odd

Berry connection   Berry curvature

$$\mathbb{Z}_2 = \frac{1}{2\pi}\left[\oint_{\partial\mathcal{B}^+} d\boldsymbol{k}\cdot\boldsymbol{A}(\boldsymbol{k}) - \int_{\mathcal{B}^+} d^2k\,\mathcal{F}(\boldsymbol{k})\right] \bmod 2$$

From Hsin Lin

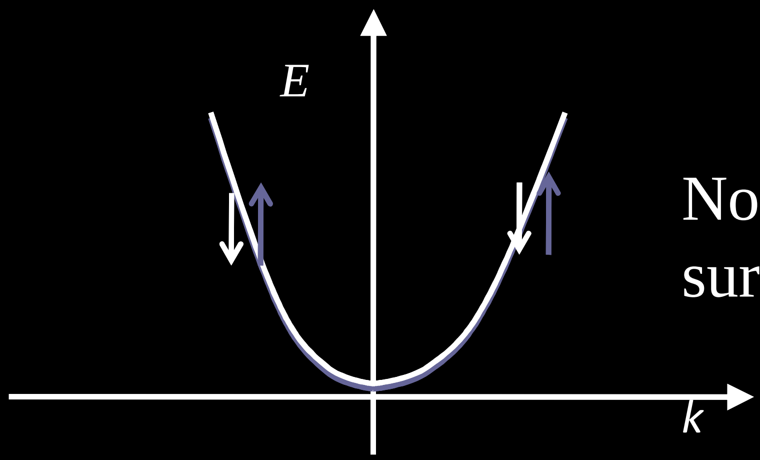
3D Topological Insulator



Odd Z2 topological Invariant dictates the existence of a Surface State with unique properties

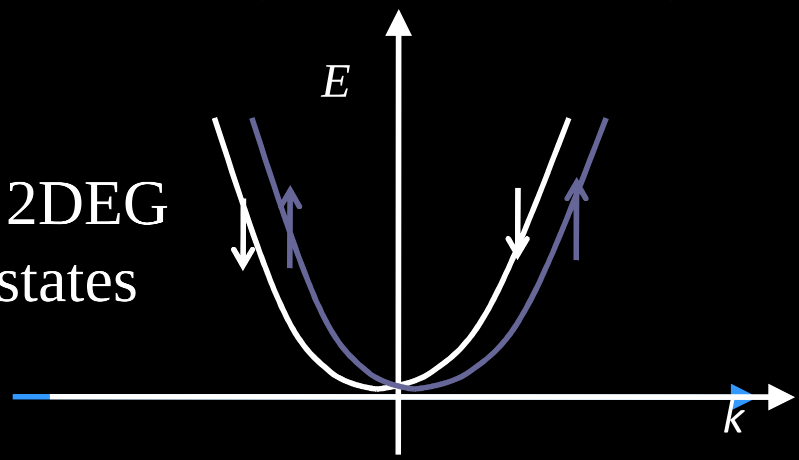
2D Surface States

Conventional surface states

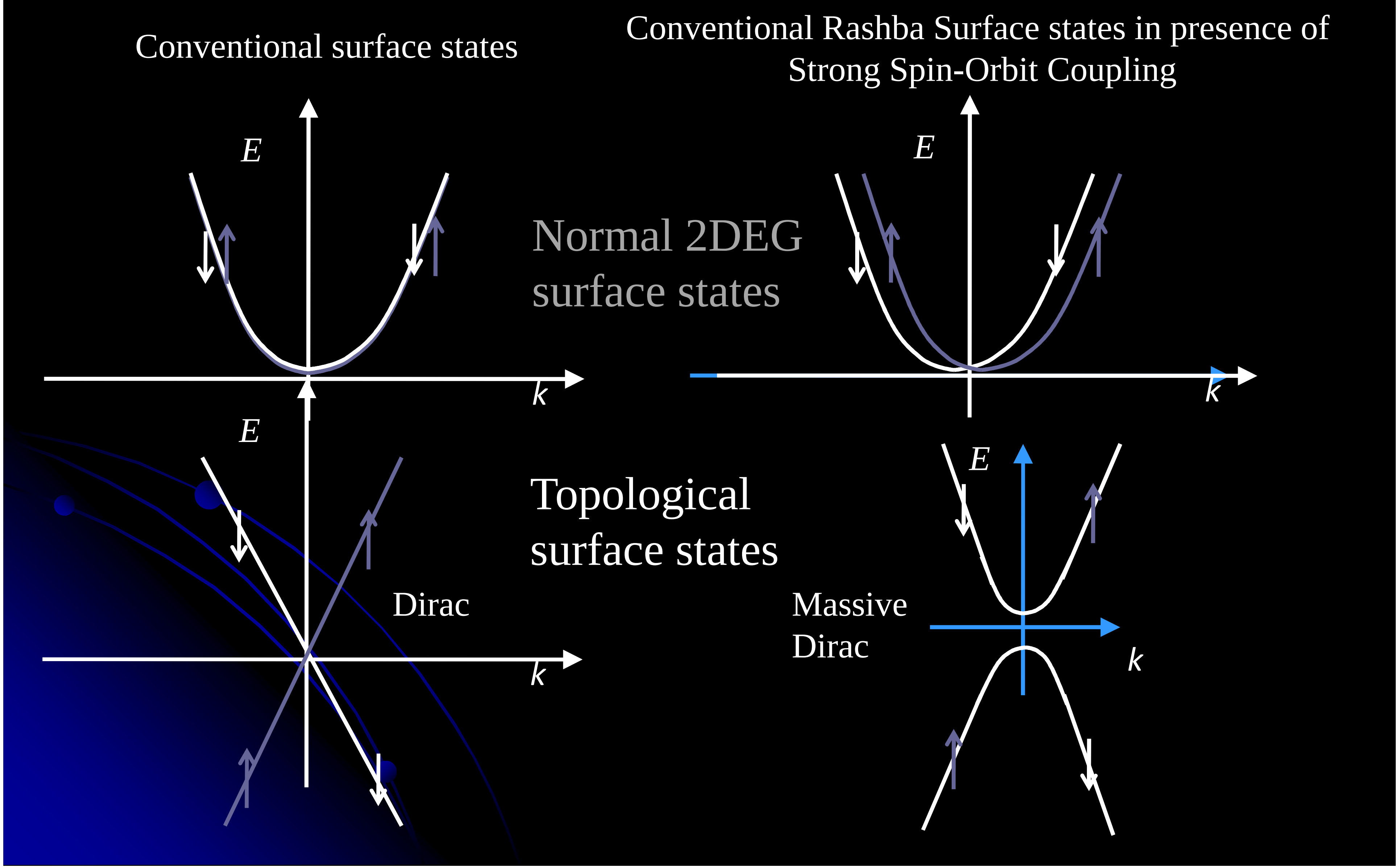


Normal 2DEG surface states

Conventional Rashba Surface states in presence of Strong Spin-Orbit Coupling

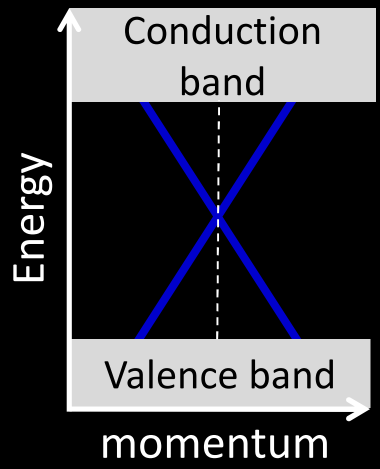


2D Surface States

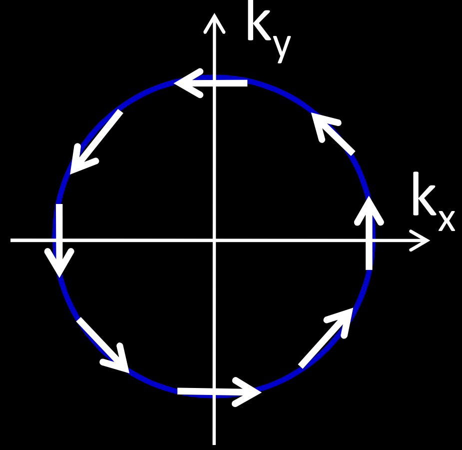


Topological surface states

(I) Linear band dispersion



(III) Chiral spin texture

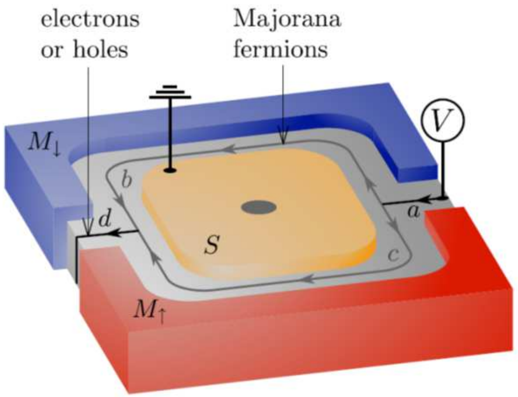


(II) Dirac node protected by:

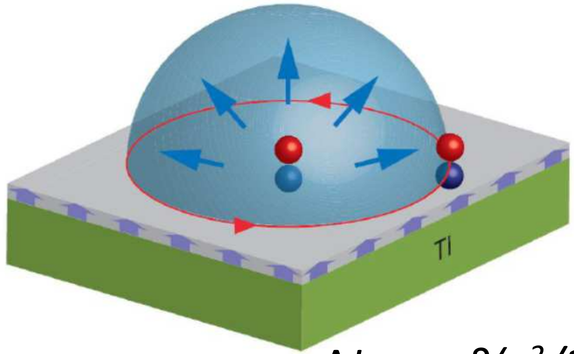
- time-reversal symmetry -- Z_2 topological insulators ($Bi_2Se(Te)_3$, $TlBi(S_{1-x}Se_x)_2$, ...)
- crystalline symmetries -- Topological crystalline insulators ($Pb_{1-x}Sn_xSe(Te)$)

3D Topological insulators : Playground for new physics

Majorana fermions
Topological quantum computing

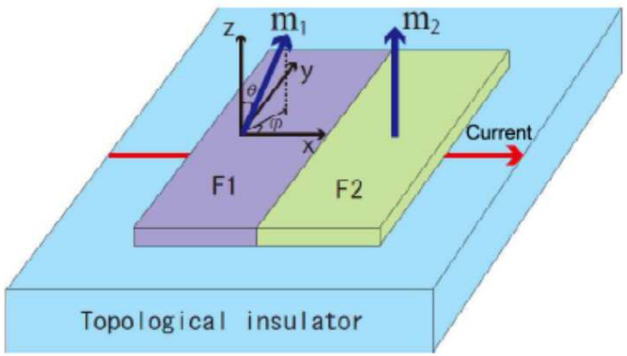


Axion electrodynamics

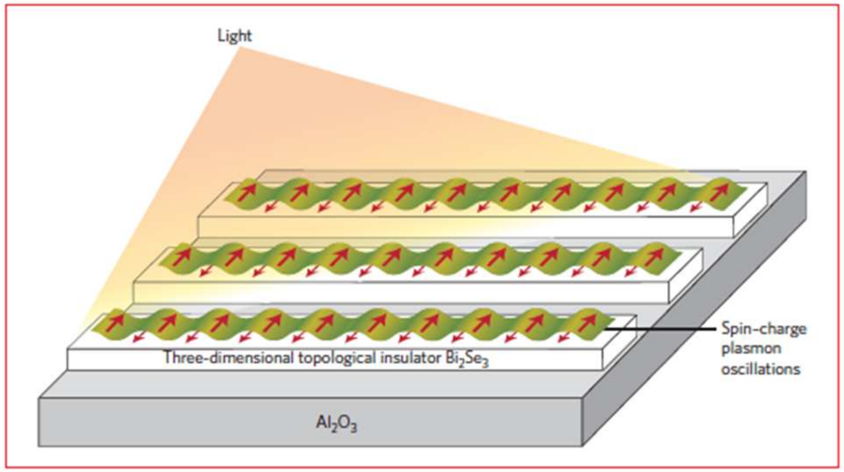


$$\Delta L_{axion} = \vartheta(e^2/2\pi hc)\mathbf{B}\cdot\mathbf{E}$$

Low power spintronics



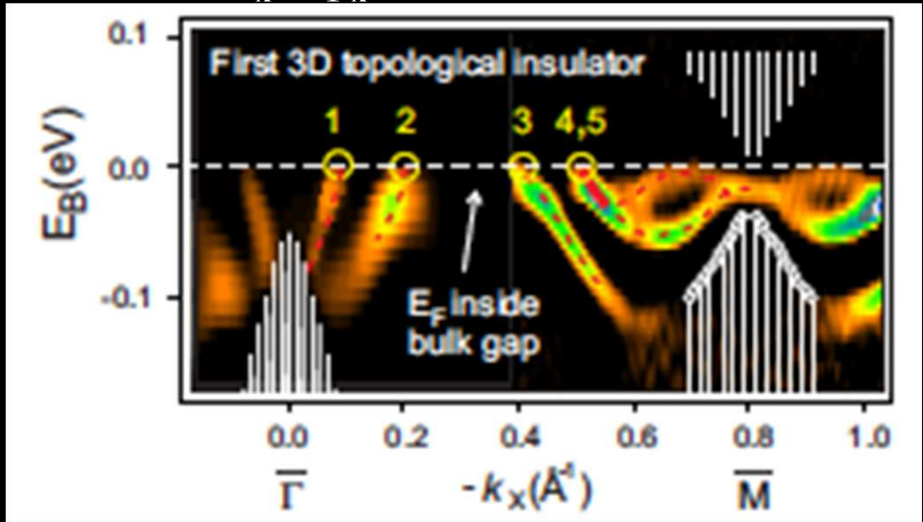
Spin Plasmons



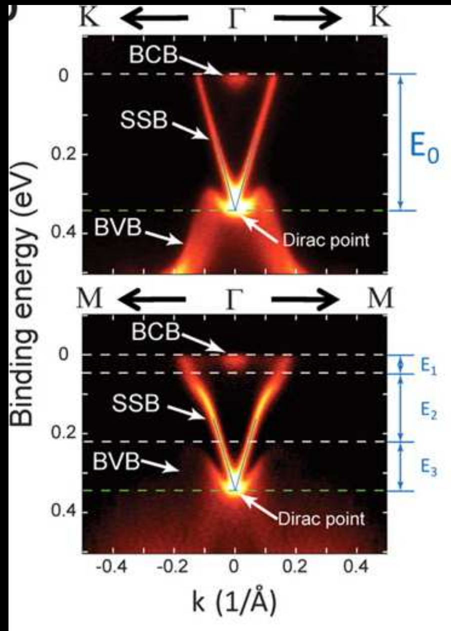
Courtesy David Hsieh

3D Topologically Non-Trivial Families

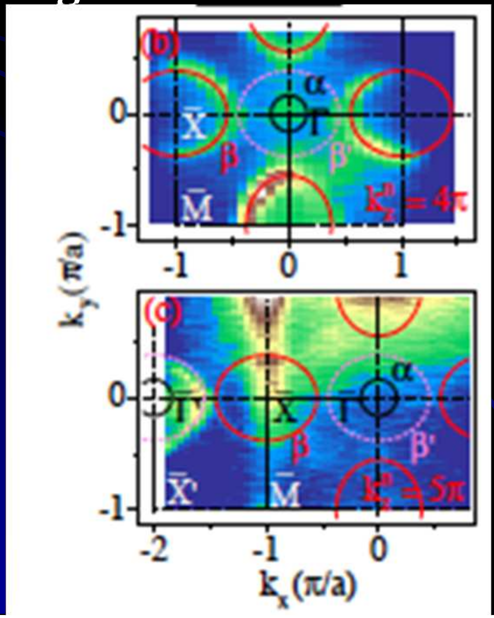
2008: Bi_xSb_{1-x}



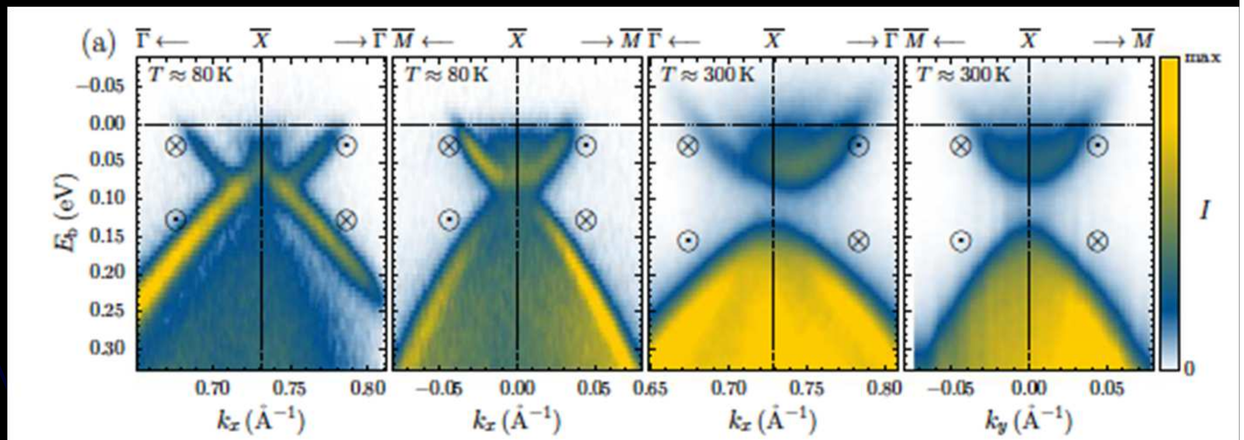
Prototypical TI
Bi_2Se_3/Bi_2Se_3



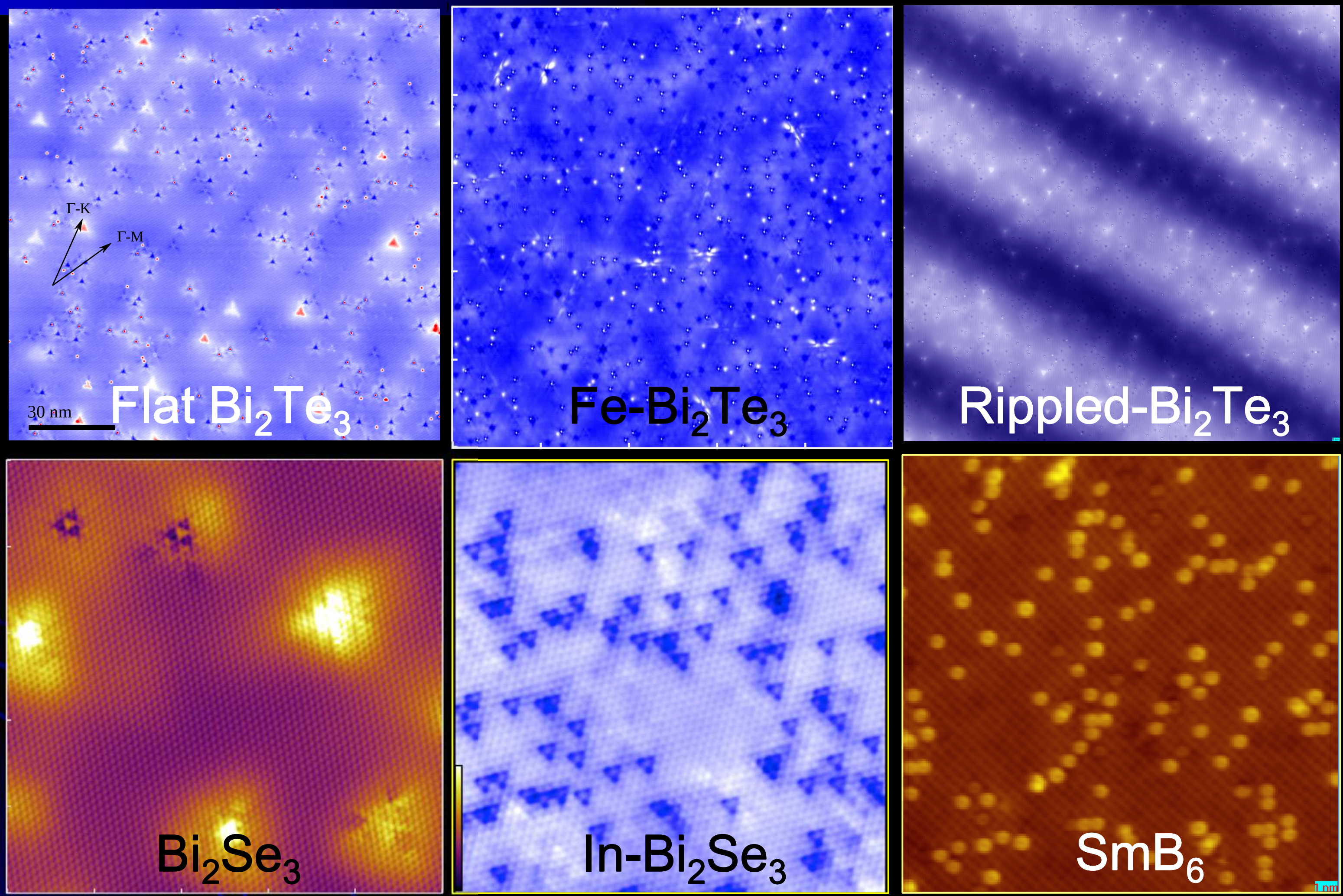
Topological Kondo insulator candidate SmB_6



Topological Crystalline Insulator: Pb/SnSe



Z_2 3D-TI materials probed at Boston College



Z_2 TI

Y. Okada *et al*. Phys. Rev. Lett. (2011)
Y. Okada *et al*. Phys. Rev. Lett. (2012)
Y. Okada *et al*. Nature Commn. (2013)

STM is also very powerful for TCIs

Y. Okada *et al*. Science (2013)
Ilija Zeljkovic and Y. Okada *et al*. (archive)

Essential Ingredients for Topological Material with Dirac Surface States

- Band Inversion
- Insulating Gap
- Symmetry protecting the Dirac Node (Time-Reversal/Mirror/Lattice or crystalline symmetry, etc)
- Spin-Orbit Coupling
  $H_{SO} = \lambda\, L \cdot S$

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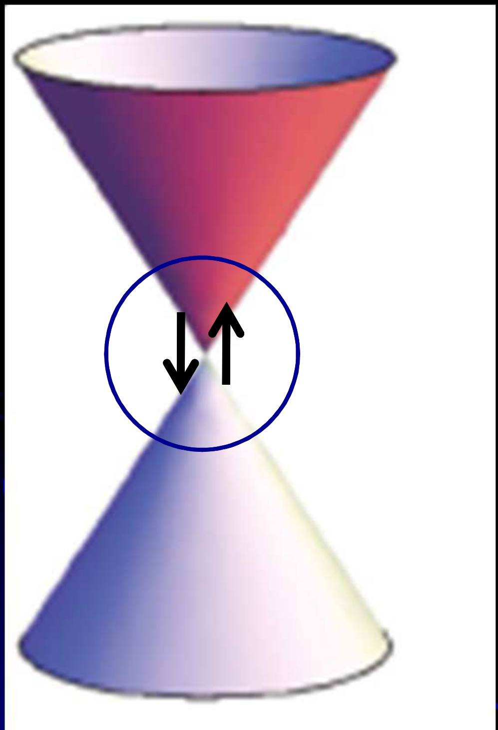
Role of Symmetry protecting Dirac Point

Two fold degeneracy at Dirac point is protected by a Symmetry



(Eigenstates of the Symmetry Group)

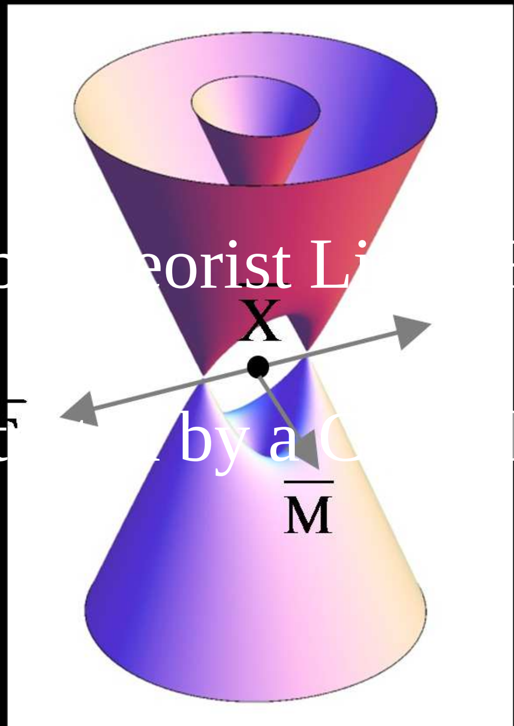
As long as, perturbations don’t break the global symmetry, degeneracy cannot be lifted.

Conventional Topological Insulators: Dirac node protected by Time-Reversal symmetry.

Topological Crystalline Insulators: Degeneracy is protected by Crystalline Symmetries

New Dirac Material: Topological Crystalline Insulators

Predicted b eorist Li Fu in 2011



Dirac point prot by a C lline Symmetry

Liang Fu, "Topological Crystalline Insulators", Phys. Rev. Lett. 106, 106802 (2011)

In 2012: ARPES studies suggested Pb/Sn Se and Pb/Sn Te classes are potentially TCI

$Pb_{1-x}Sn_xSe$: Promising TCI class of materials

Transition from a TCI to a trivial phase may form the basis of a thermal 'switch'

High mobility (30,000 $cm^2 V^{-1} s^{-1}$ for Pb0.77Sn0.23Se at liquid-helium temperatures)

Extraordinary tunability by alloying and temperature: ferromagnetism, superconductivity, ferroelectricity ...

Potential device applications: tunable electronics, spintronics and spin-plasmonics

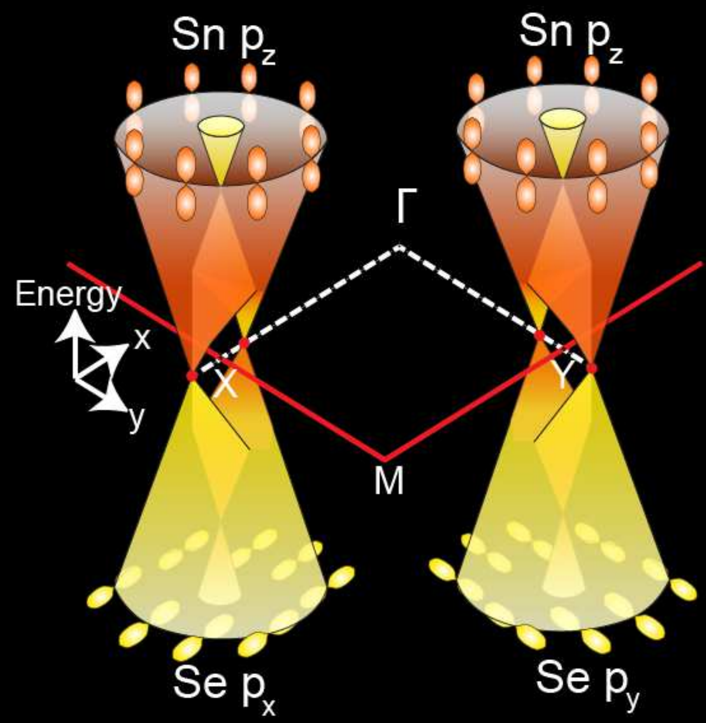
System: Topological crystalline insulators

Distinct from other topological materials:

- Multiple Dirac cones within the BZ

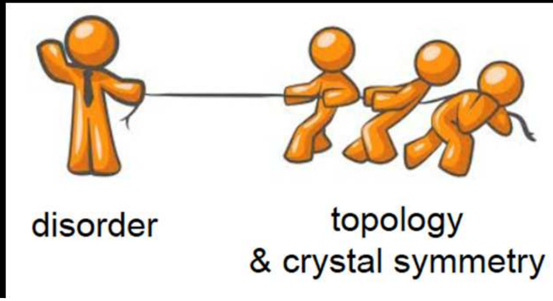
- Theoretical prediction: Vastly different orbital character below and above the Dirac point

- Theoretical prediction: Surface states topologically protected by mirror symmetry



System: Topological crystalline insulators

- Band Inversion known
- Is the dispersion linear near the `Dirac point’? (Do we have massless Dirac electrons?)
- Are the surface states topologically protected by mirror symmetry? (Can you open up a gap by breaking mirror symmetry?)
- Disorder necessarily violates crystal symmetry (Do we still have Massless Fermions?)
- Are TCI surface states robust against strong disorder?



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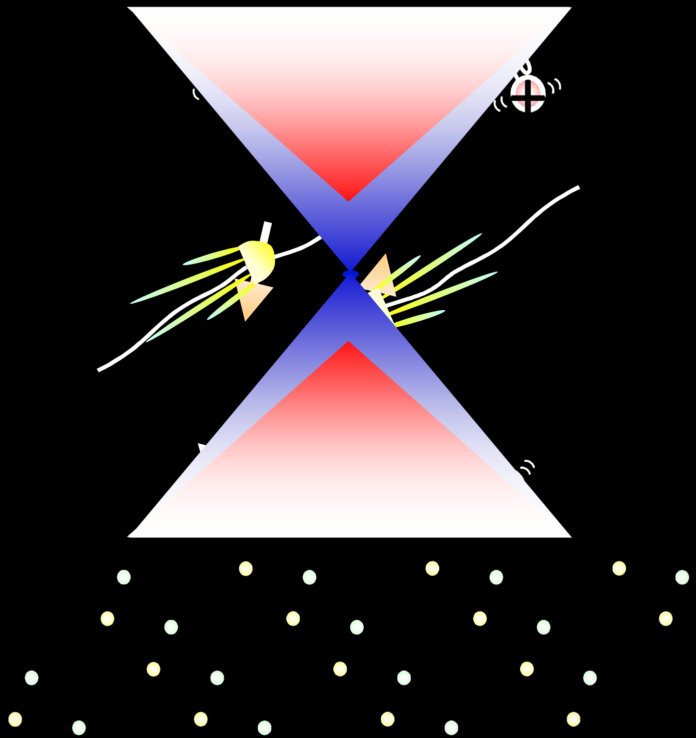
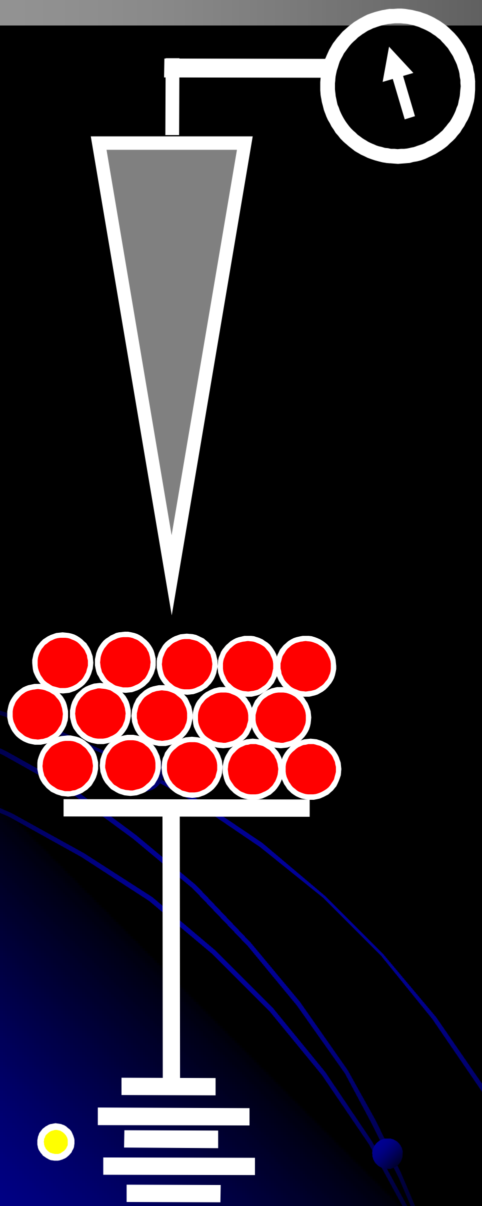
STM studies of Dirac Materials: Topological Crystalline Insulators



- Quasiparticle interference-Fourier transform spectroscopy
- Landau level Spectroscopy

Scanning Tunneling Microscopy (STM)

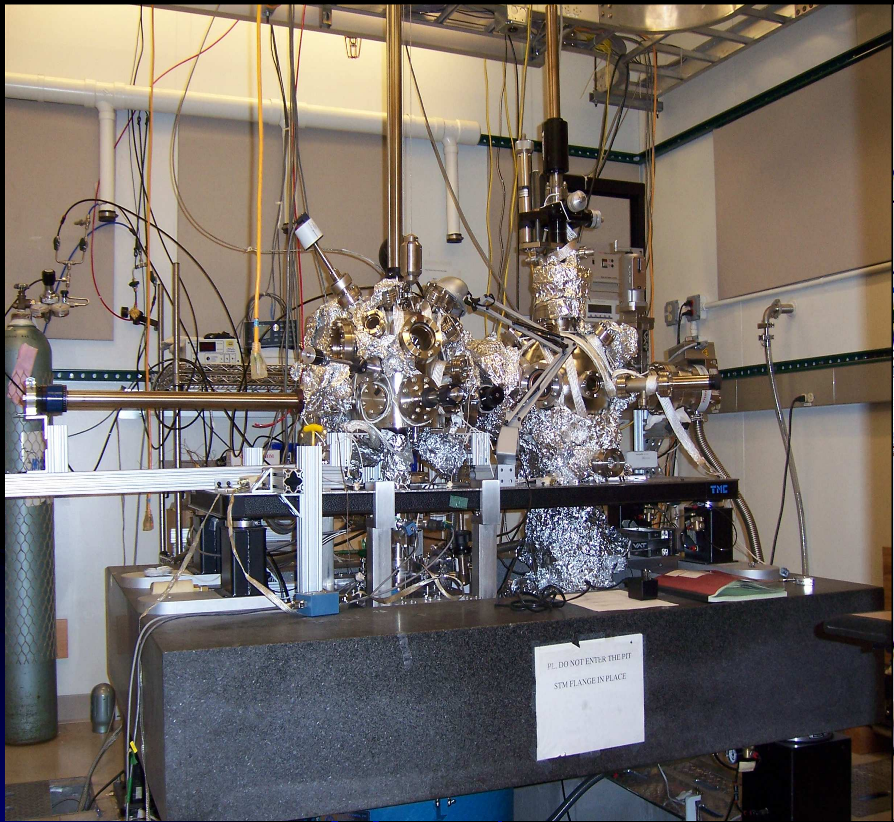


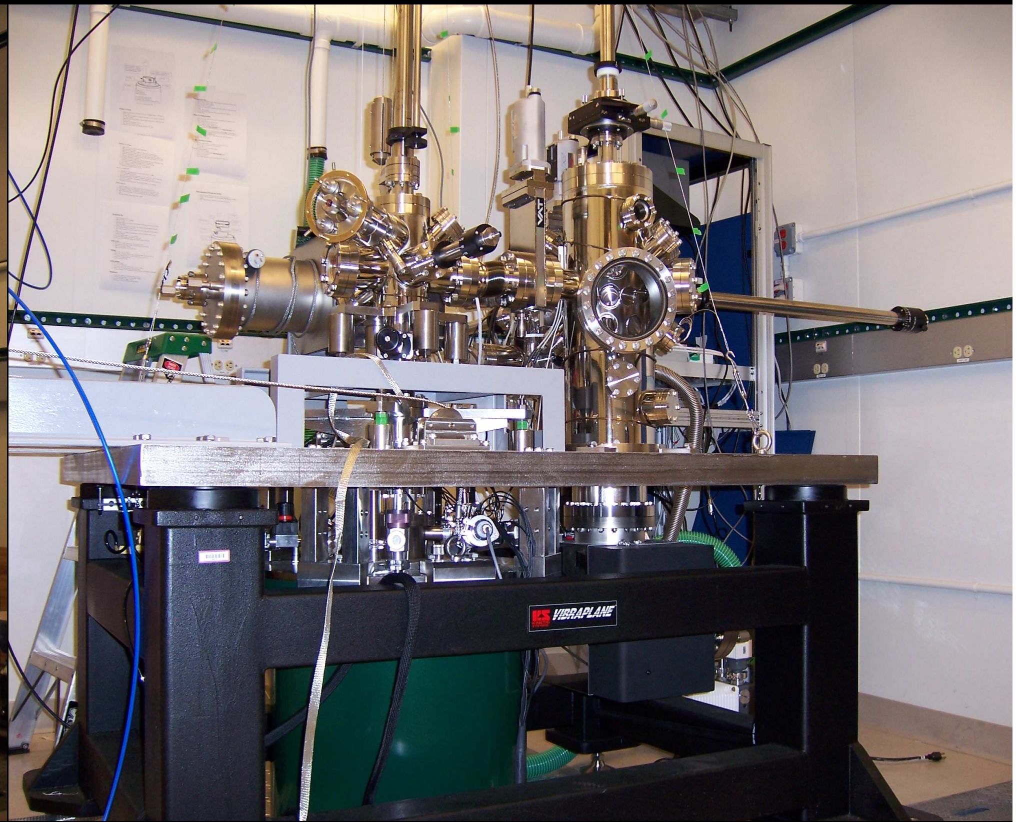
The STM measures the number of electrons that tunnel across the Vacuum barrier (tunnel current) at a fixed real space position

Imaging Atoms

Precision Spectroscopy

STMs at Boston College

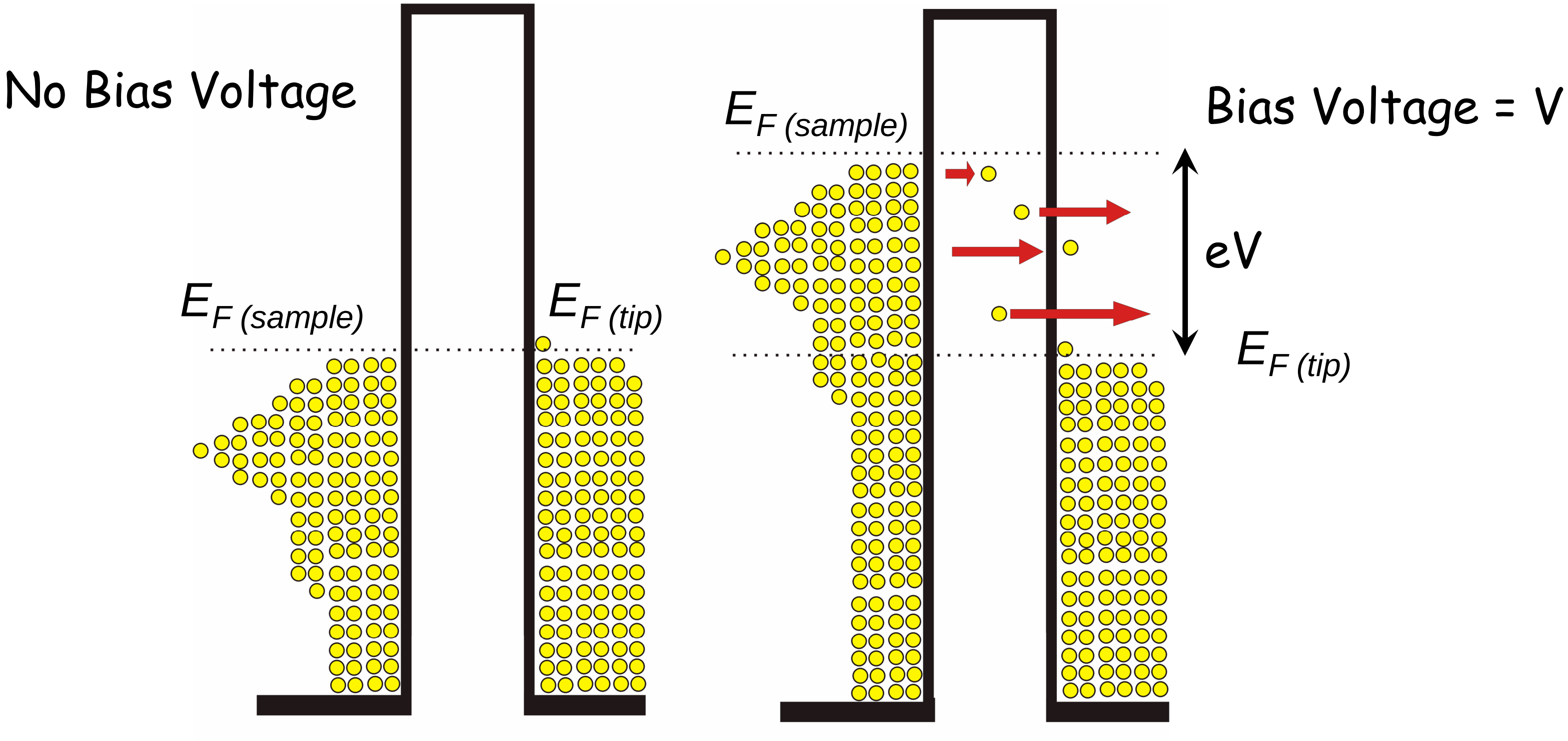




Homebuilt STM

Custom Unisoku STM

STM Technique: Imaging

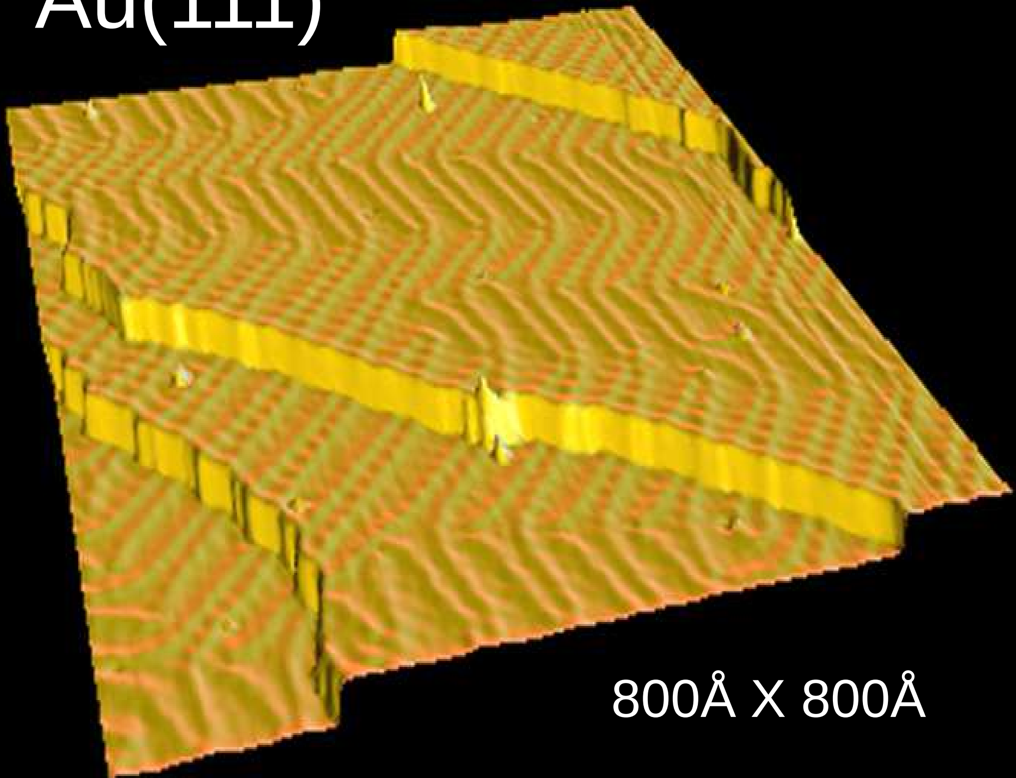


$$I(eV_b) = \int_0^{eV_b} \sum_n \left|\Psi_n(r)\right|^2 \delta(eV - E_n)\, dV$$

$$I(eV_b) \approx \int_0^{eV_b} LDOS(E)dE \sim exp\ (-2kd)$$

I ~ exp (-2kd) ⇒ Measure *I(x, y)* to obtain image

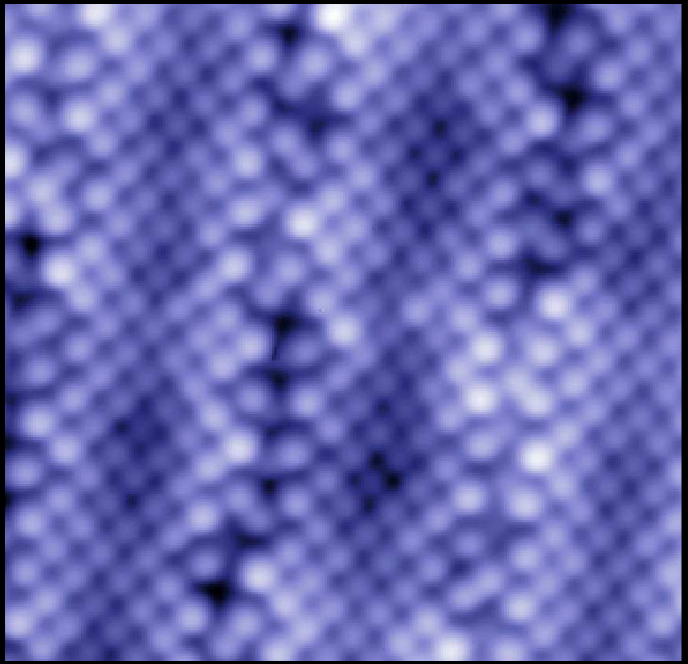
Au(111)



800Å X 800Å

W. Chen, V. Madhavan, T. Jamneala, M.F. Crommie
Phys. Rev. Lett. 80, 1469 (1998)

$Bi_2Sr_2CaCu_2O_{8+d}$



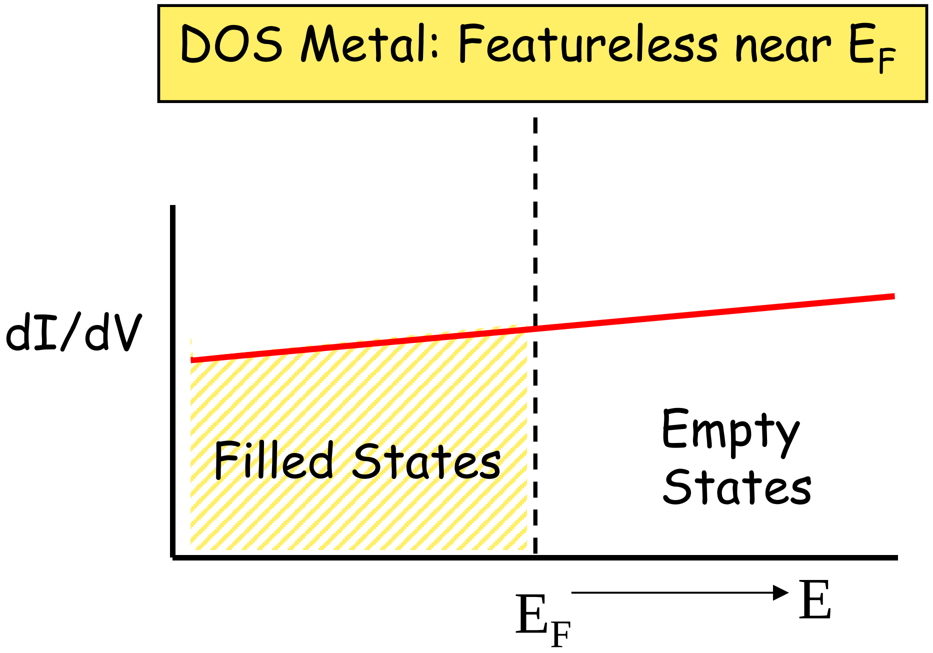
64Å X 64Å

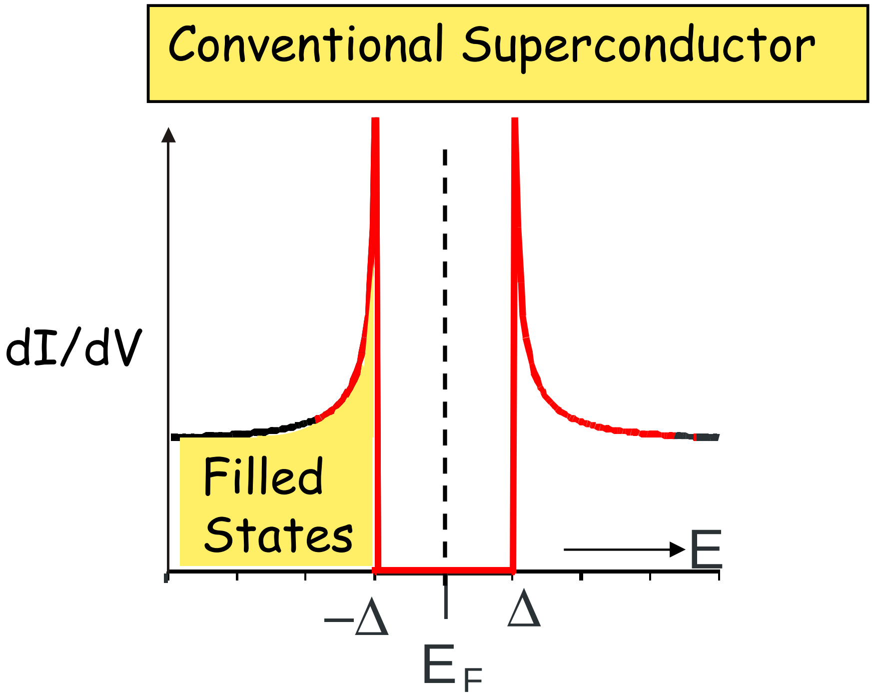
STM Technique: Spectroscopy

$$I(eV_b) = \int_0^{eV_b} \sum_n \left|\Psi_n(r)\right|^2 \delta(eV - E_n)\,dV \quad \Rightarrow \quad I(eV_b) = \int_0^{eV_b} LDOS\,dV$$

STM Spectroscopy

Measure *dI/dV* to obtain density of states





Quasiparticle interference-Fourier transform spectroscopy

Band structure, gaps, resonances, phonons, spin etc

$dI/dV\,(r, eV) \;\square\; \Psi^2{}_S\,(r, eV)\;\; DOS_S\,(r, eV)$

Interference patterns , dispersion, charge ordering

Using Interference Patterns to obtain momentum space (k-space) information

Surface state electrons

Free electrons

$$E_K = \frac{\hbar^2 k_E{}^2}{2m} \qquad \psi(x,E) = Ae^{ik_E x}$$

$$\psi^2(x,E) = Ae^{ik_E x}A^* e^{-ik_E x} = A^2$$

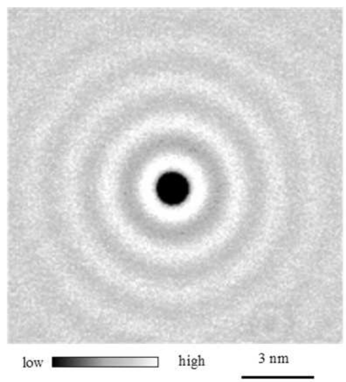
$$\boxed{\psi^2(x,E) = const}$$

Scattered states

$$\psi = \psi_i(x,E) + \psi_r(x,E) = A(e^{ik_E x} + e^{-ik_E x})$$

$$\psi^2(x,E) = A(e^{ik_E x} + e^{-ik_E x})A^*(e^{ik_E x} + e^{-ik_E x}) = 2A^2 \cos 2k_E x$$

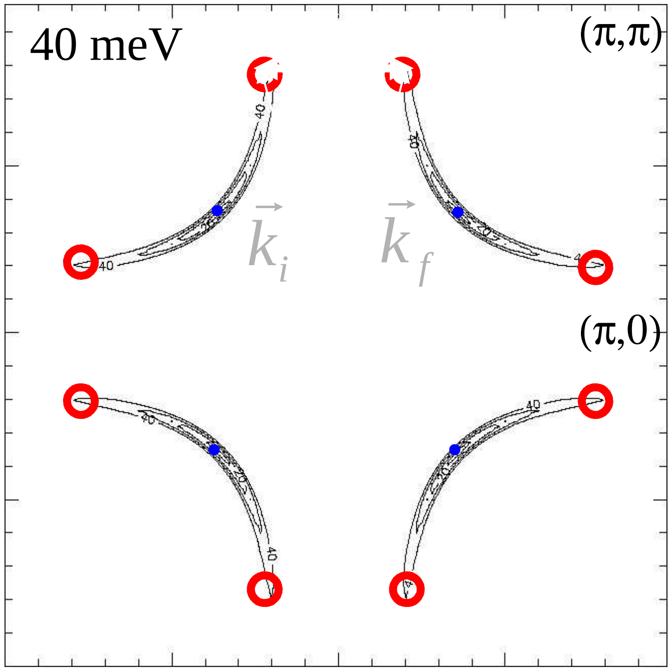
$$\boxed{\psi^2(x,E) \propto \cos 2k_E x = waves!}$$



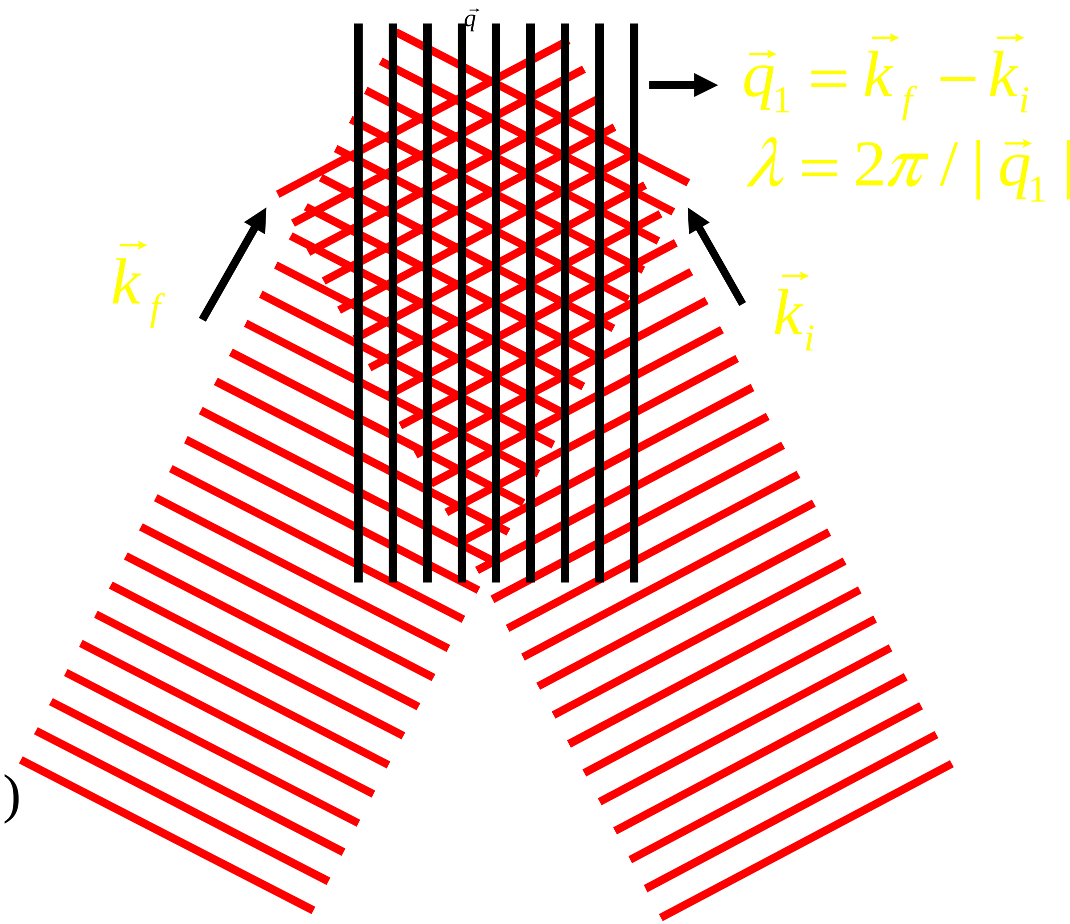
In general: $\psi^2(x,E) \propto \cos(\vec{k}_i - \vec{k}_f)x$

Scattering between $\vec{k} - space$ states (allowed states in momentum space) produces LDOS interference patterns in real-space with wavelength where q_1 is the scattering vector

$\vec{k} - space$



Real-space

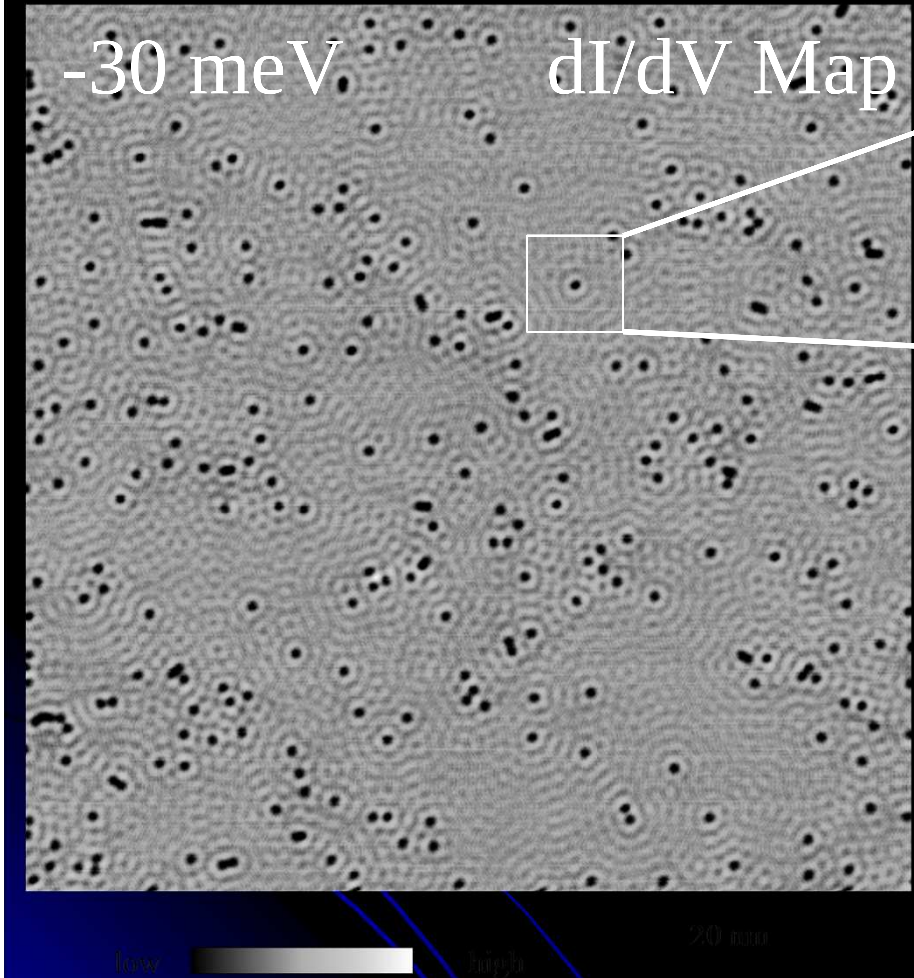


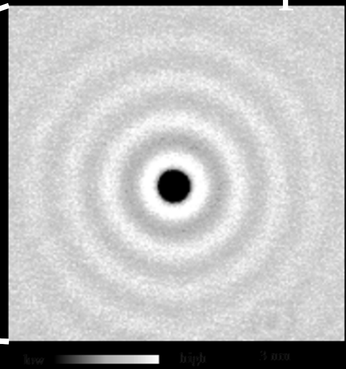
$$dI/dV(r,E) \propto LDOS(r,E) \propto \psi^2(r,E)$$

$$\psi^2(r,E) \propto \cos \vec{q}.\vec{r} = waves!$$

Slide courtesy: Seamus Davis

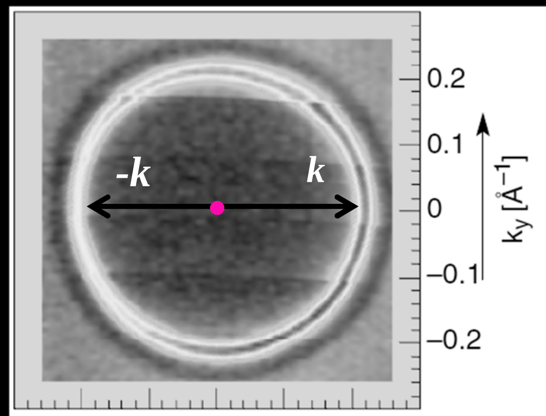
Cu(111): Parabolic 2D surface state band





Y. Okada et al.

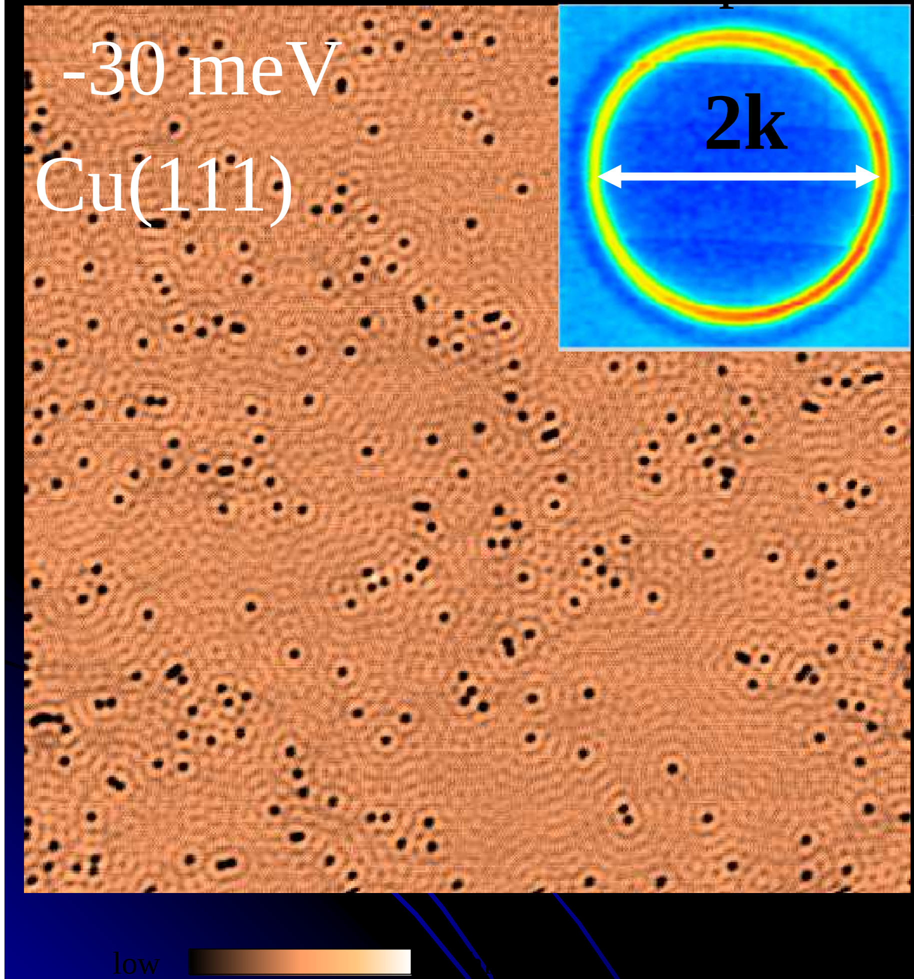
ARPES k-space allowed states

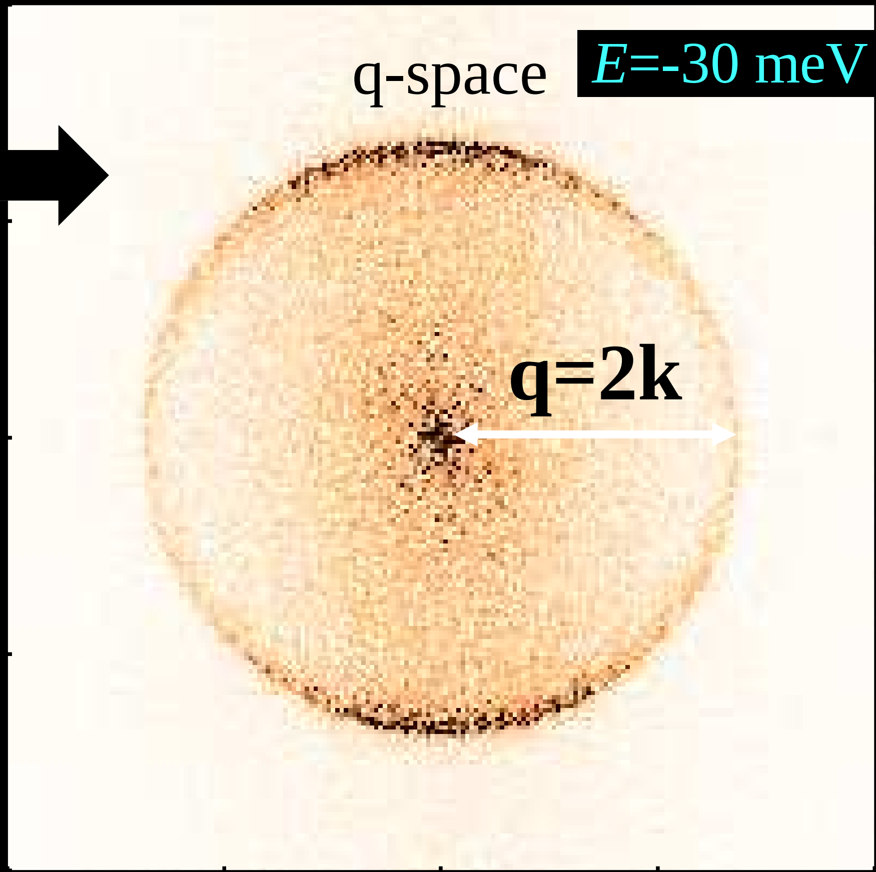


Huffner et al. New J. Phys. (2005)

q(E) ⟺ k(E)

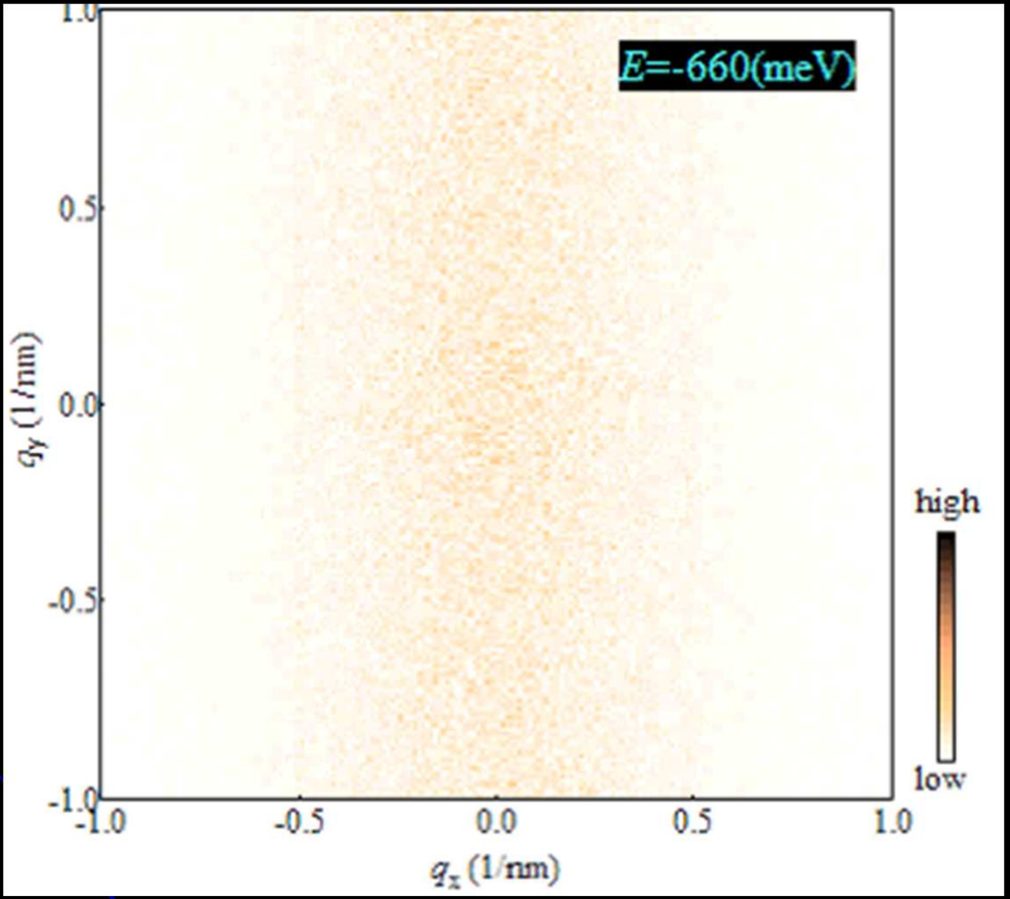
q space reflects momentum (k-space) allowed states

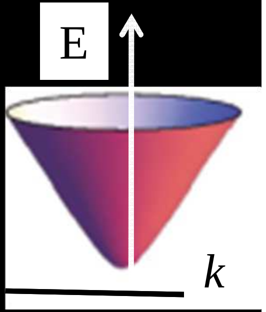




q(E) ⟺ k(E)

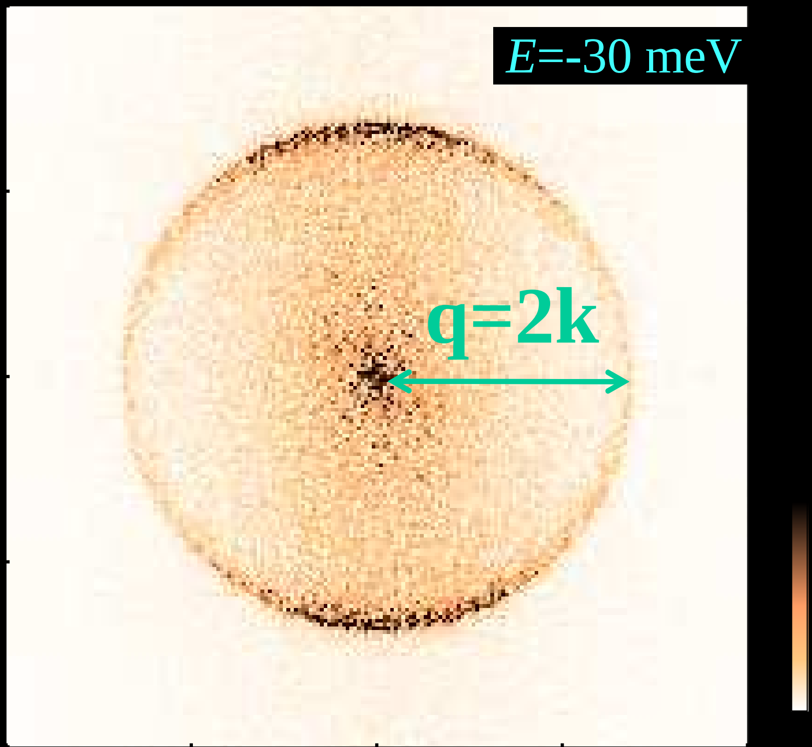
Cu(111)

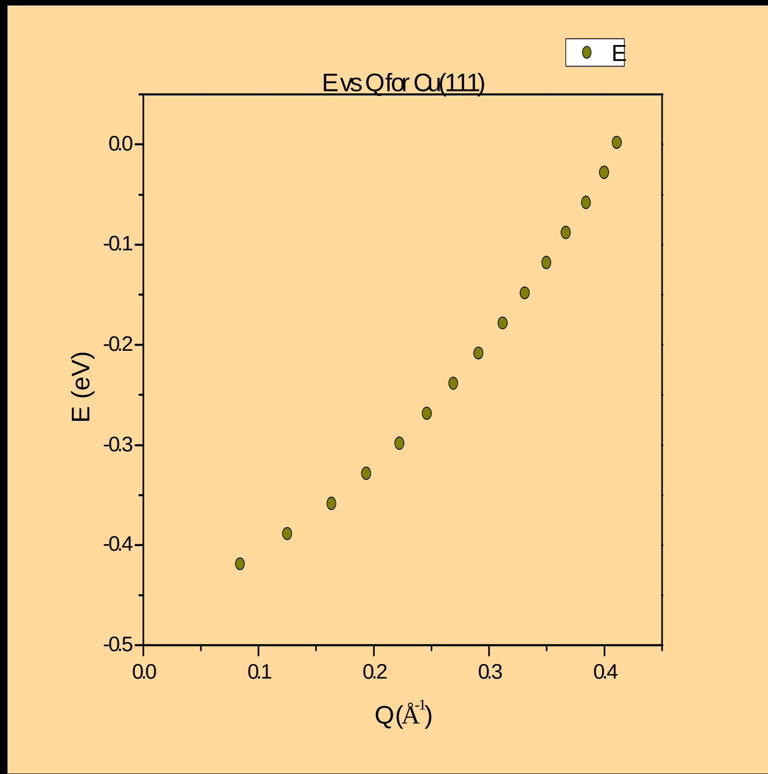




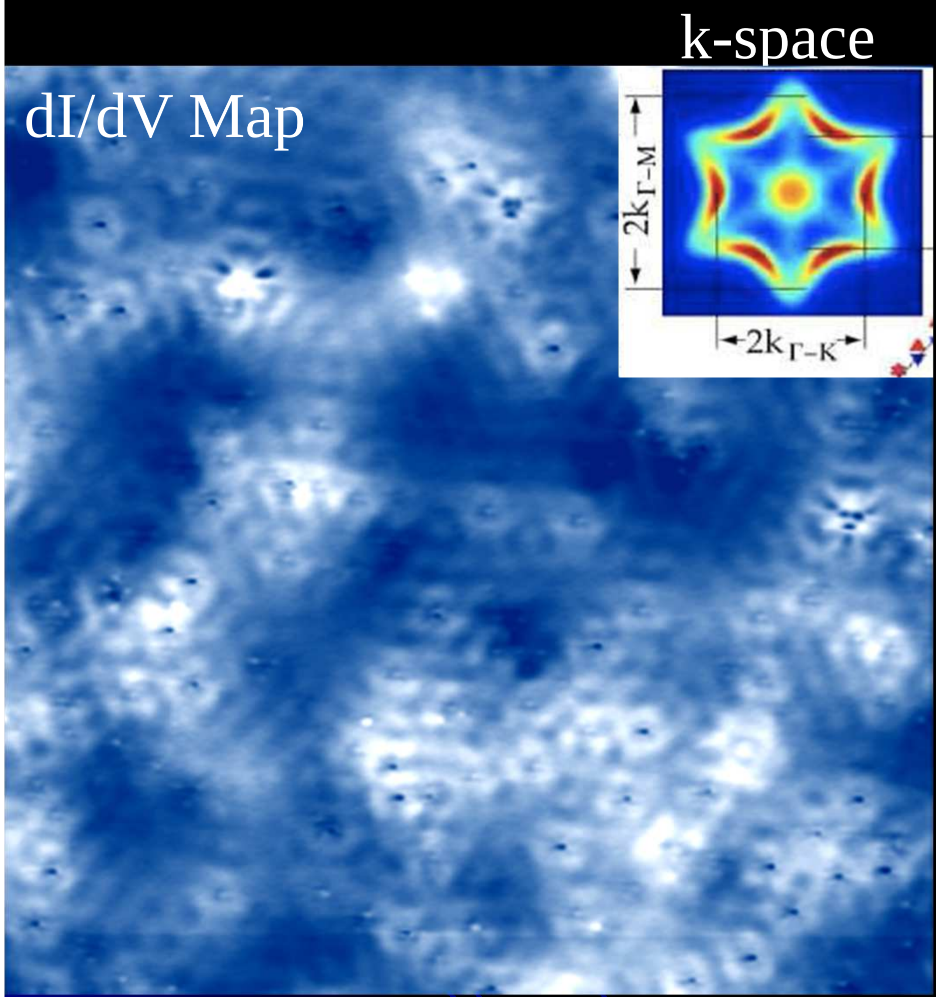
Cu(111)

Dispersion

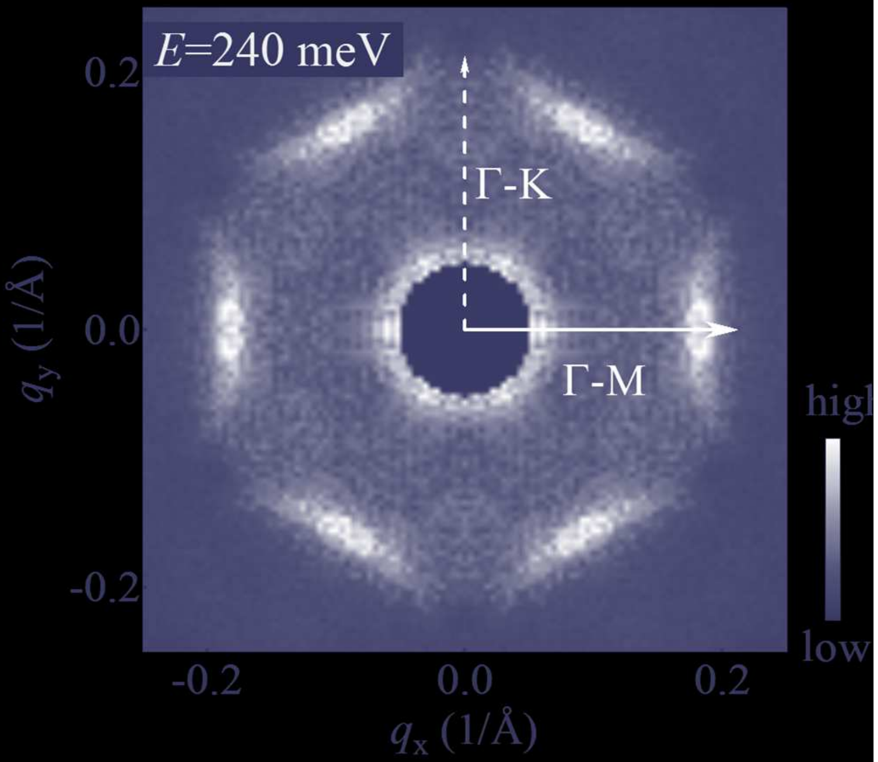




Bi_2Te_3: q space reflects momentum (k-space)

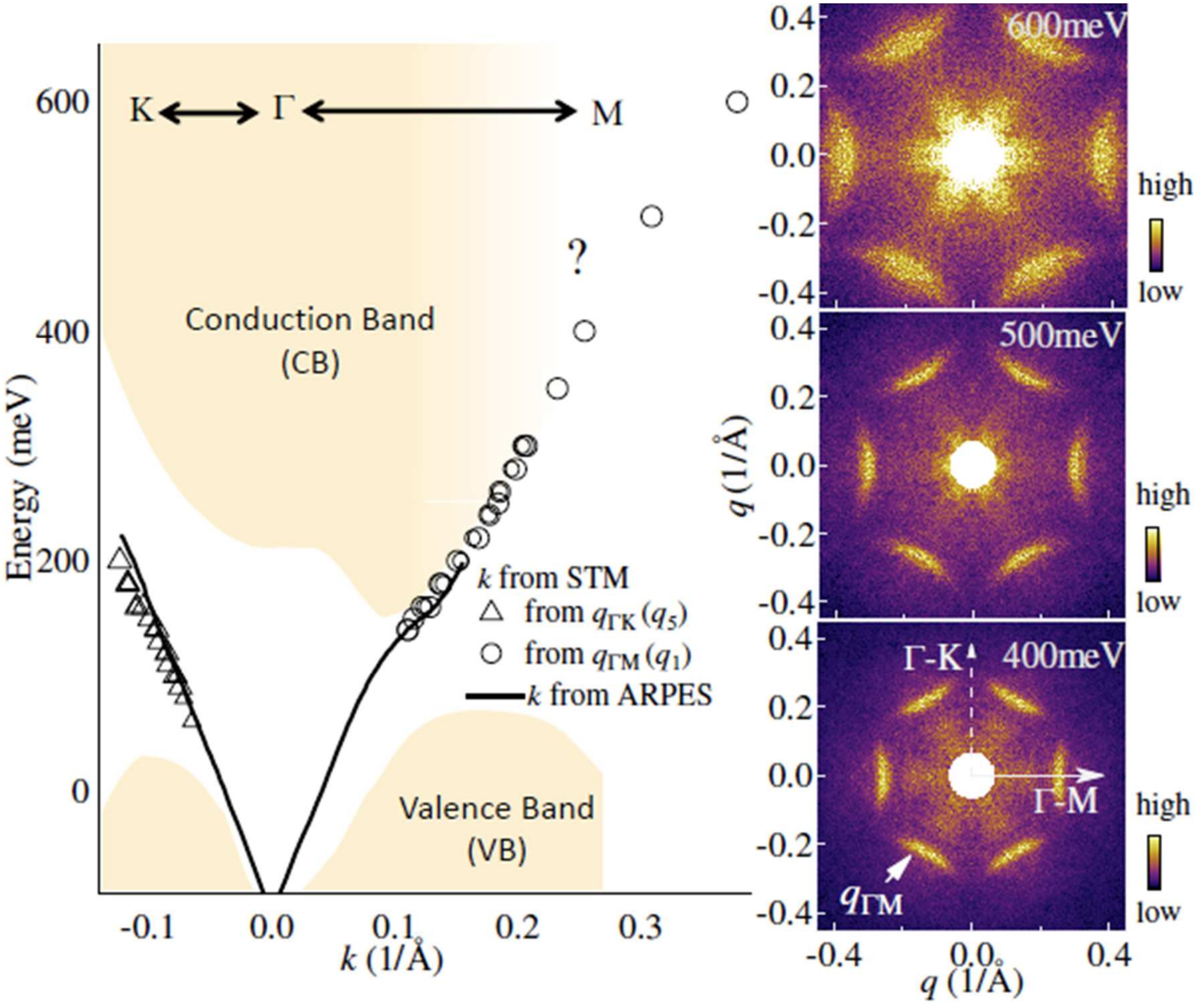


Fourier Transform
q-space



q(E) ⟷ k(E)

Surface State dispersion from Interference Patterns

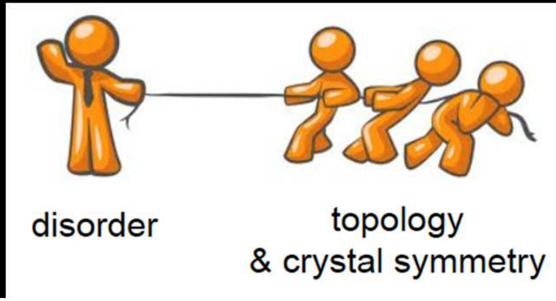


Outline

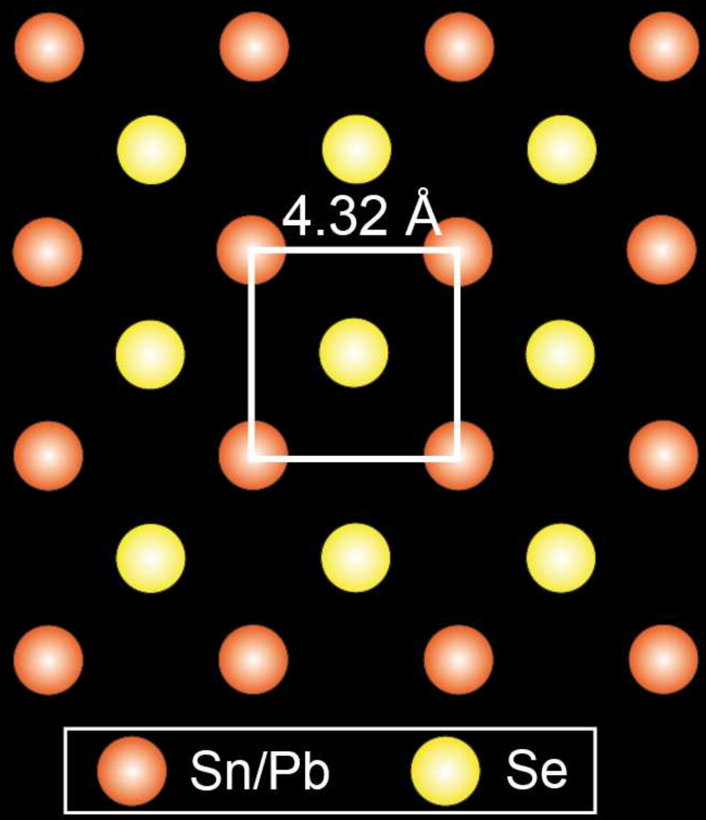
- The System: Topological Insulators
  - Unique properties of Conventional Z2 Topological Insulators
  - New Topological Material: Topological Crystalline Insulator
- The Technique: Scanning Tunneling Microscopy
  - Interference Patterns
  - Landau Level Spectroscopy
- The Experiment and Results: Breaking Mirror Symmetry to impart Mass to Massless Dirac Fermions
- Outstanding Questions and the Future

System: Topological crystalline insulators

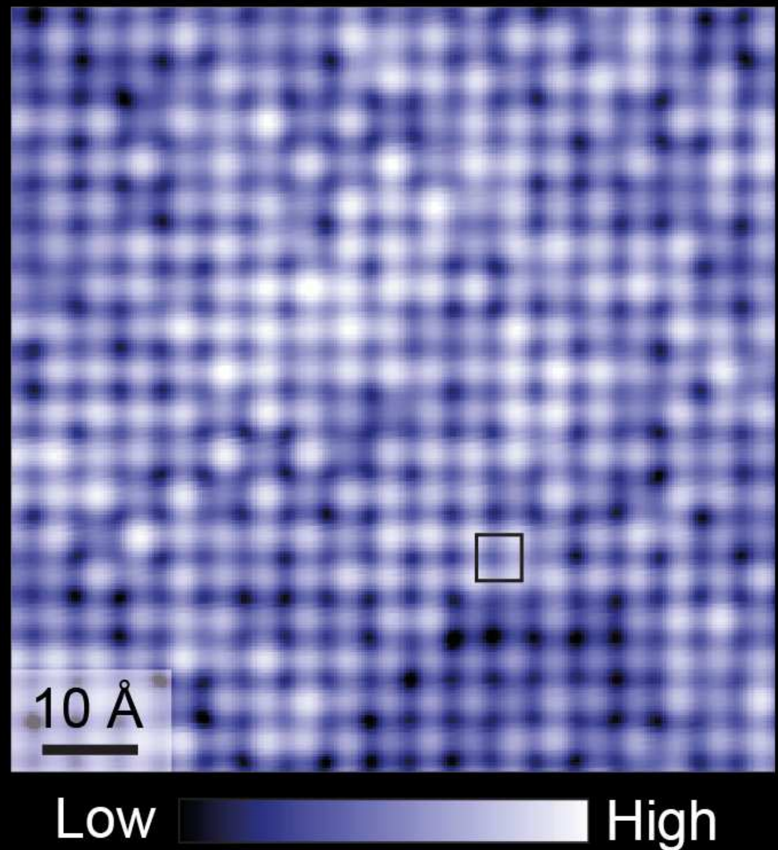
- Band Inversion known
- Is the dispersion linear near the `Dirac point'? (Do we have massless Dirac electrons?)
- Are the surface states topologically protected by mirror symmetry? (Can you open up a gap by breaking mirror symmetry?)
- Disorder necessarily violates crystal symmetry (Do we still have Massless Fermions?)
- Are TCI surface states robust against strong disorder?



(001) surface structure of $Pb_{1-x}Sn_xSe$

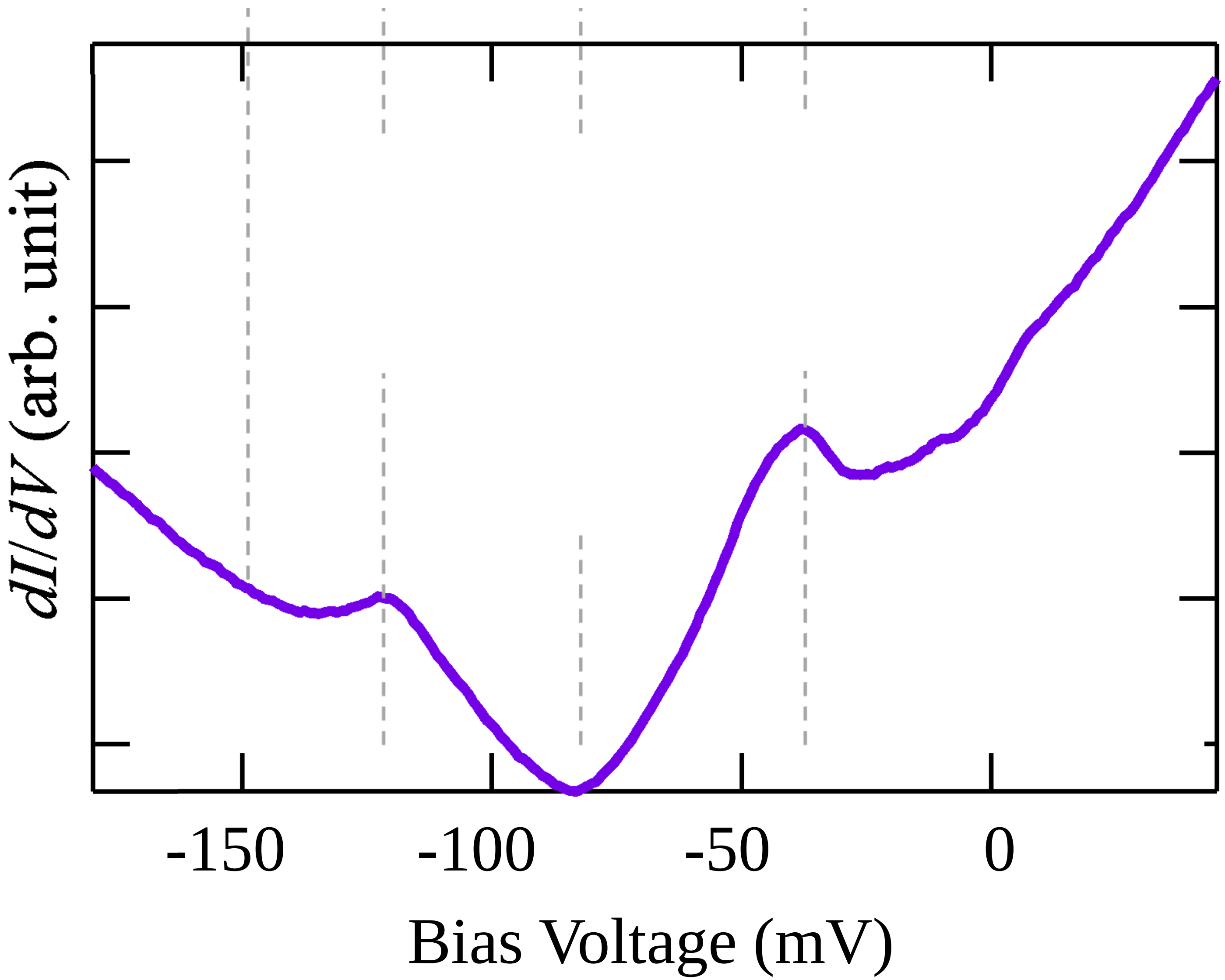


STM topograph (10 pA, -100 mV)



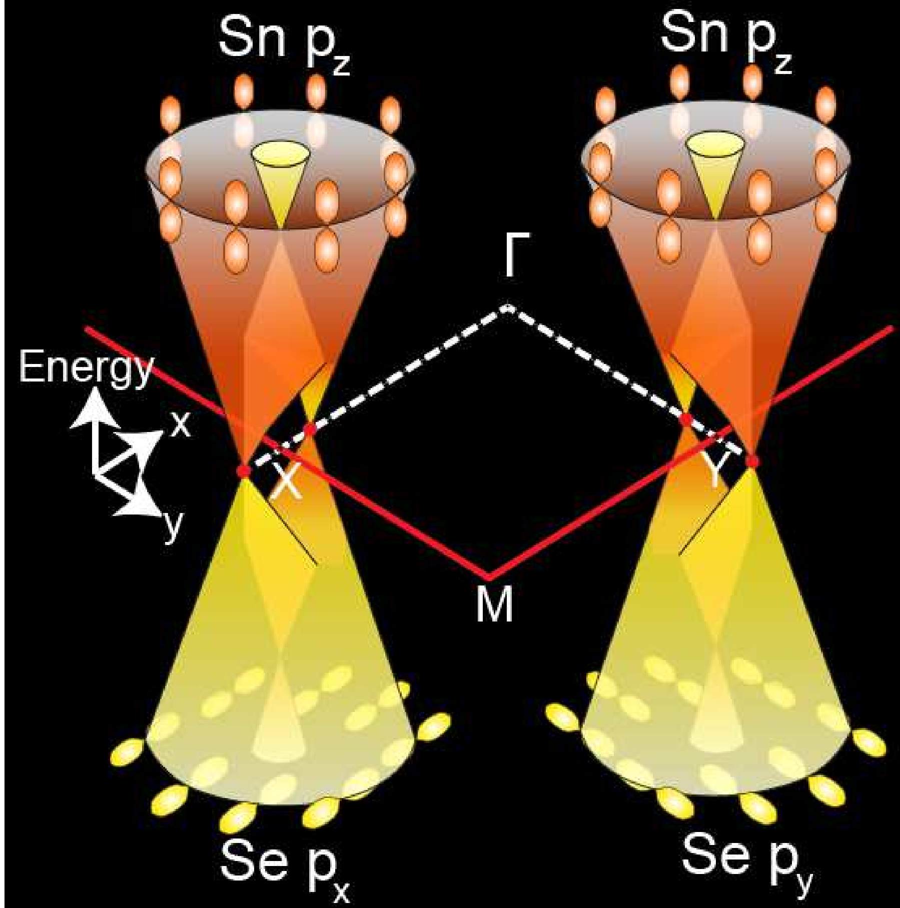
Only image the Pb/Sn sublattice

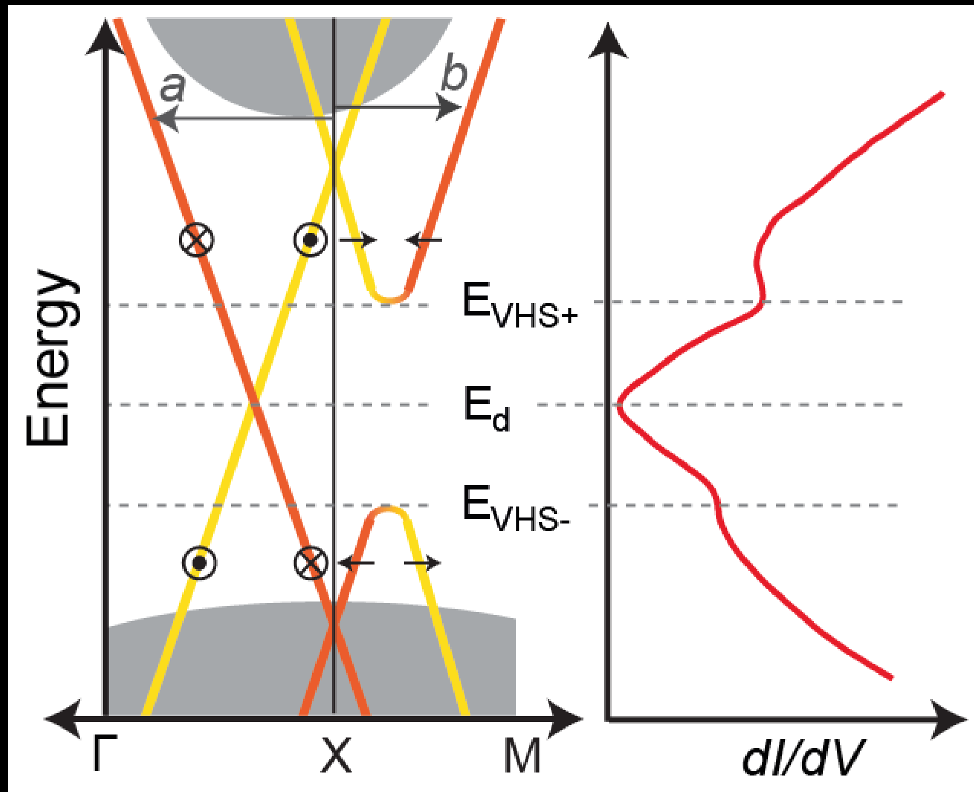
Density of states from STM spectroscopy



Okada *et al. Science* (2013)

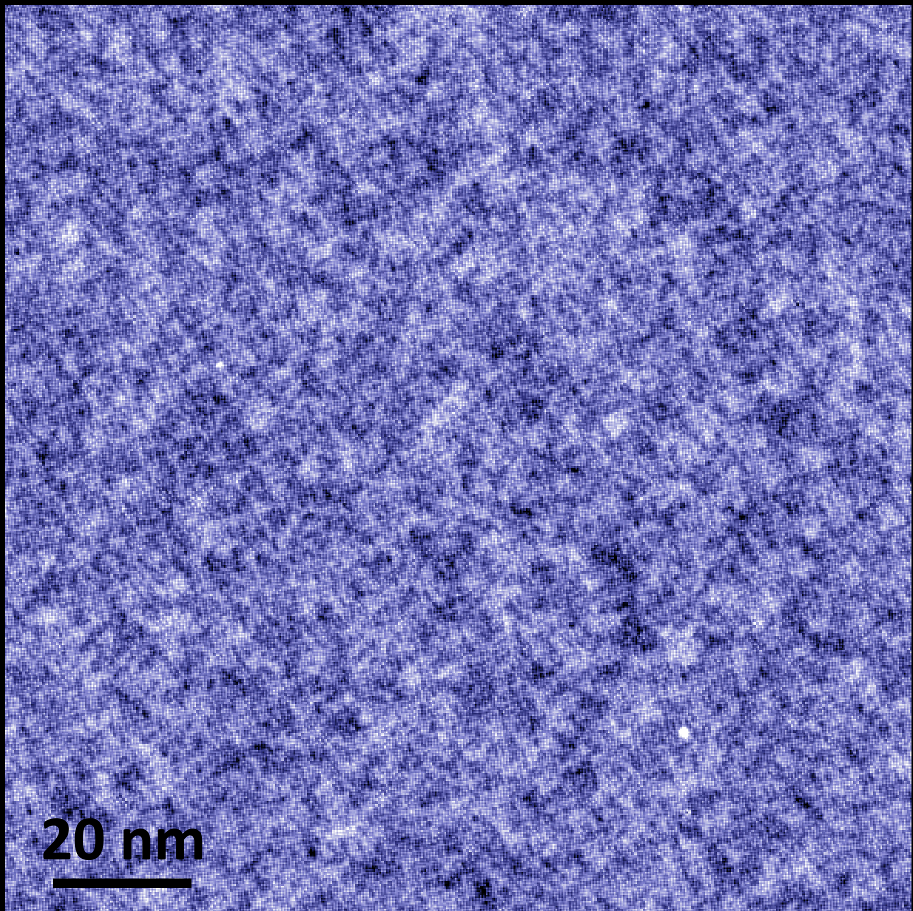
Band structure of $Pb_{1-x}Sn_xSe$ (001) surface



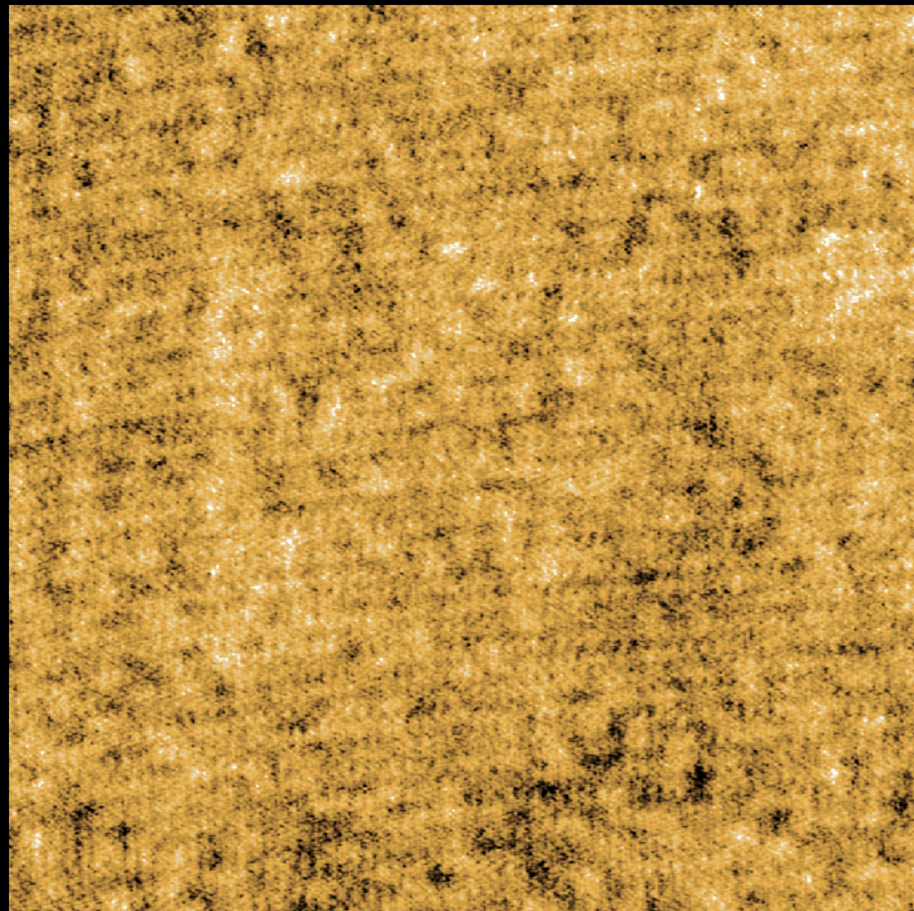


dI/dV conductance maps of $Pb_{0.63}Sn_{0.37}Se$

STM topograph

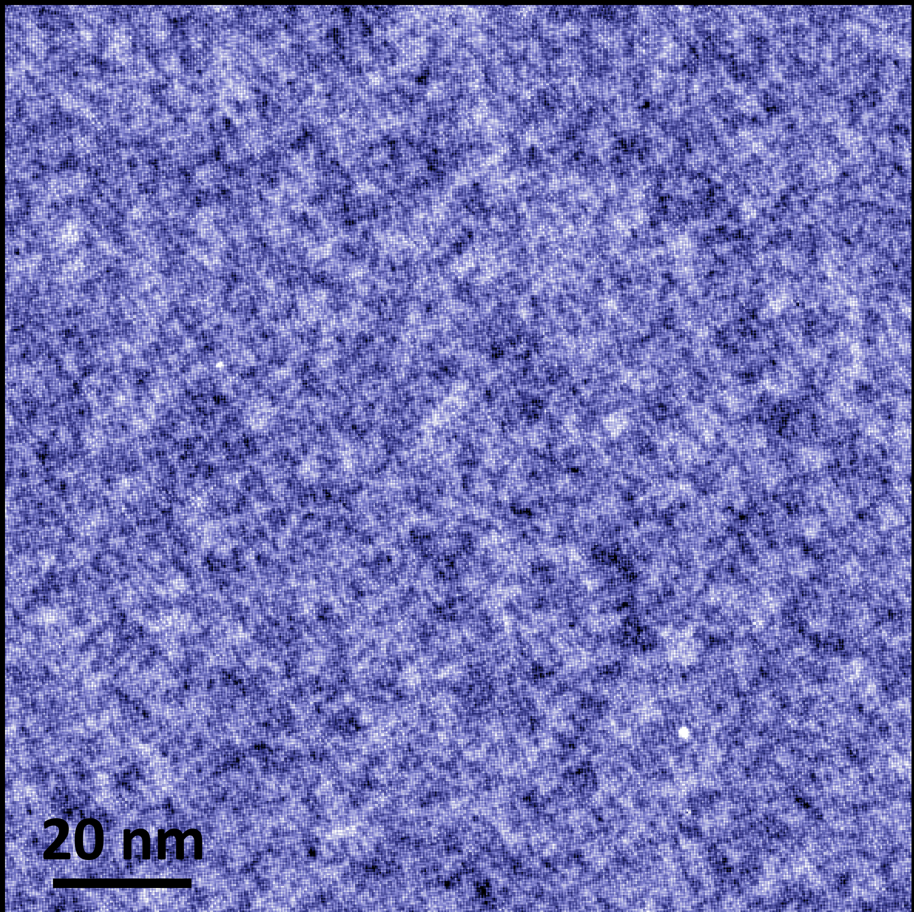


dI/dV conductance map (-15 mV)

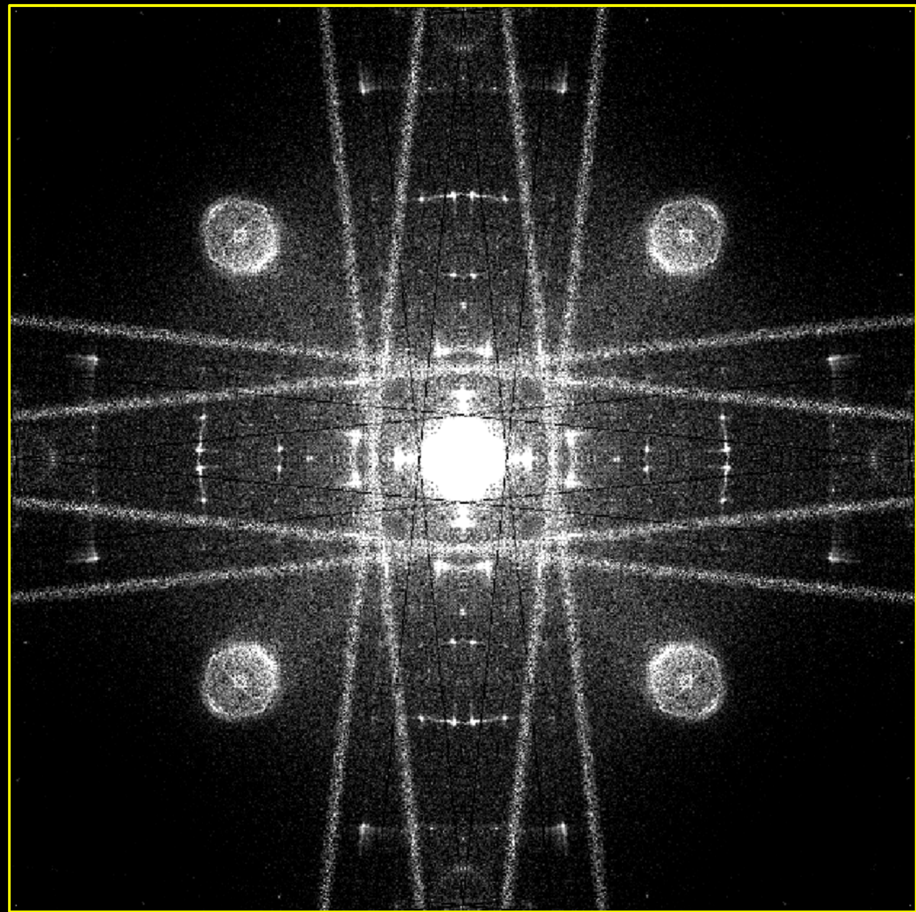


dI/dV conductance maps of $Pb_{0.63}Sn_{0.37}Se$

STM topograph



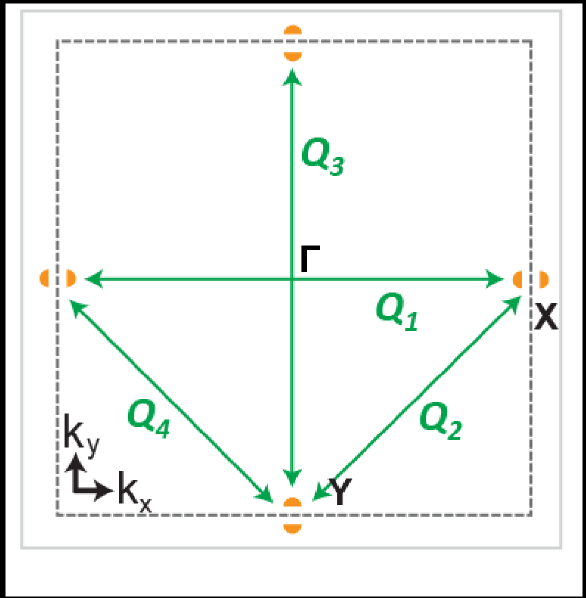
Fourier Transform of
dI/dV conductance map (-15 mV)

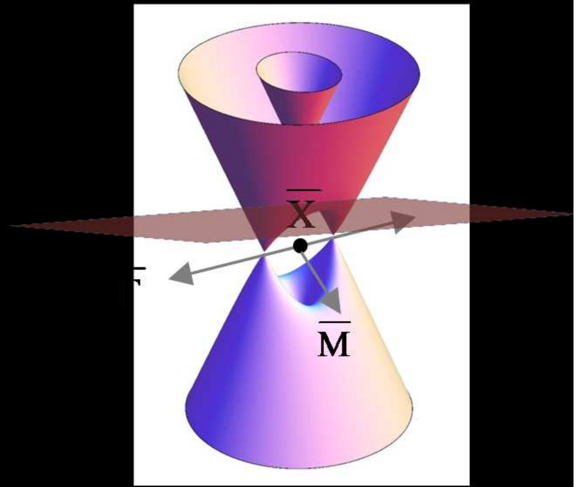


Expected QPI pattern

Constant-energy contours
(CECs)

~ Dirac point



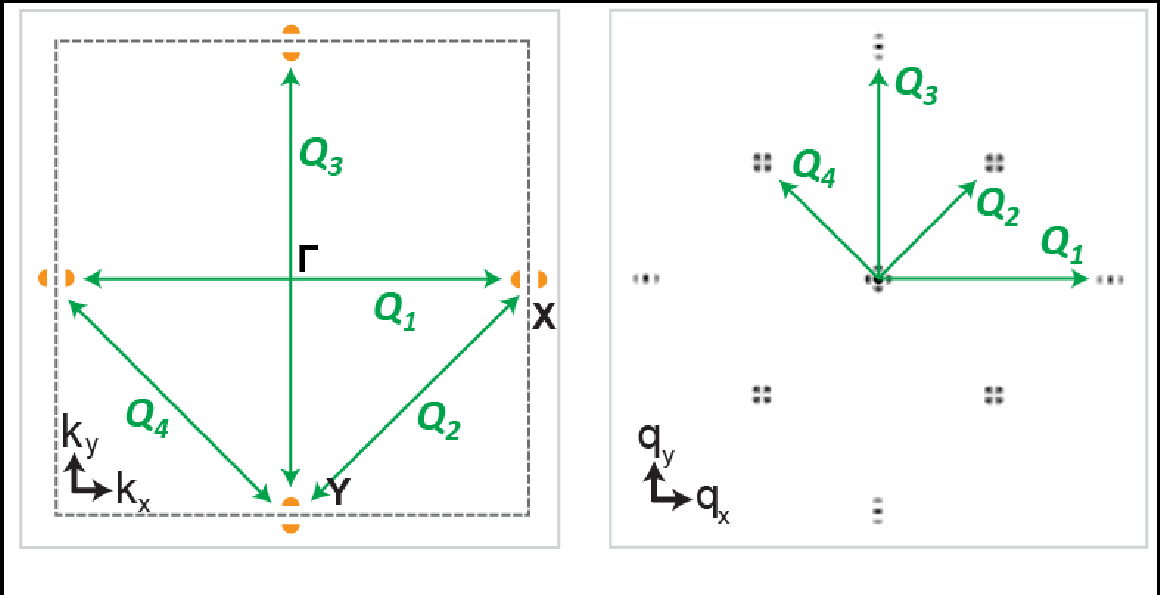


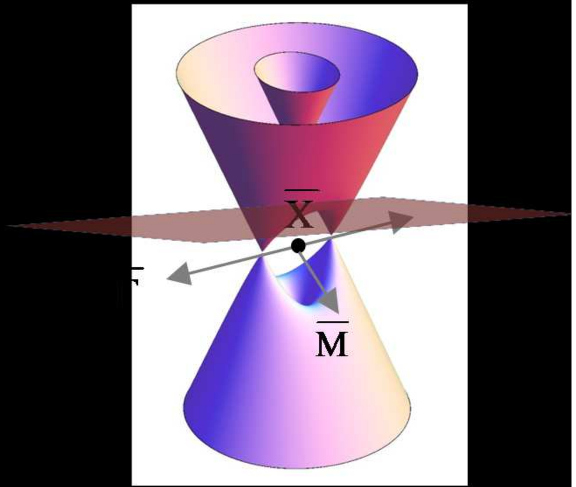
Expected QPI pattern

~ Dirac point

Constant-energy contours (CECs)

Autocorrelation of CECs



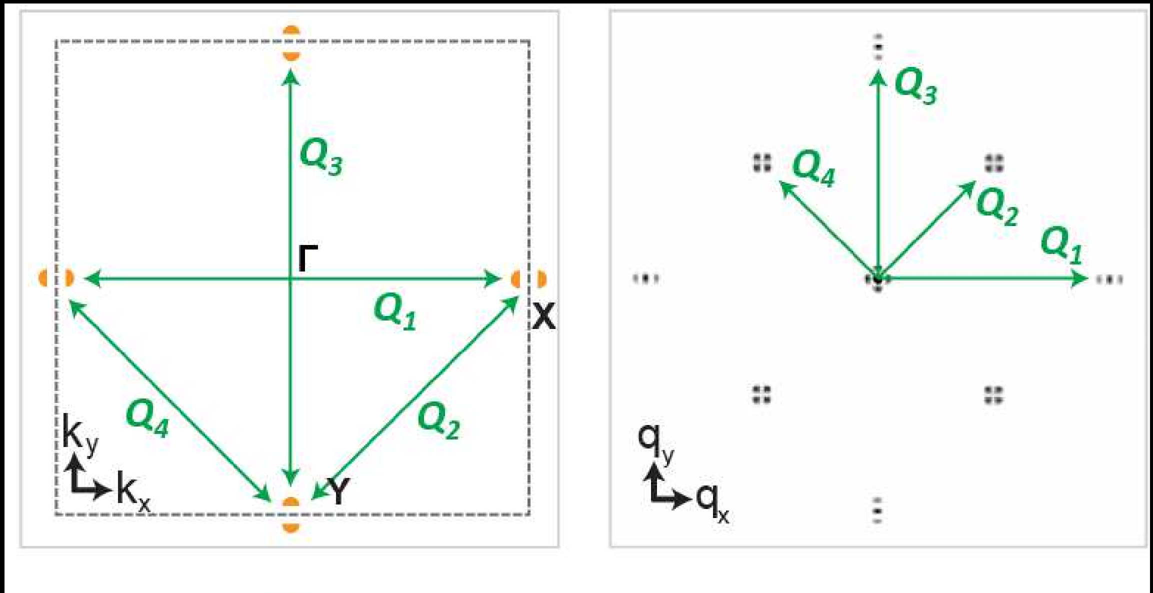


Expected QPI pattern

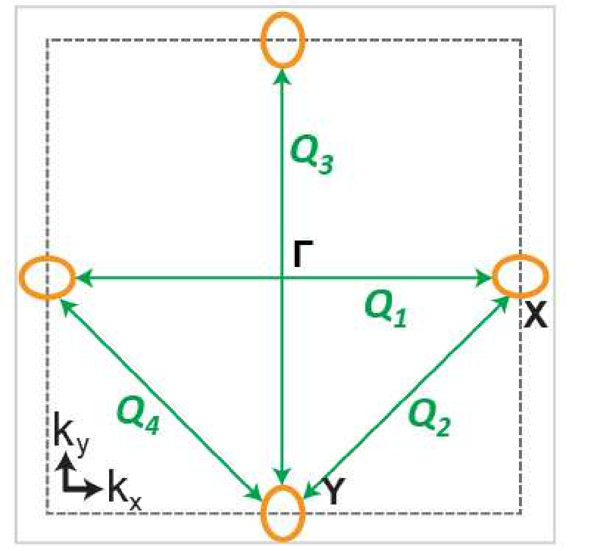
Constant-energy contours (CECs)

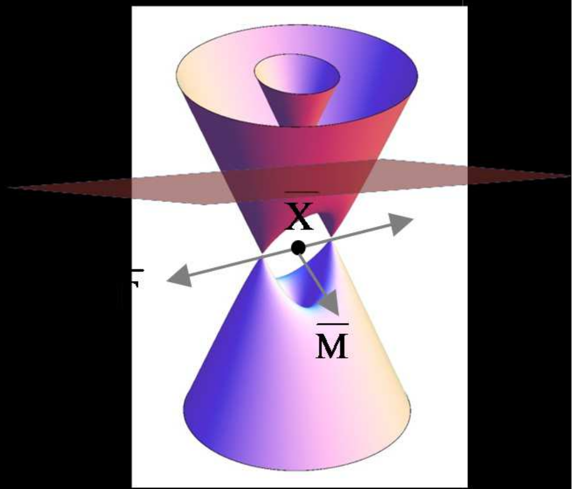
Autocorrelation of CECs

~ Dirac point



Away from Dirac point

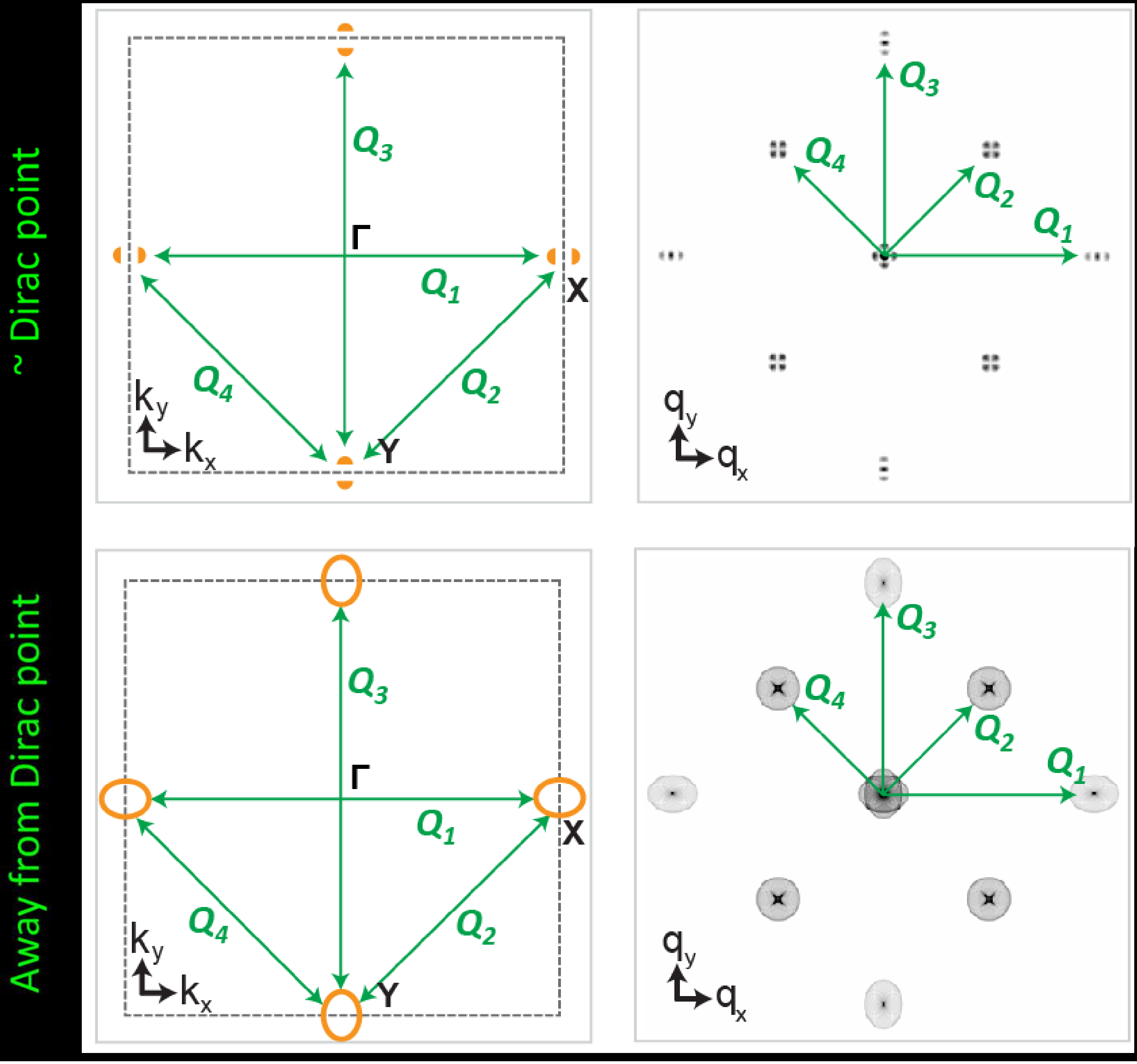


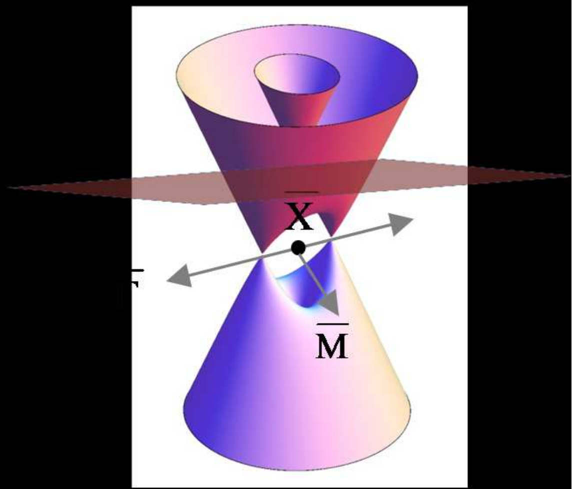


Expected QPI pattern

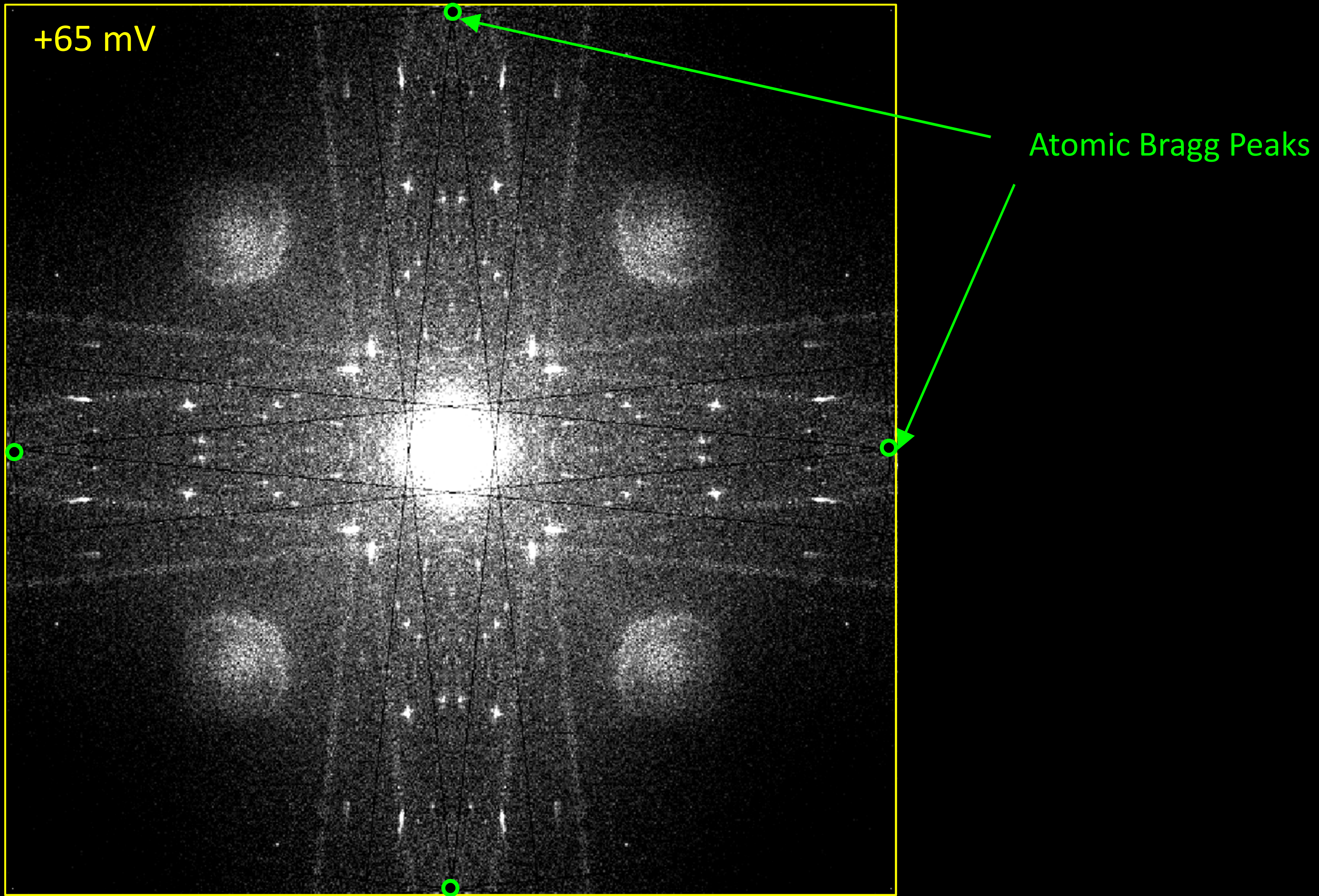
Constant-energy contours (CECs)

Autocorrelation of CECs

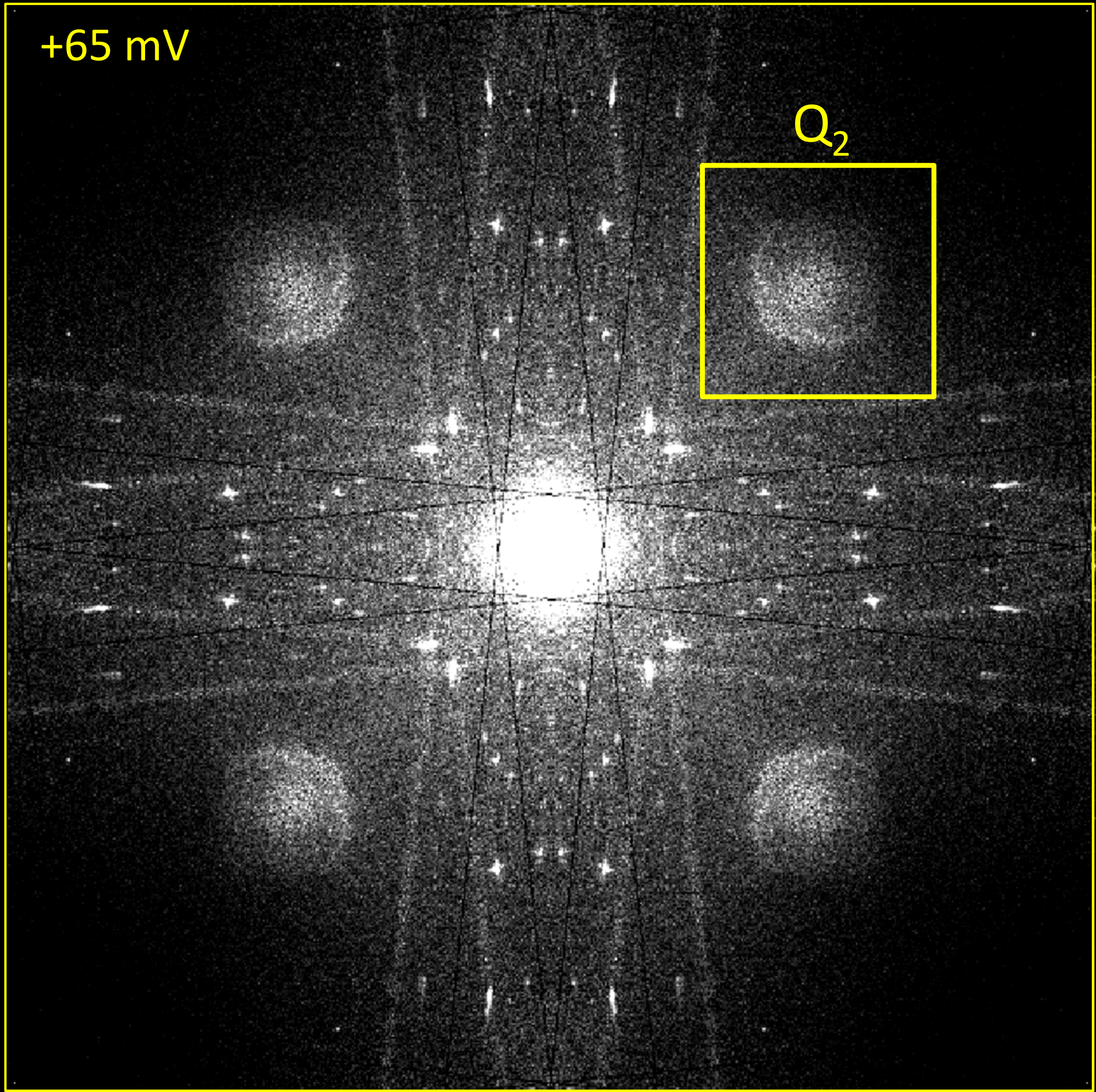


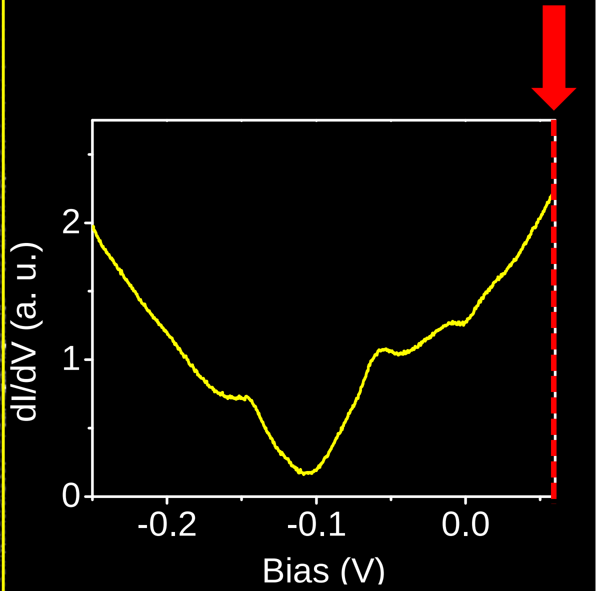


QPI Fourier transform patterns

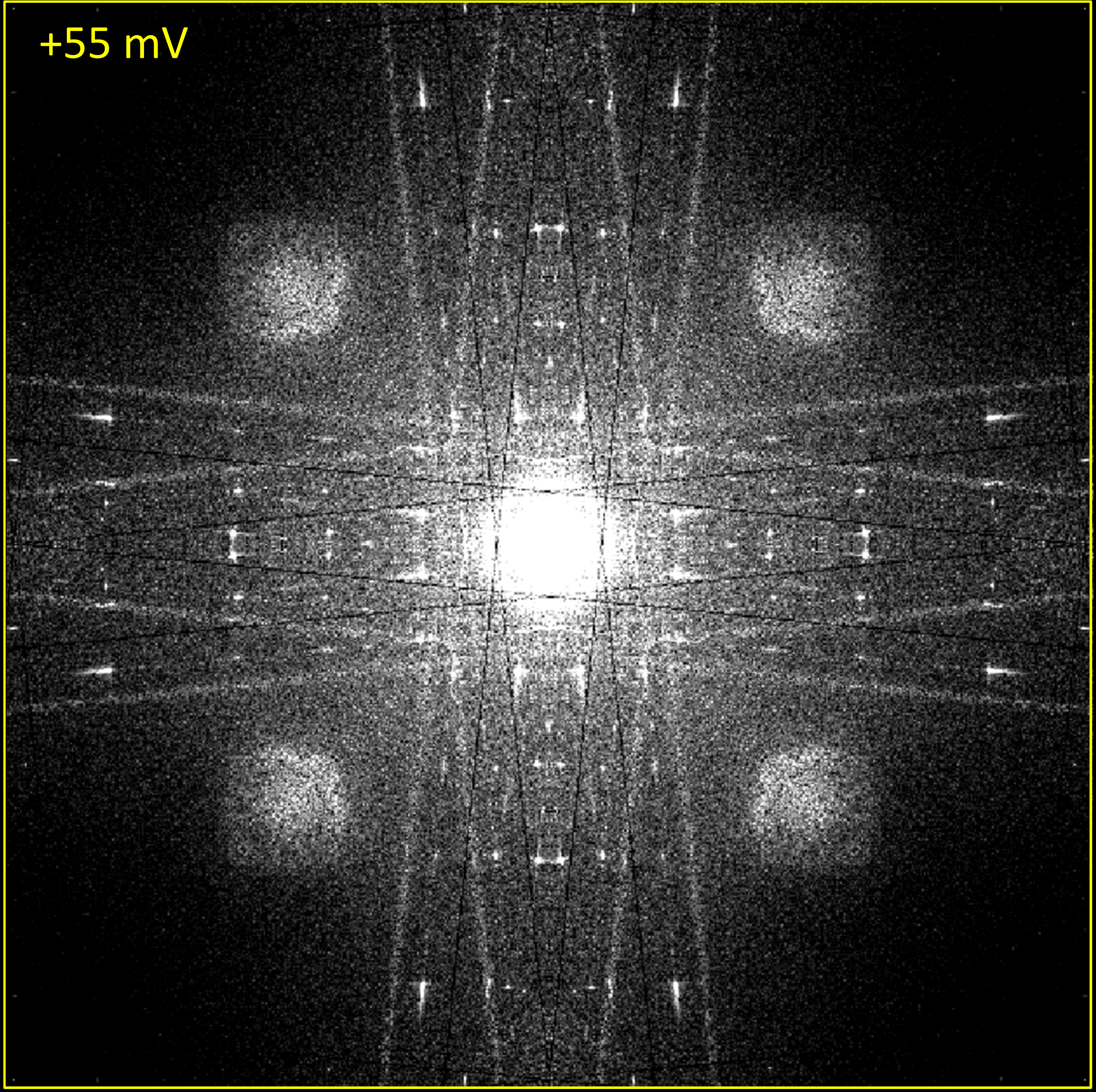


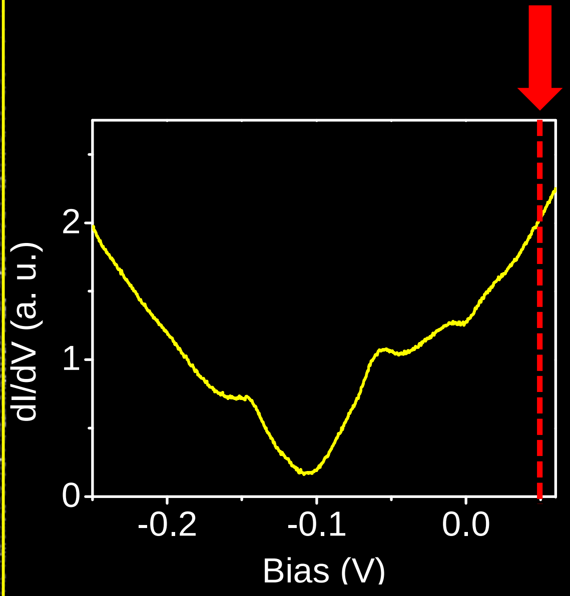
QPI Fourier transform patterns



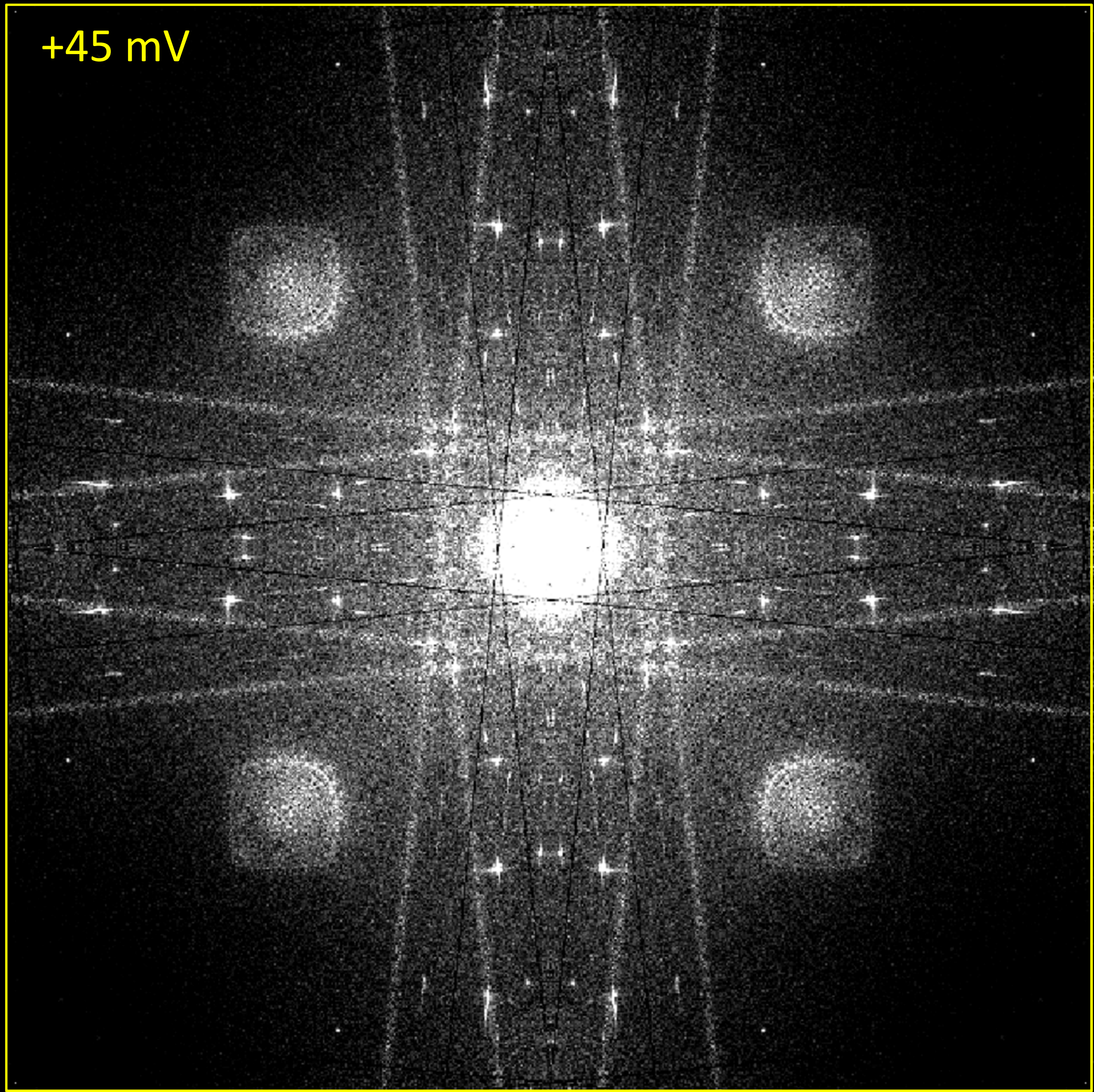


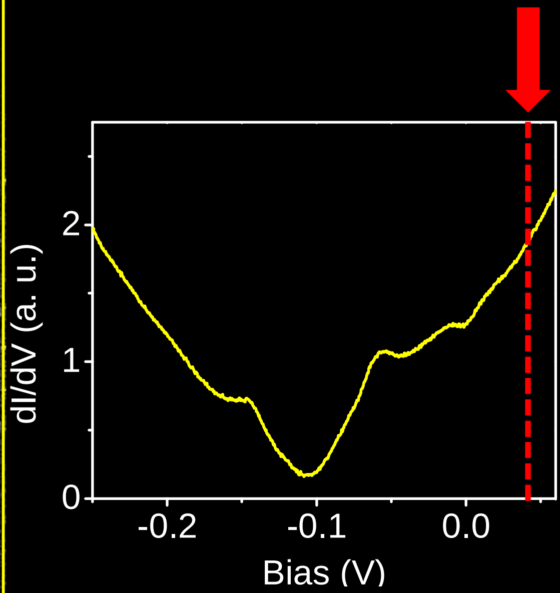
QPI Fourier transform patterns



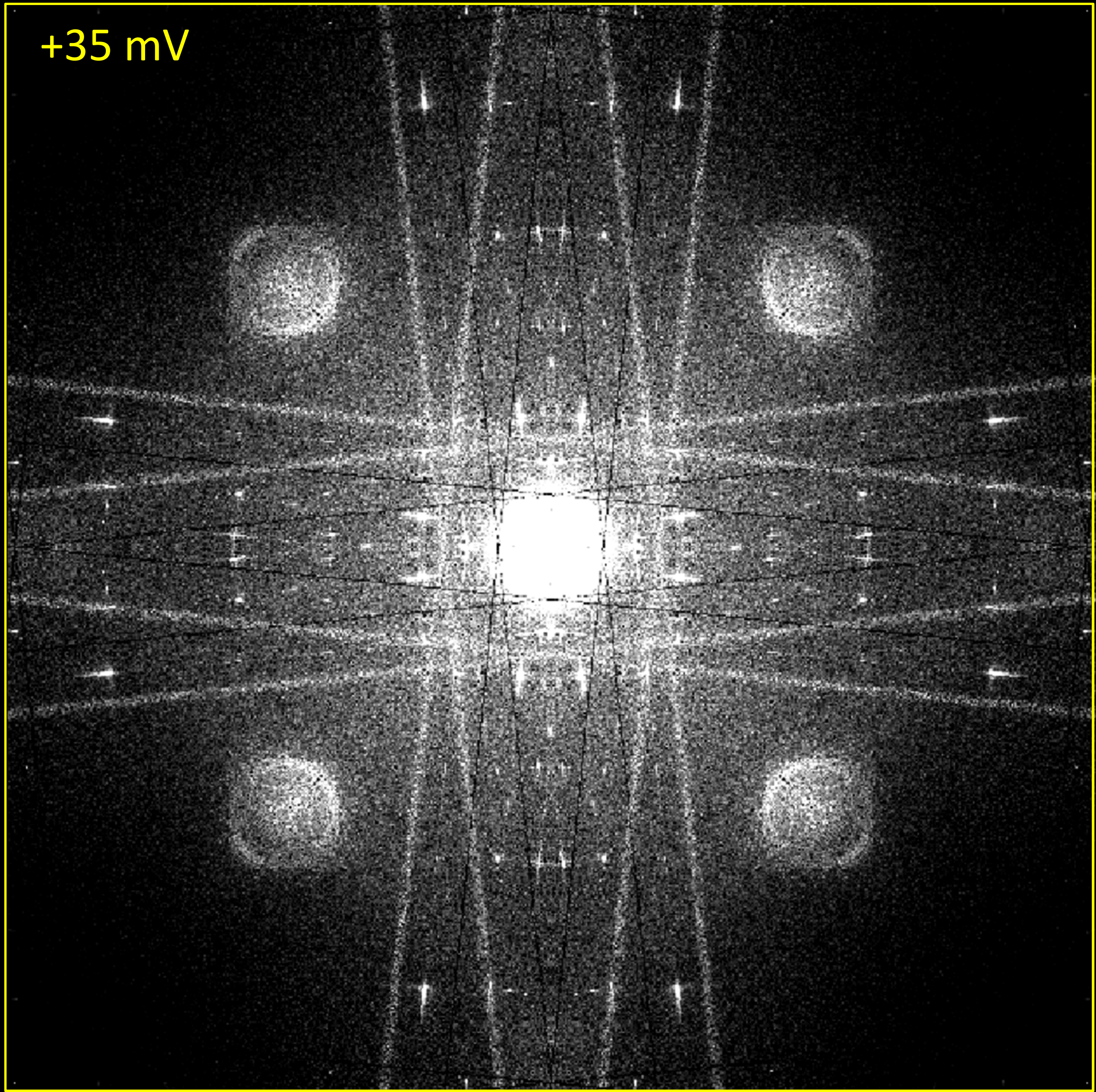


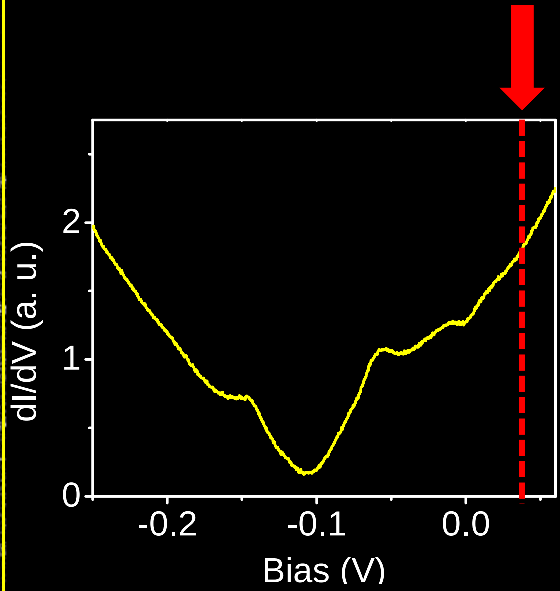
QPI Fourier transform patterns



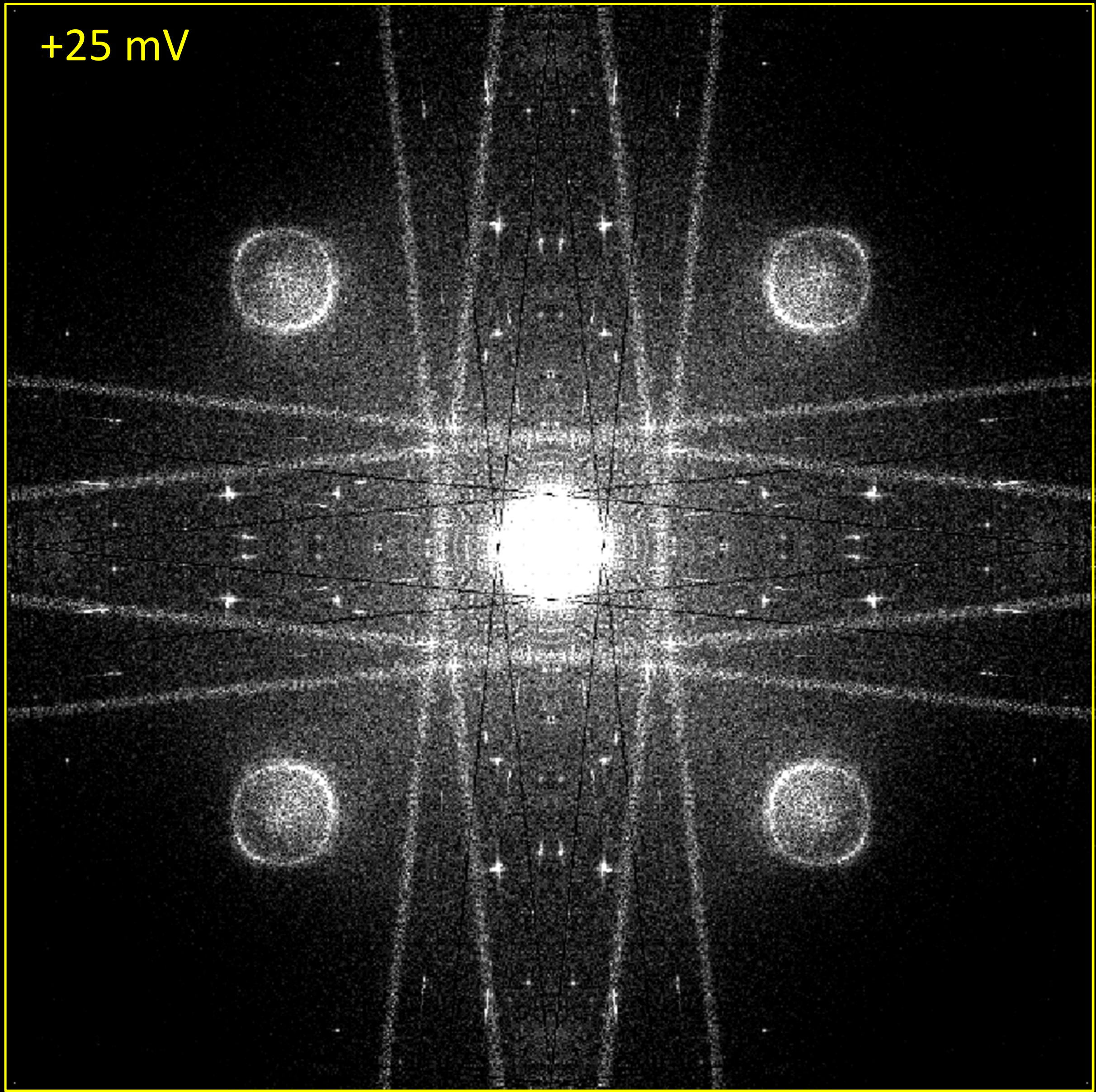


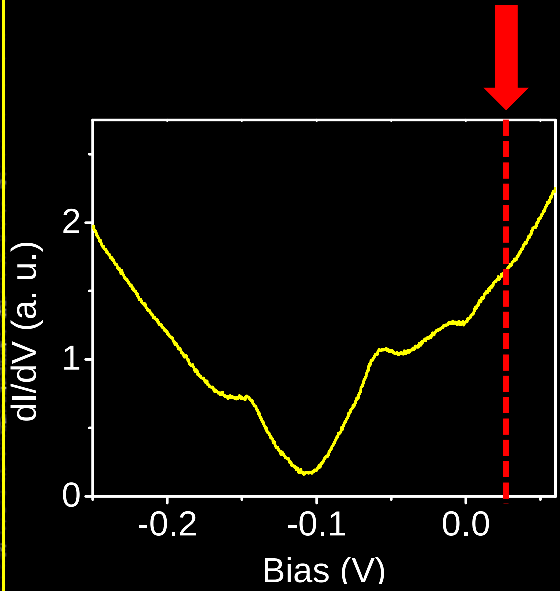
QPI Fourier transform patterns



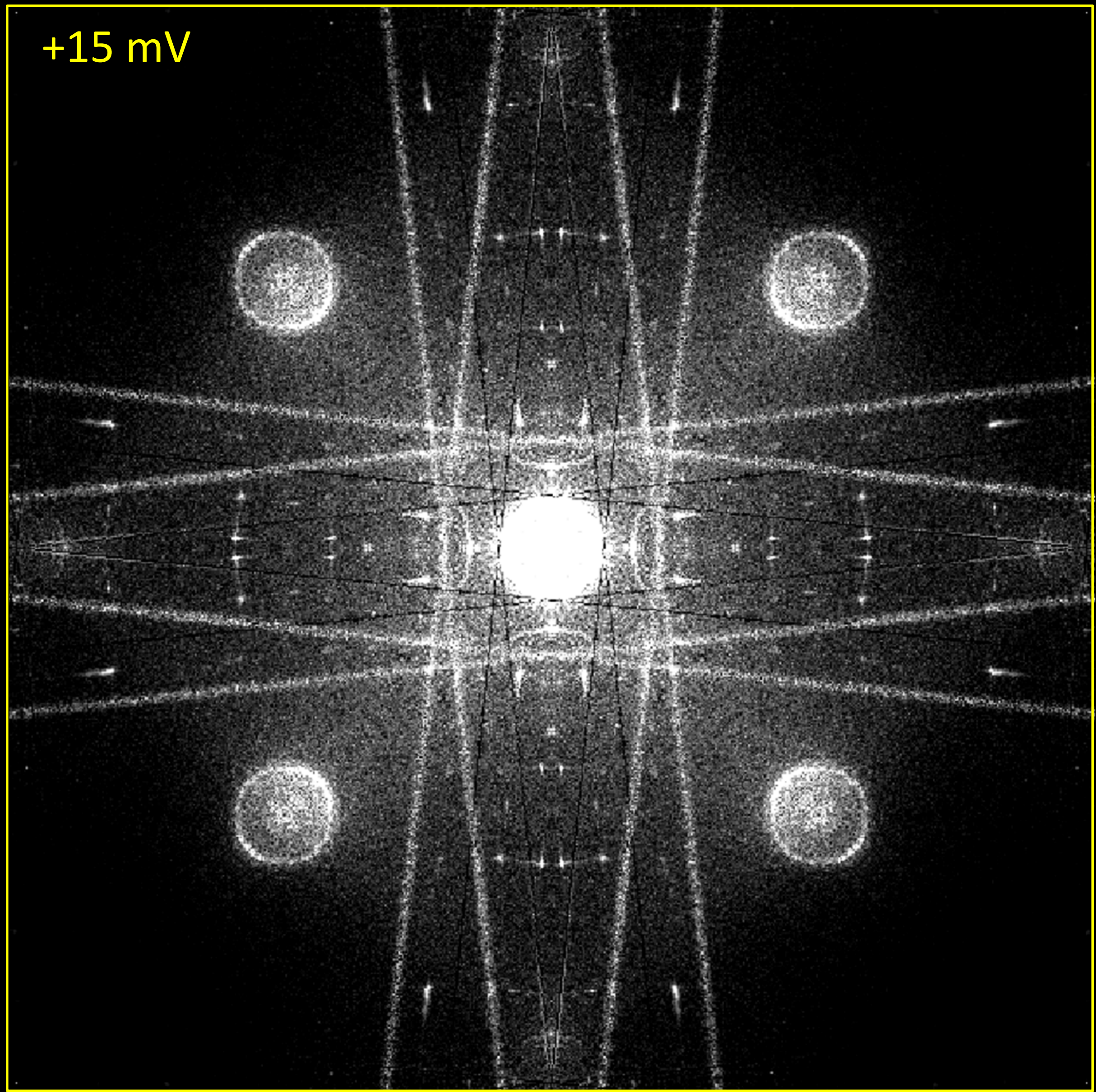


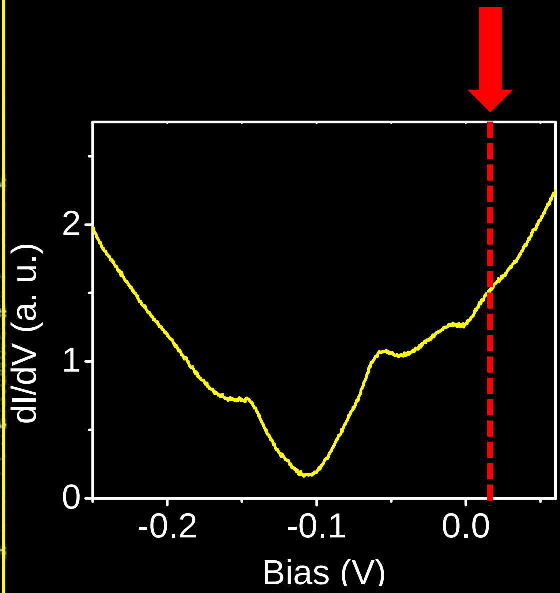
QPI Fourier transform patterns



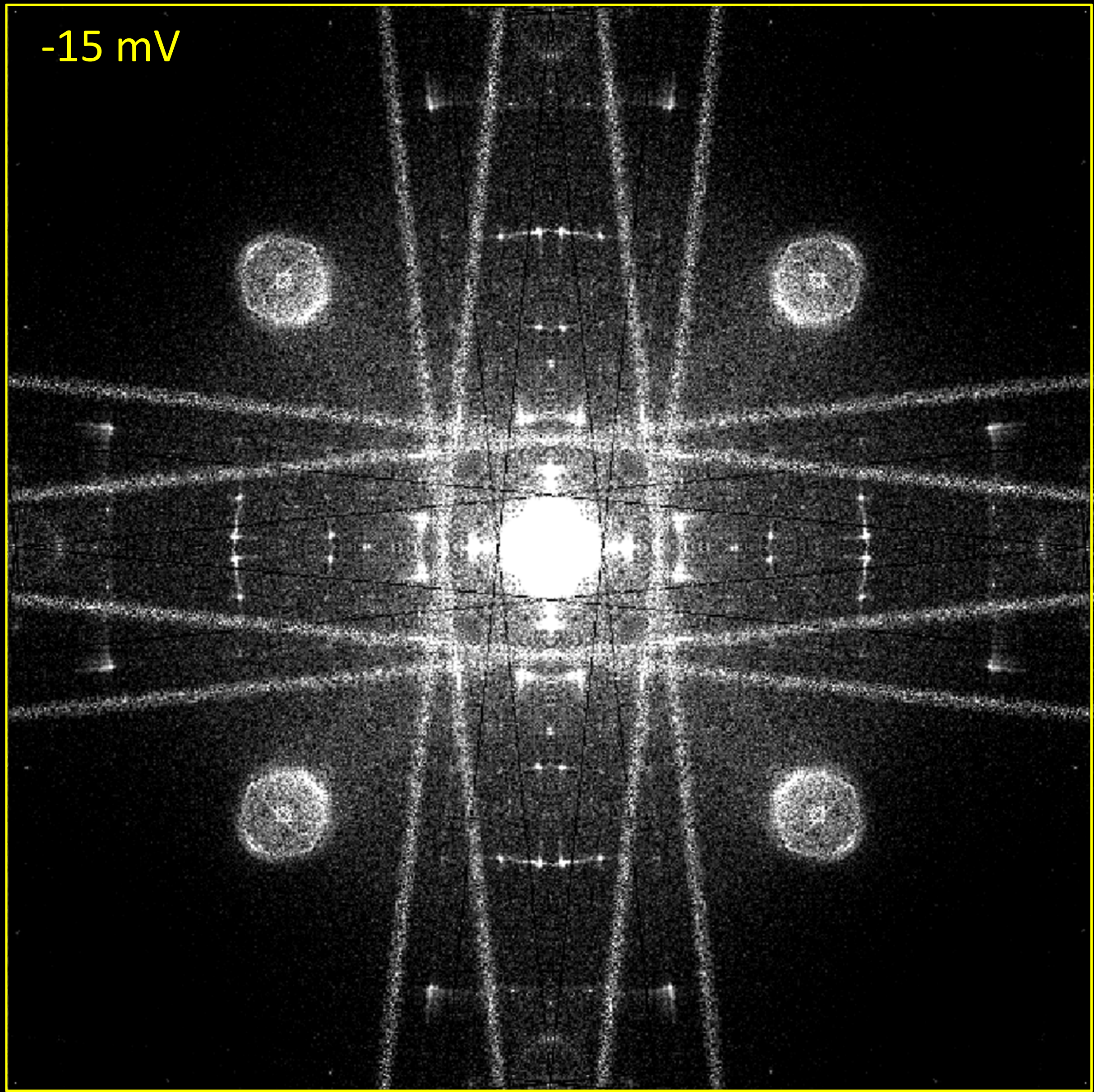


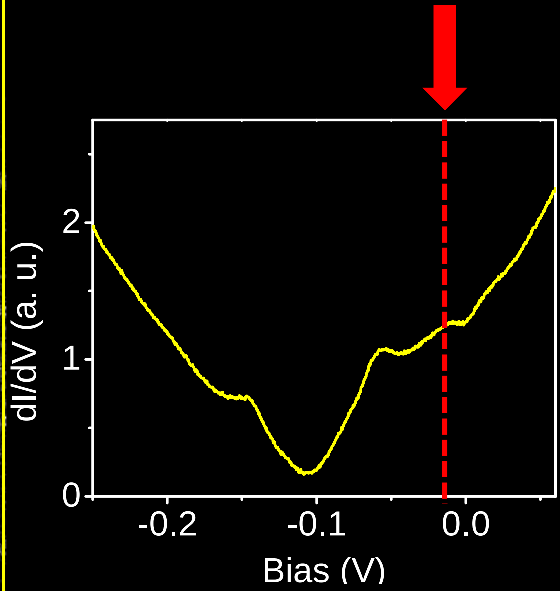
QPI Fourier transform patterns



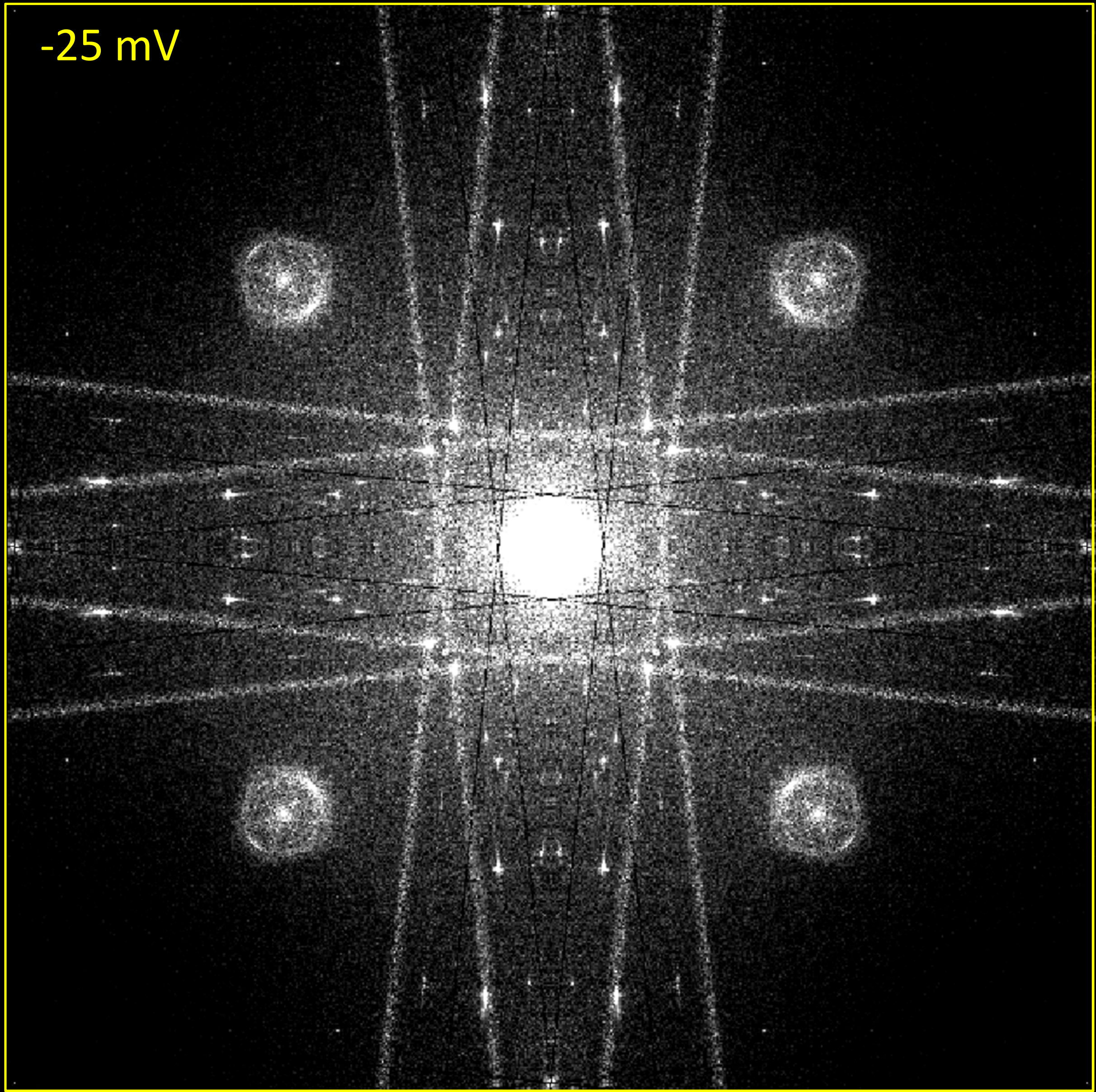


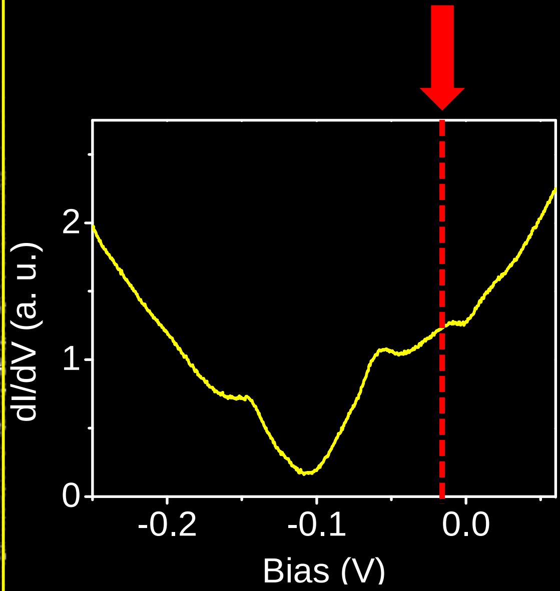
QPI Fourier transform patterns



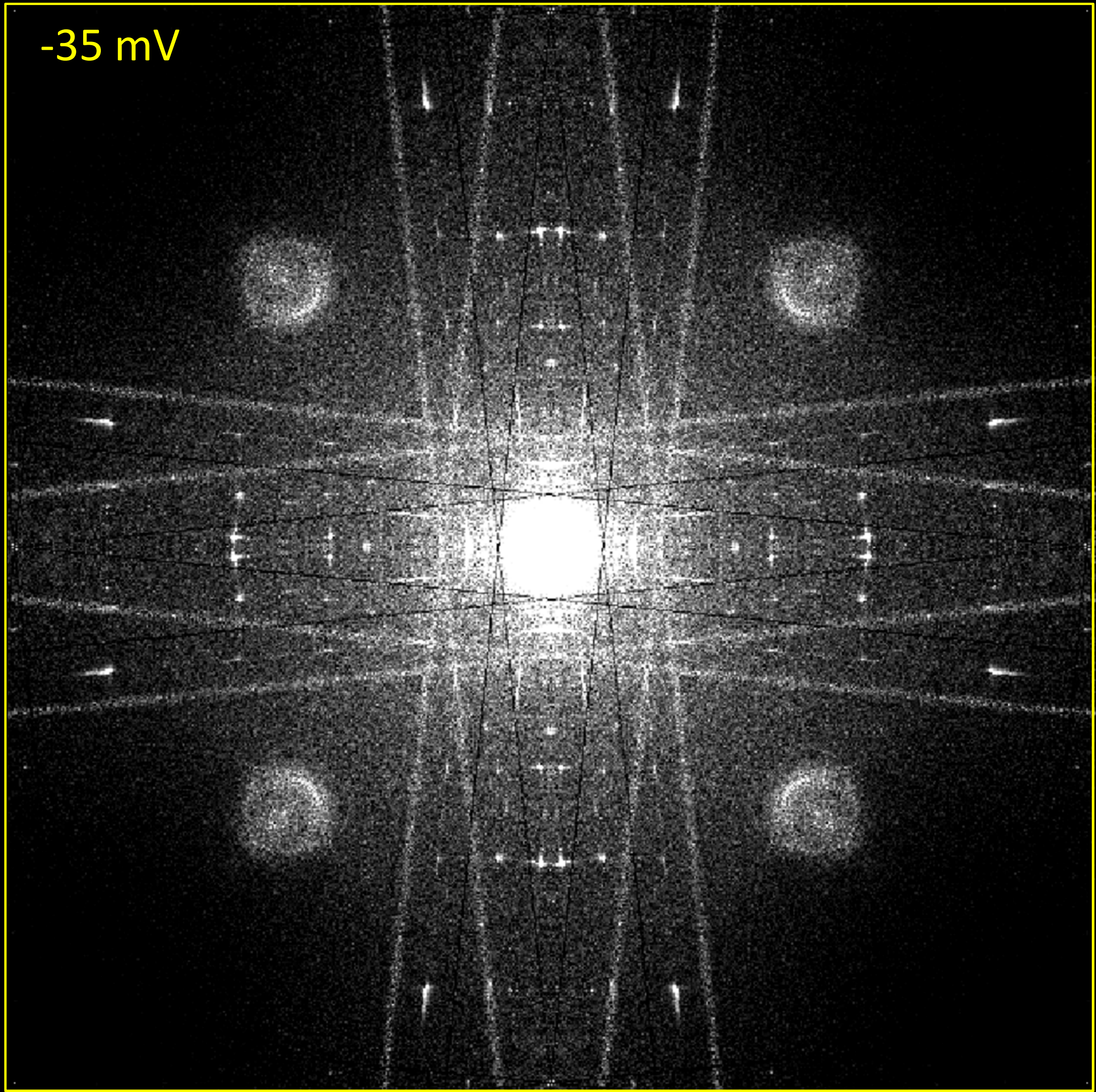


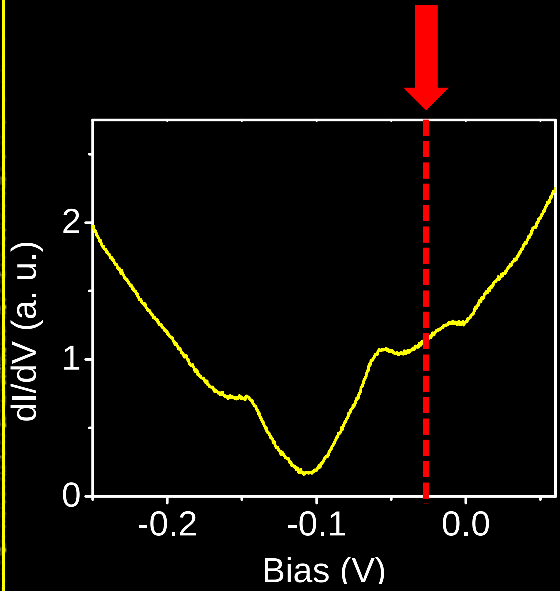
QPI Fourier transform patterns



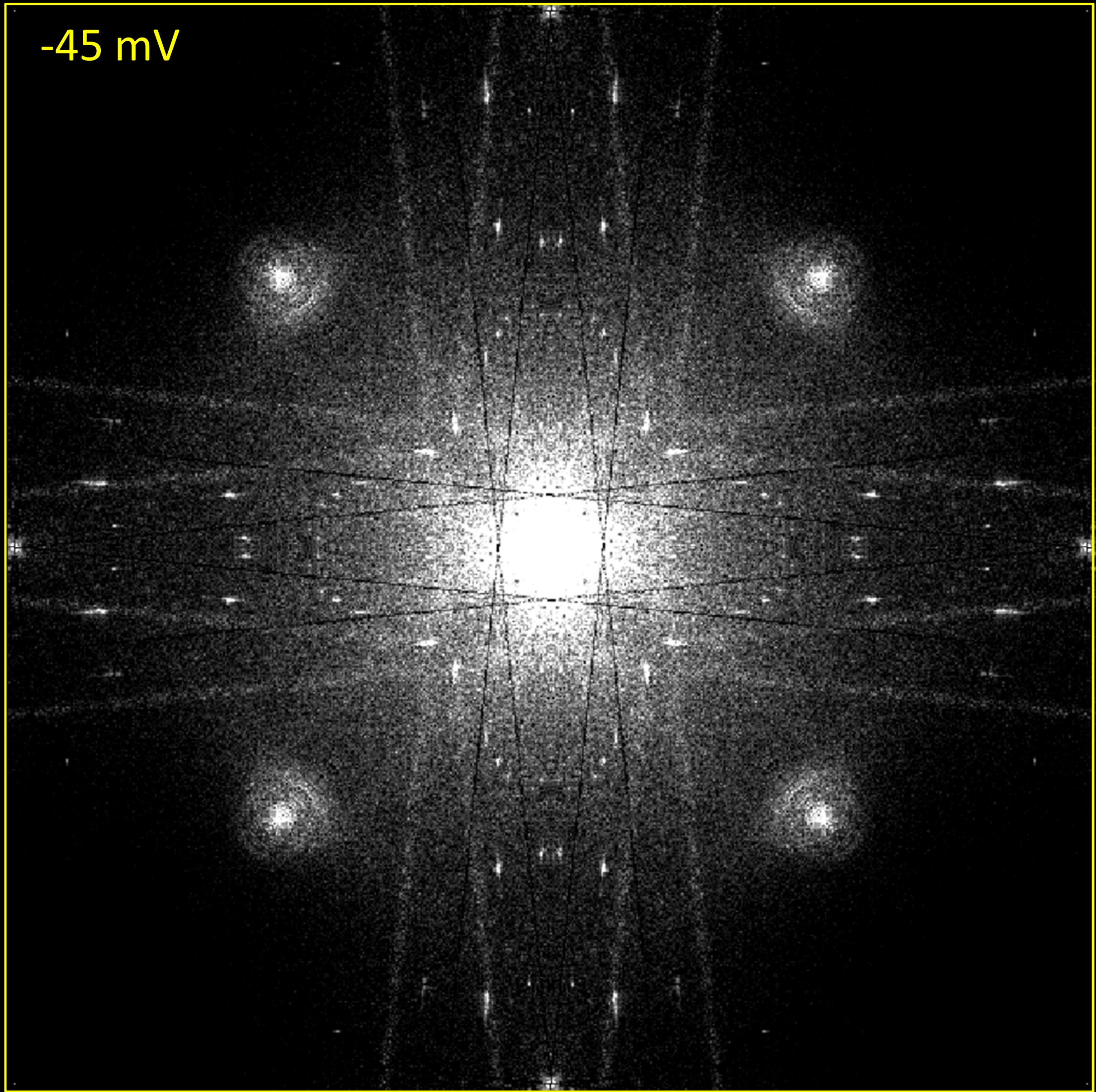


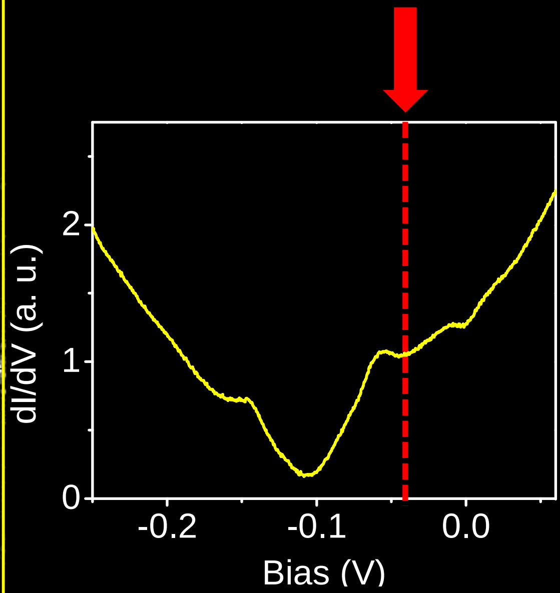
QPI Fourier transform patterns



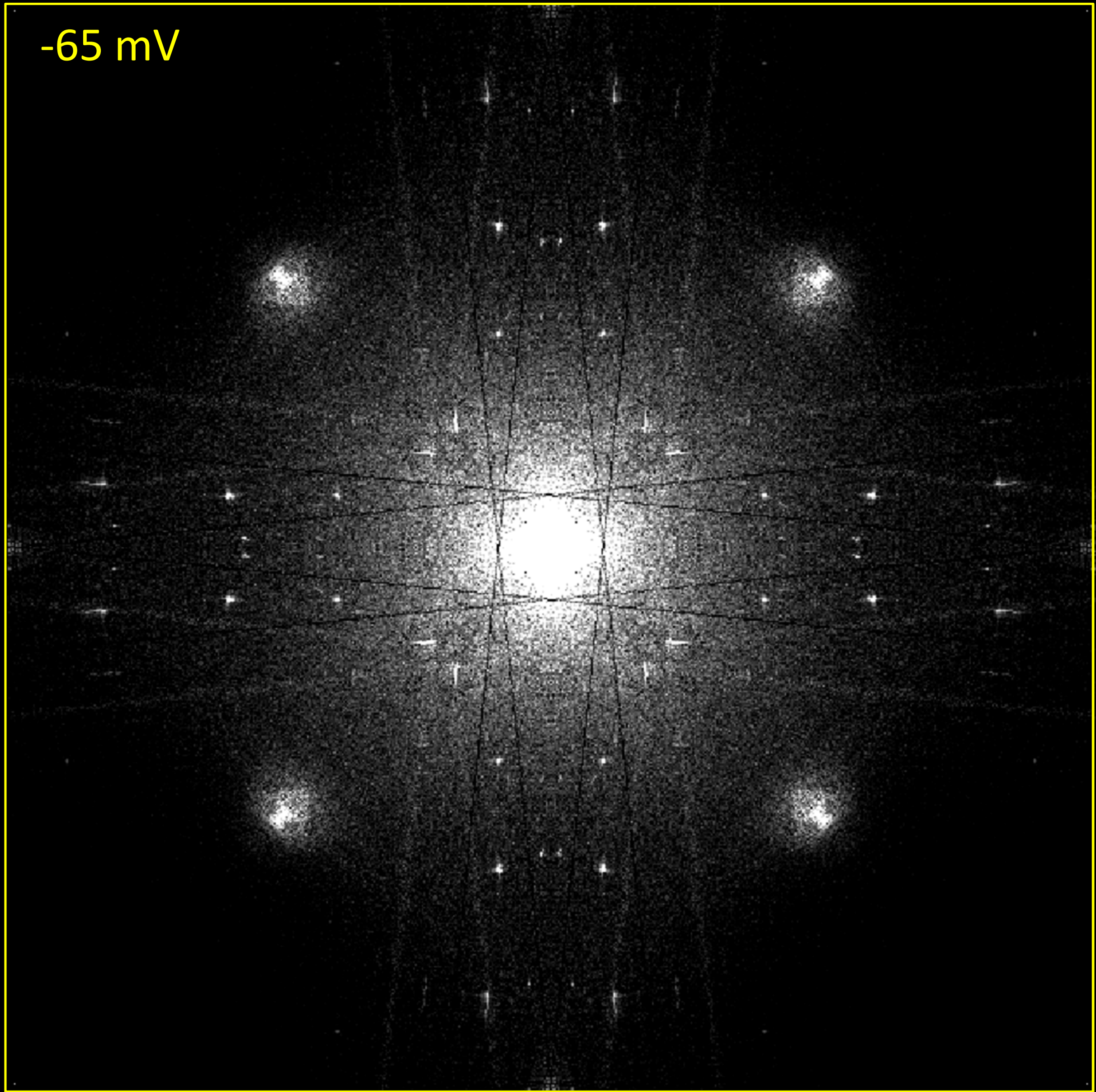


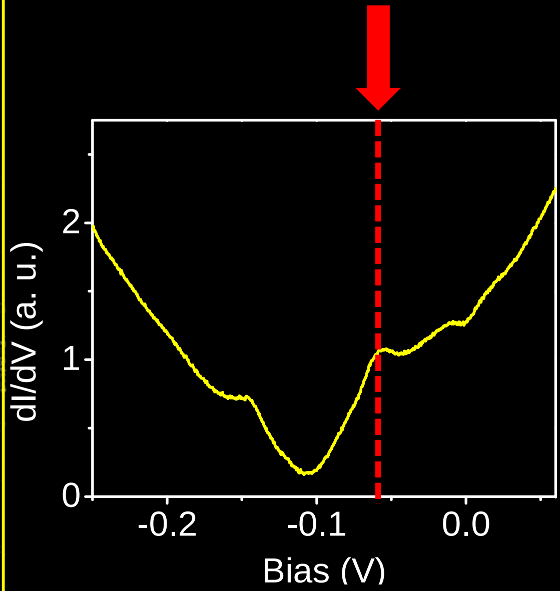
QPI Fourier transform patterns



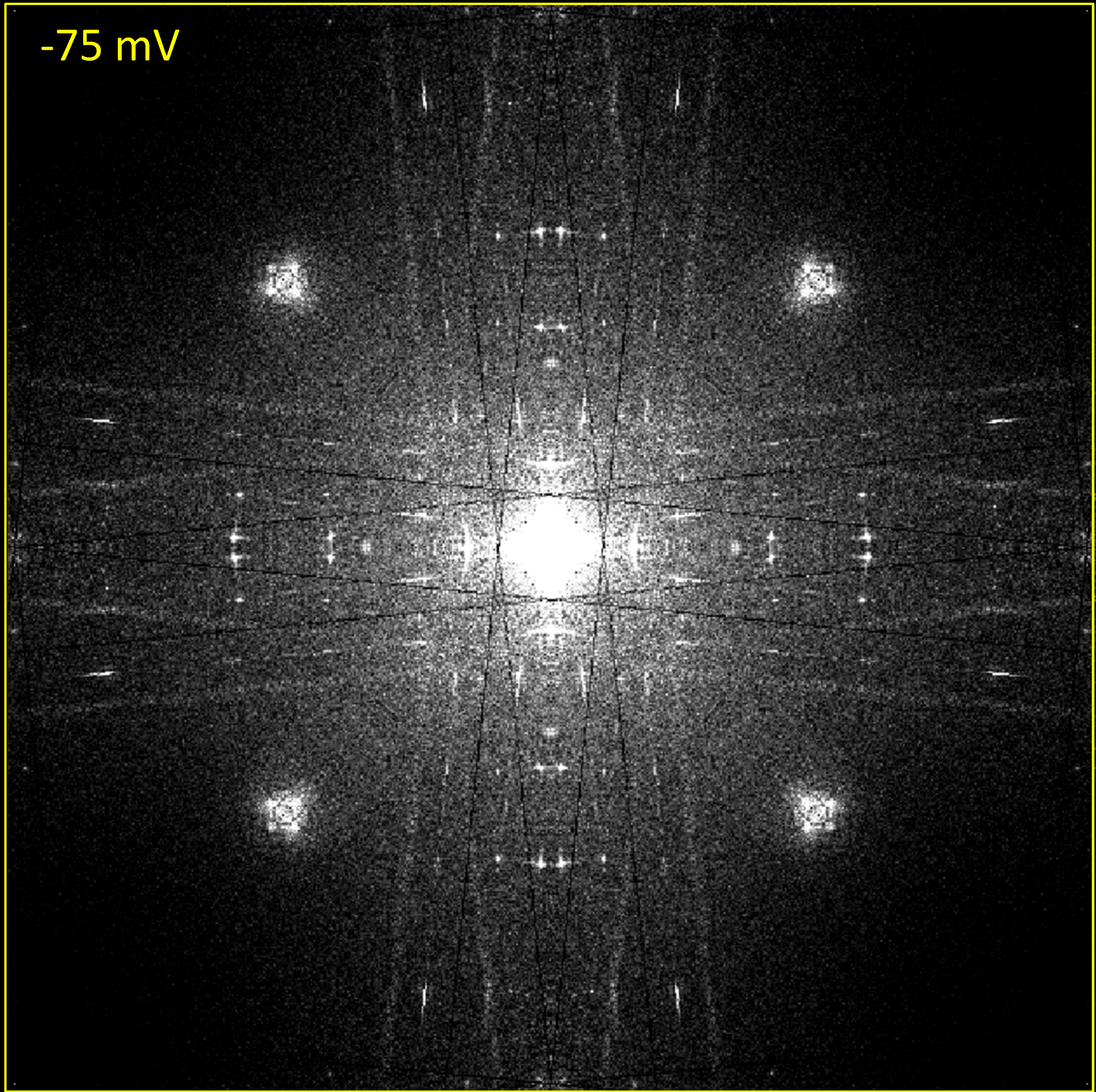


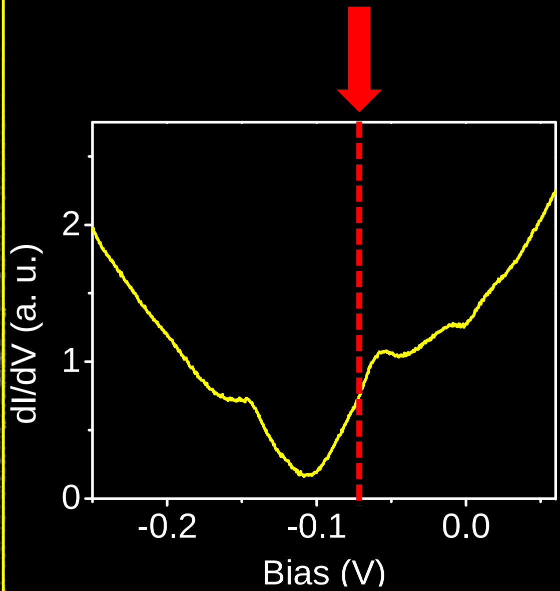
QPI Fourier transform patterns



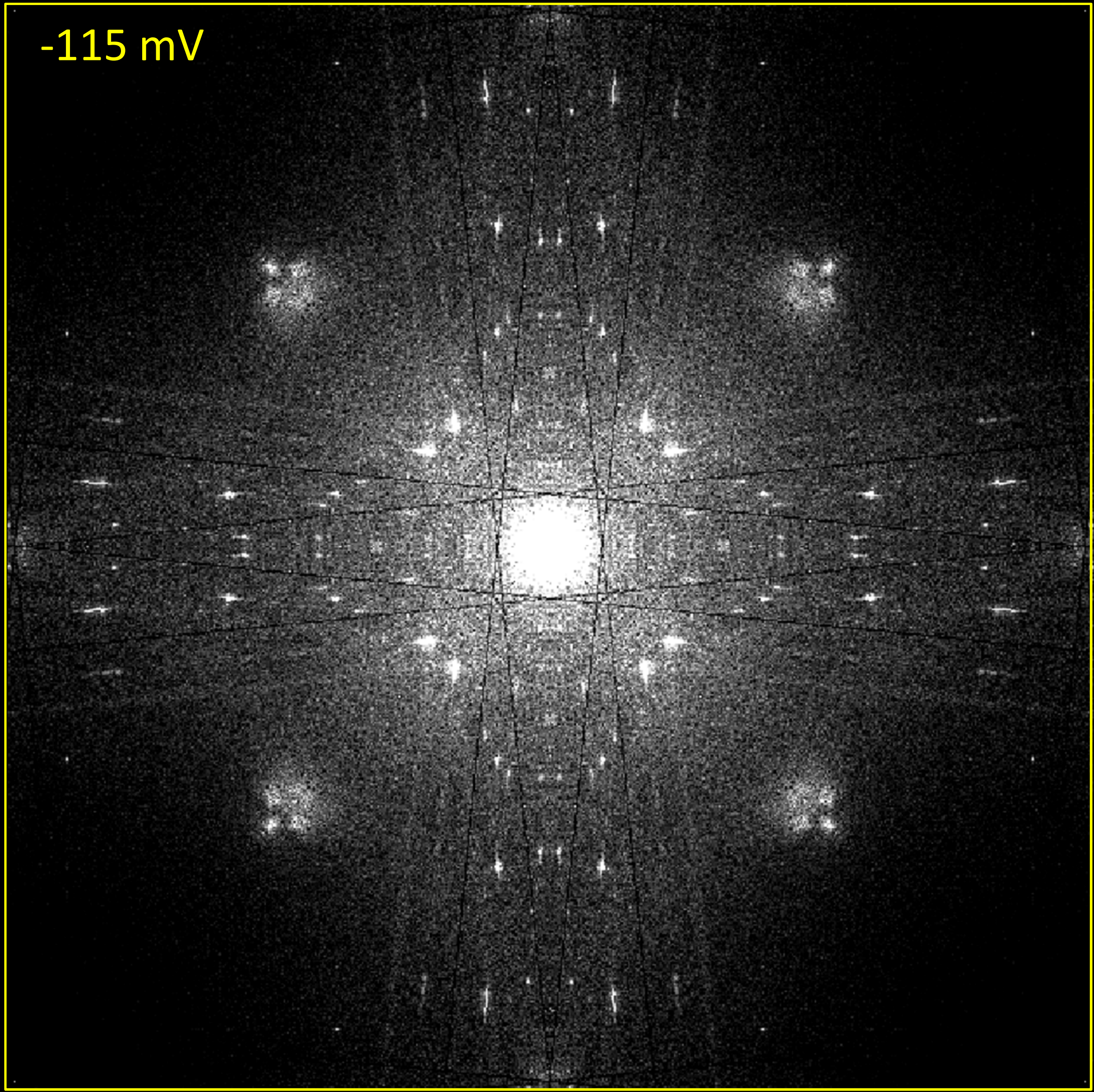


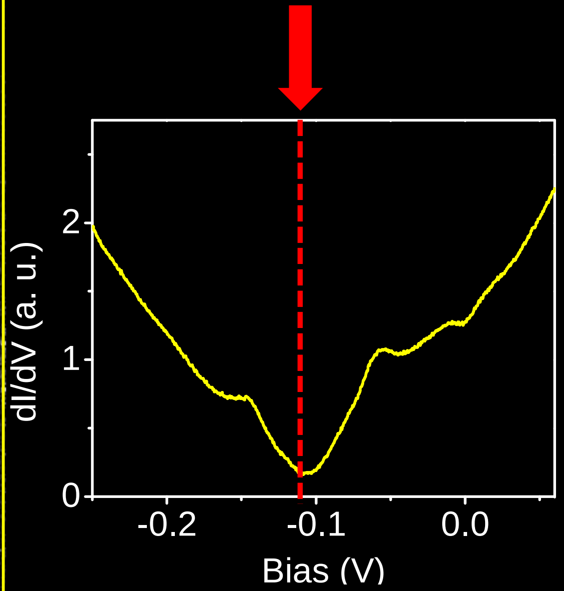
QPI Fourier transform patterns



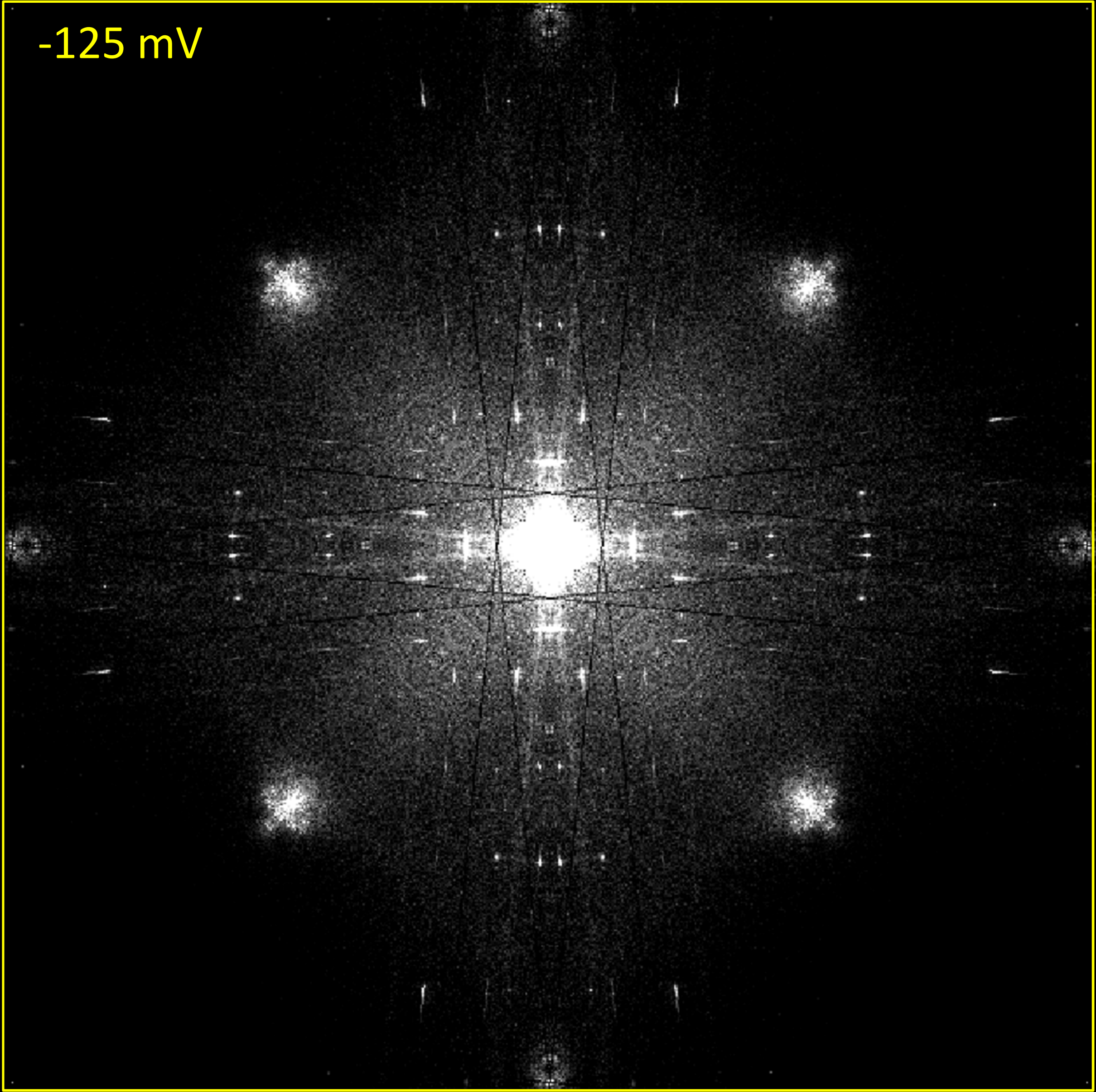


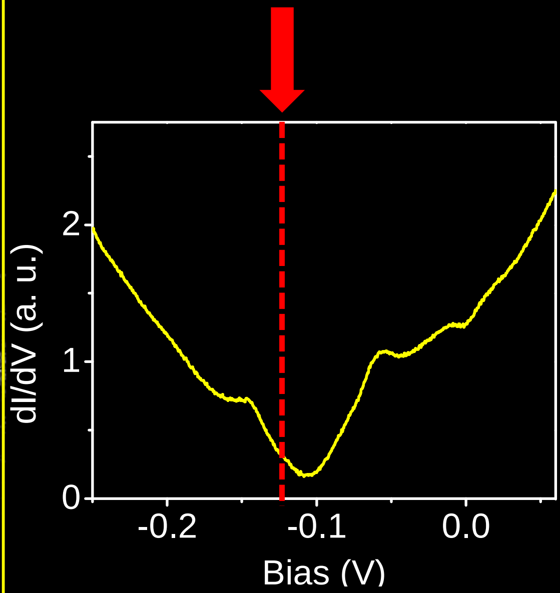
QPI Fourier transform patterns



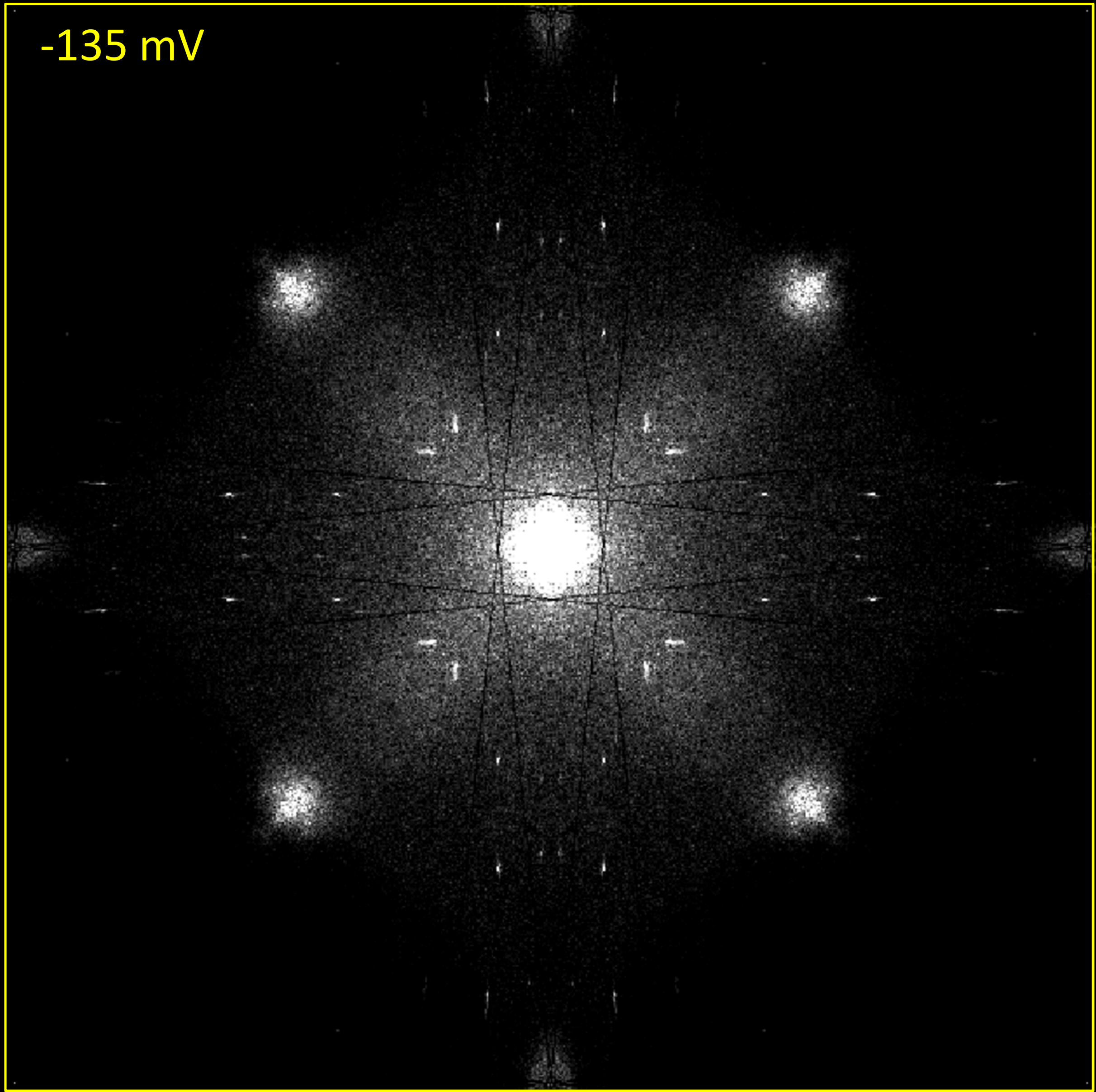


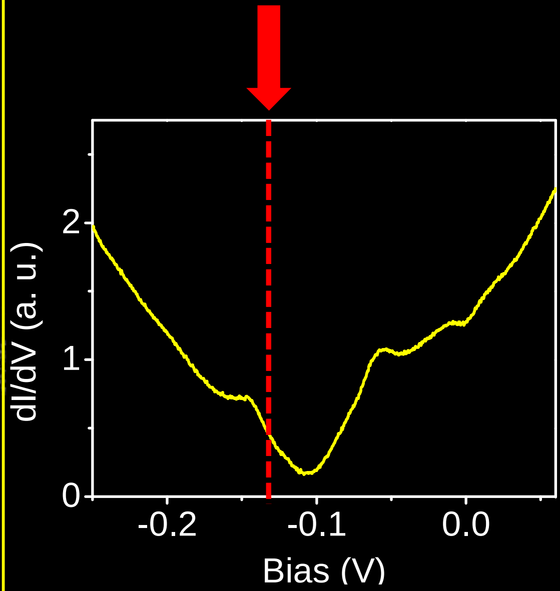
QPI Fourier transform patterns



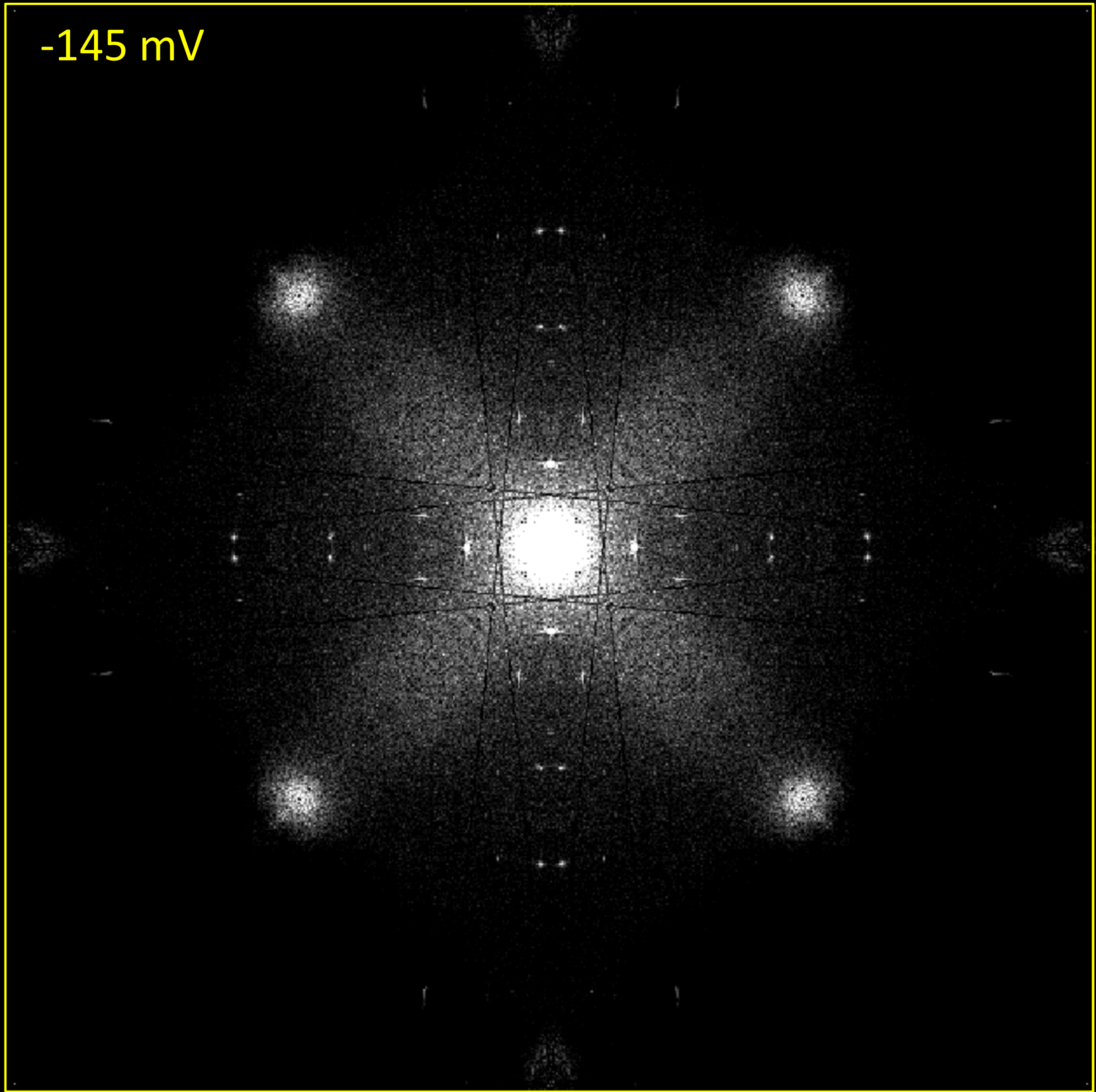


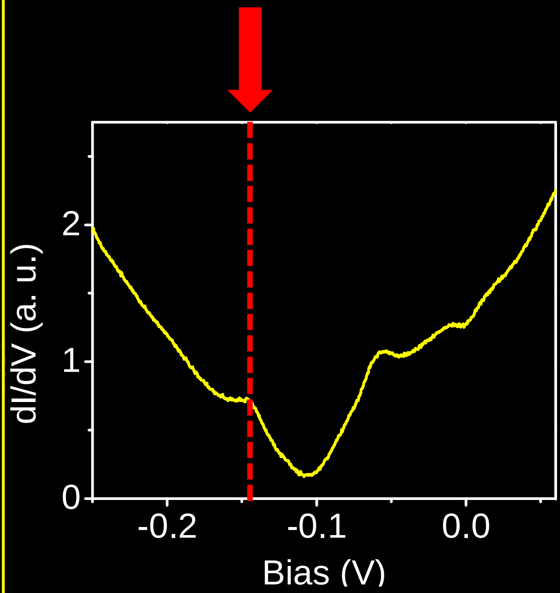
QPI Fourier transform patterns



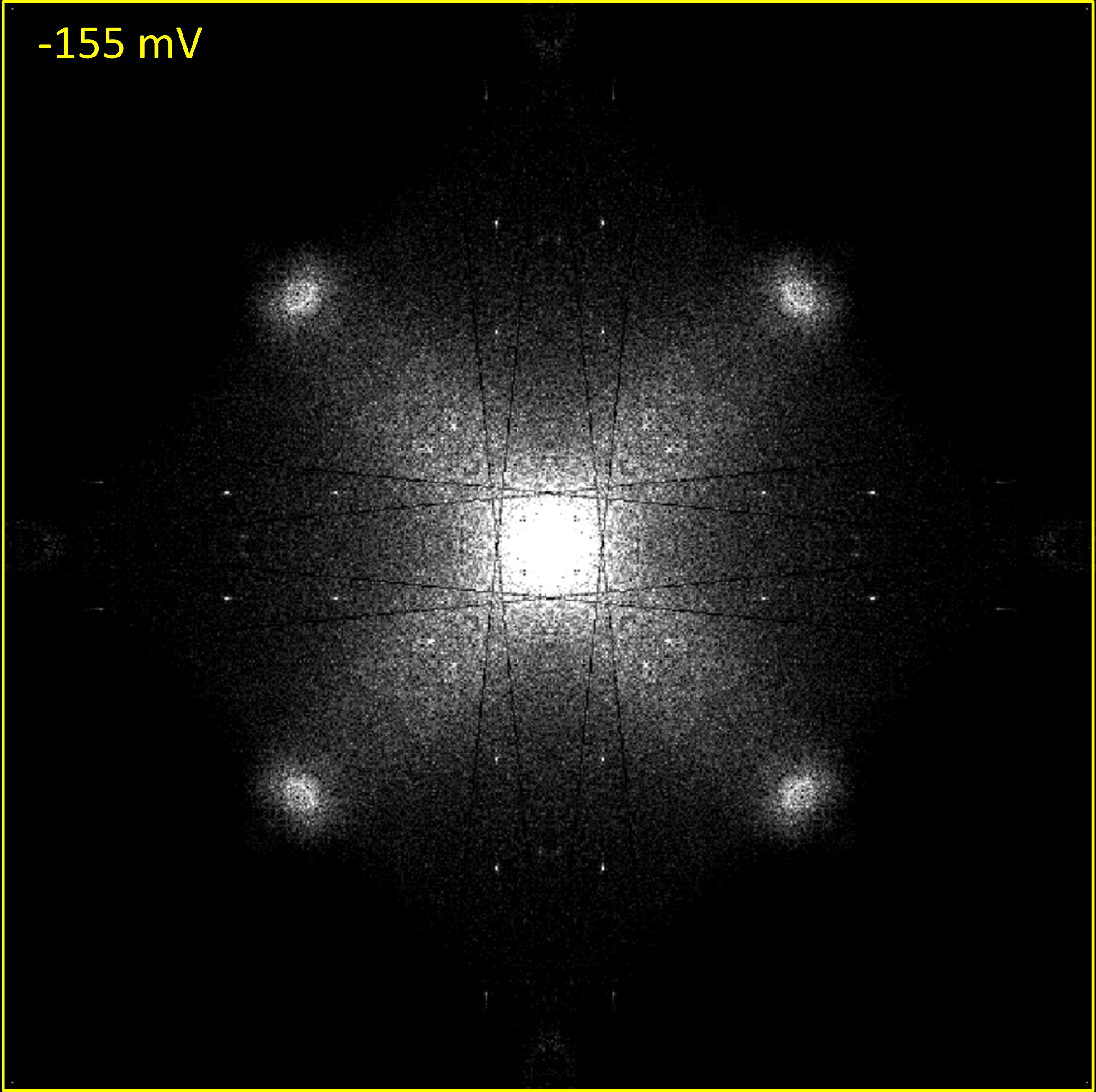


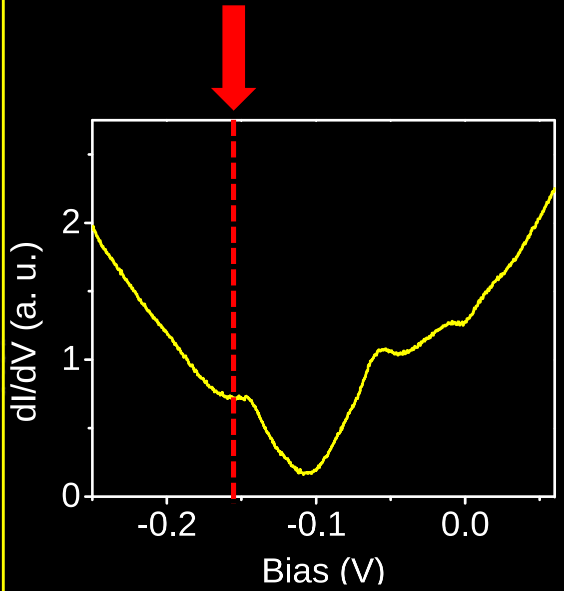
QPI Fourier transform patterns



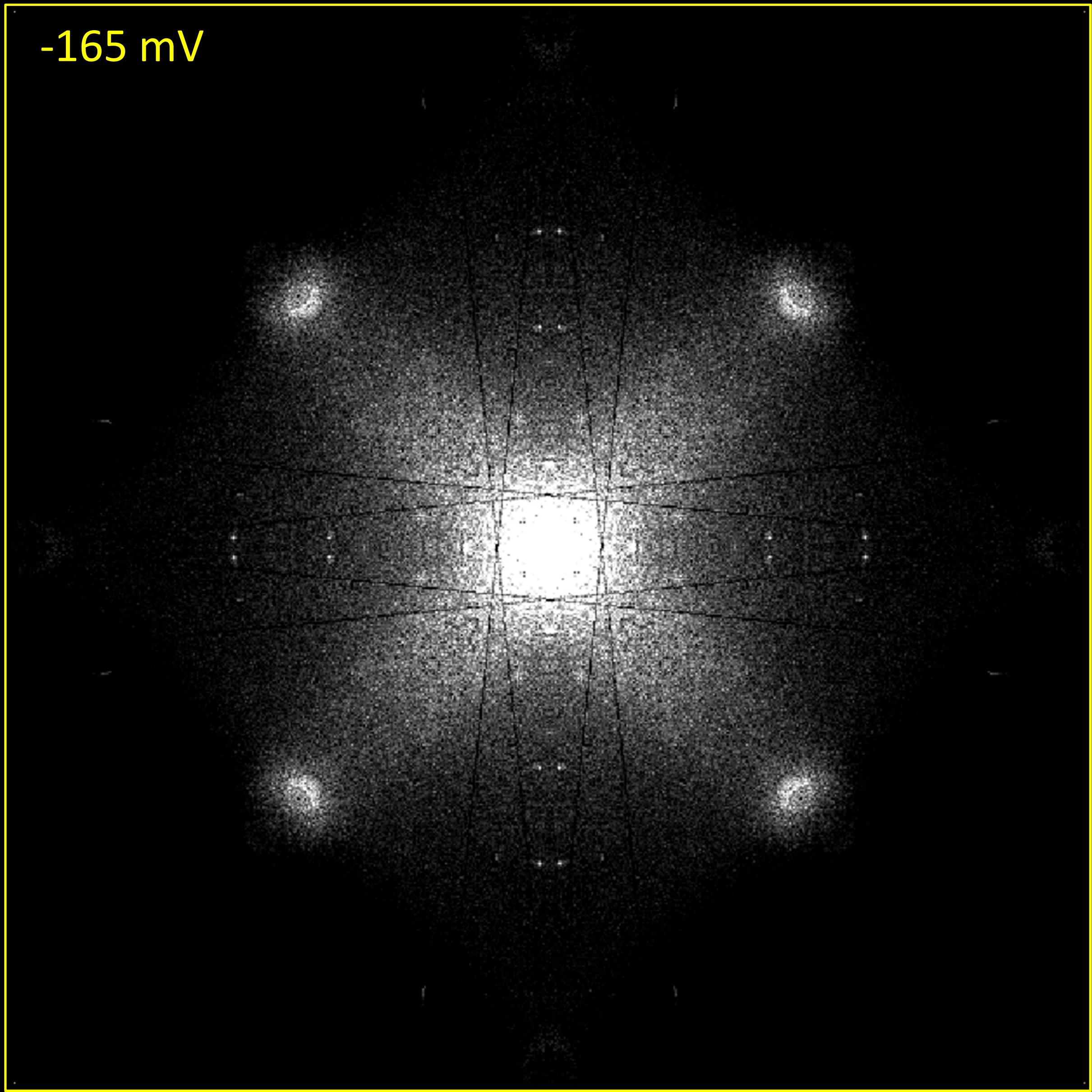


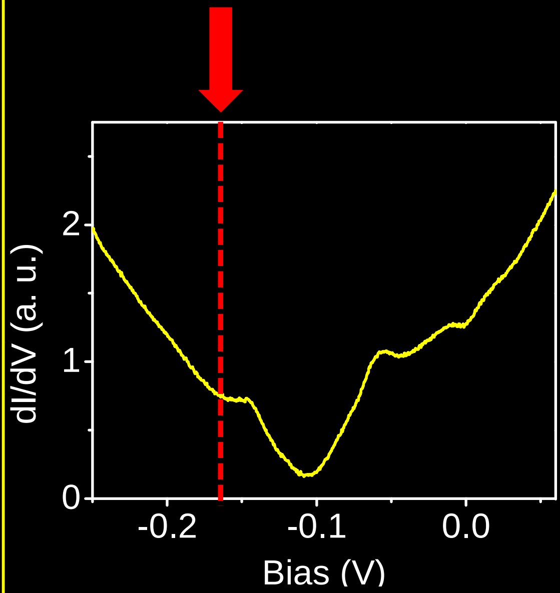
QPI Fourier transform patterns



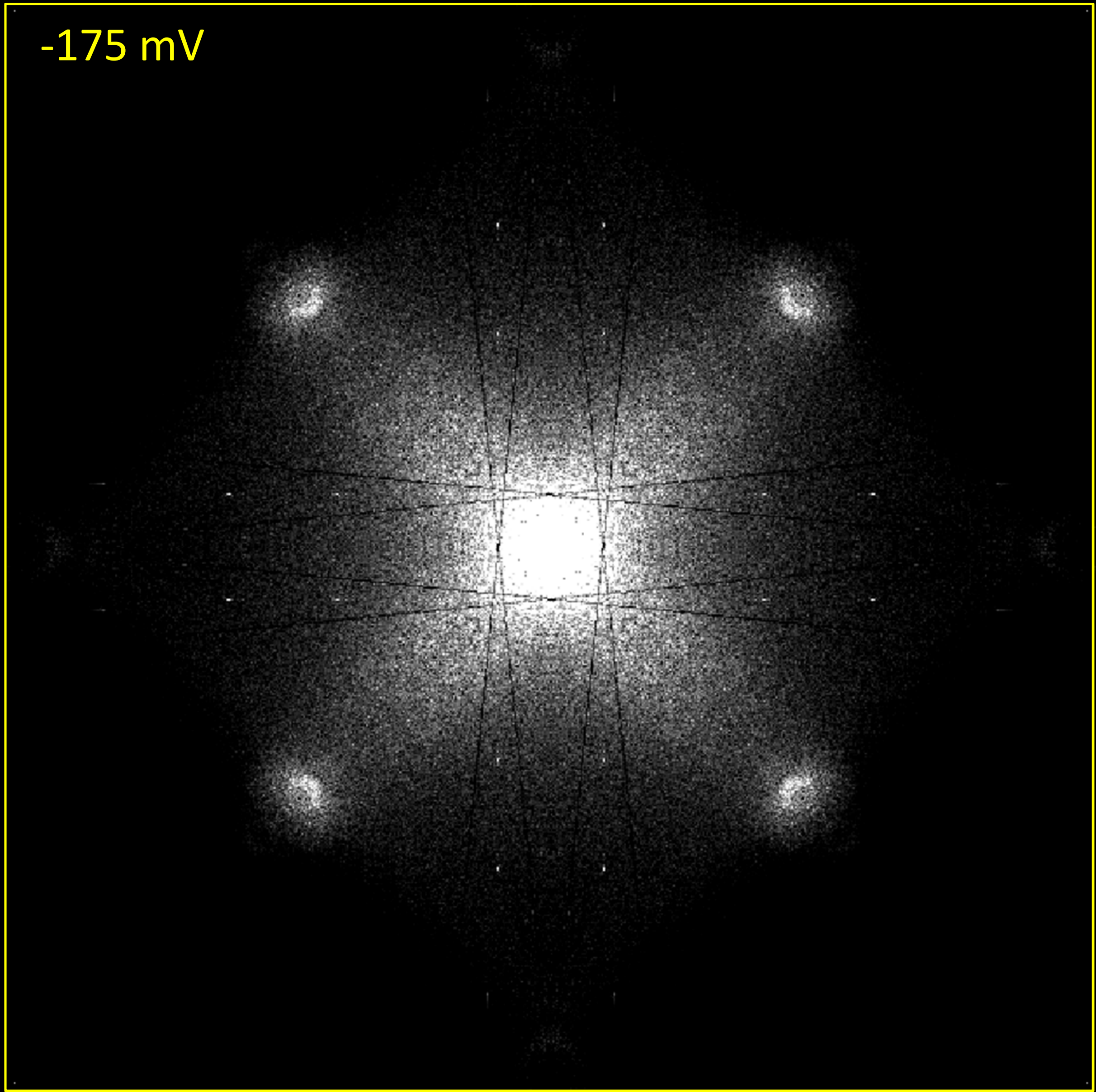


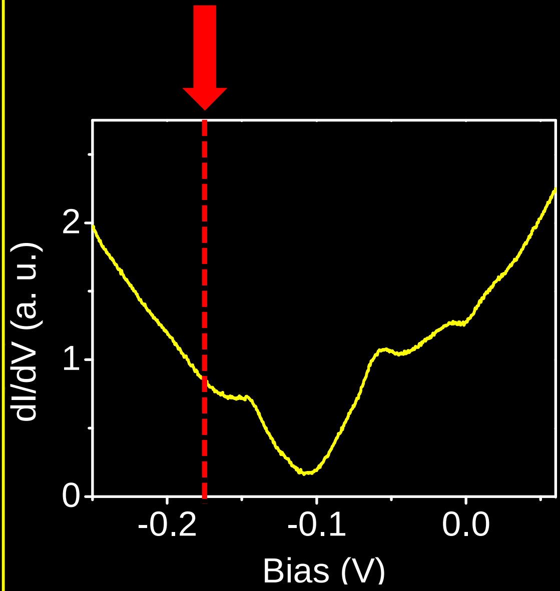
QPI Fourier transform patterns



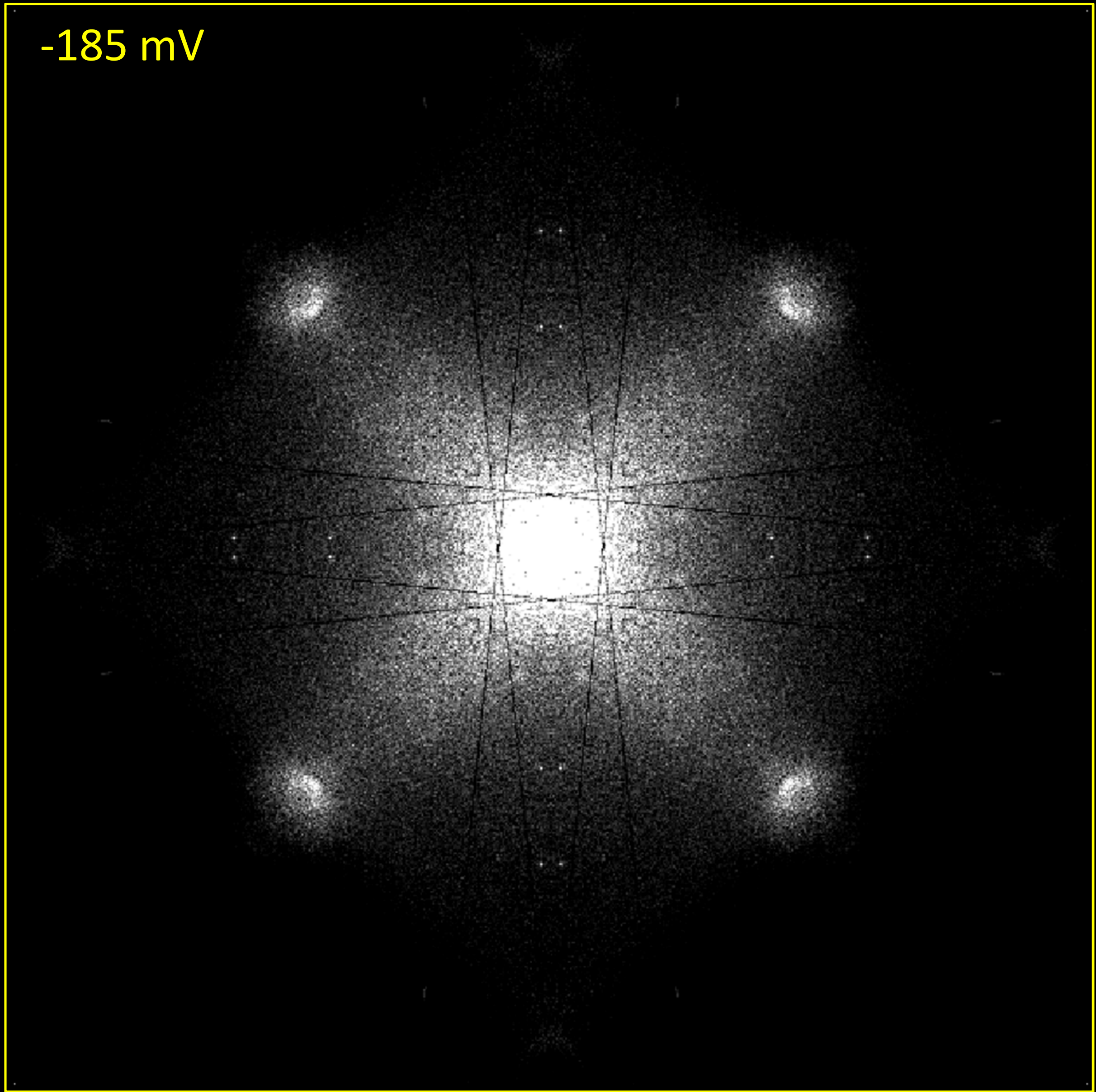


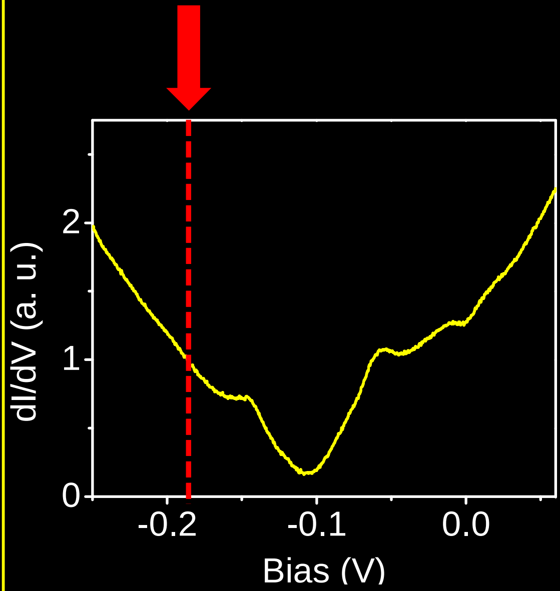
QPI Fourier transform patterns



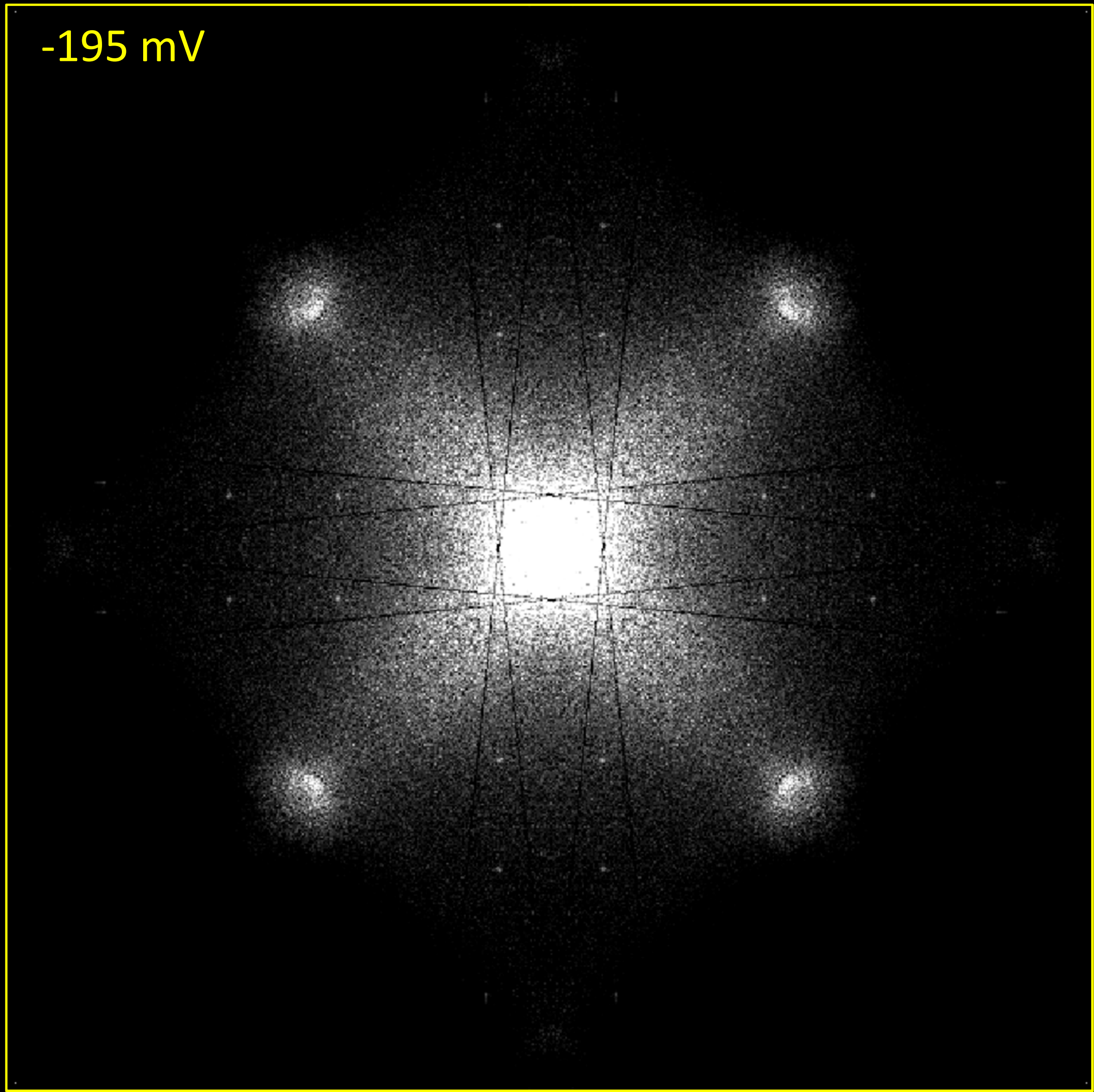


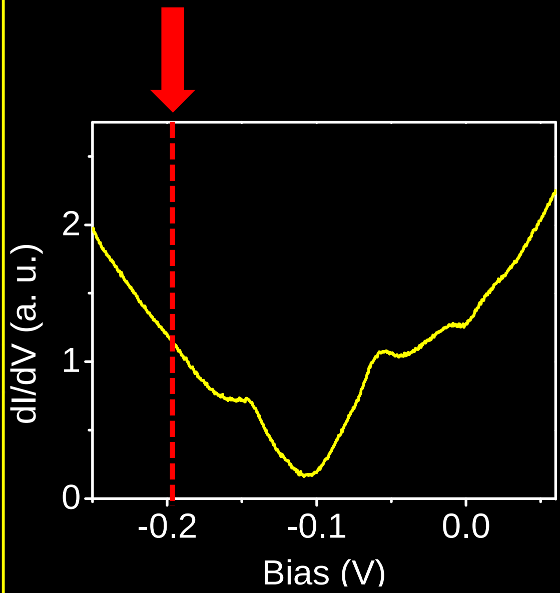
QPI Fourier transform patterns





QPI Fourier transform patterns





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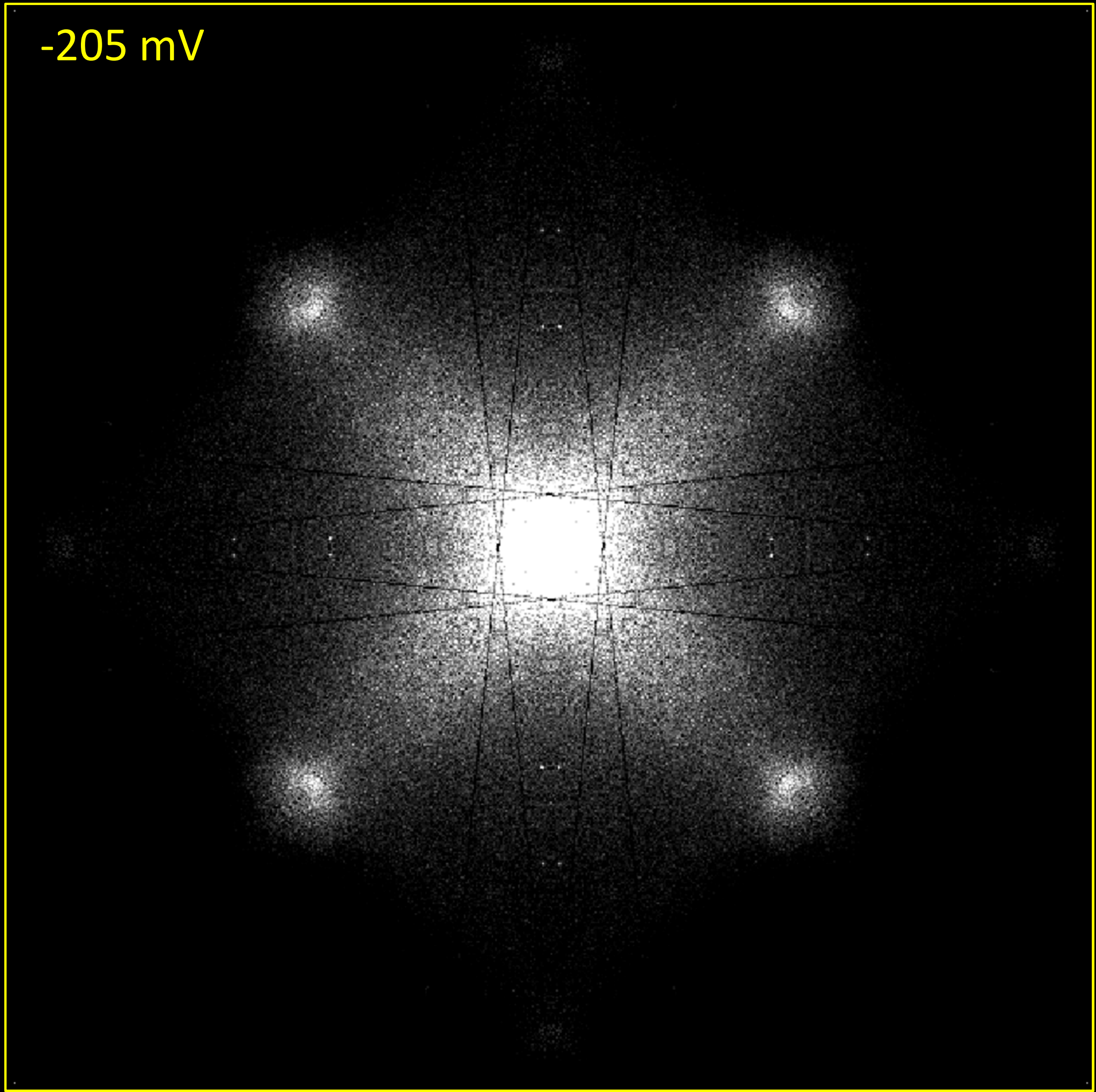

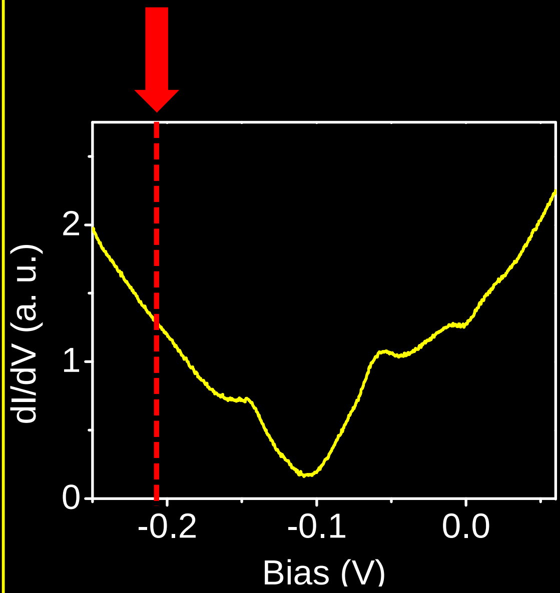

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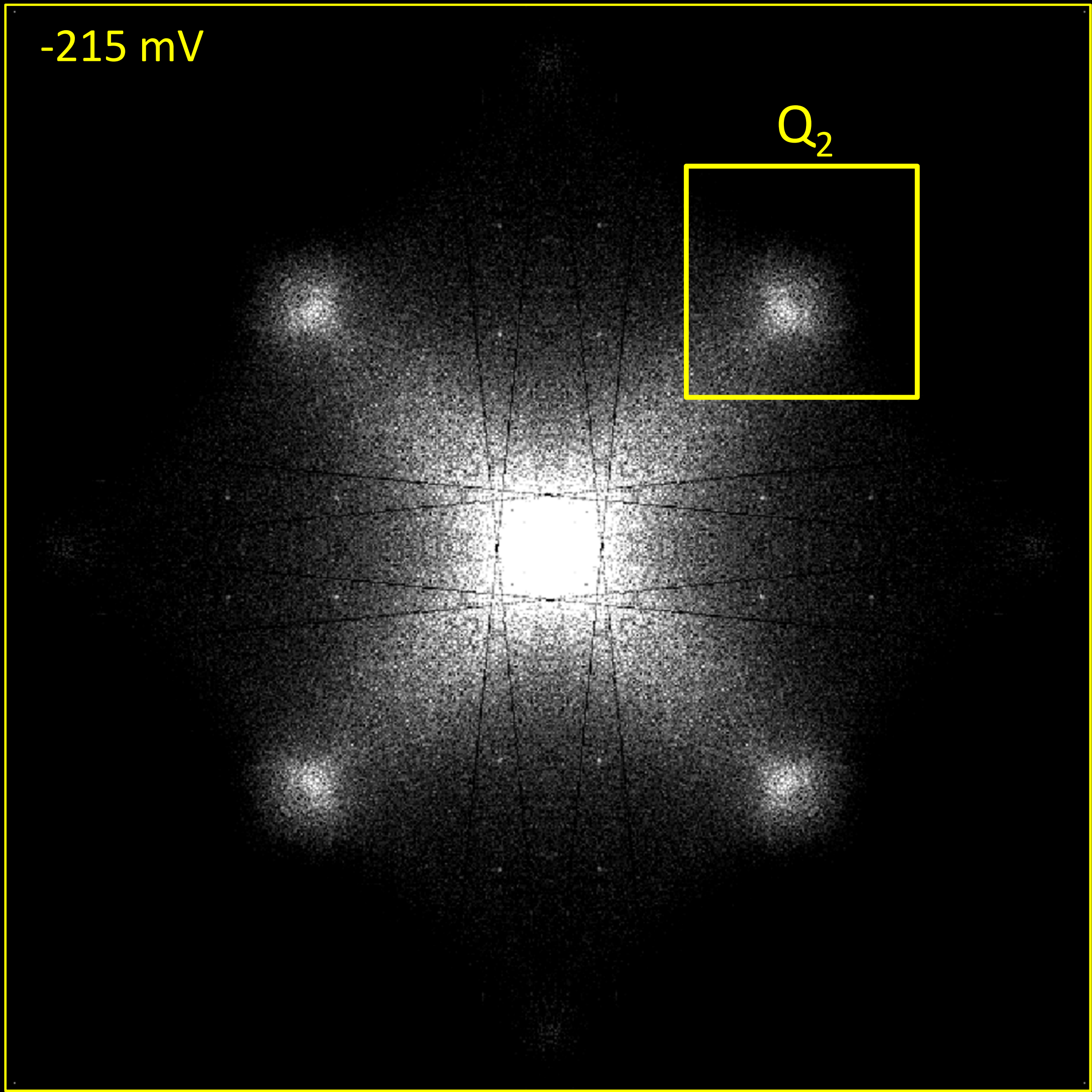

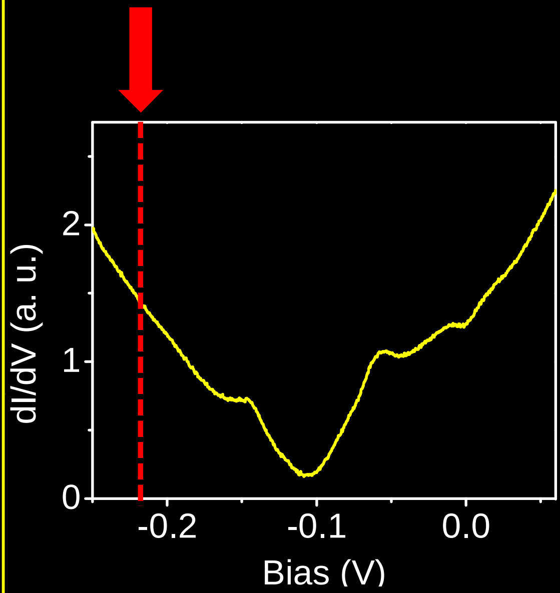

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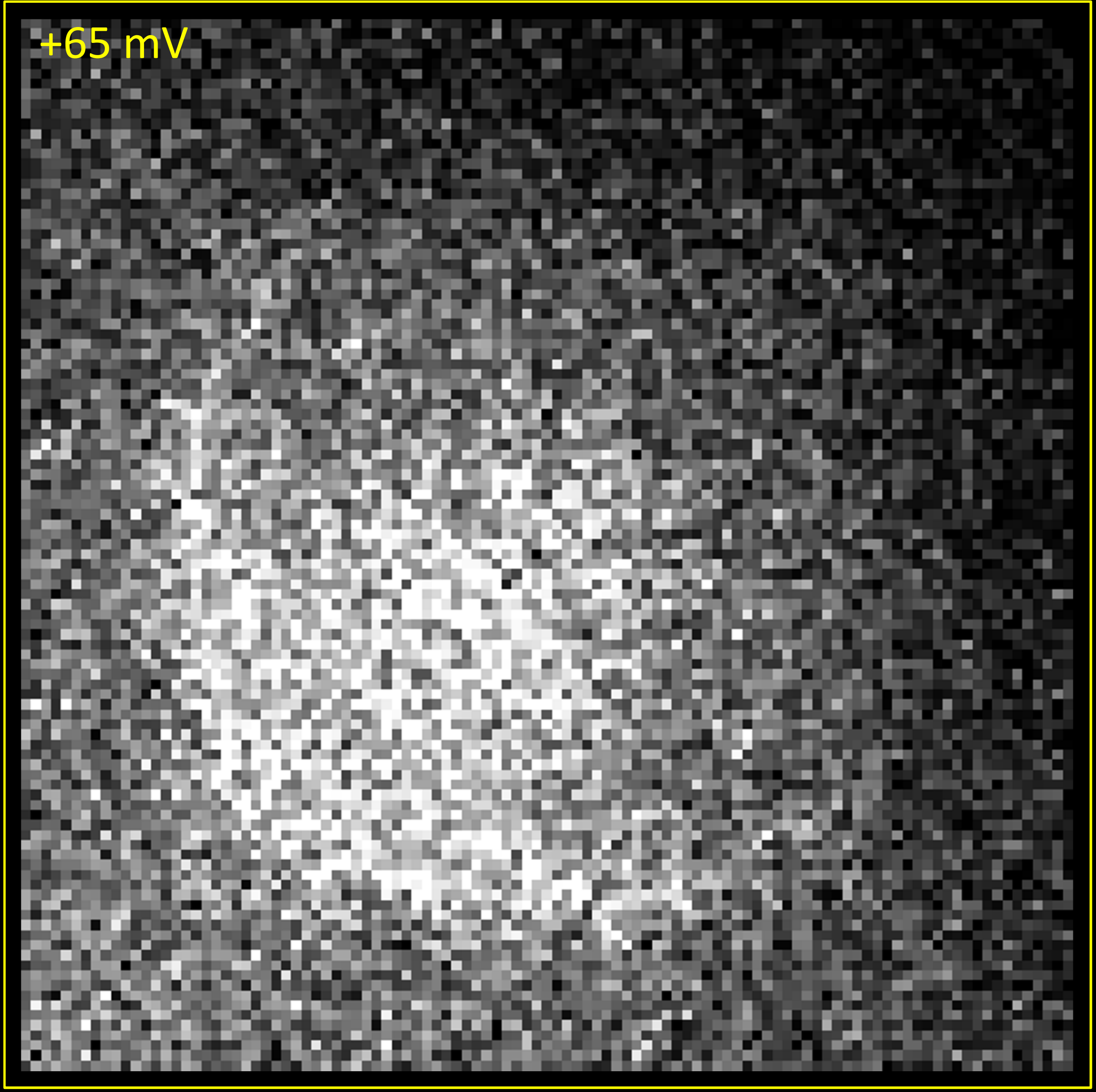

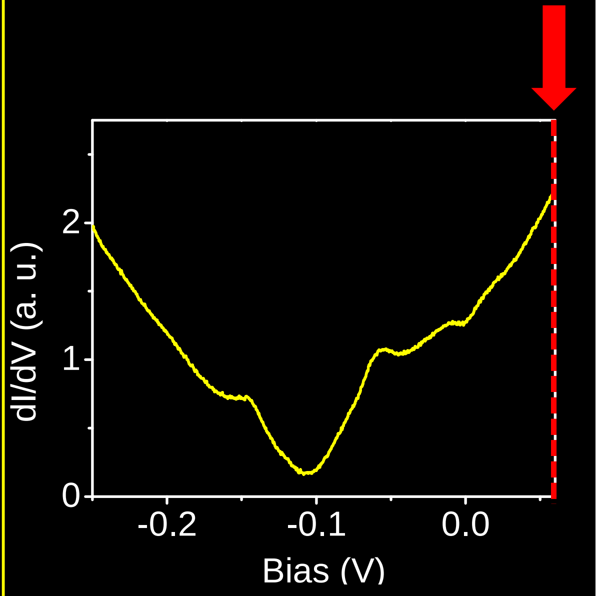

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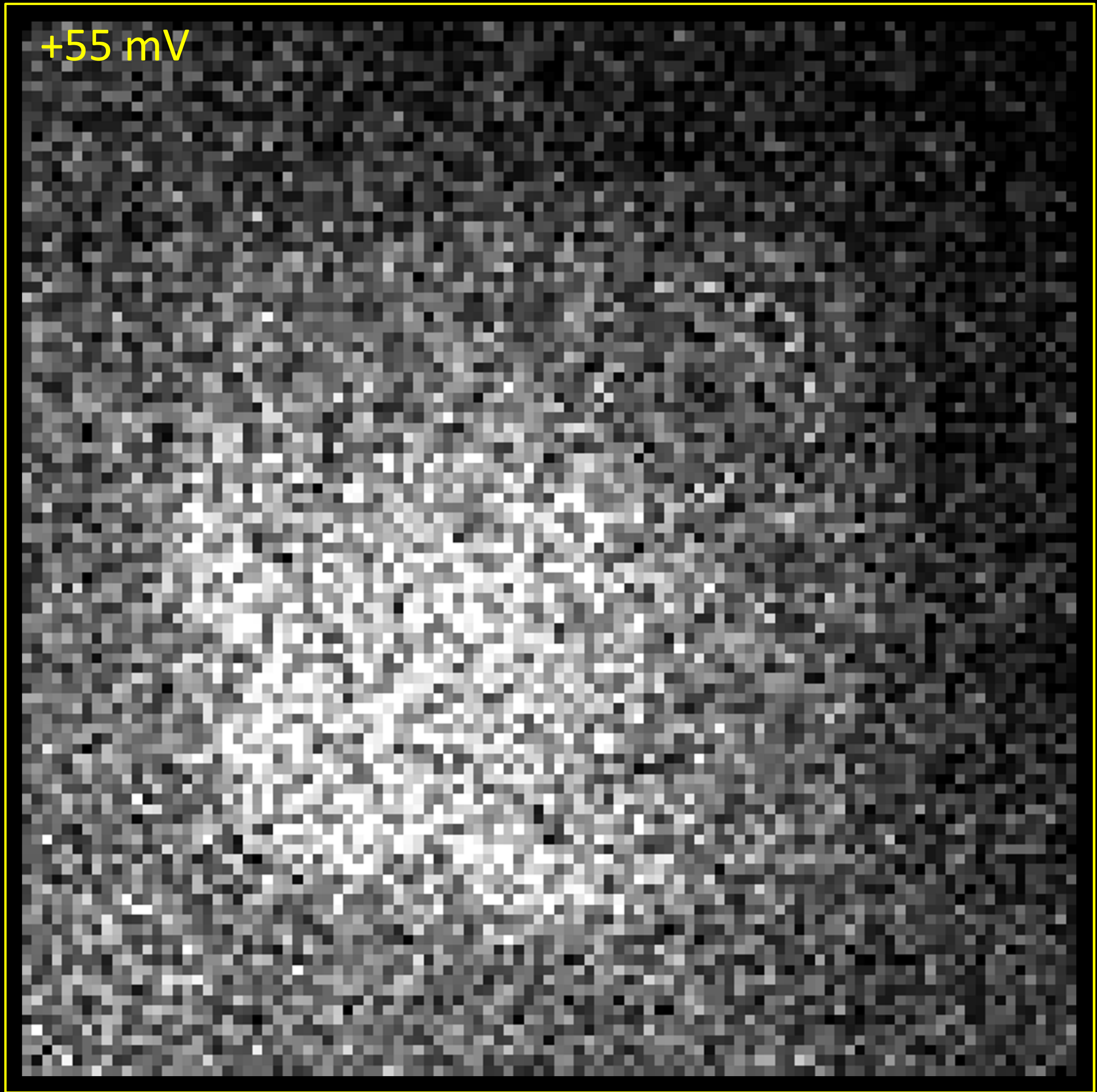

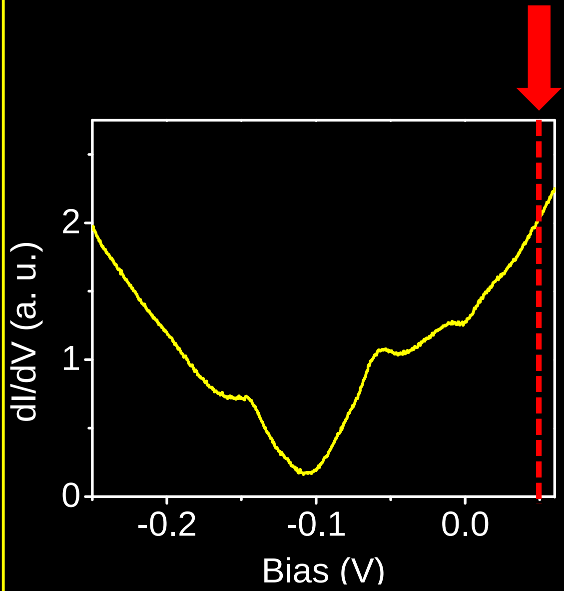

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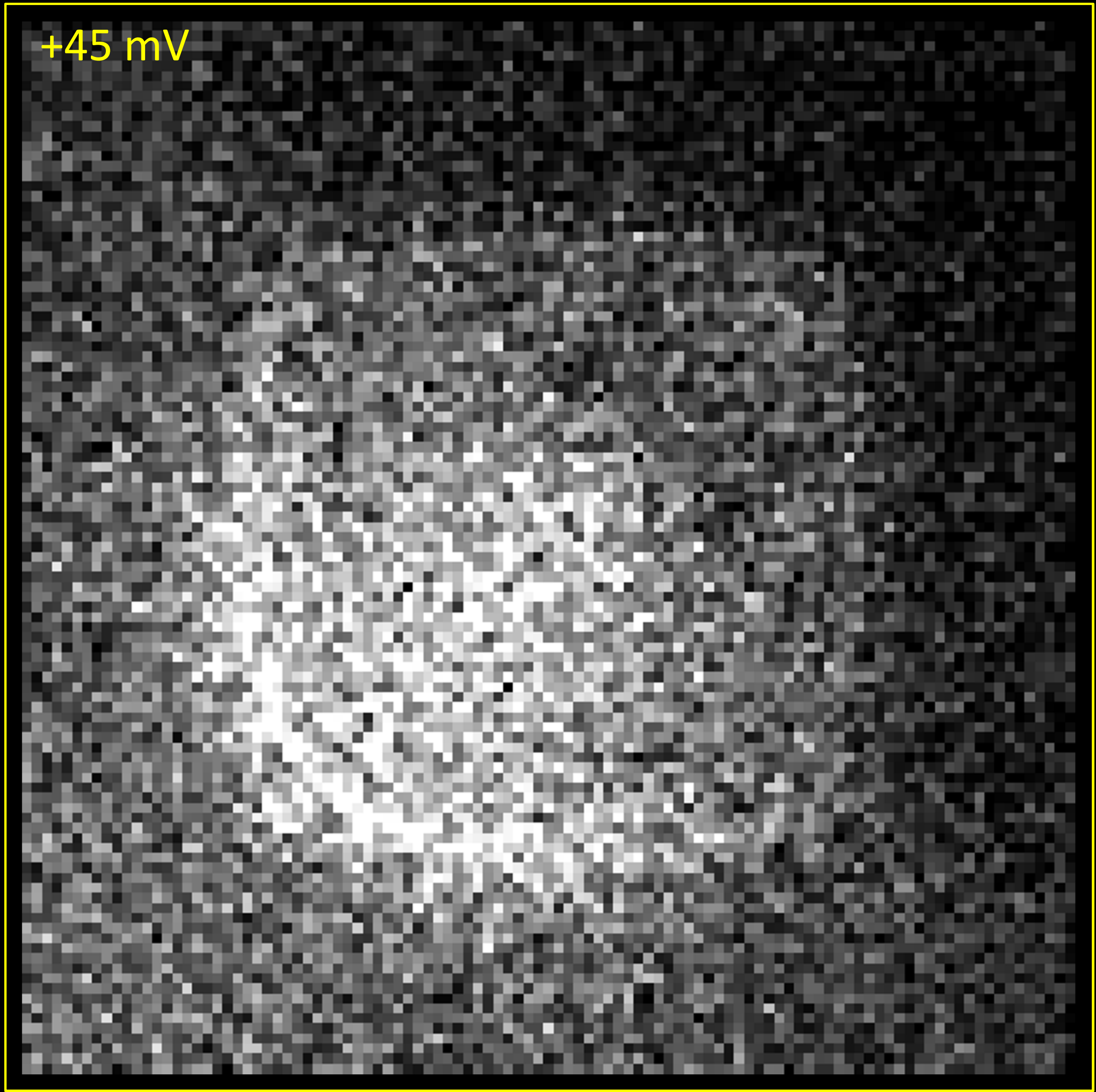

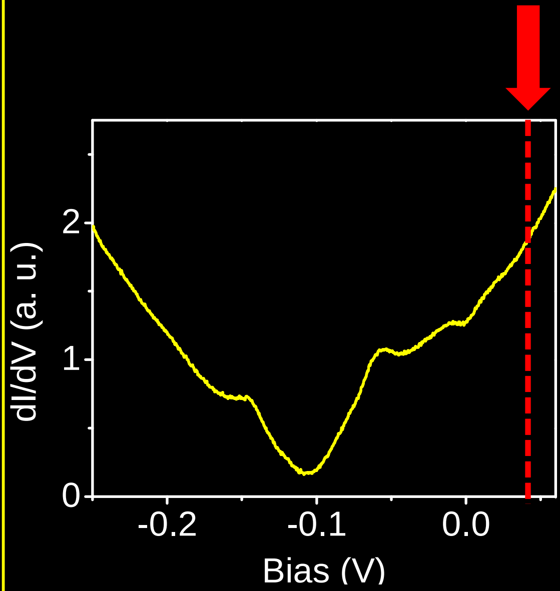

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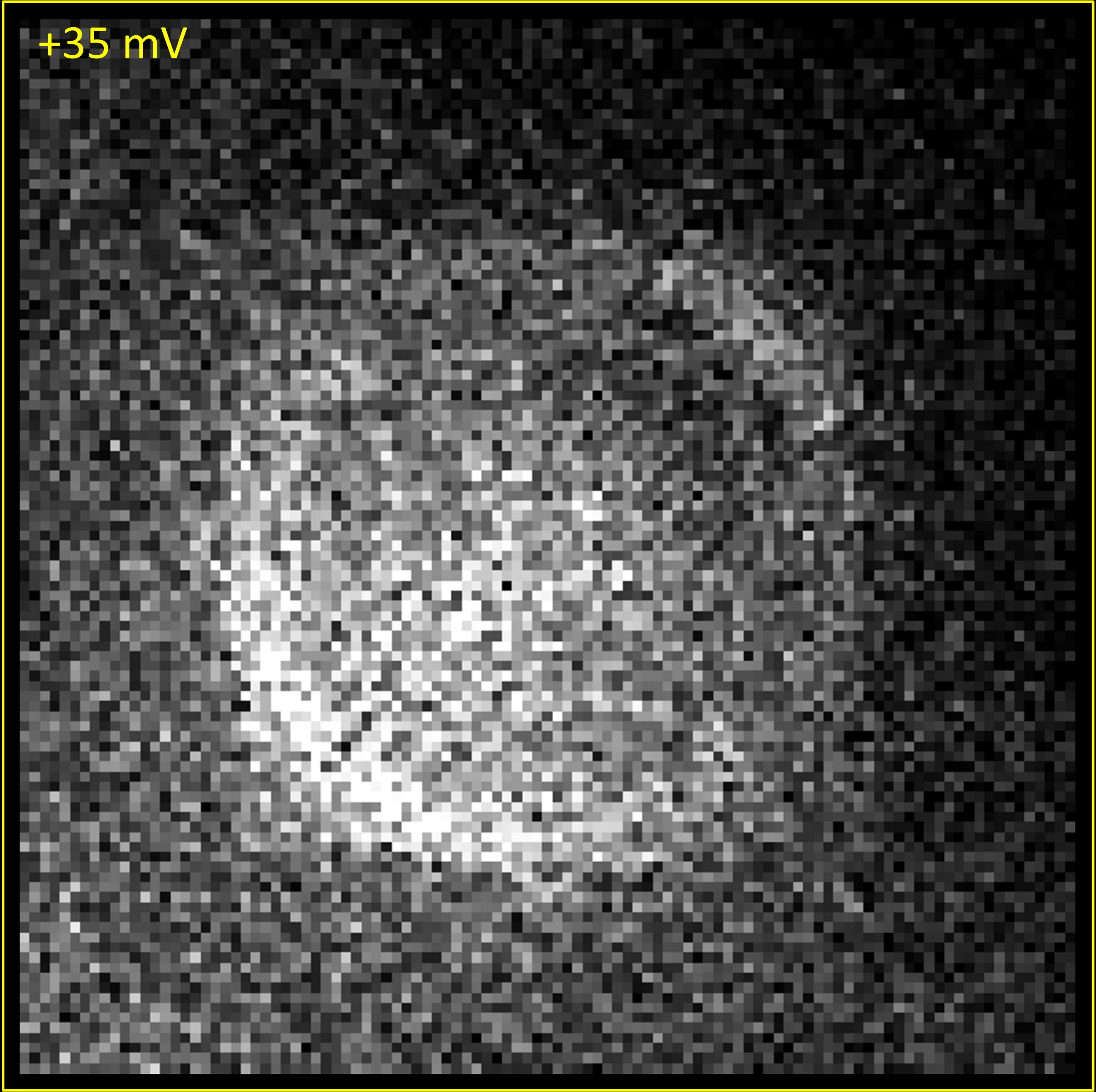
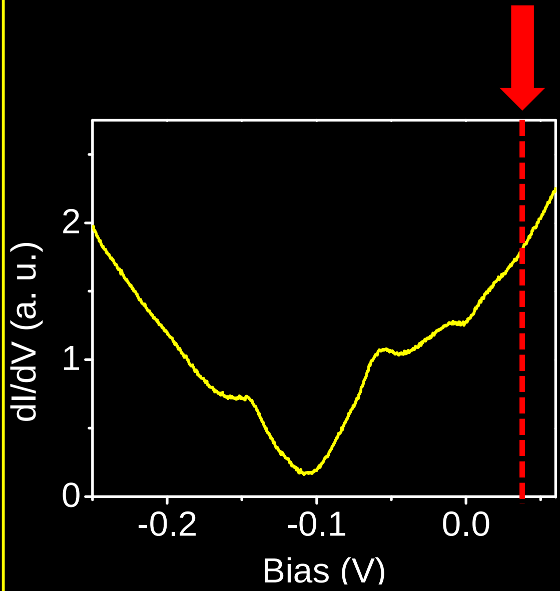
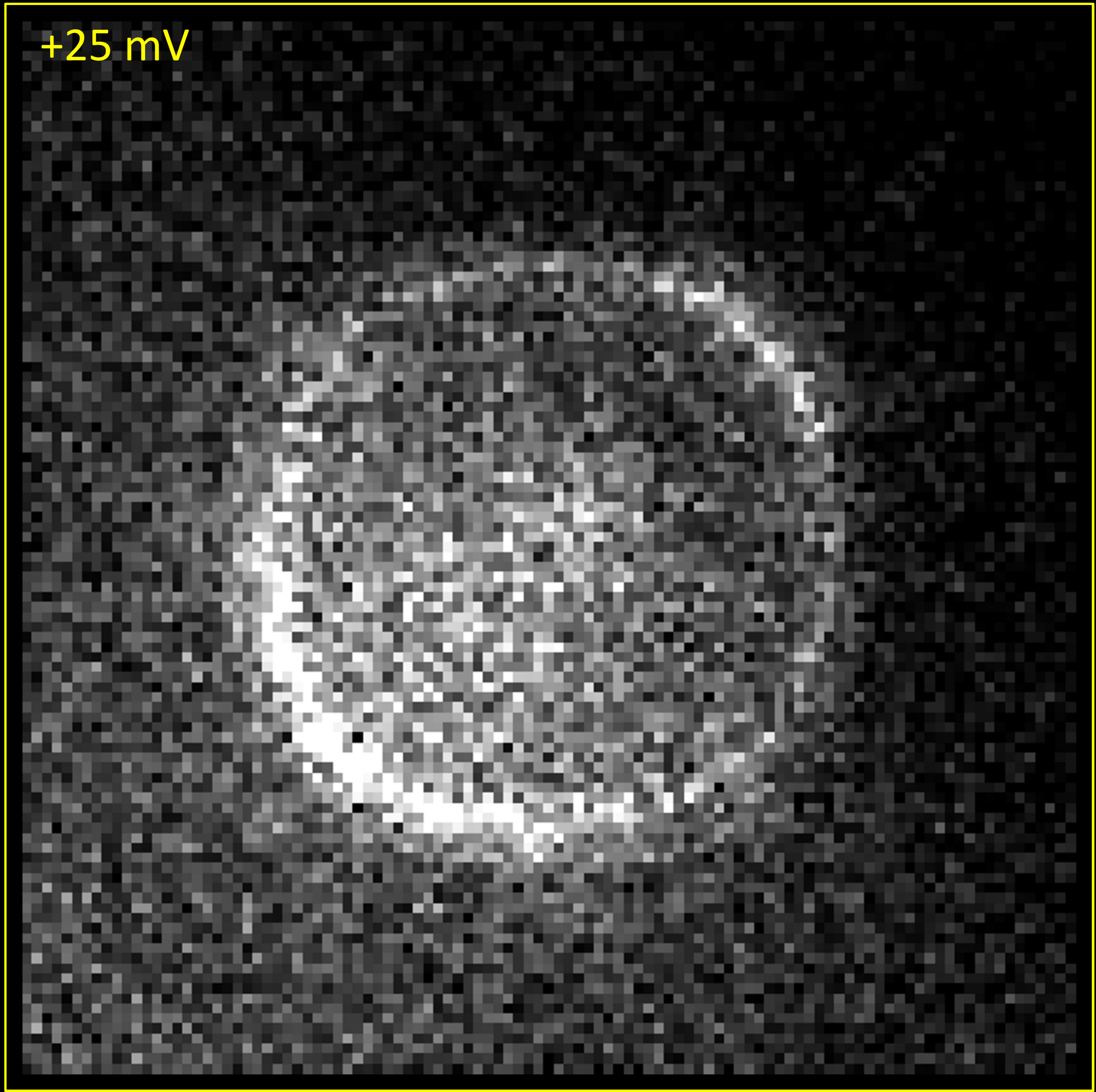
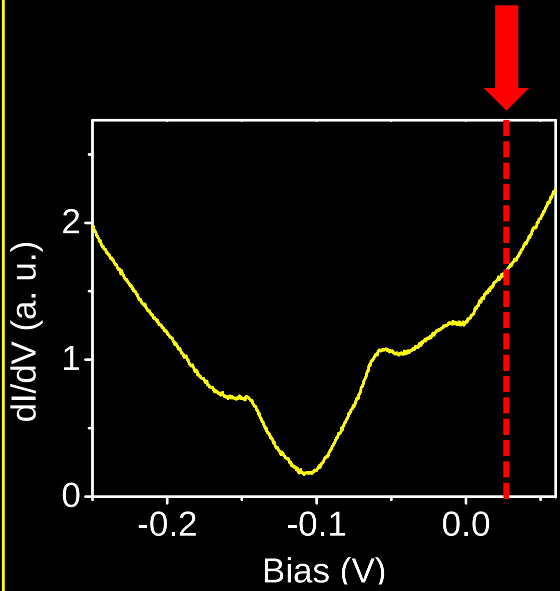




QPI Fourier transform patterns





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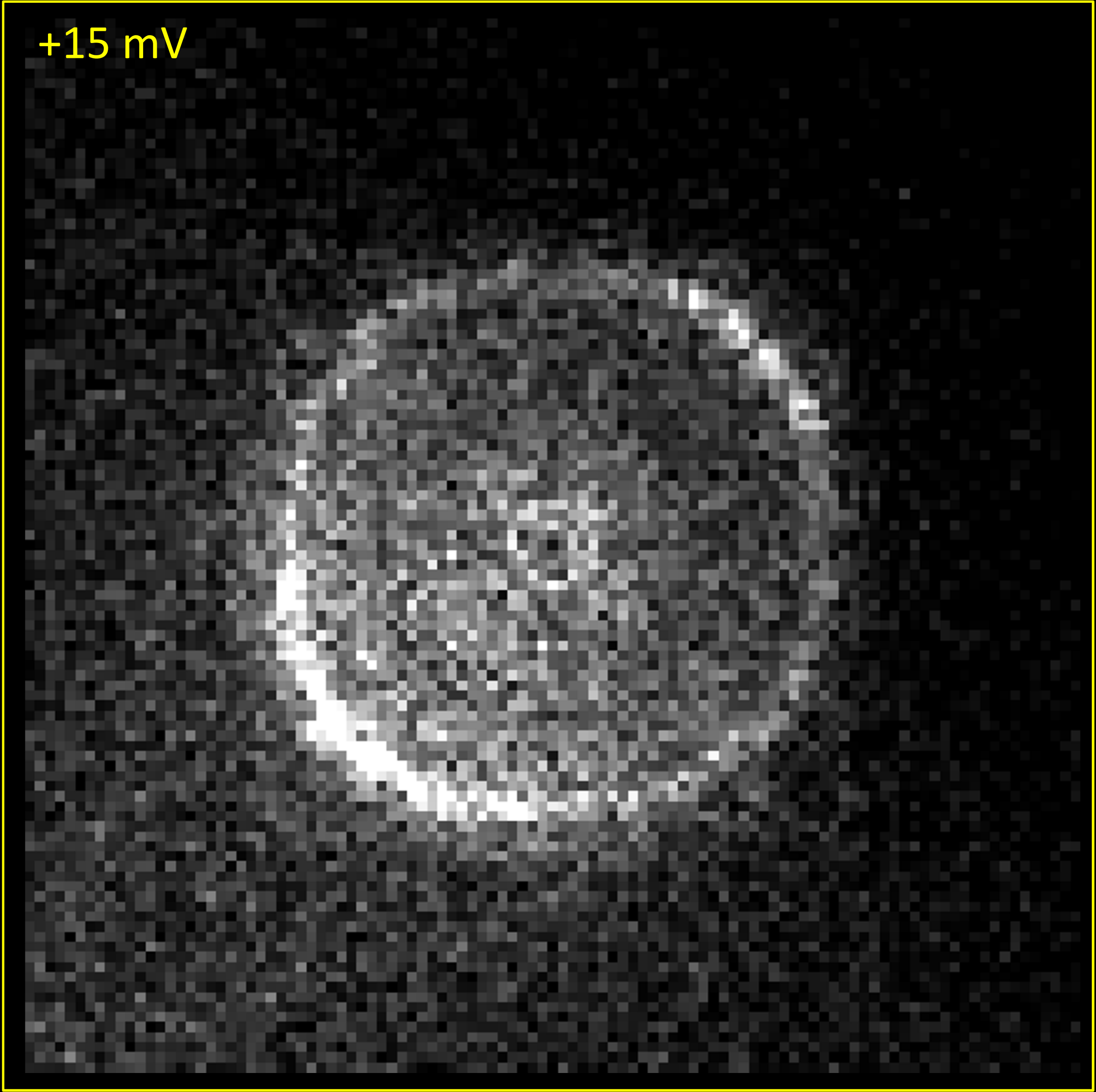

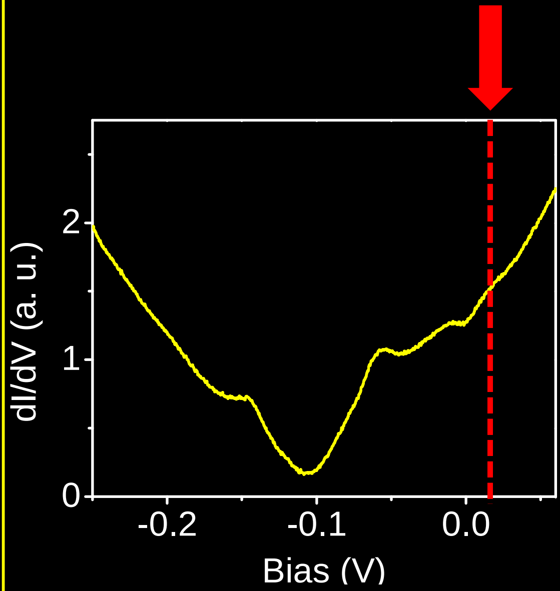

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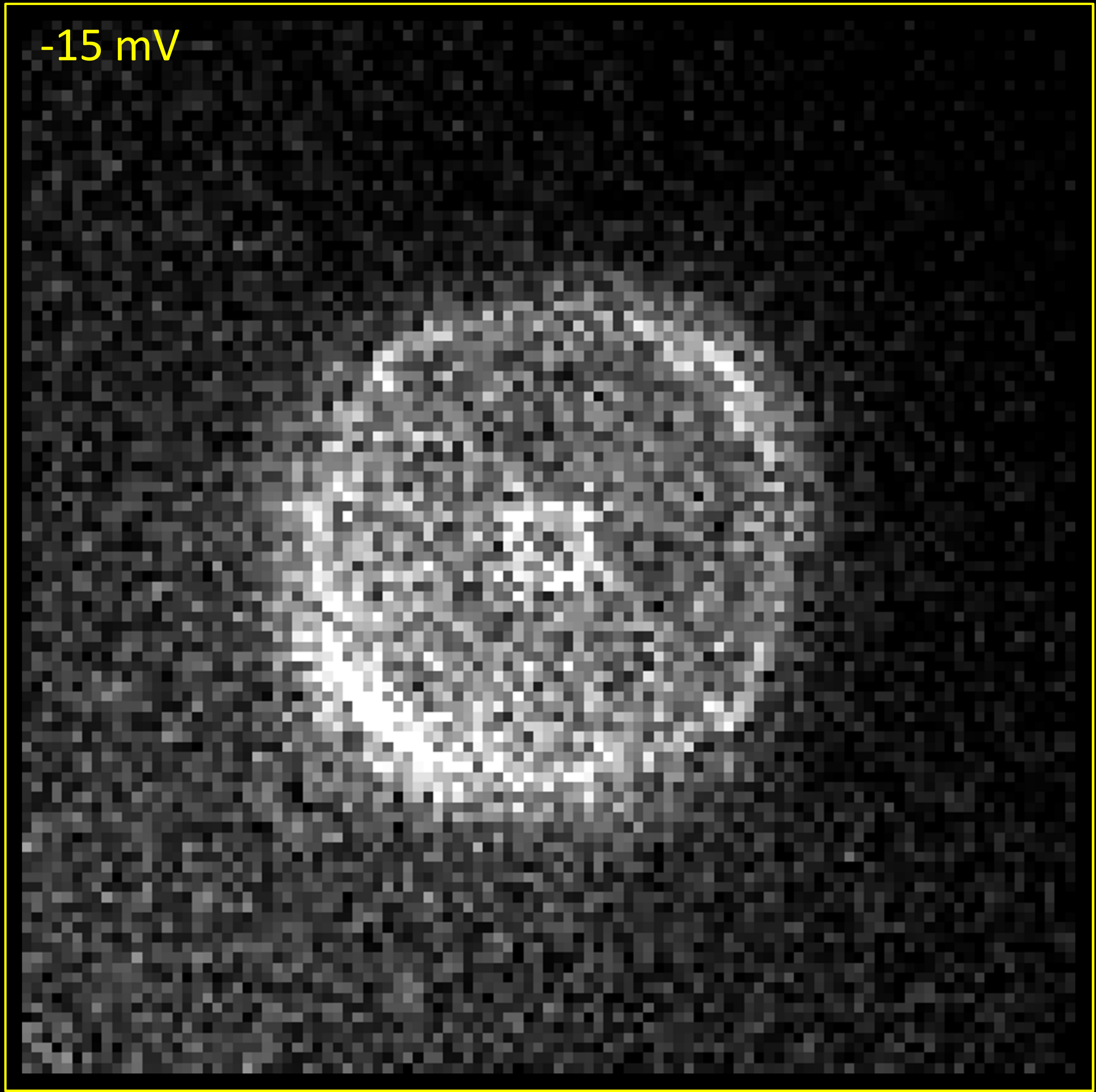

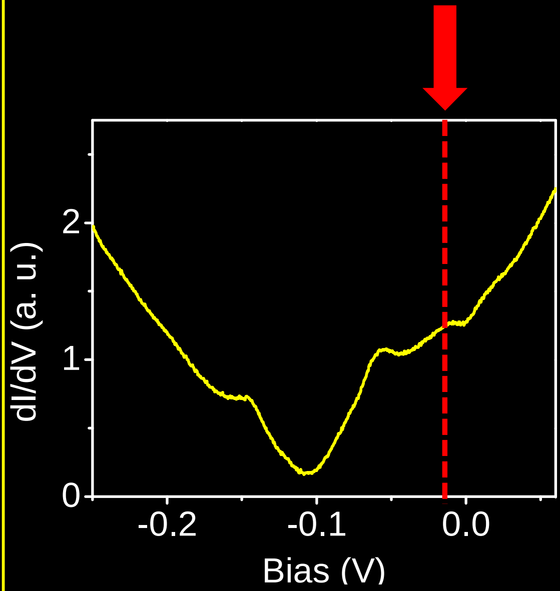

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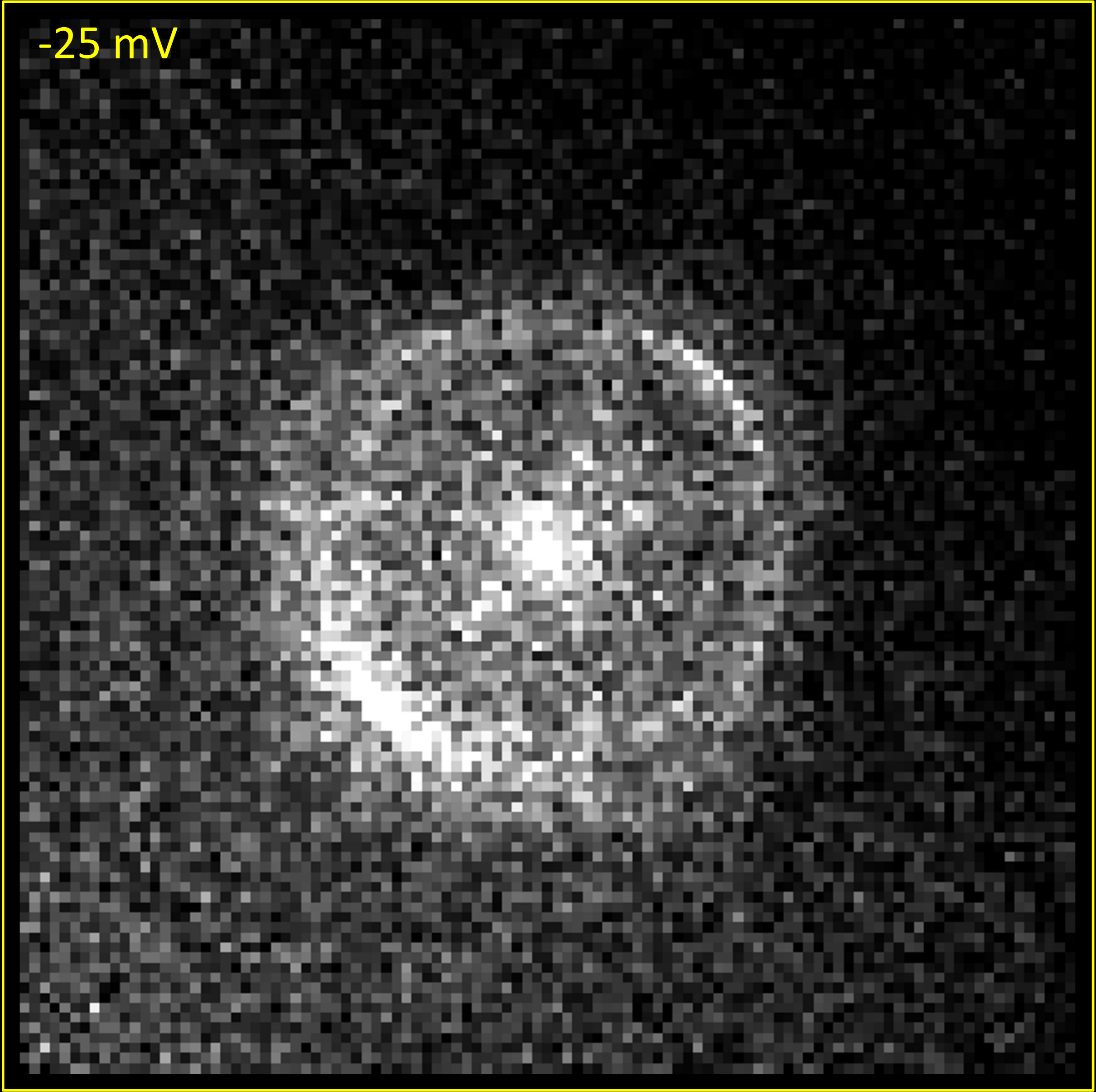

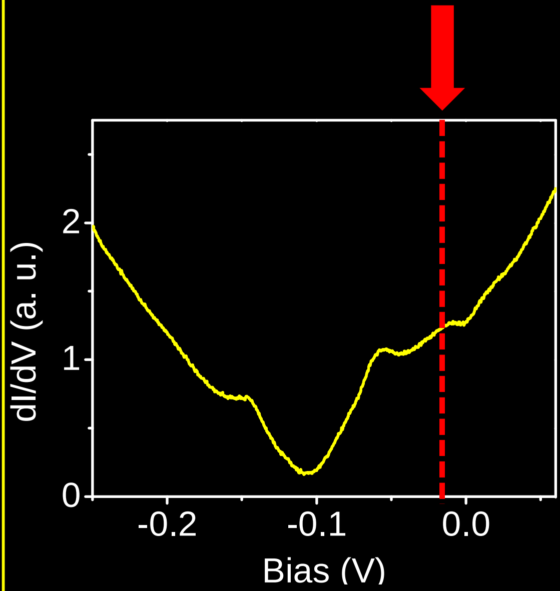

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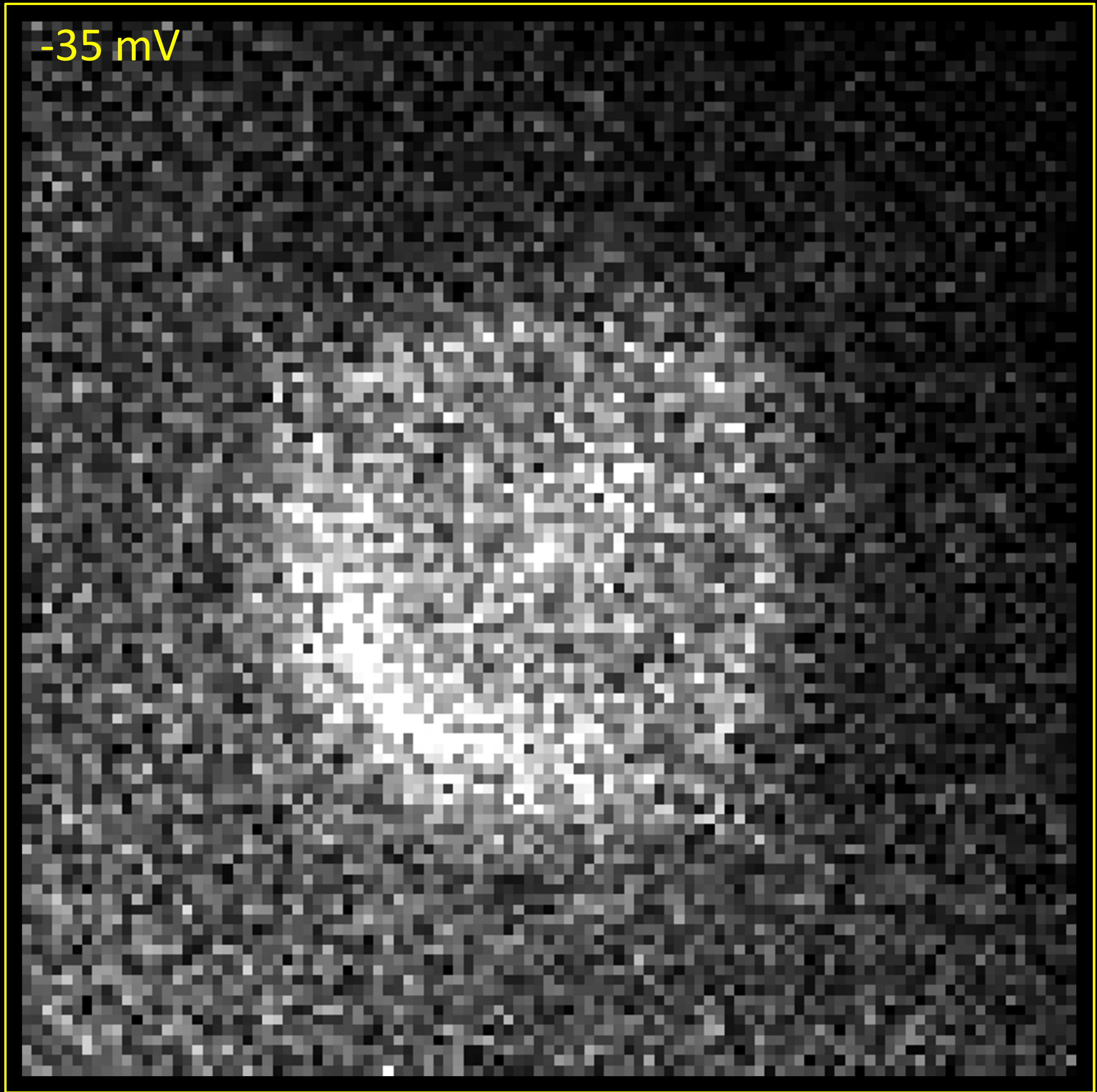

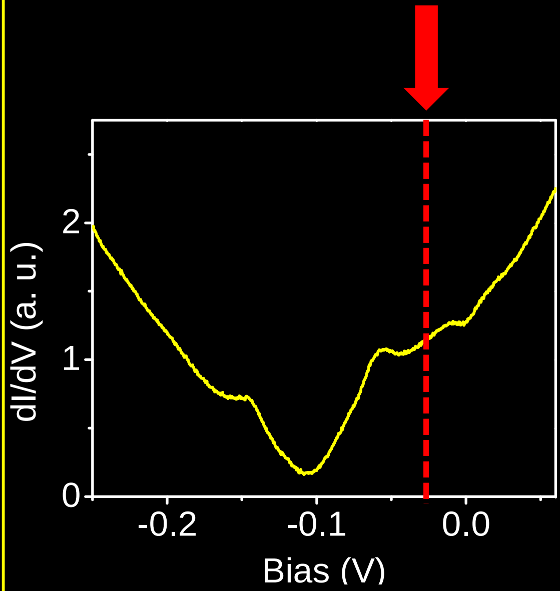

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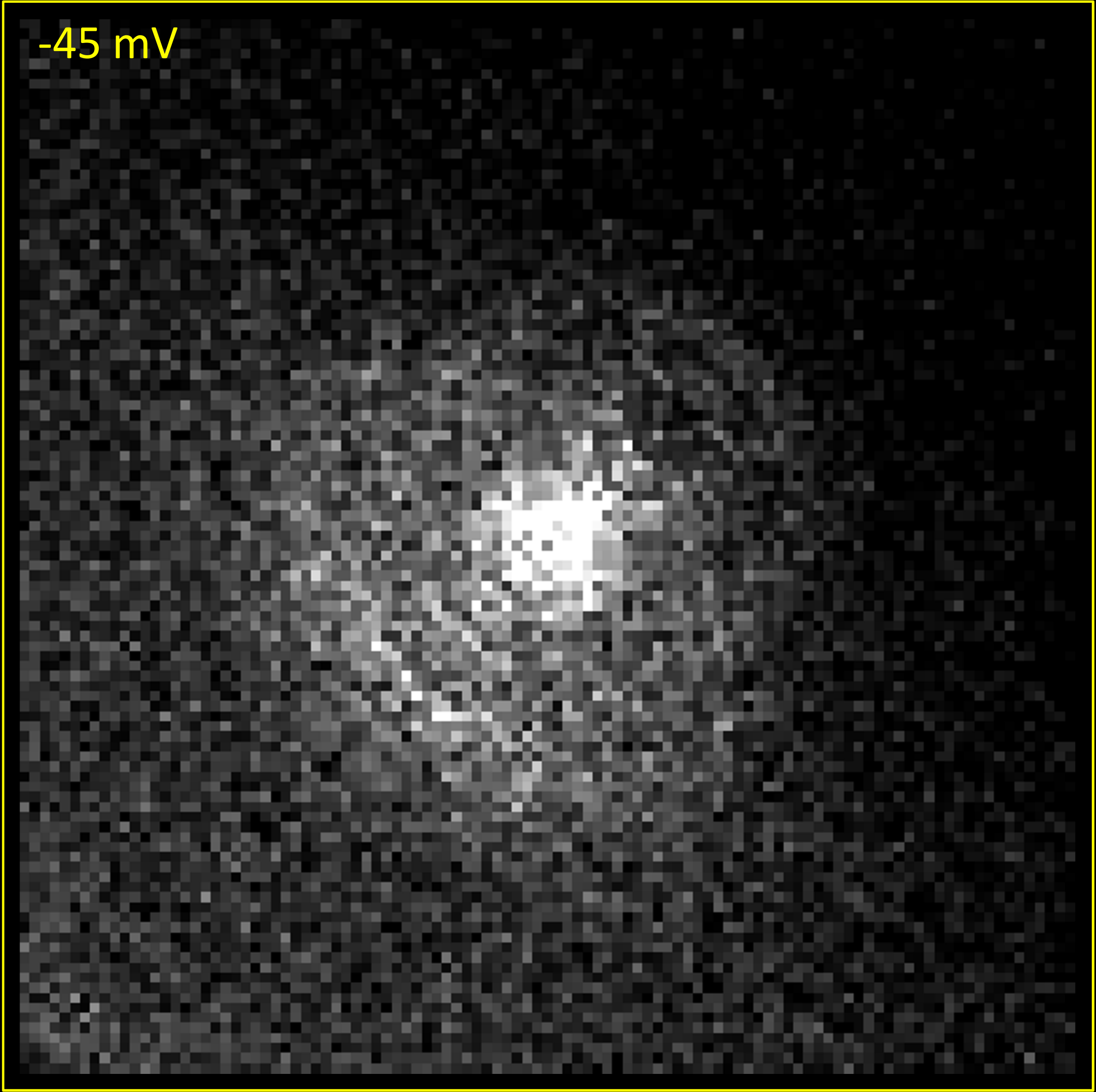

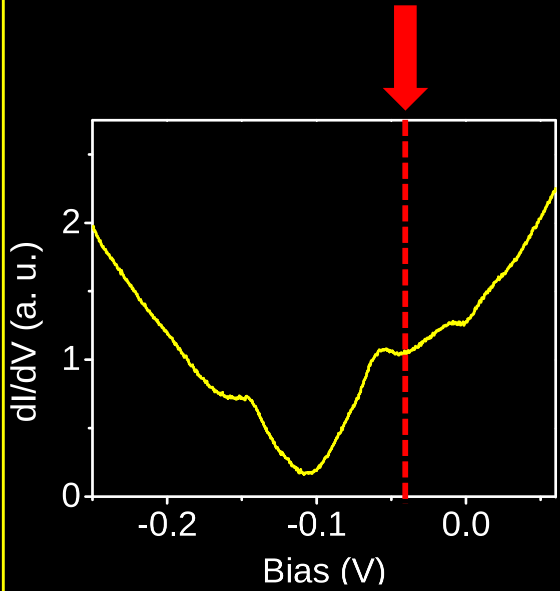

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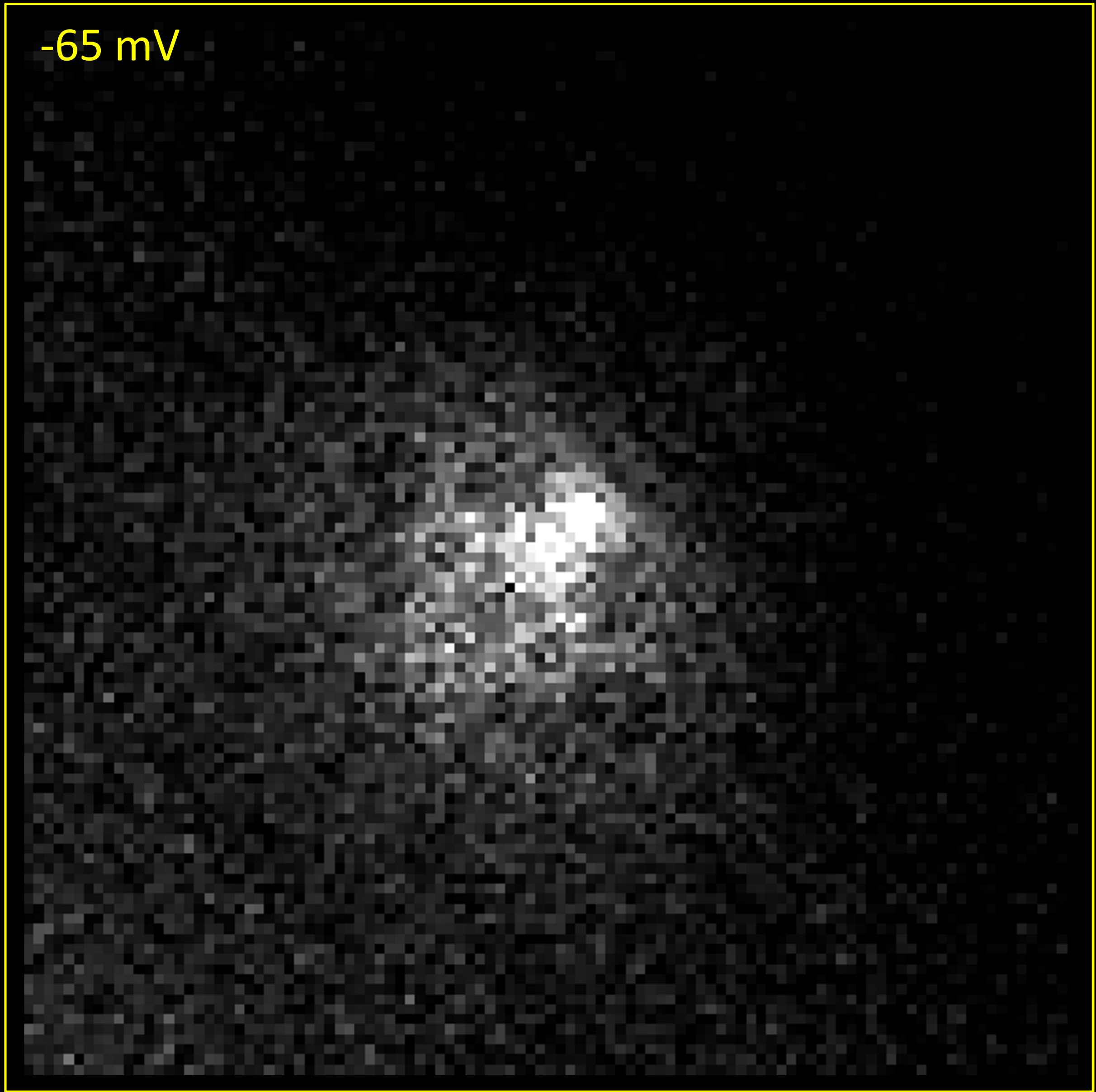

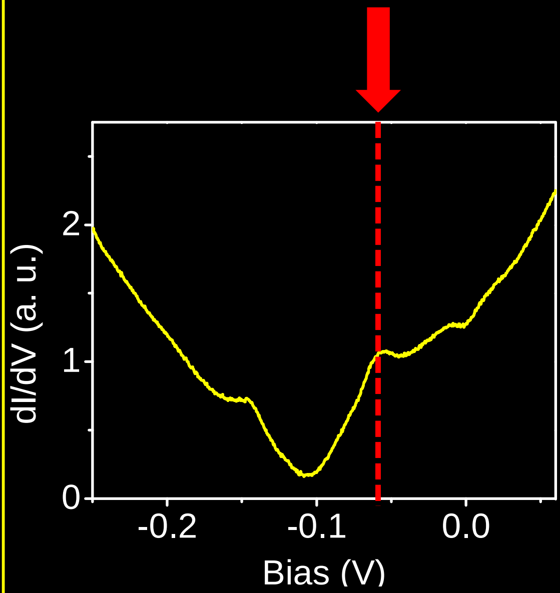

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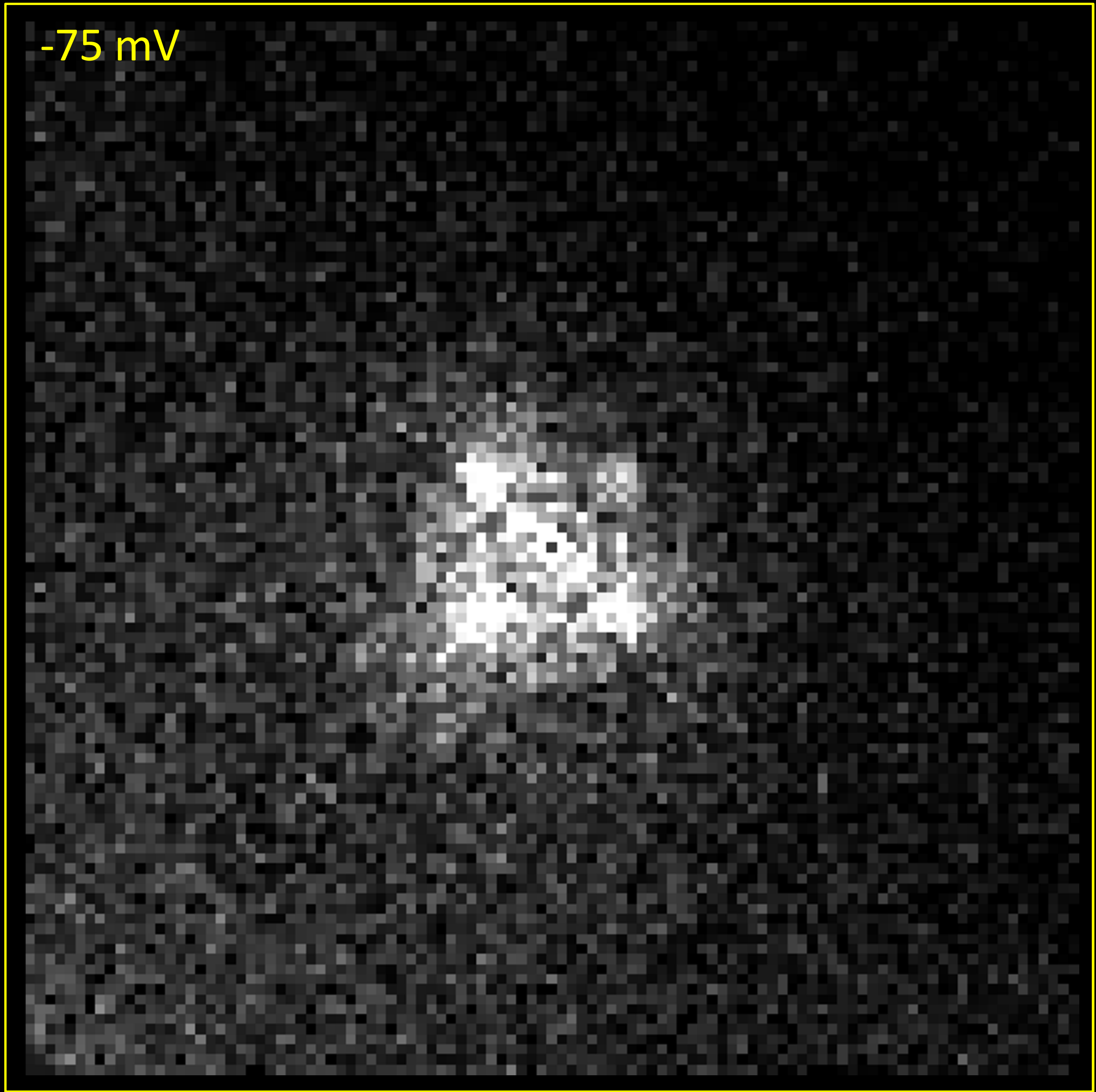

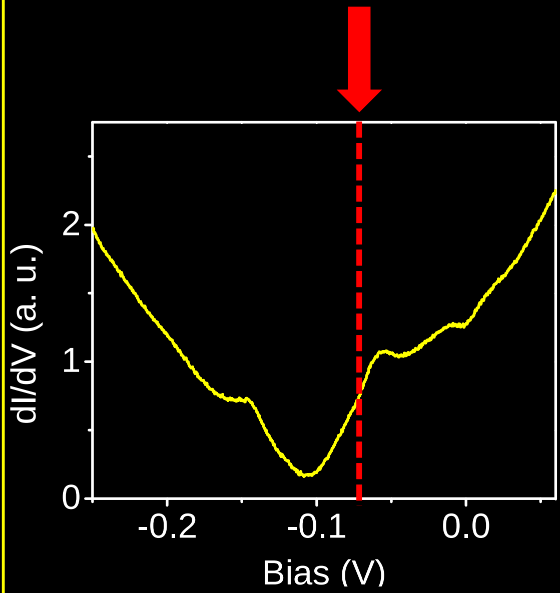

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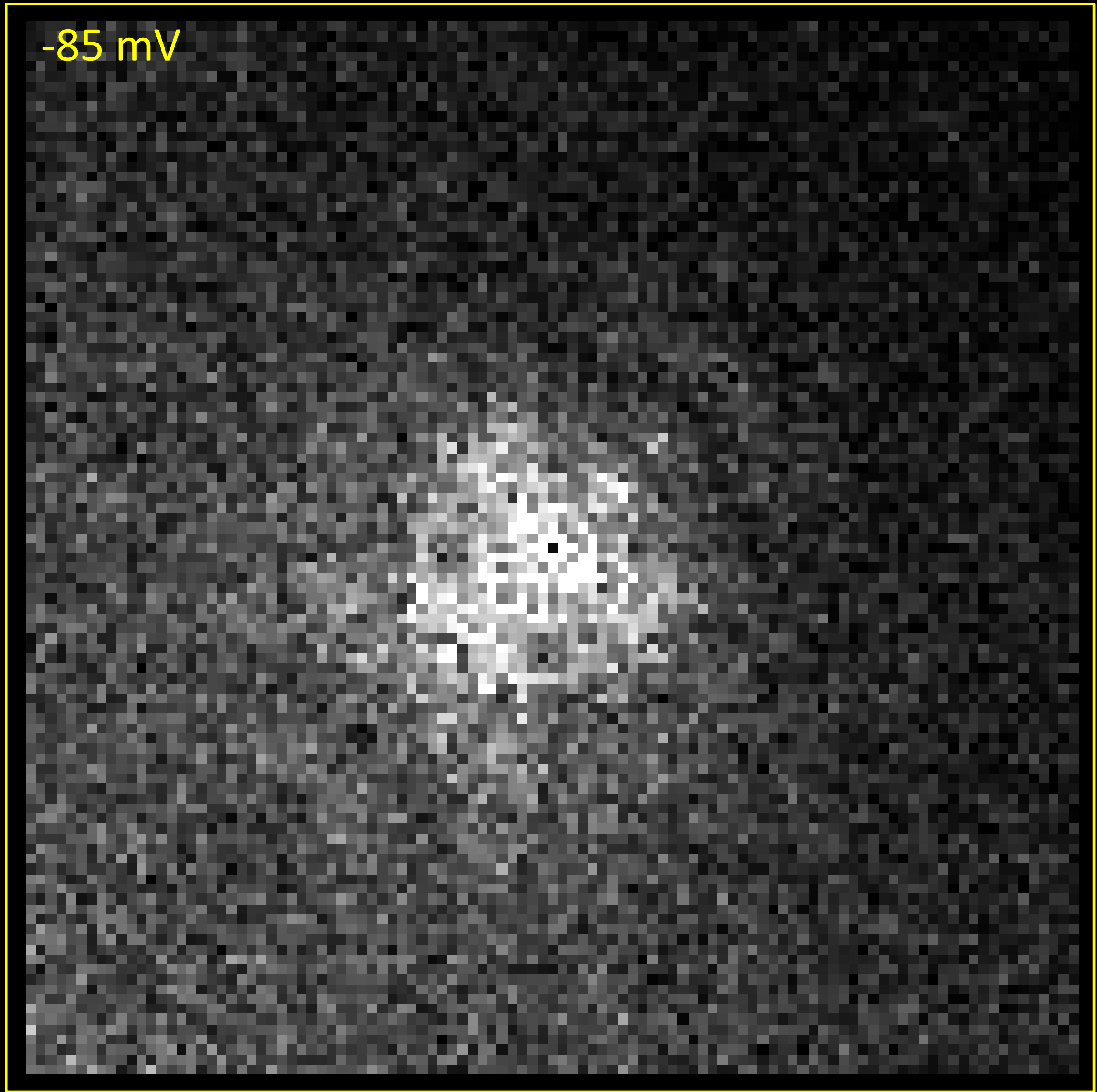

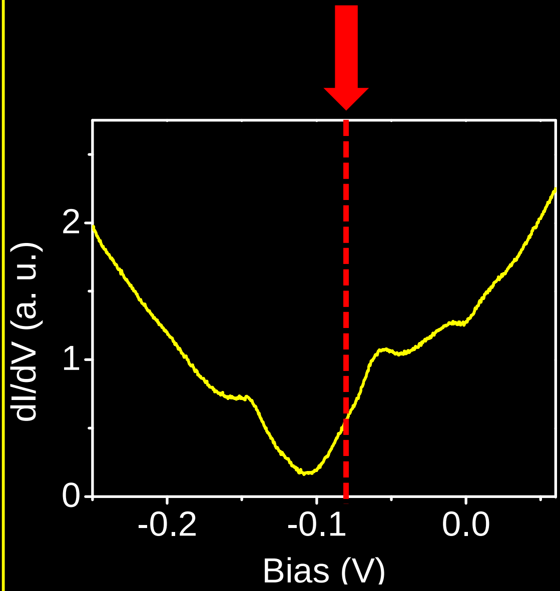

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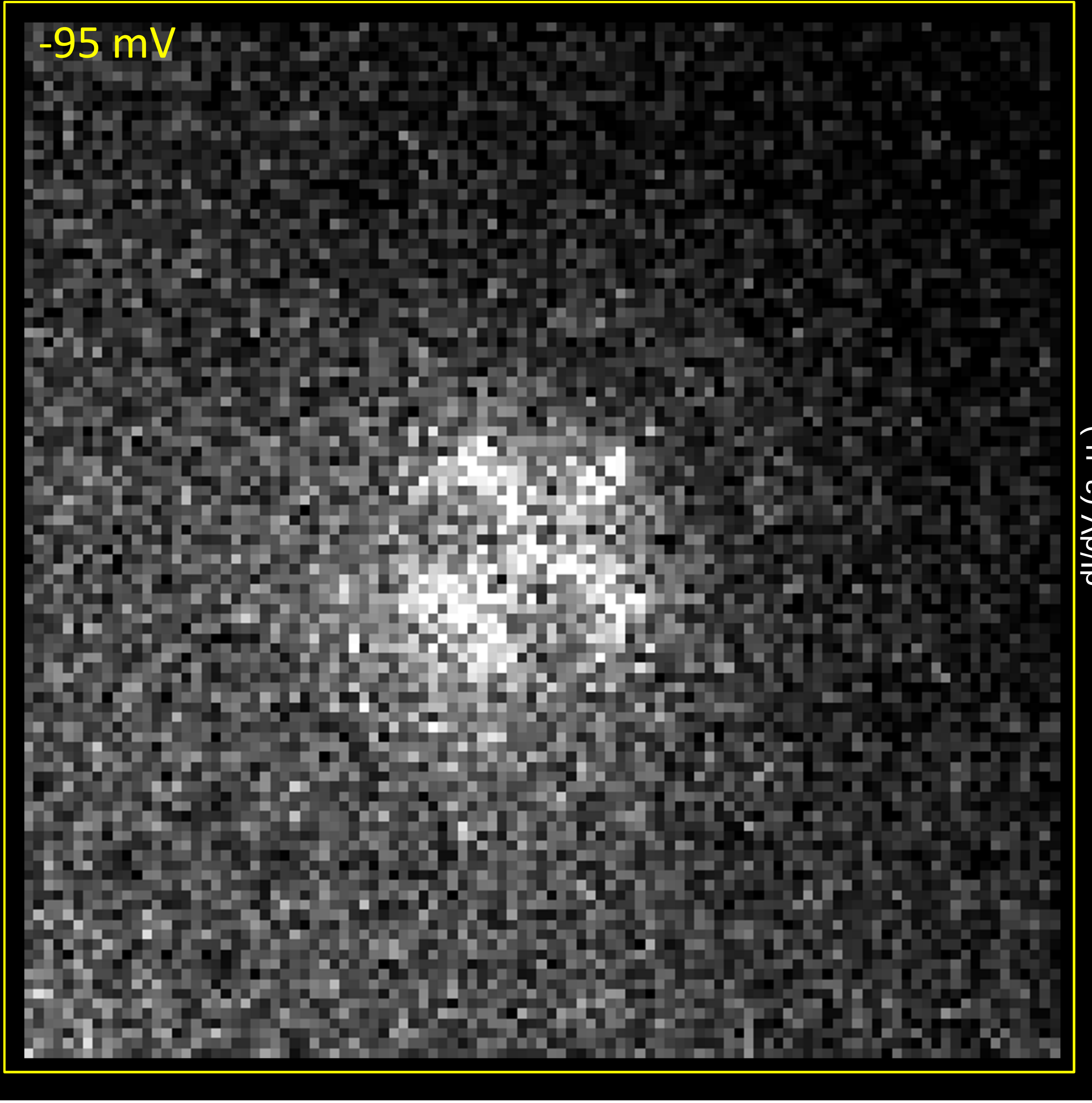

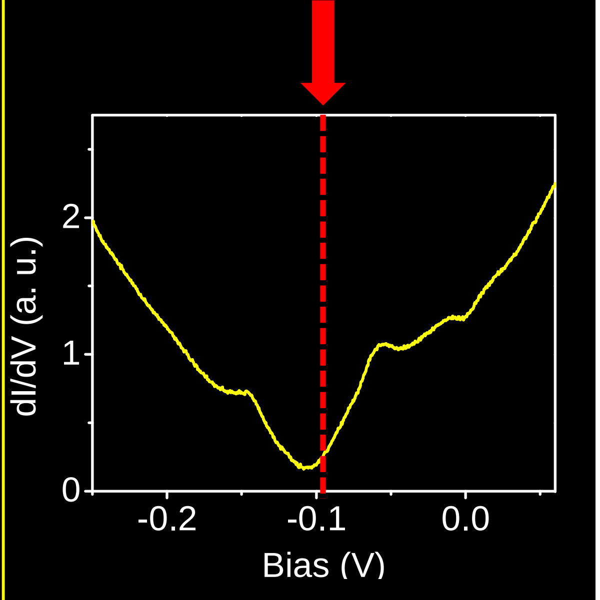

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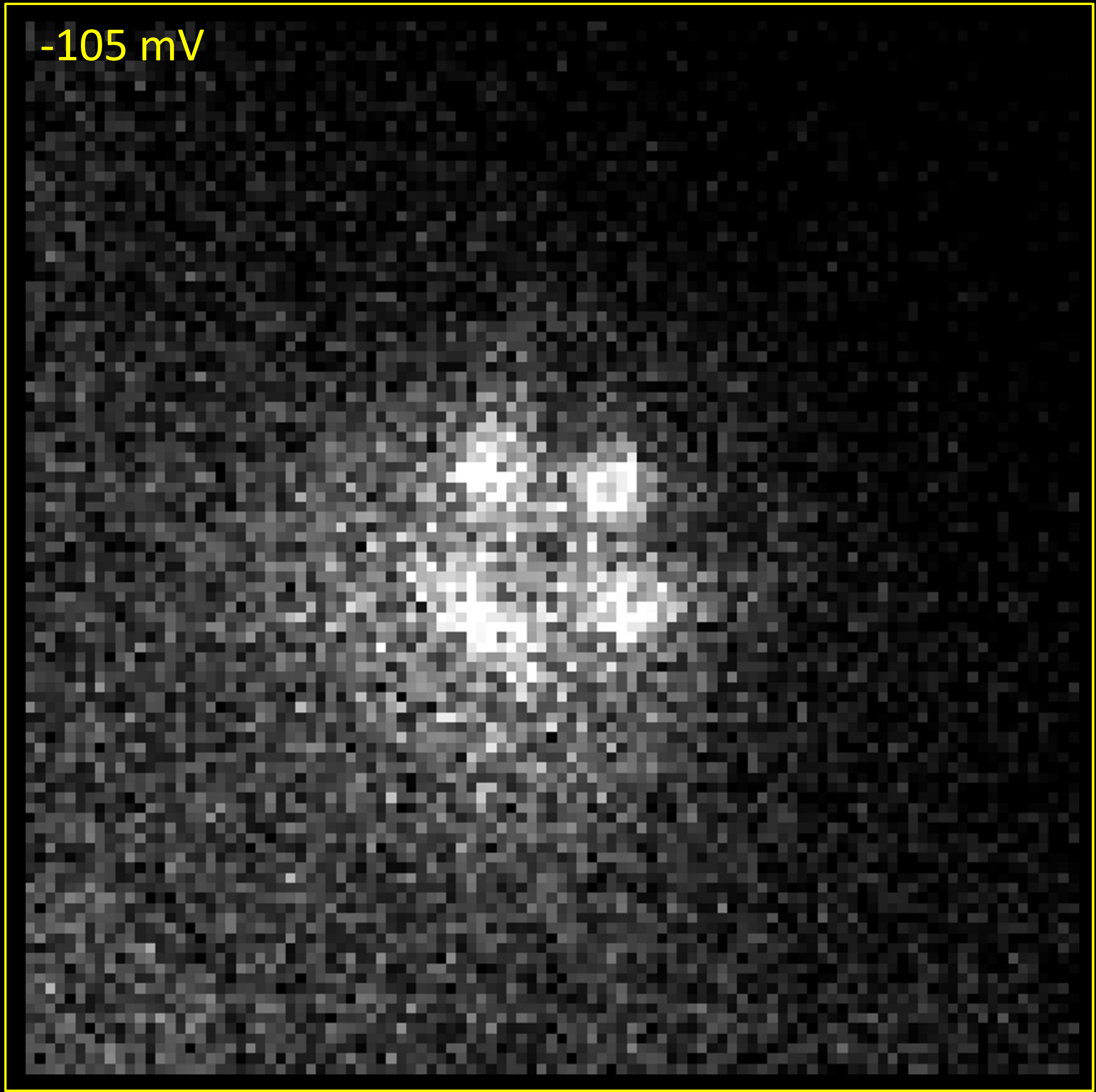

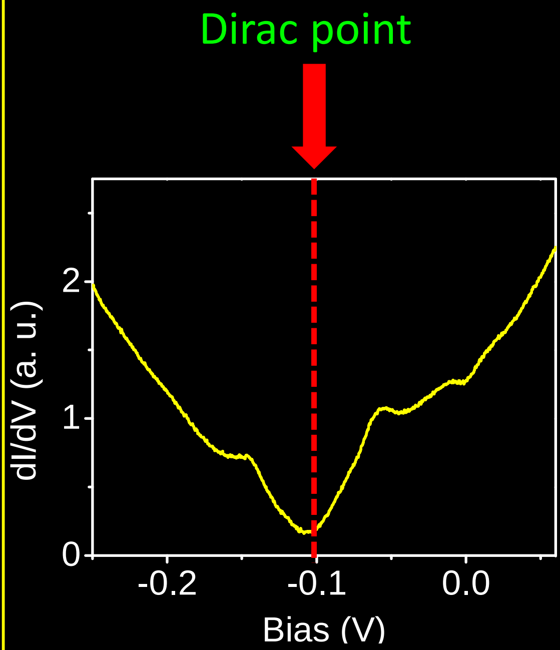

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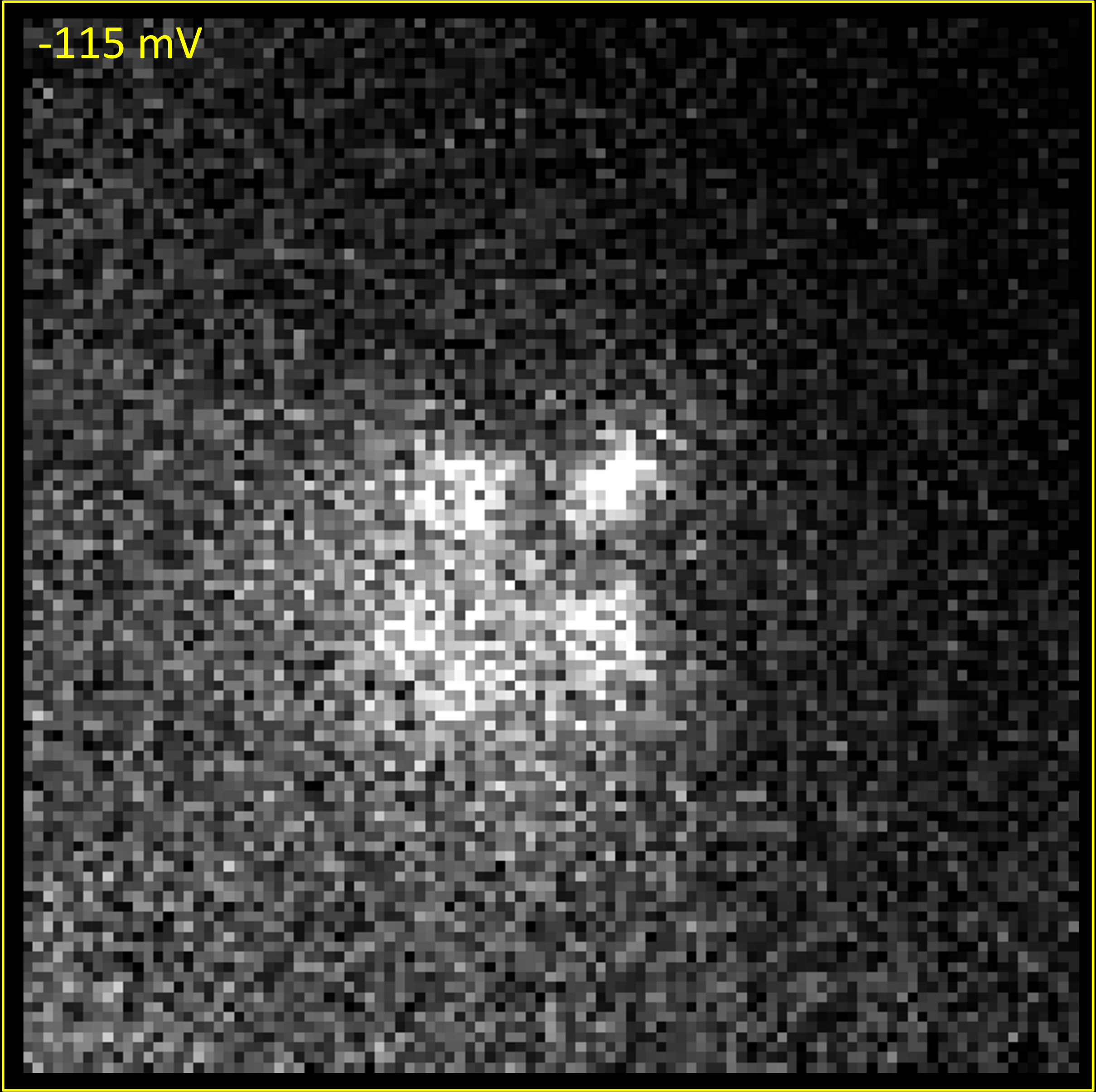

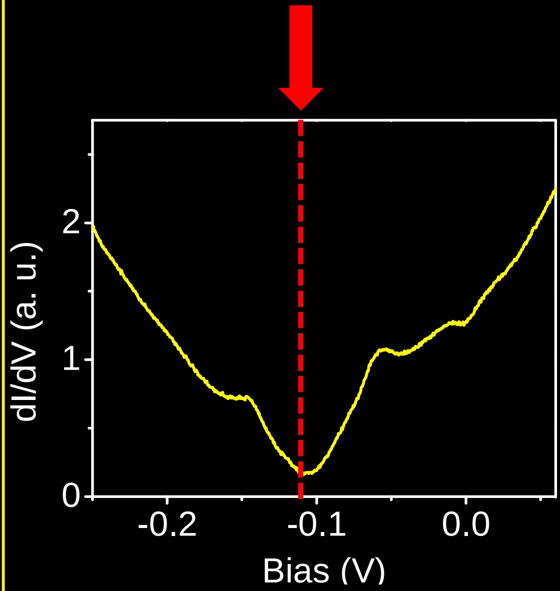

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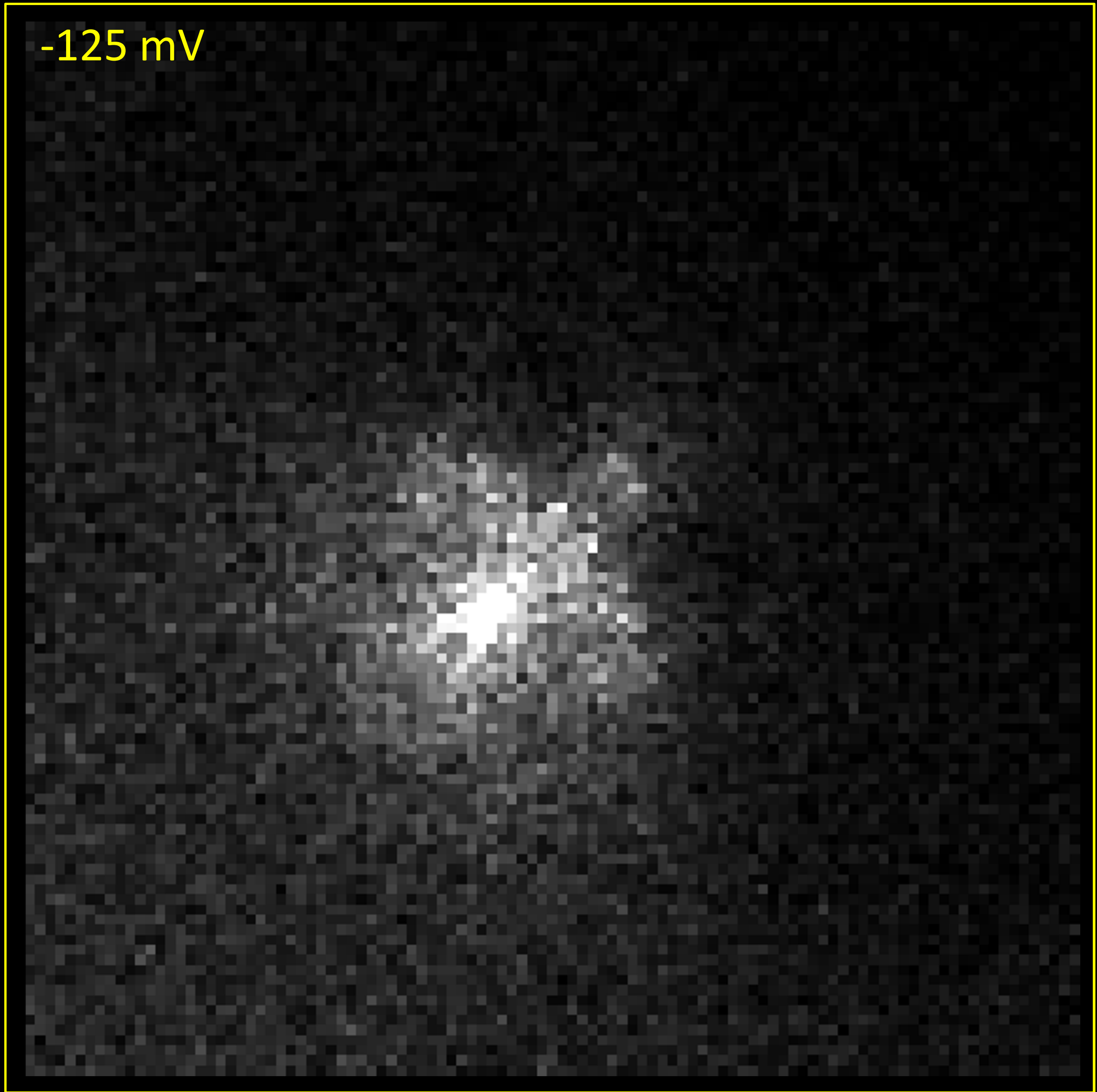

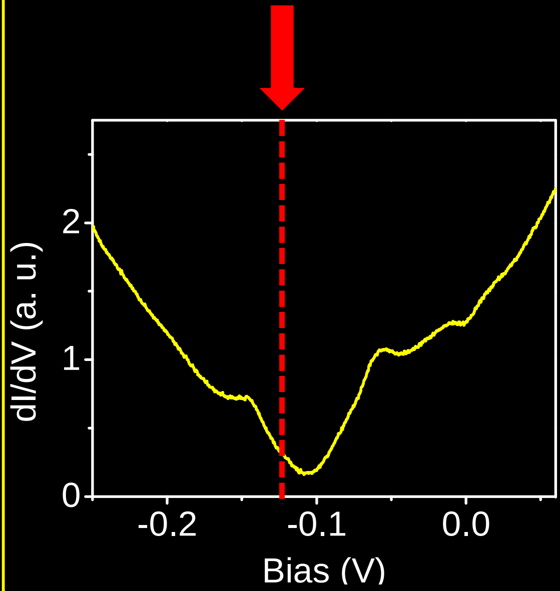

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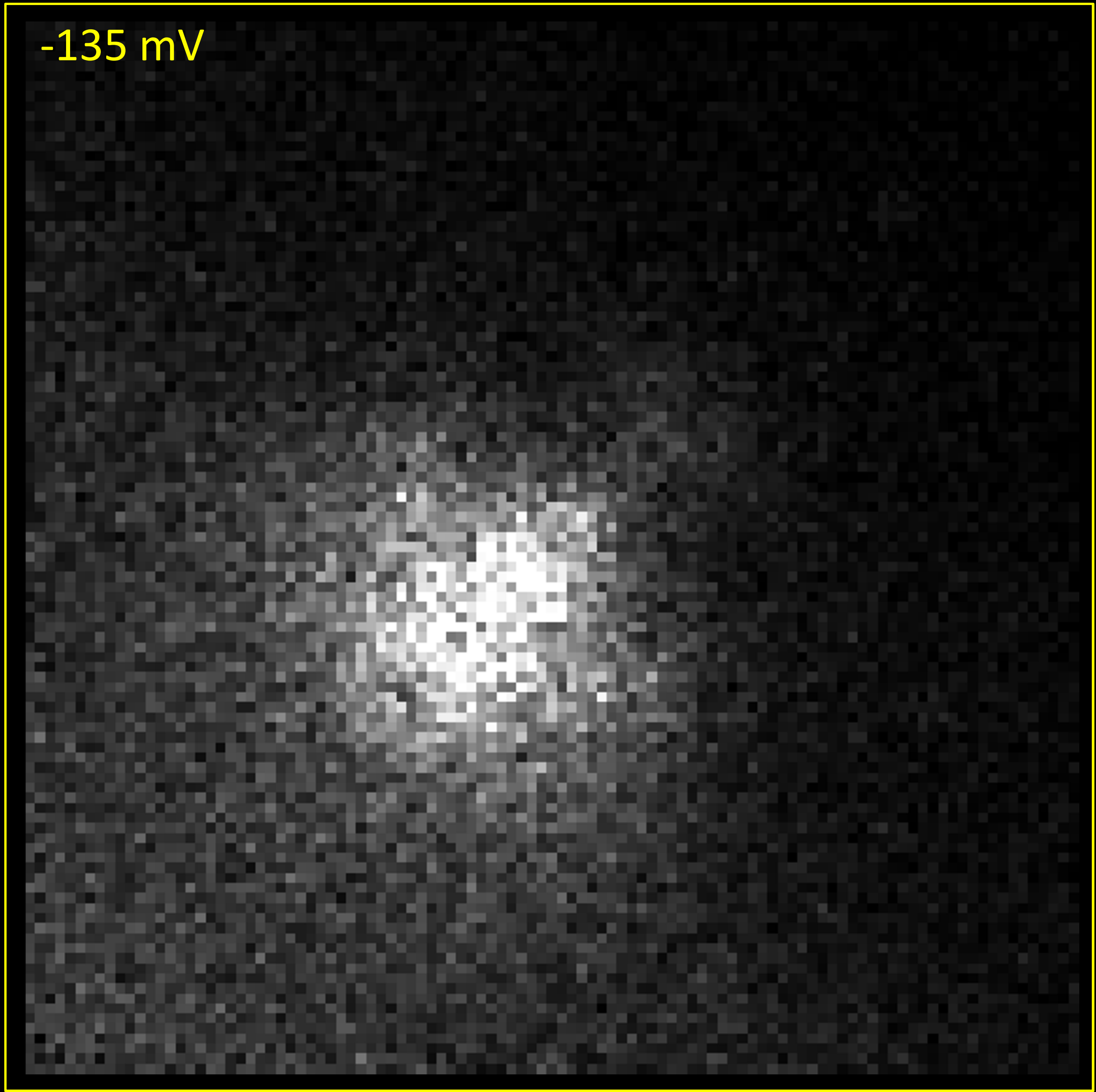

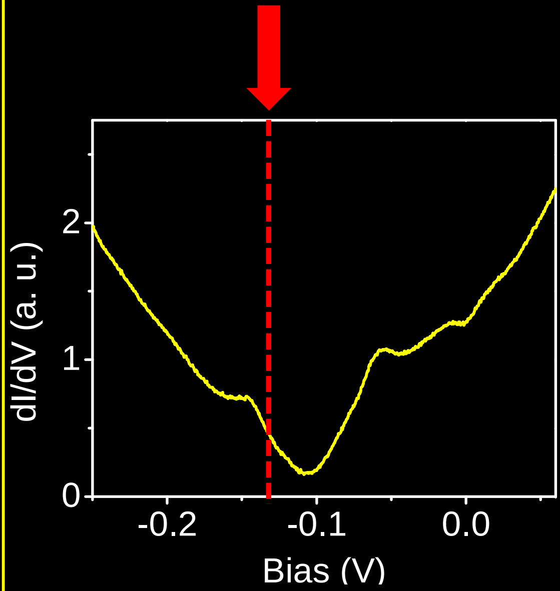

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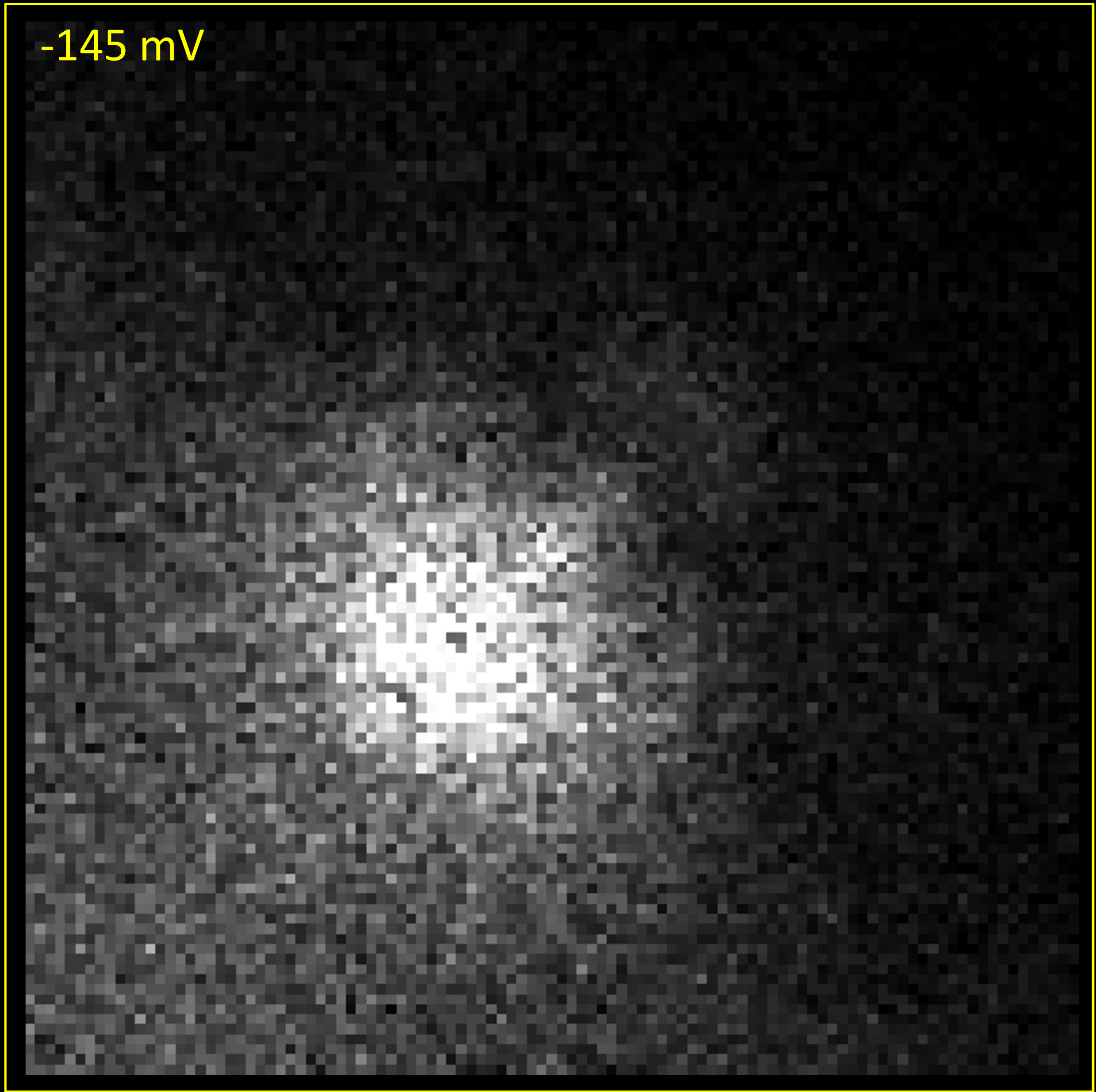

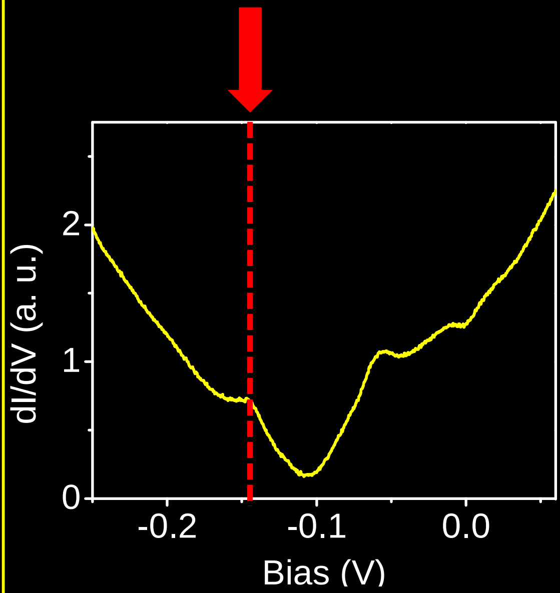

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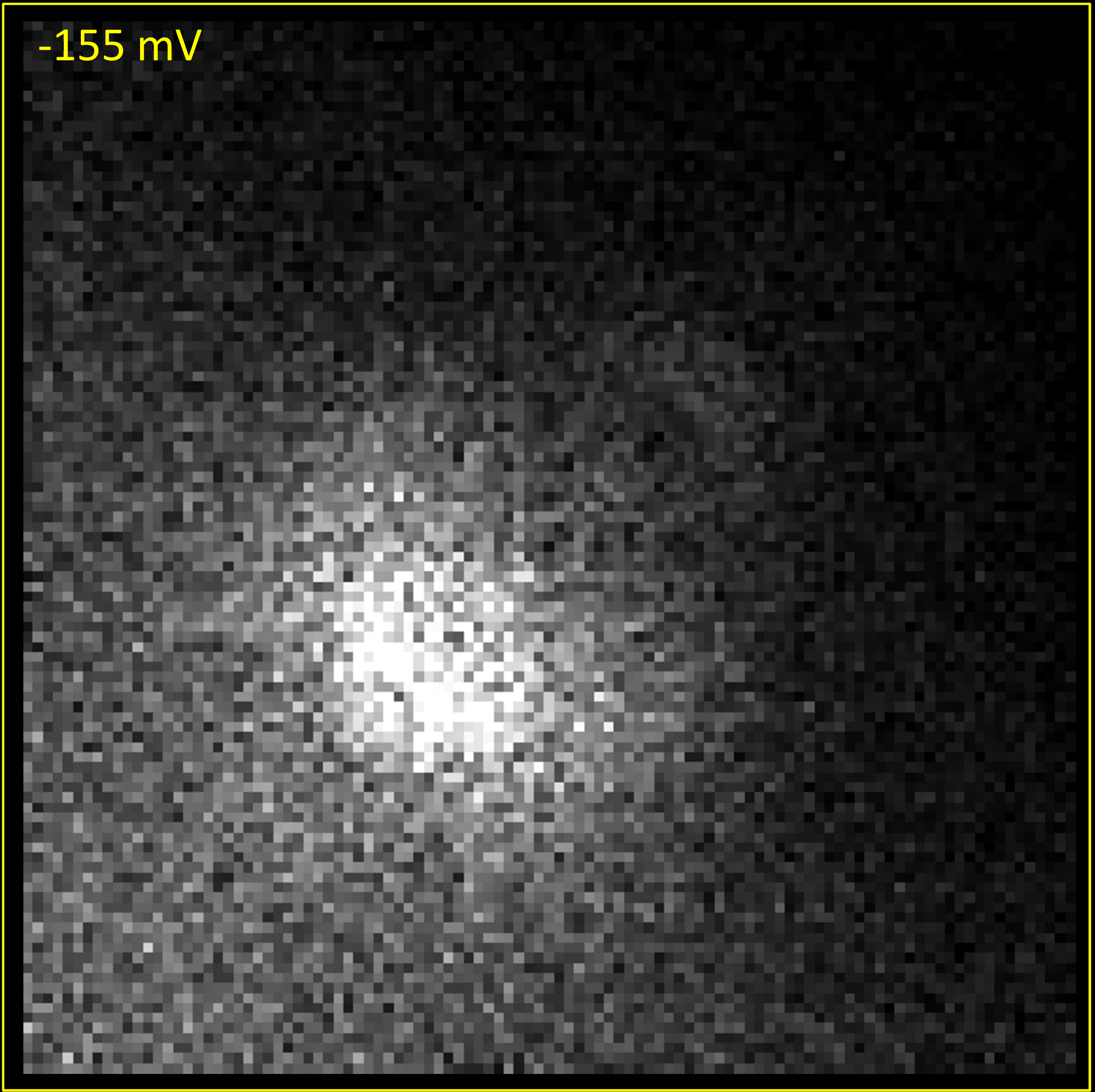

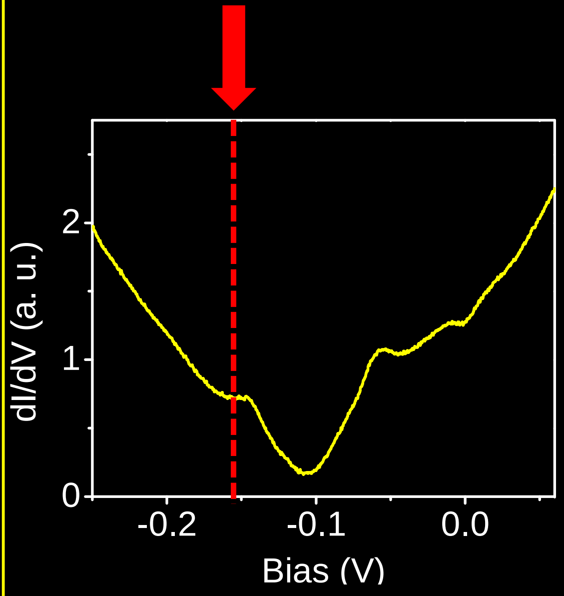

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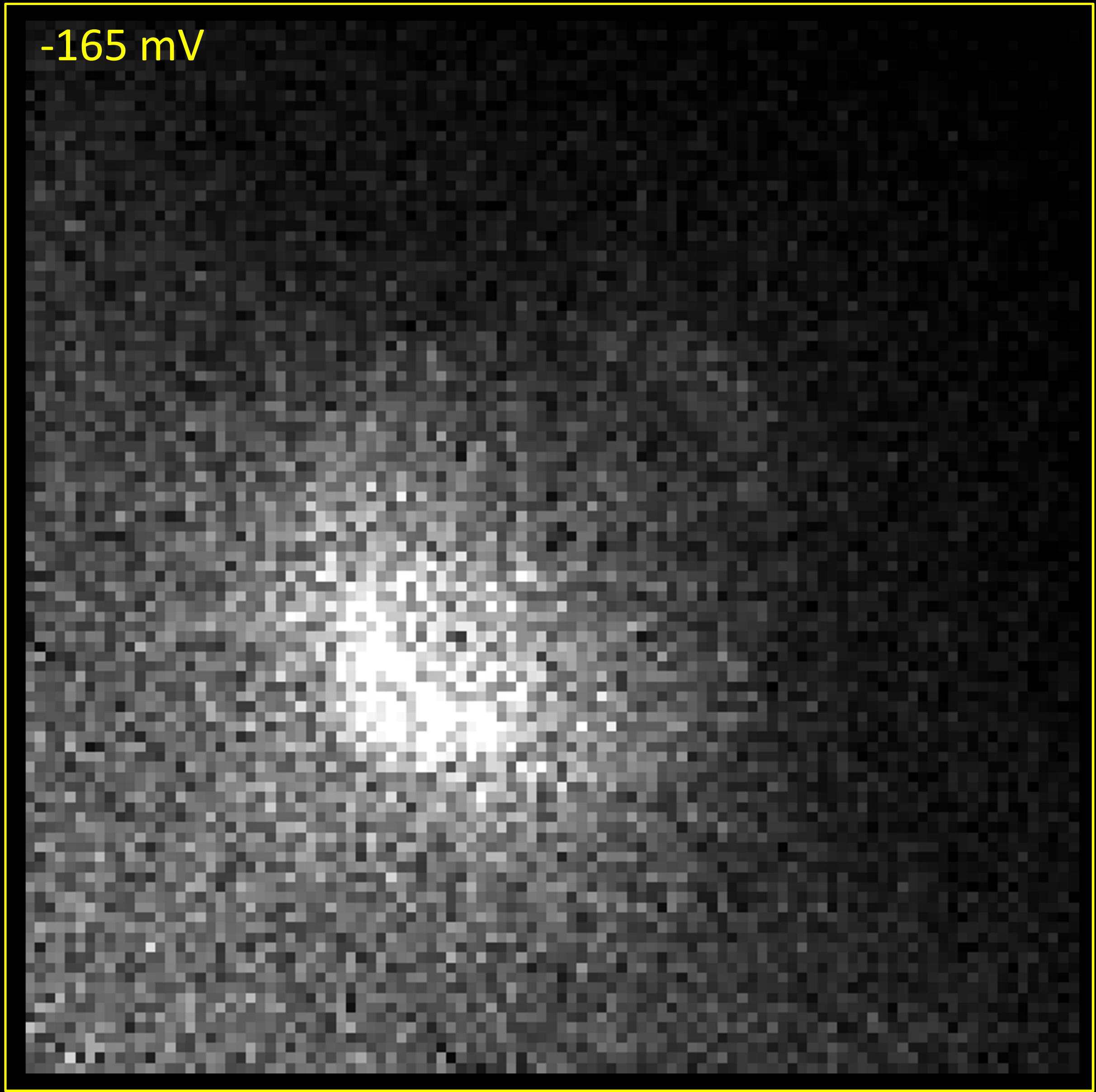
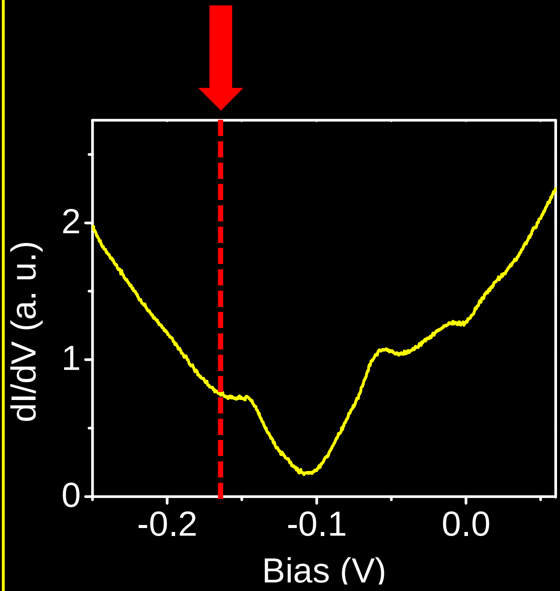
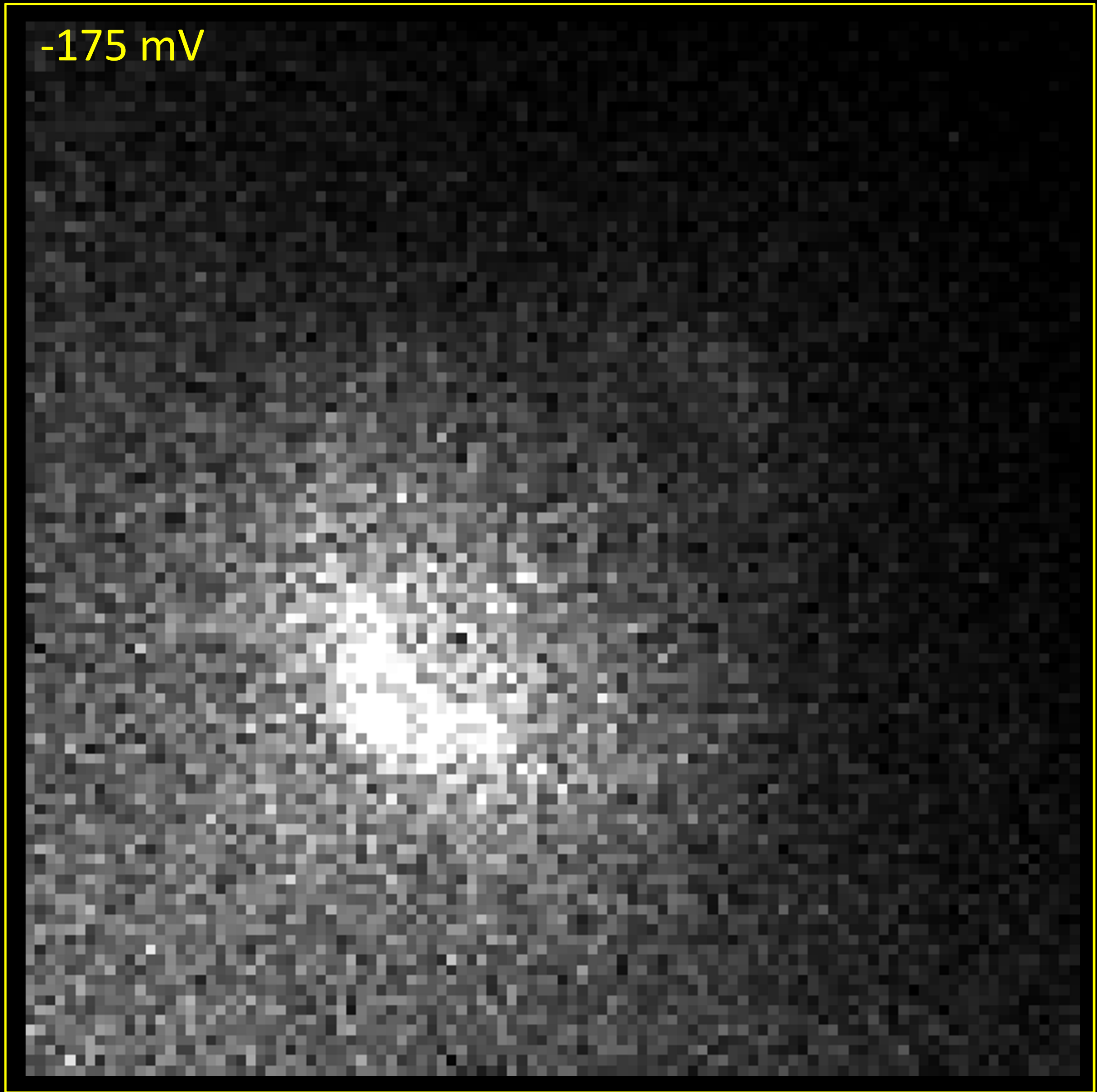
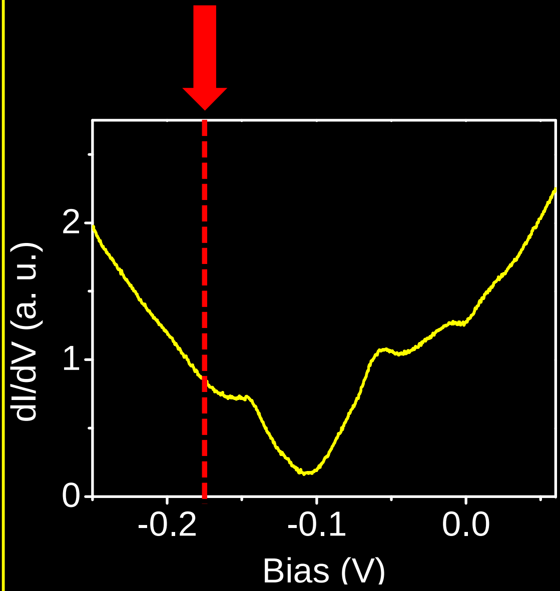


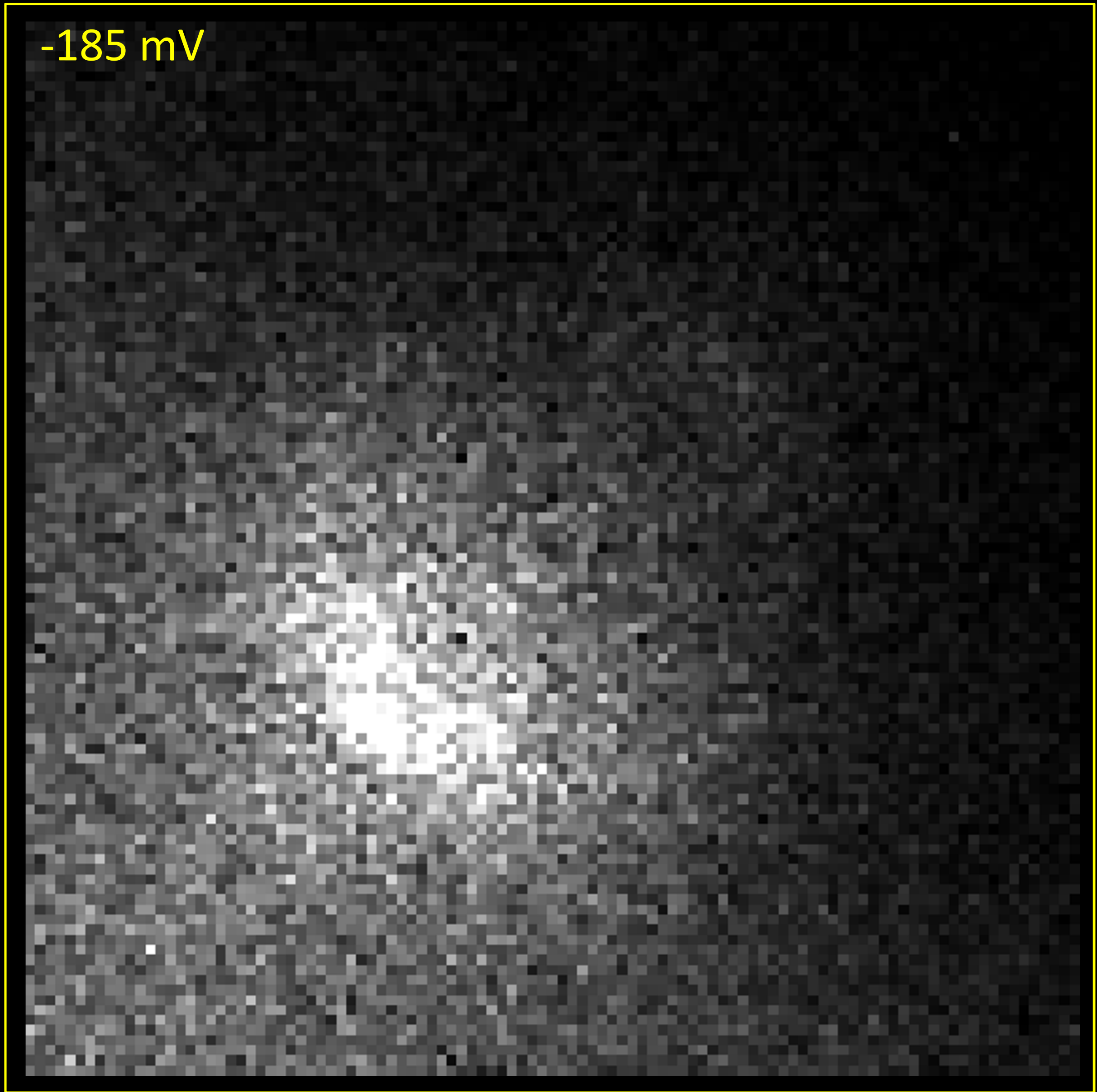


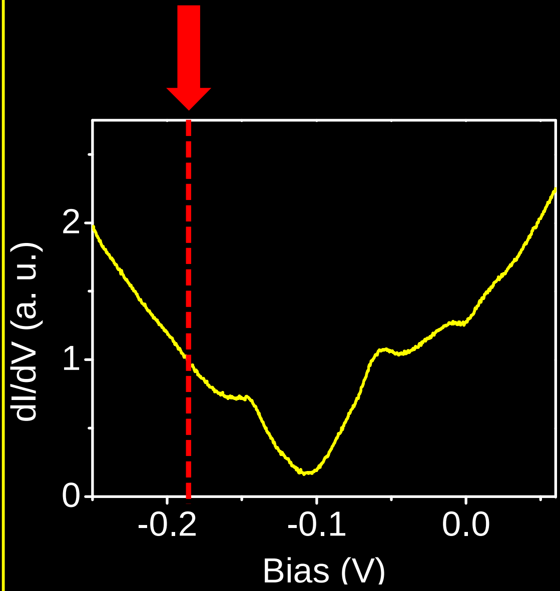
QPI Fourier transform patterns



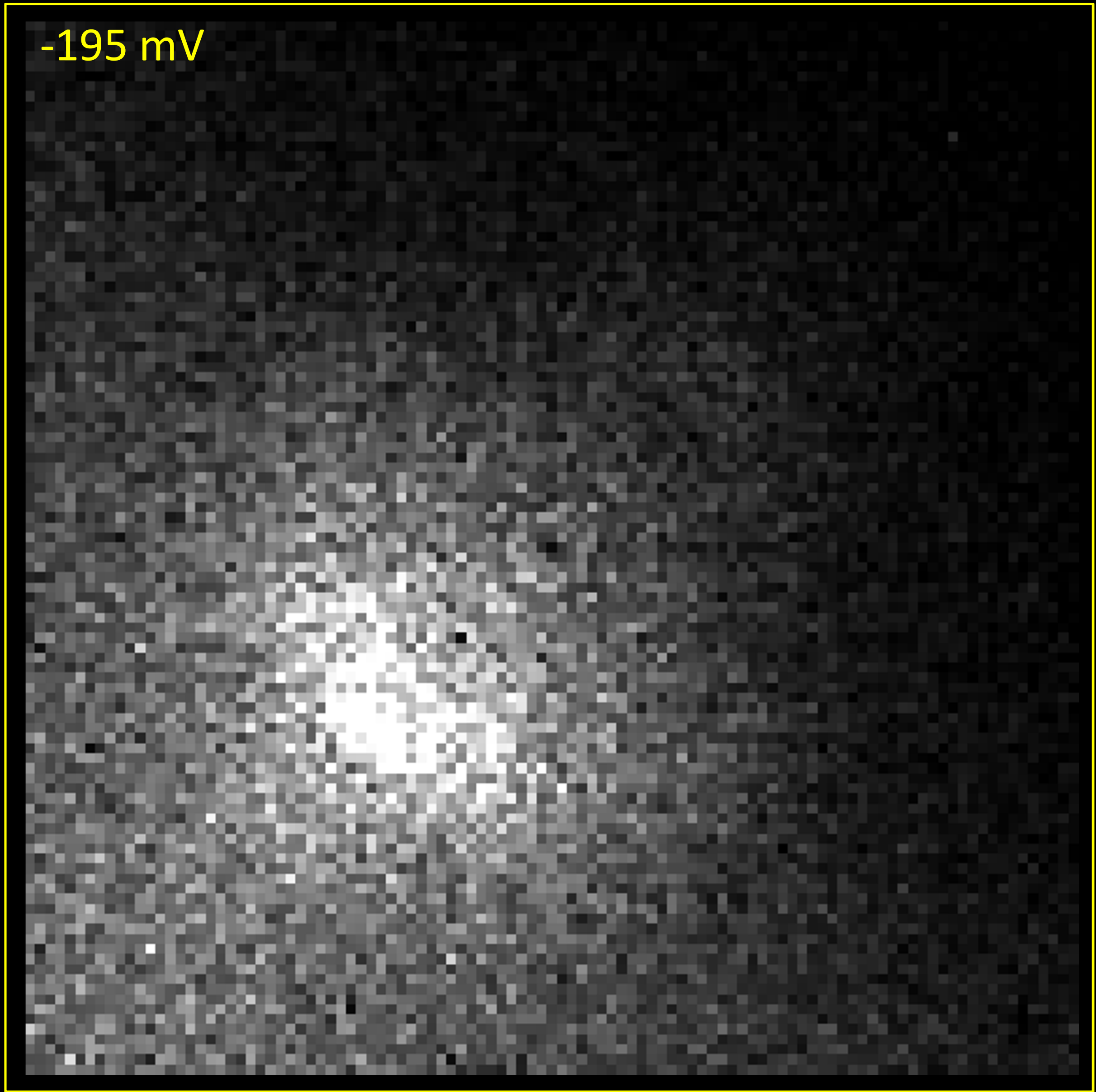


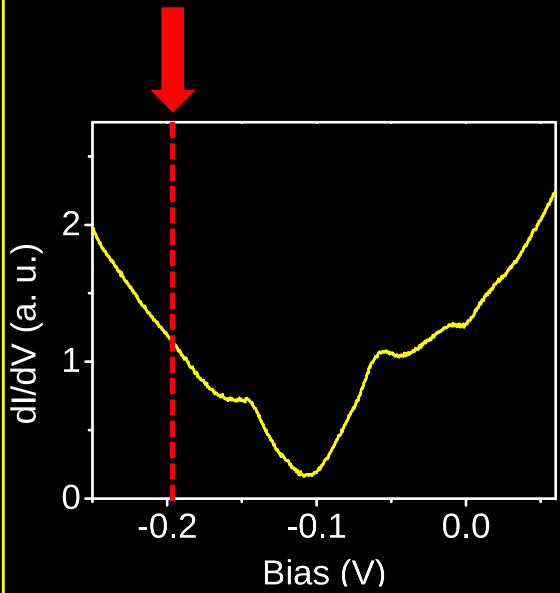
QPI Fourier transform patterns



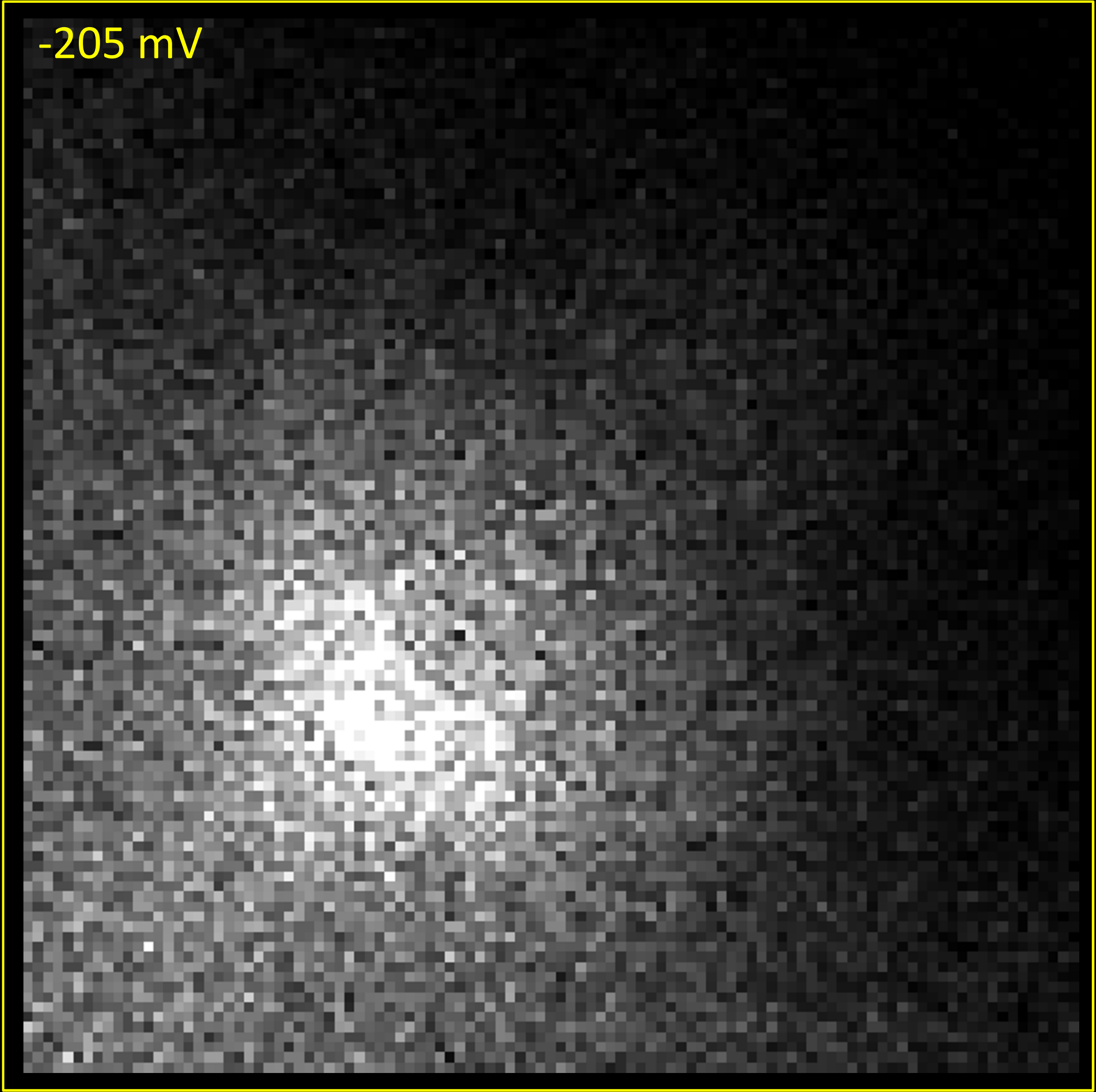


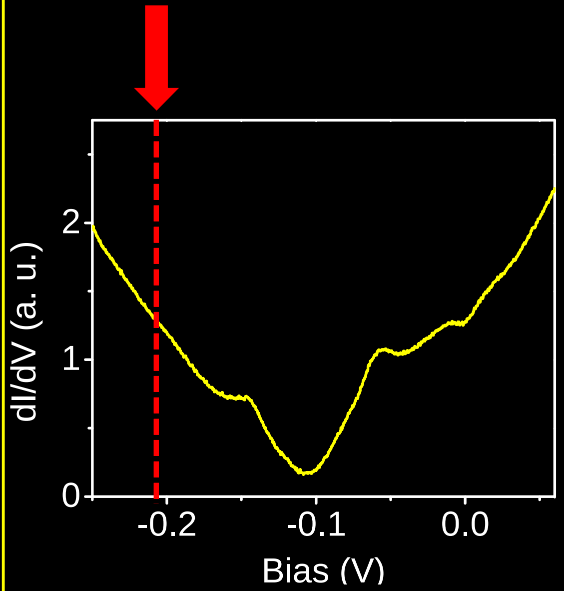
QPI Fourier transform patterns



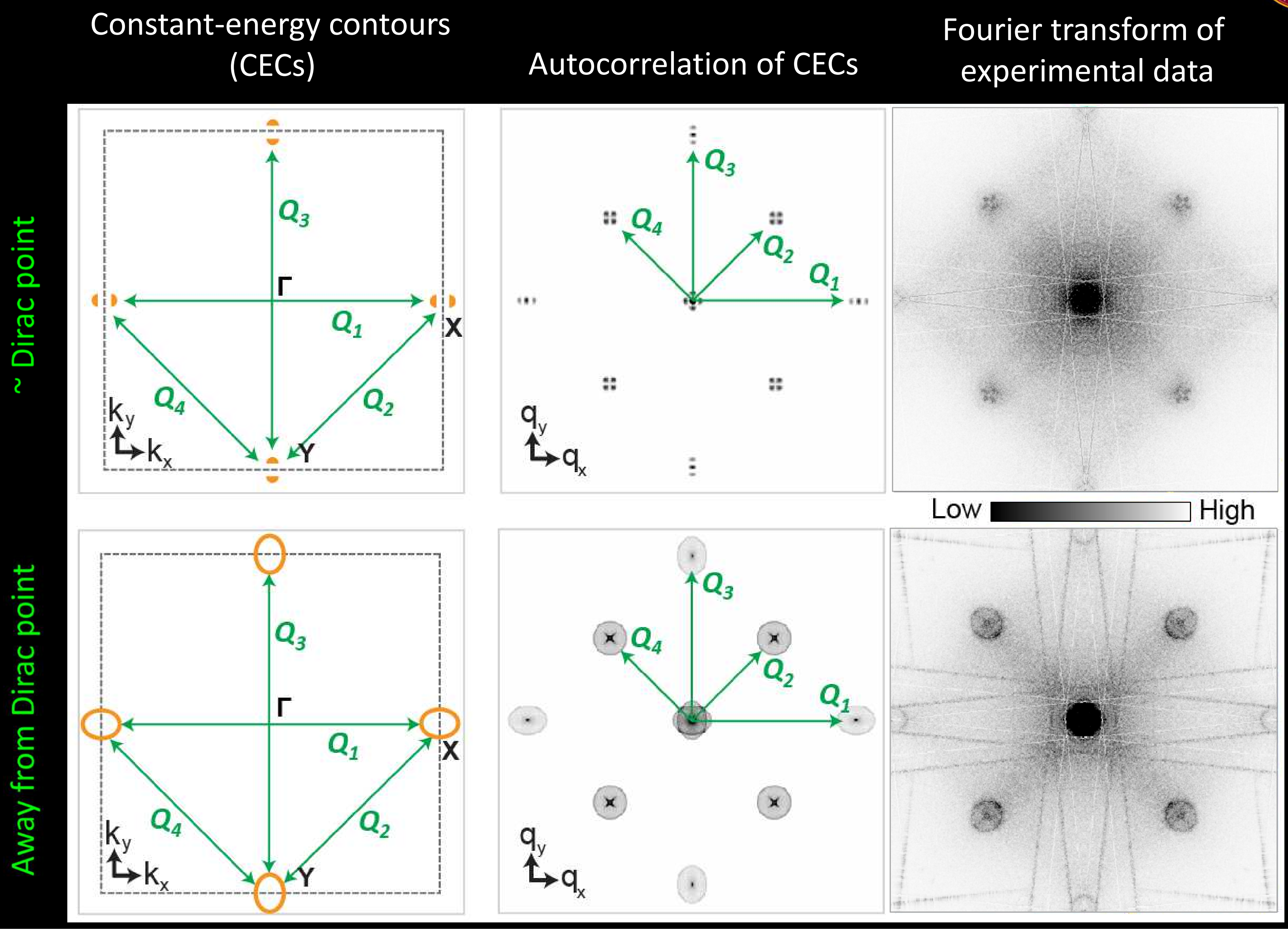


QPI Fourier transform patterns

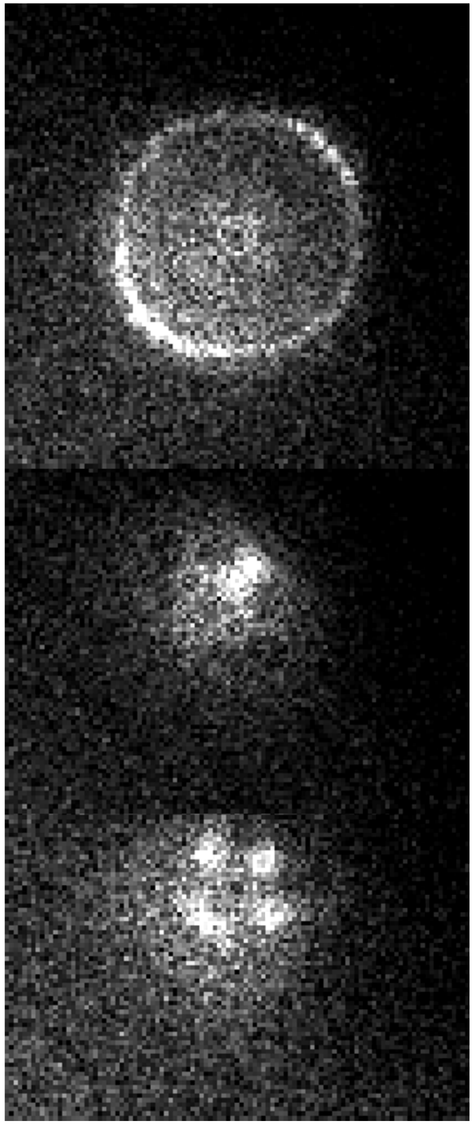


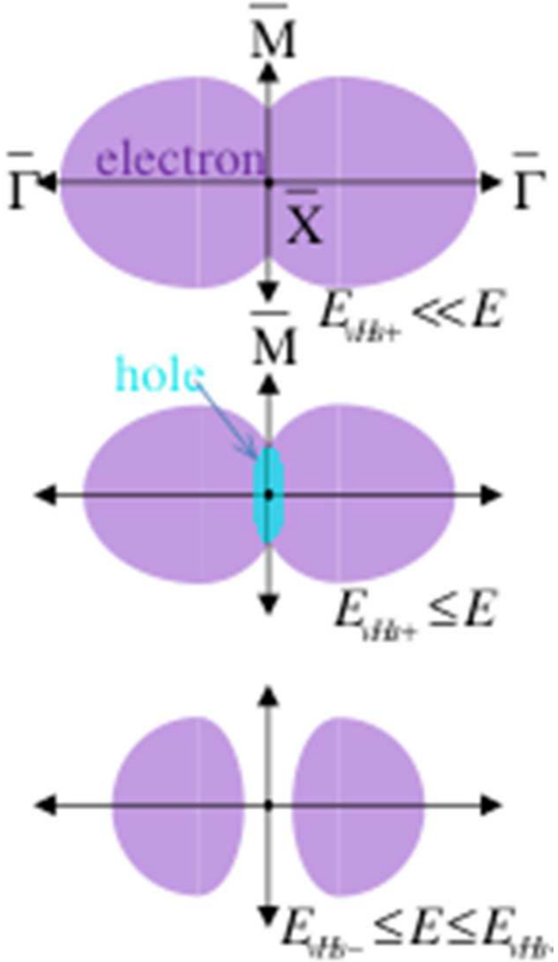


Expected QPI pattern

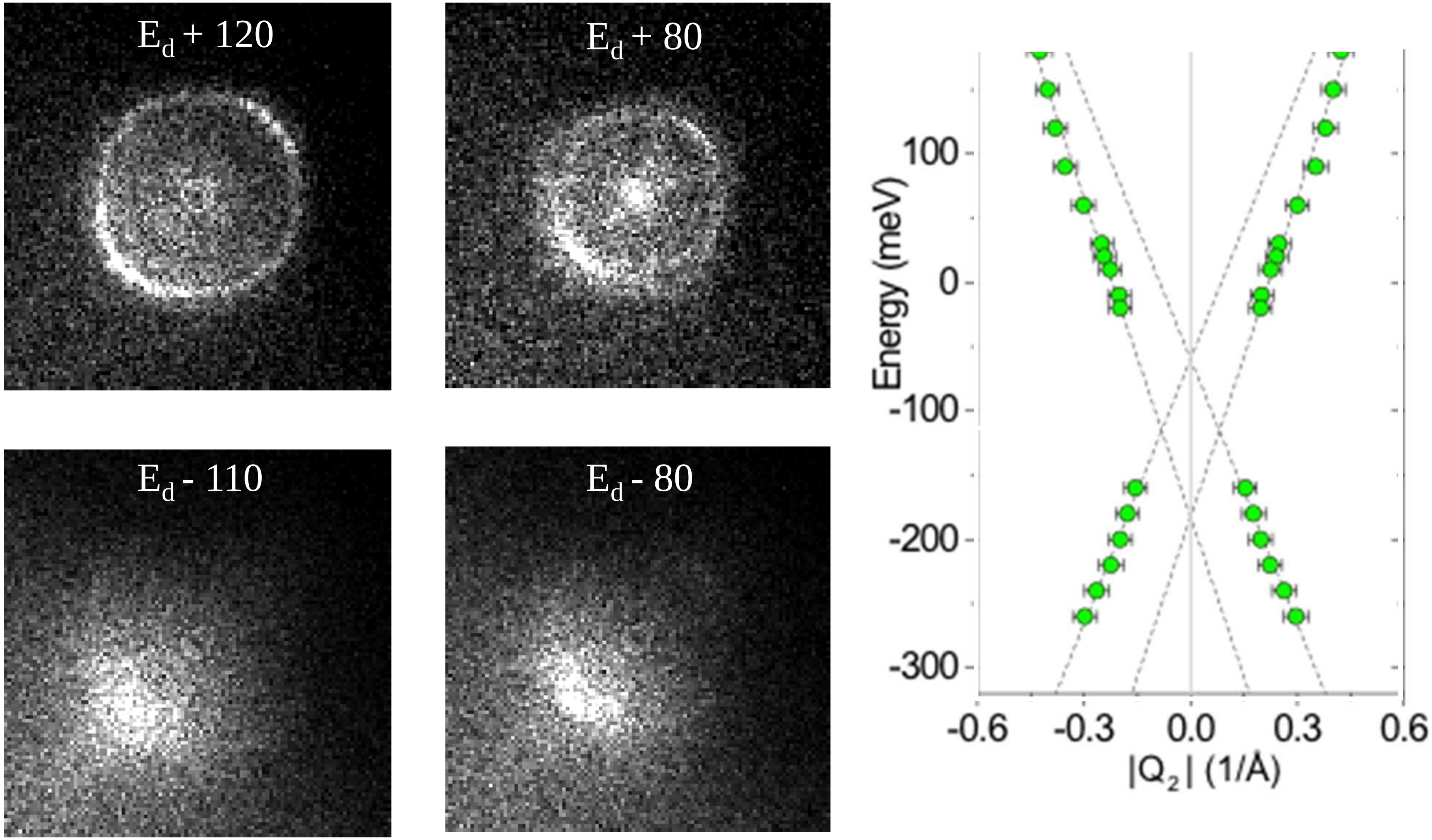


I: Lifshitz Transition



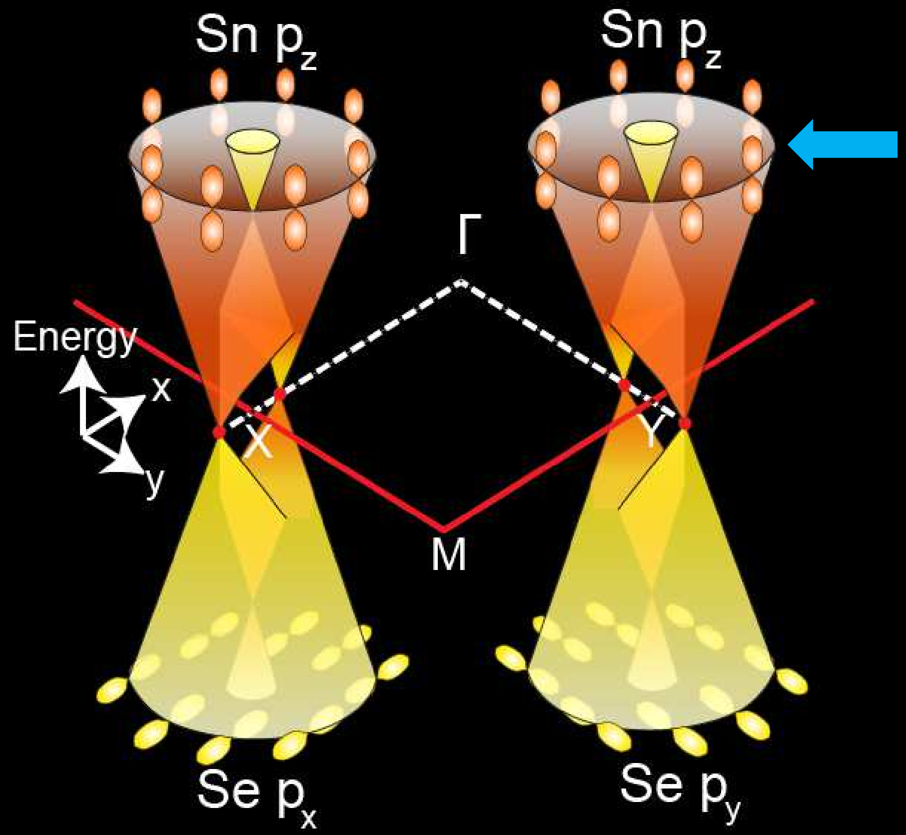


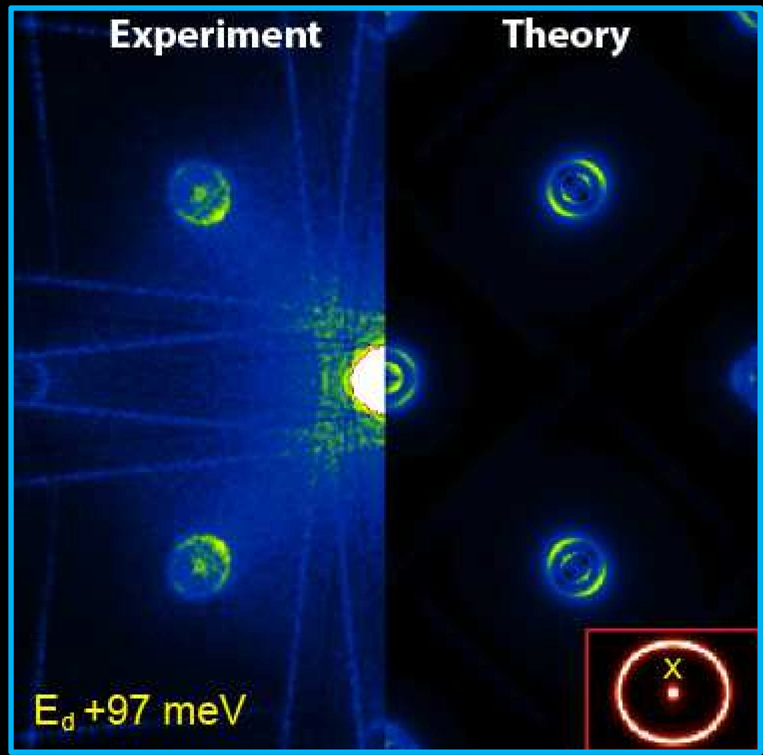
II. Particle-Hole asymmetry: Orbital Effects



Comparison to theory

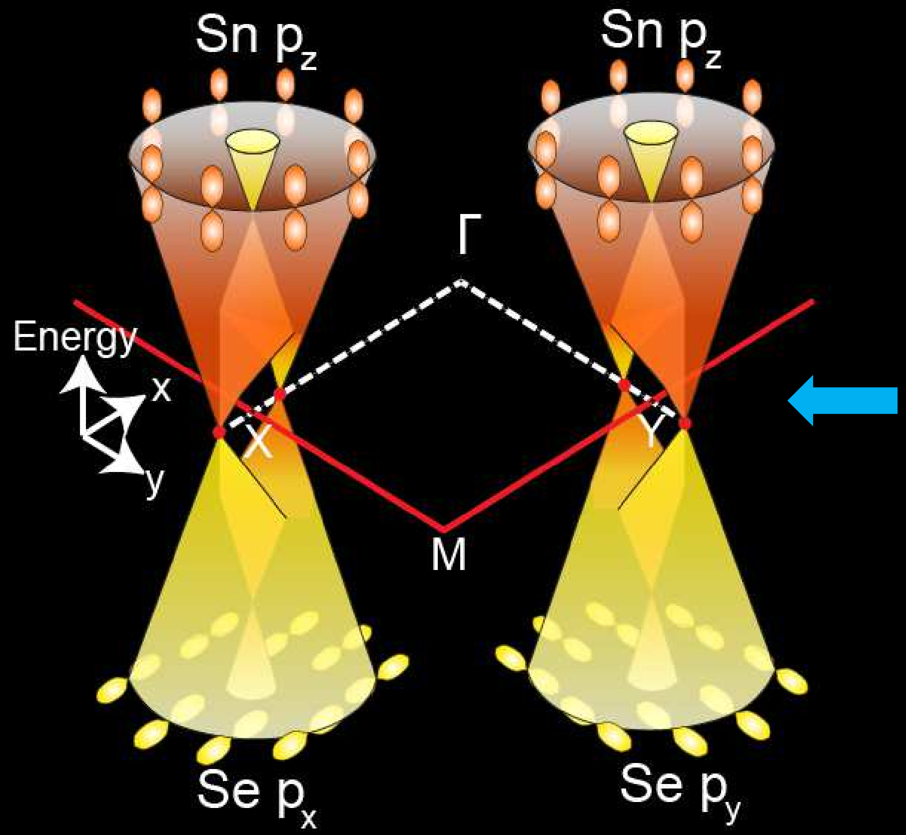
Scattering between upper Dirac cones

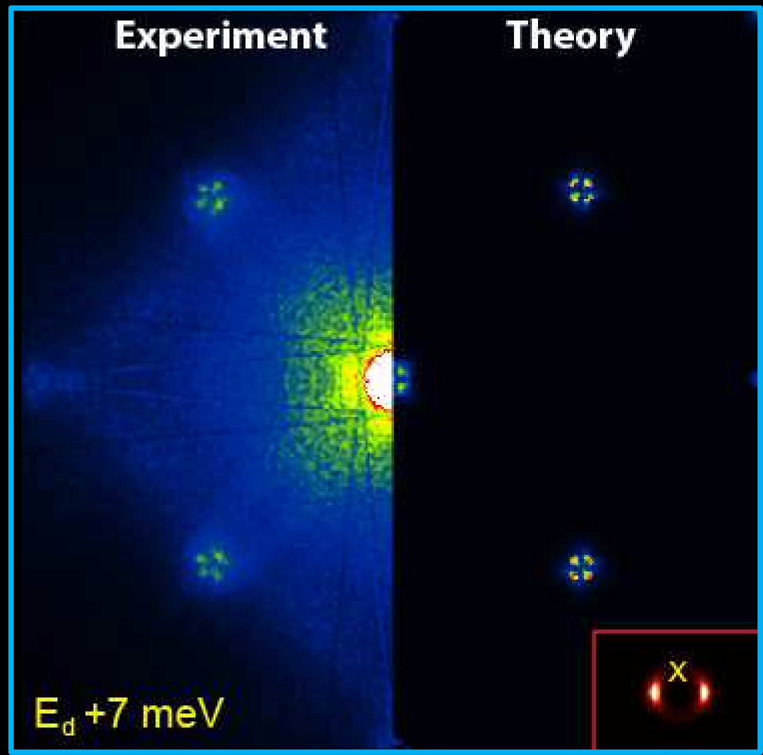




Comparison to theory

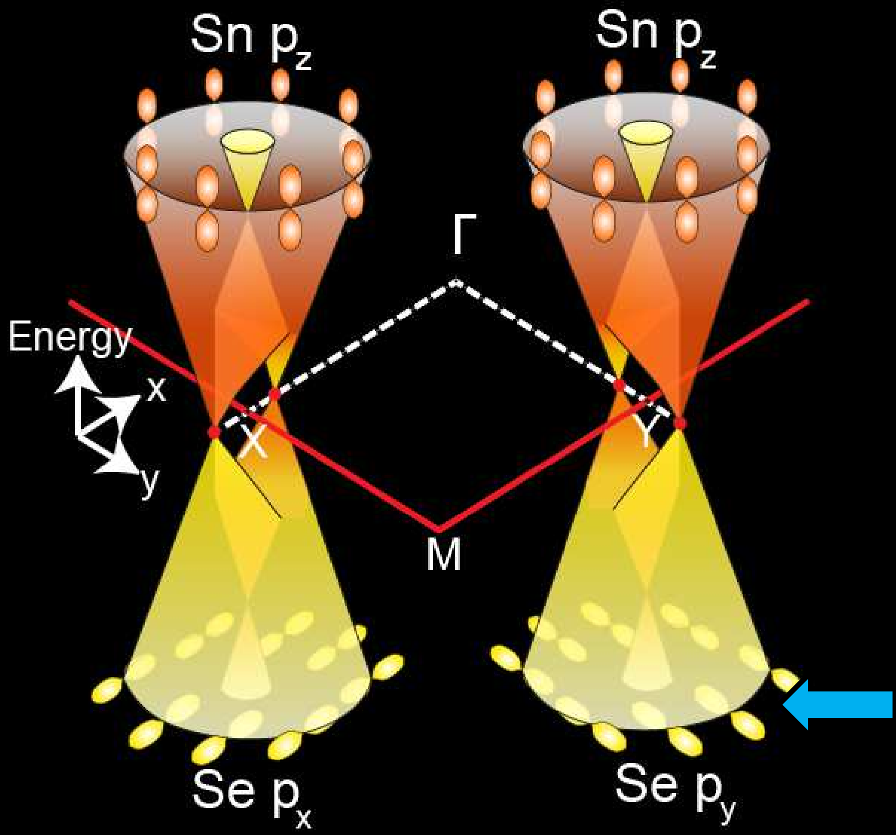
Scattering at the Dirac point

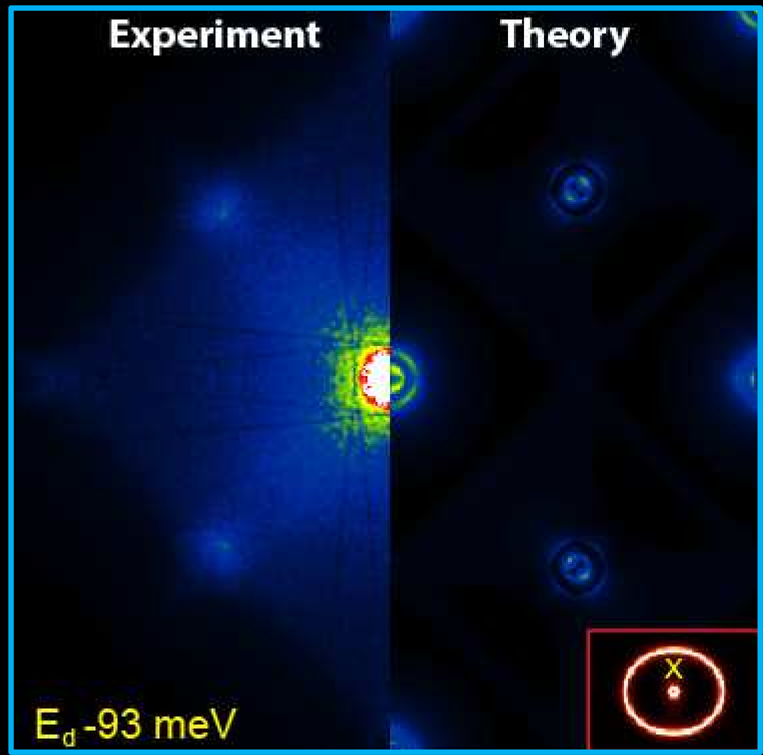




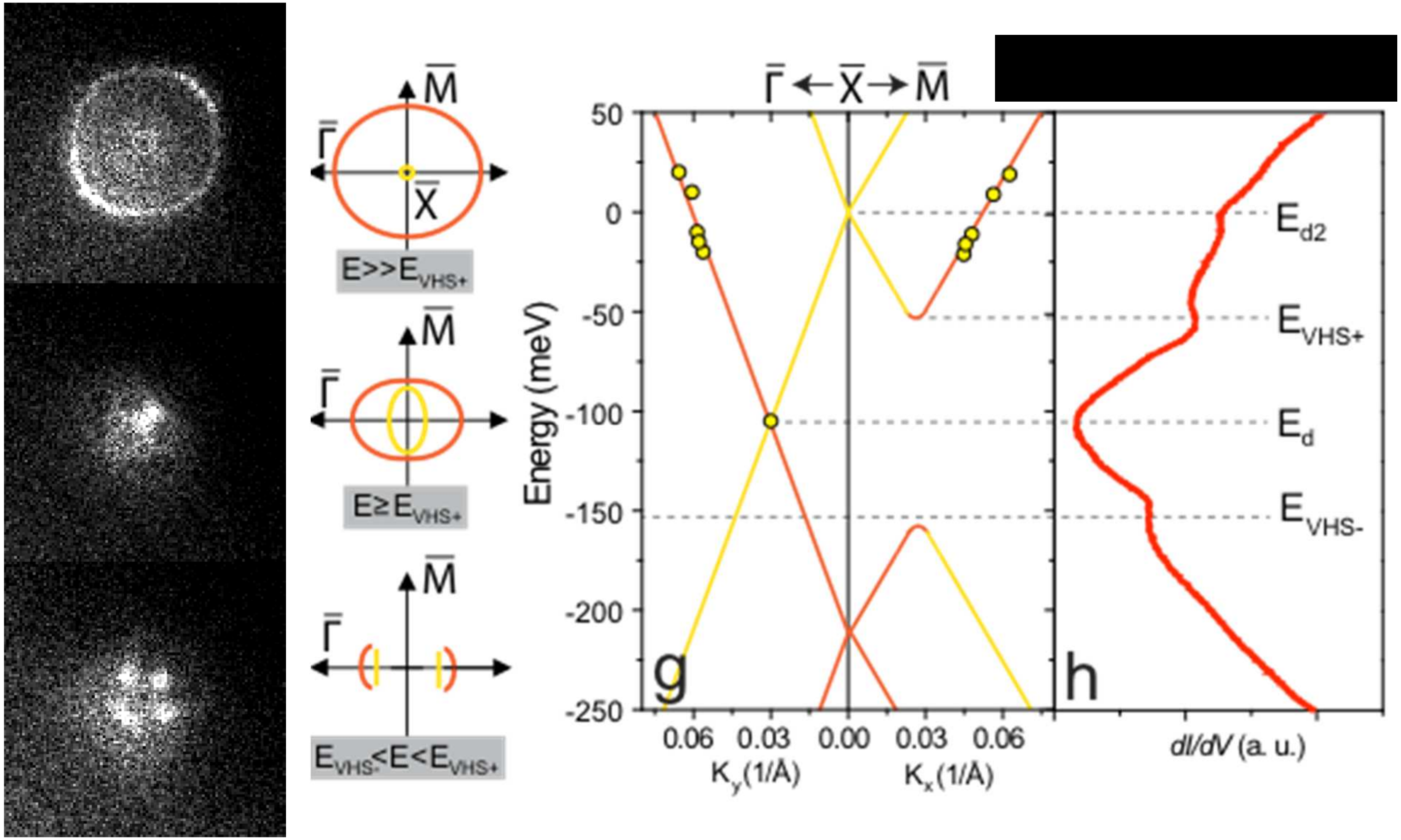
Comparison to theory

Scattering between lower Dirac point

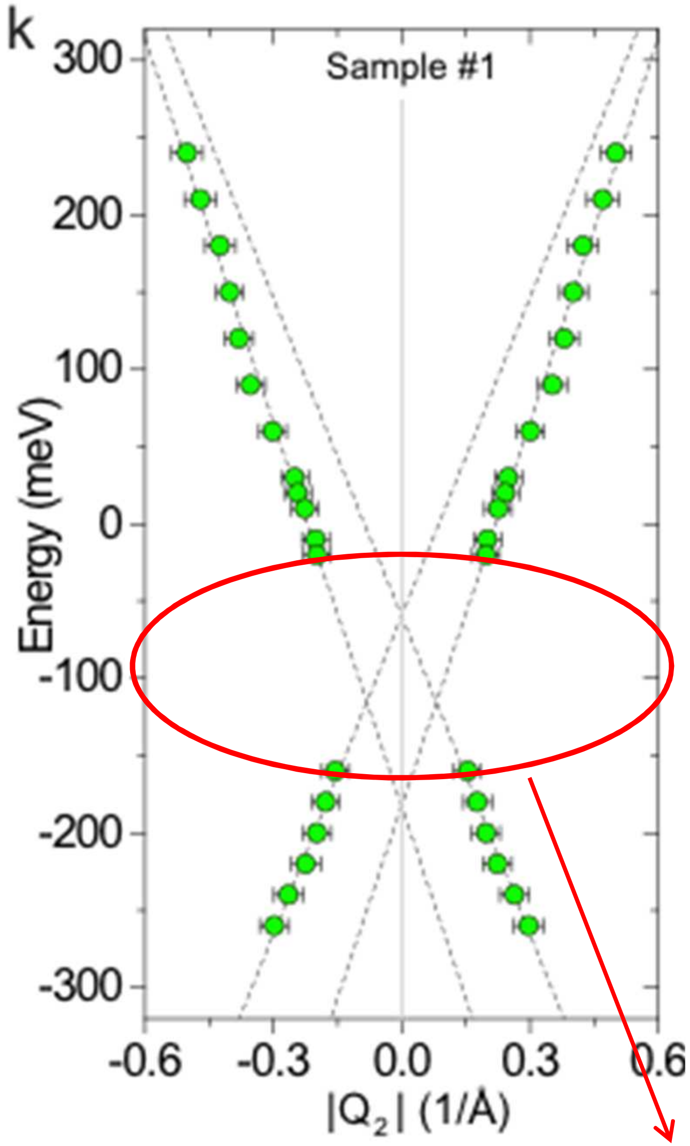




III. Dispersion



Dispersion



Cannot Access region near Dirac Point

Do we really have massless Dirac particles??

Outline

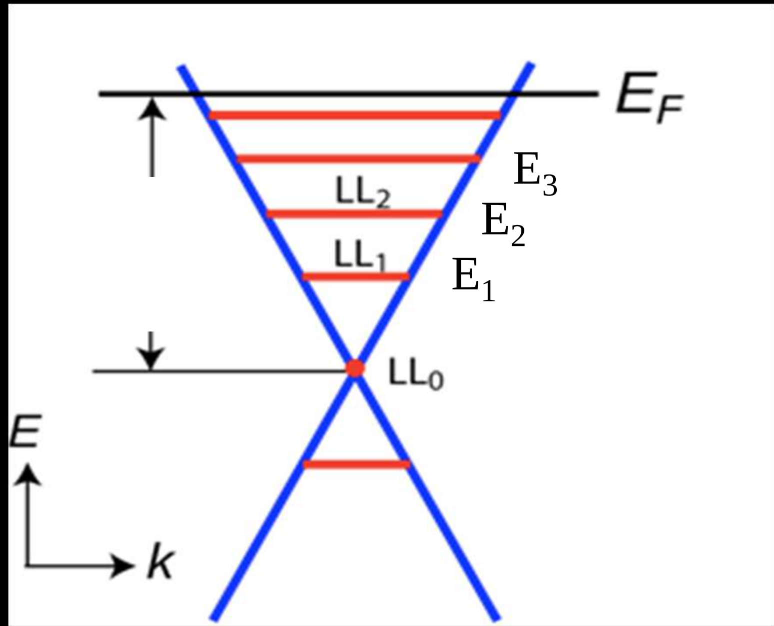
- The System: Topological Insulators
  - Unique properties of Conventional Z2 Topological Insulators
  - New Topological Material: Topological Crystalline Insulator
- The Technique: Scanning Tunneling Microscopy
  - Interference Patterns
  - Landau Level Spectroscopy
- The Experiment and Results: Breaking Mirror Symmetry to impart Mass to Massless Dirac Fermions
- Outstanding Questions and the Future

Landau level Spectroscopy

Dirac electrons in a Magnetic Field

Discrete allowed energies called Landau levels

- Low Energy Dispersion
- Effective g-factor
- Correlation Effects
- Quasiparticle lifetime



Landau Levels labeled by quantum number n
Energy is a function of n

Landau Levels

Normal 2D surface state or 2D Electron Gas

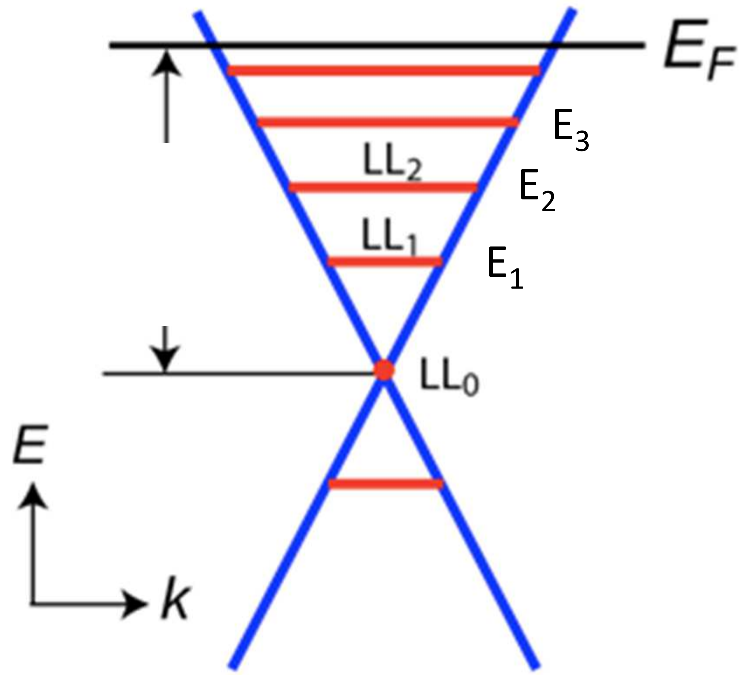
$$E_n = \frac{eB\hbar}{m^*}(n + 1/2)$$

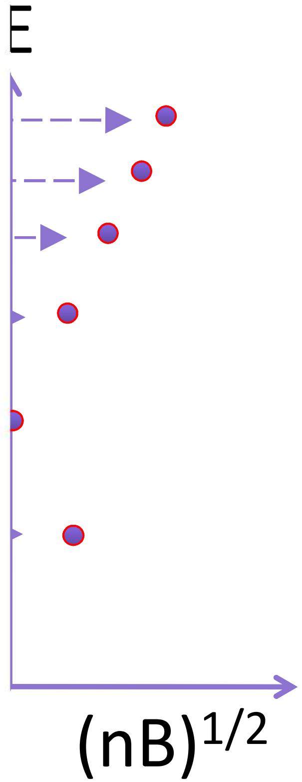
2D Surface state on Topological Insulator

$$E_n = E_D + \vartheta_F \sqrt{2eB\hbar|n|}$$

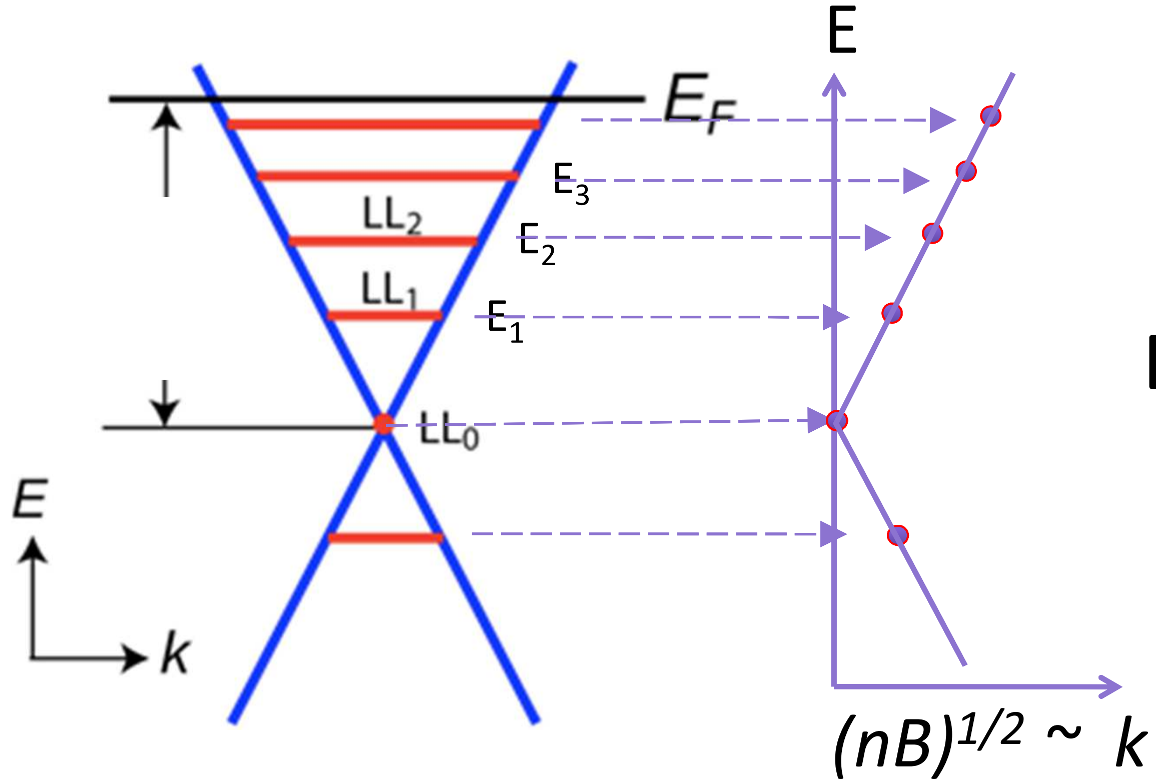
$$k^2 = \frac{2neB}{\hbar} \Rightarrow k = \sqrt{\frac{2neB}{\hbar}} \Rightarrow k \propto \sqrt{nB}$$

Landau Levels





Landau Levels



Dispersion!

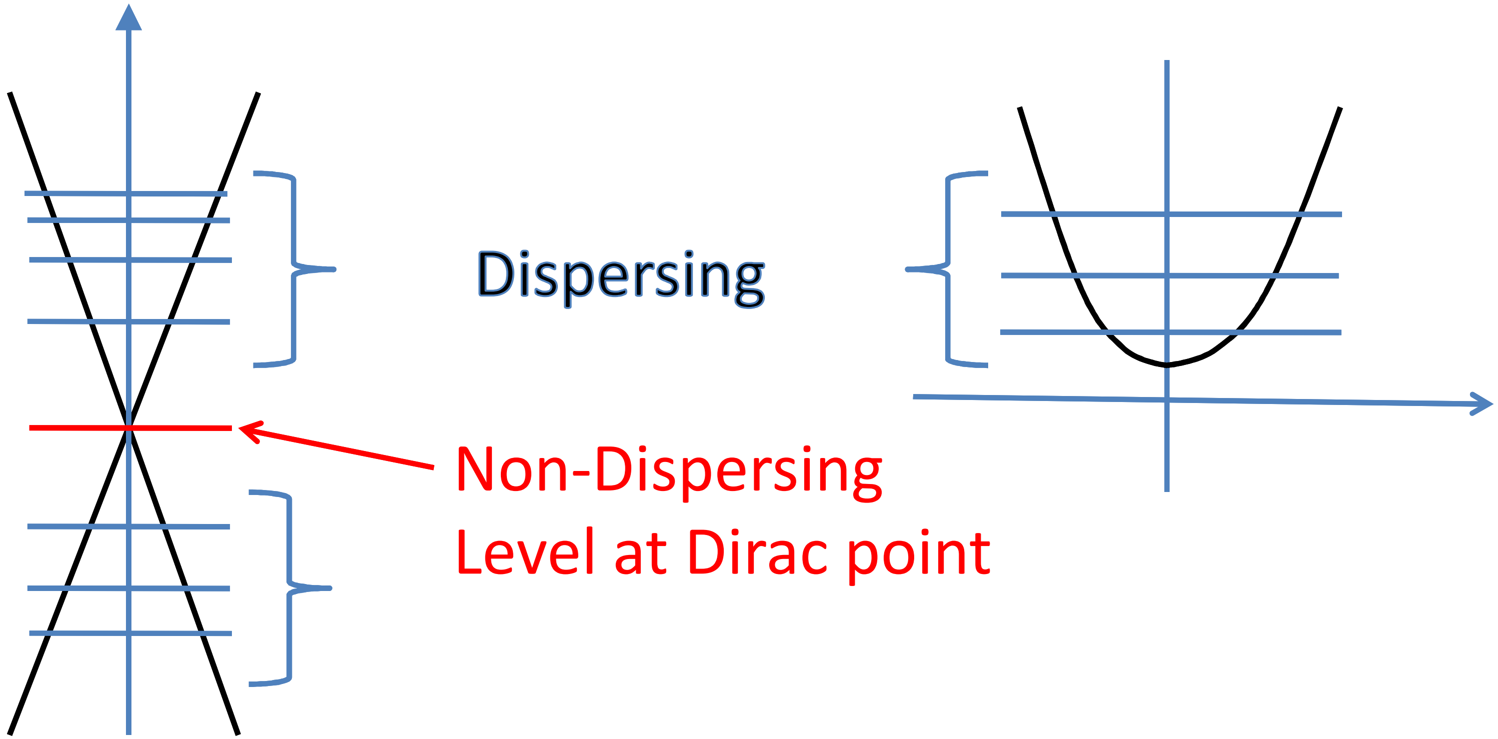
Landau level Scenarios

Massless Dirac Band

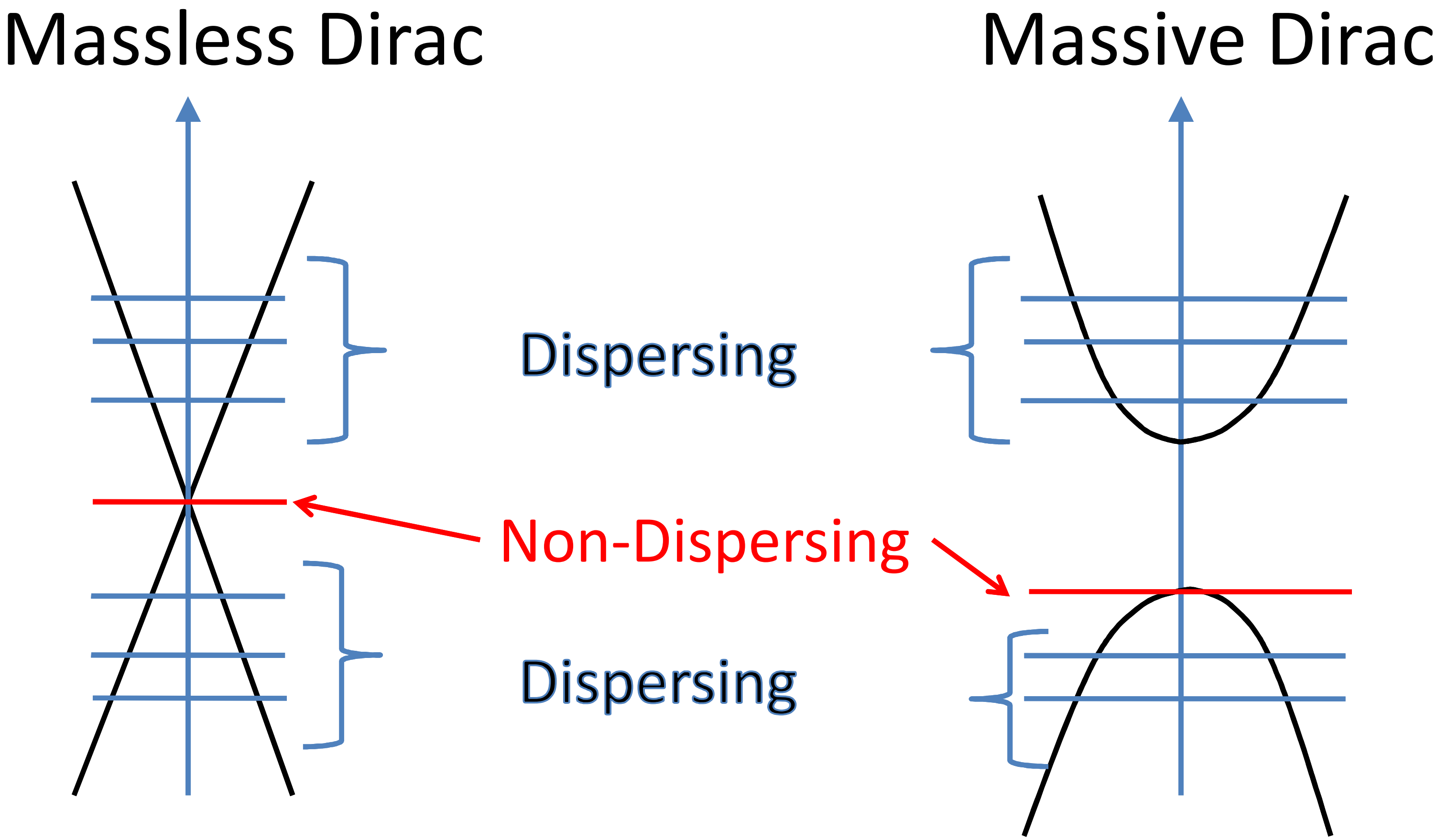
$$E_n = E_D + \vartheta_F \sqrt{2eB\hbar|n|}$$

Non-Relativistic Band

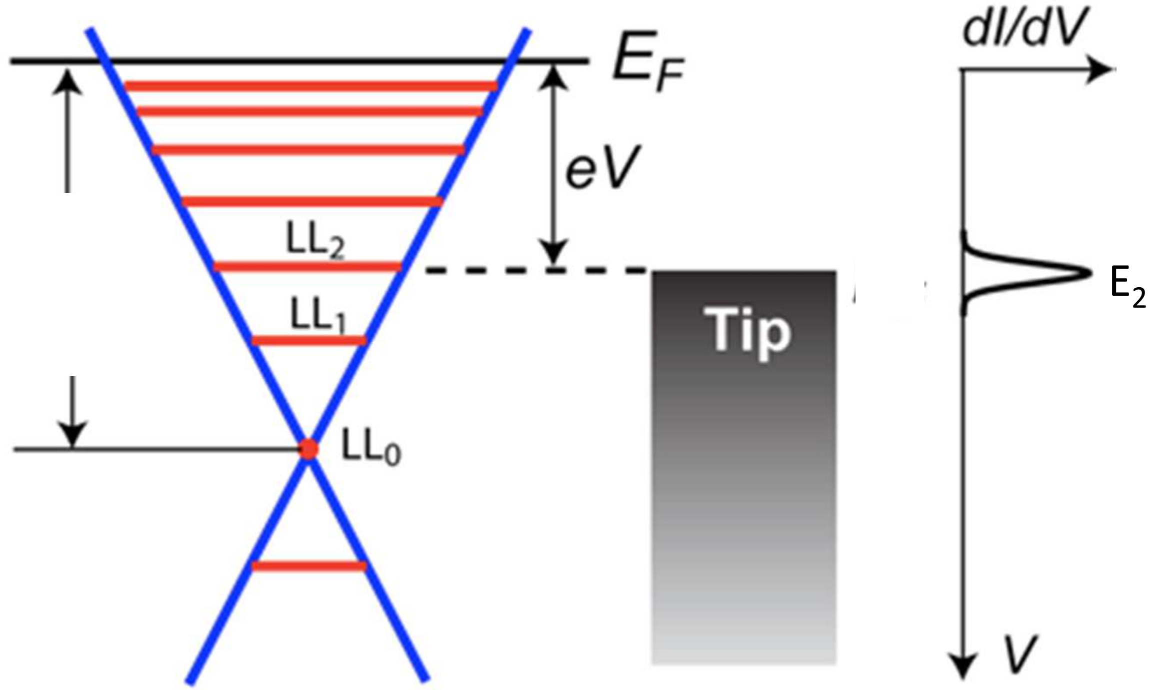
$$E_n = \frac{eB\hbar}{m^*}(n + 1/2)$$



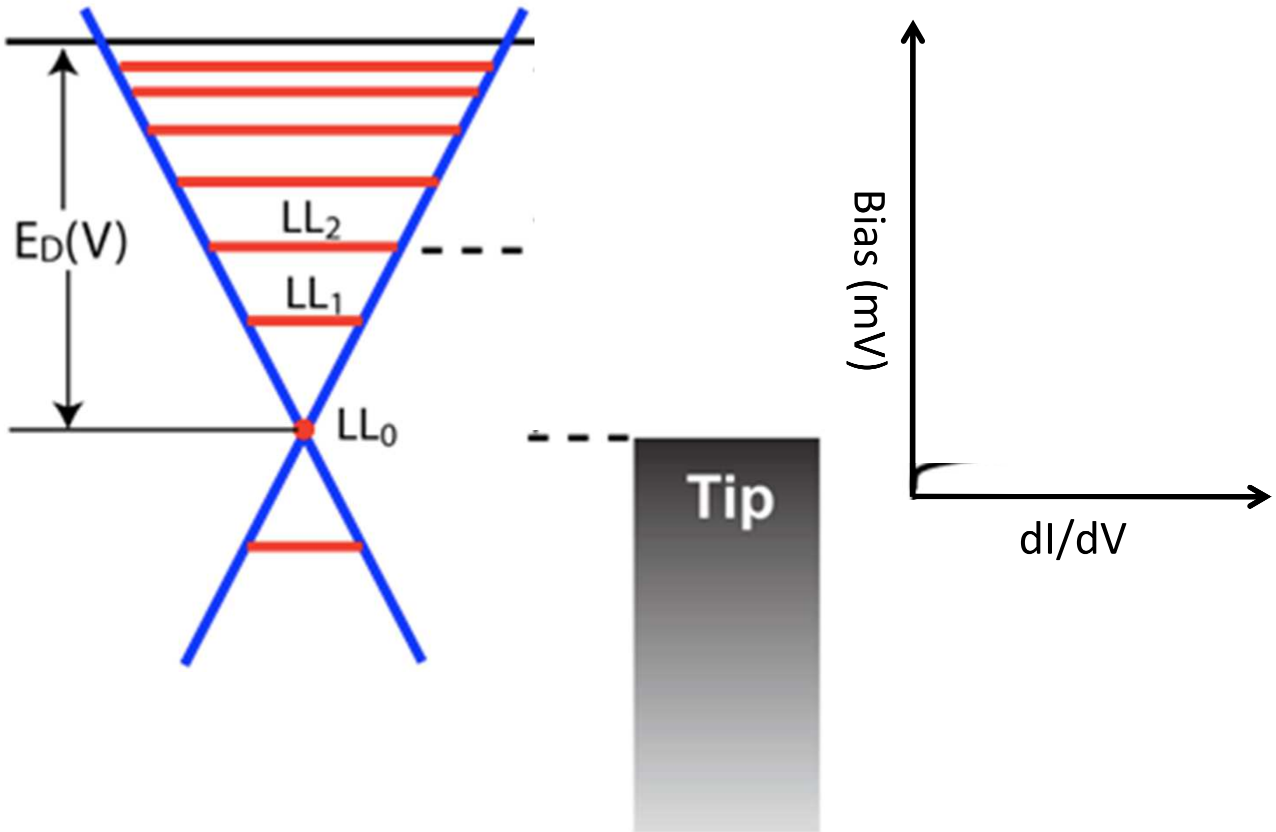
Landau level Scenarios



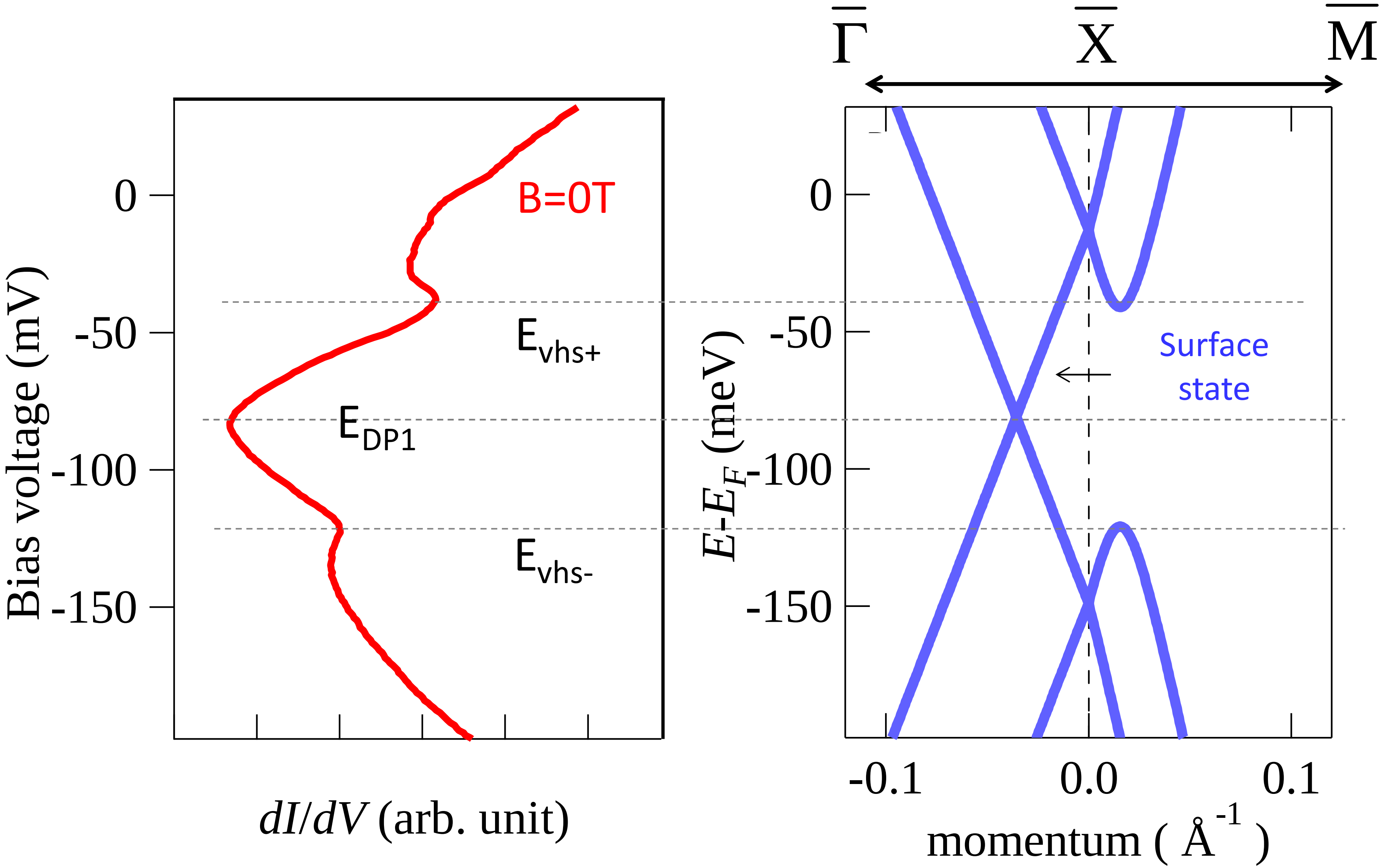
Landau Level Measurements



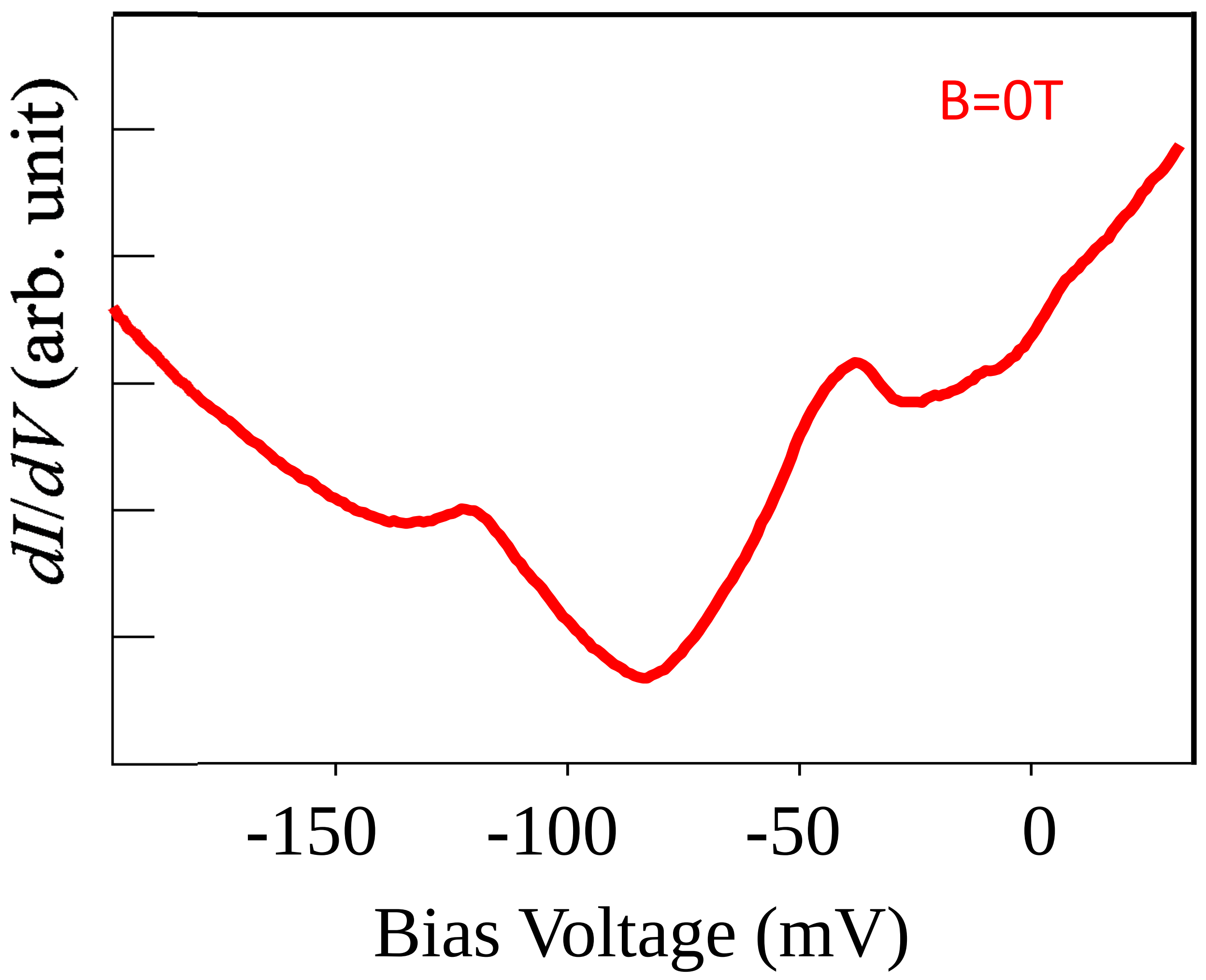
Landau Level Measurements



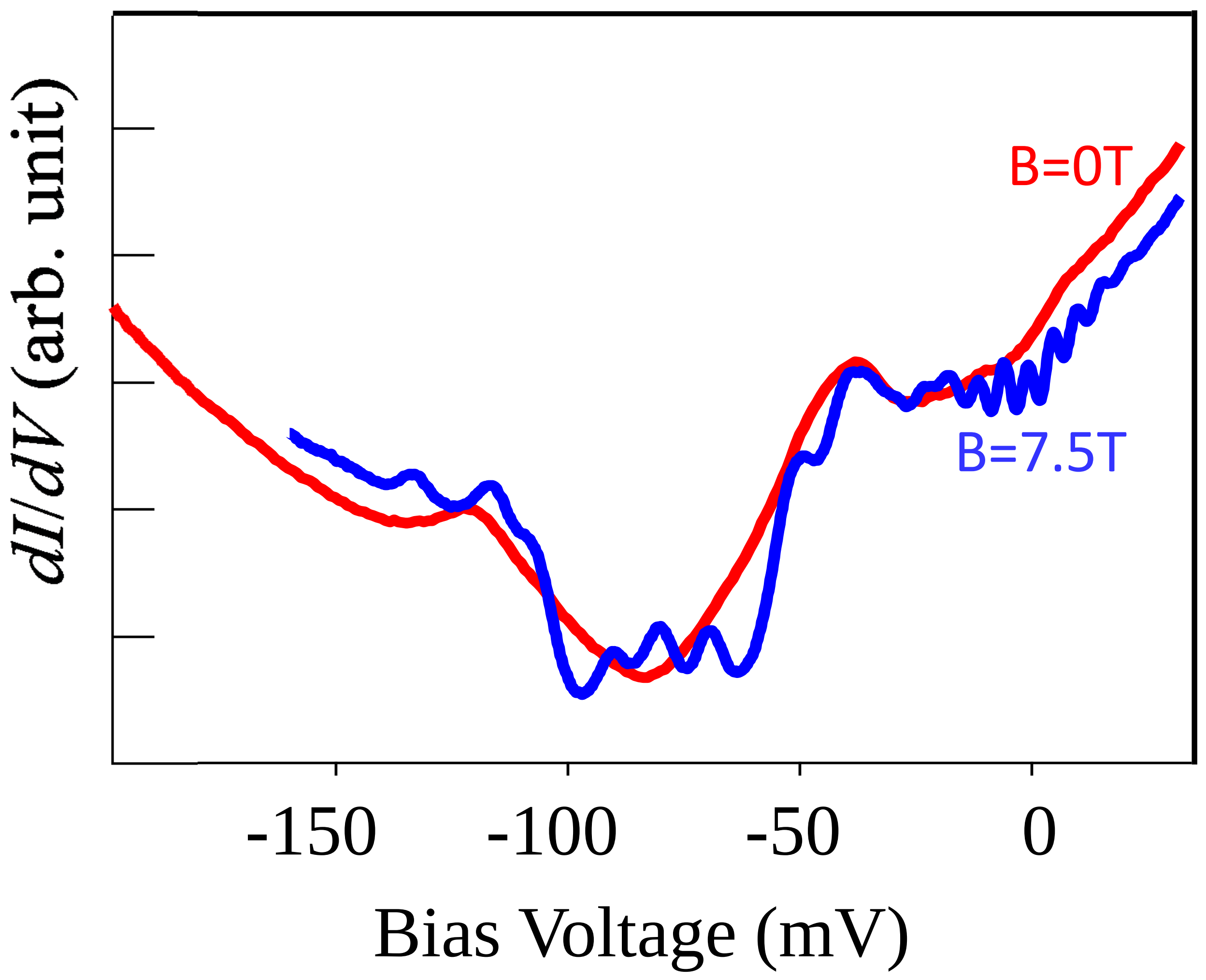
Topological Crystalline Insulators



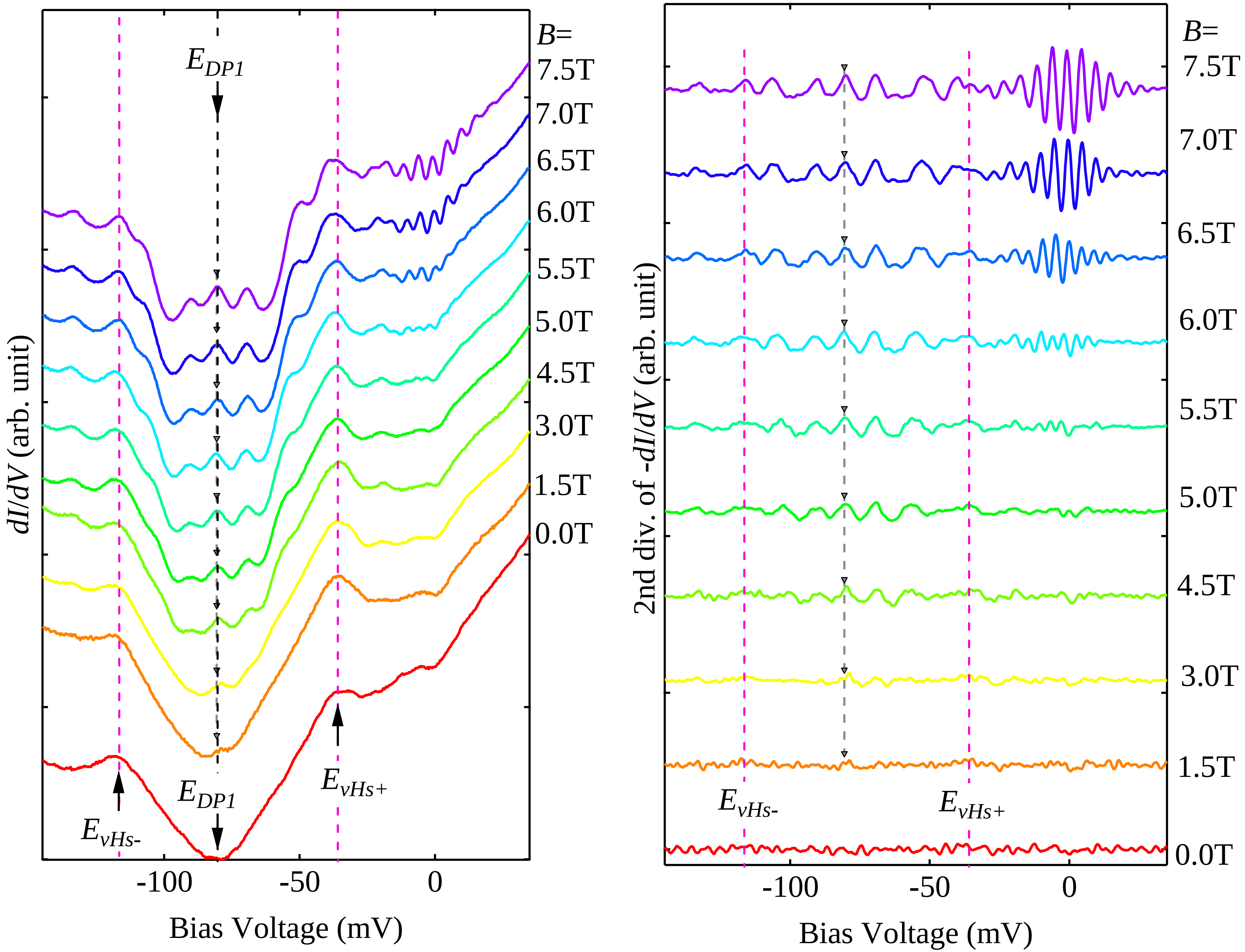
Topological Crystalline Insulators



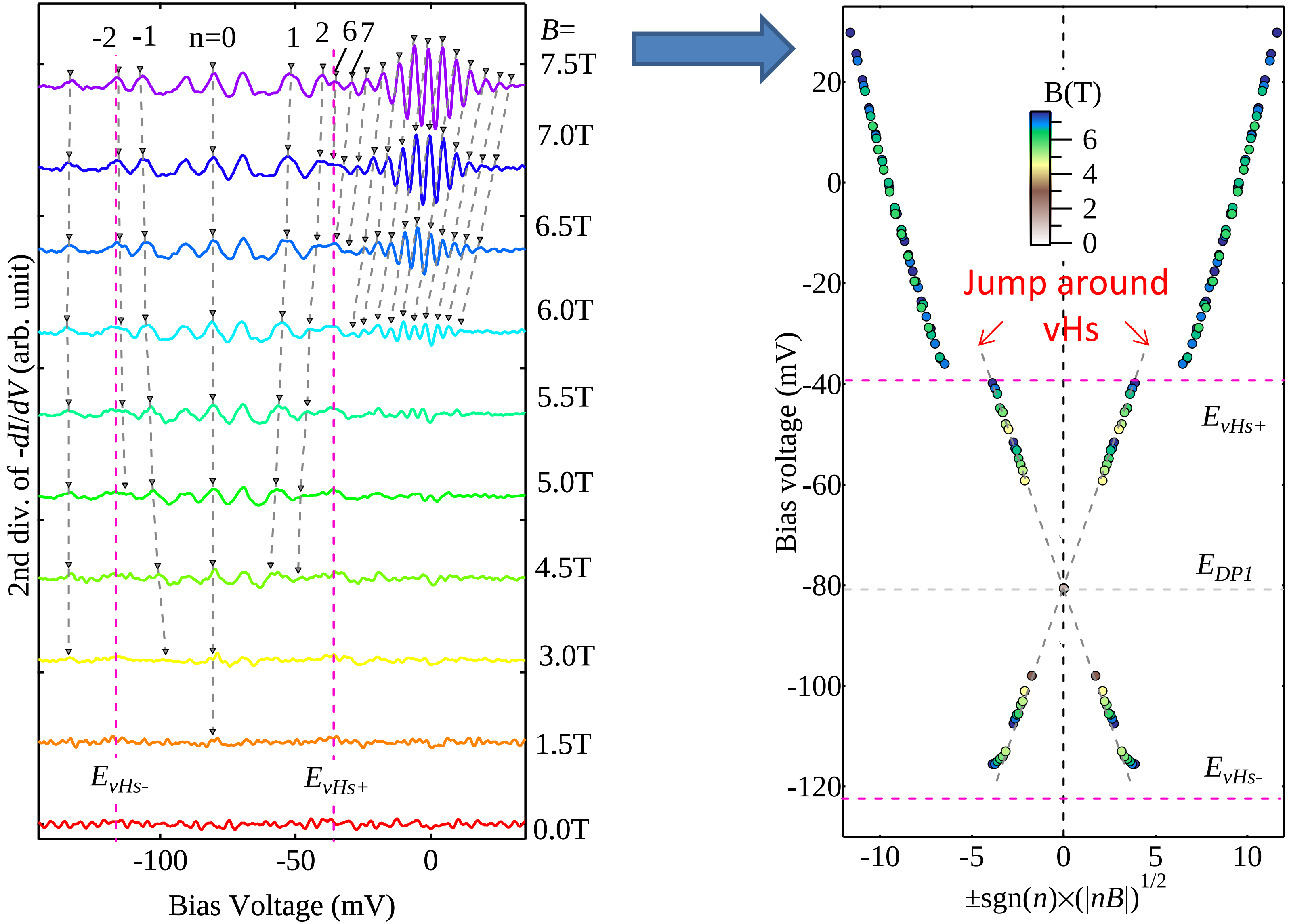
Topological Crystalline Insulators: Landau Levels



Landau Level Field Dependence

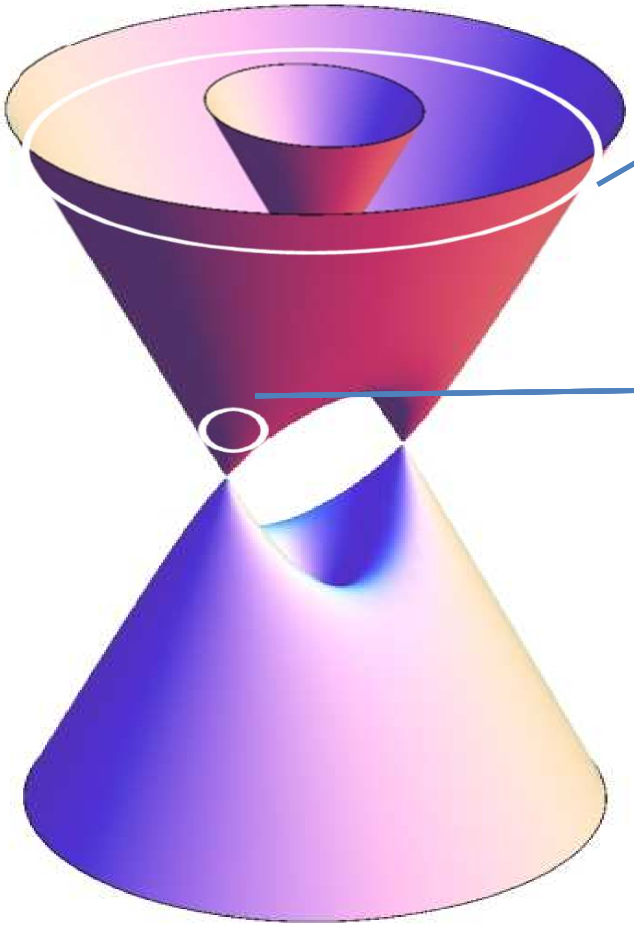


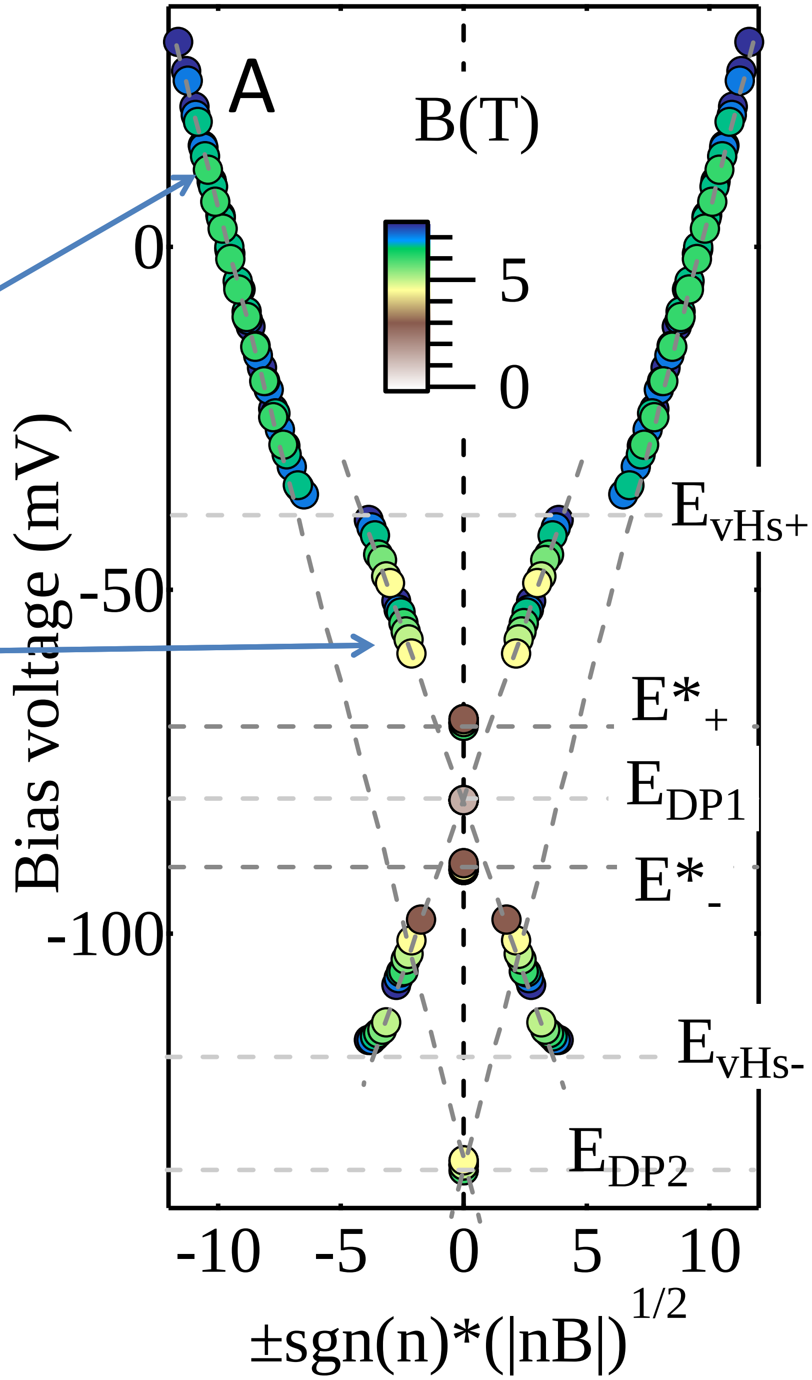
Constraint: scaling by $(nB)^{1/2}$



$$\bar{v} = \sqrt{v_1 v_2} = 2.60 \text{ eV} \cdot \text{Å}, \qquad m = 0.055 \text{ eV}$$

Landau Levels and Band Structure

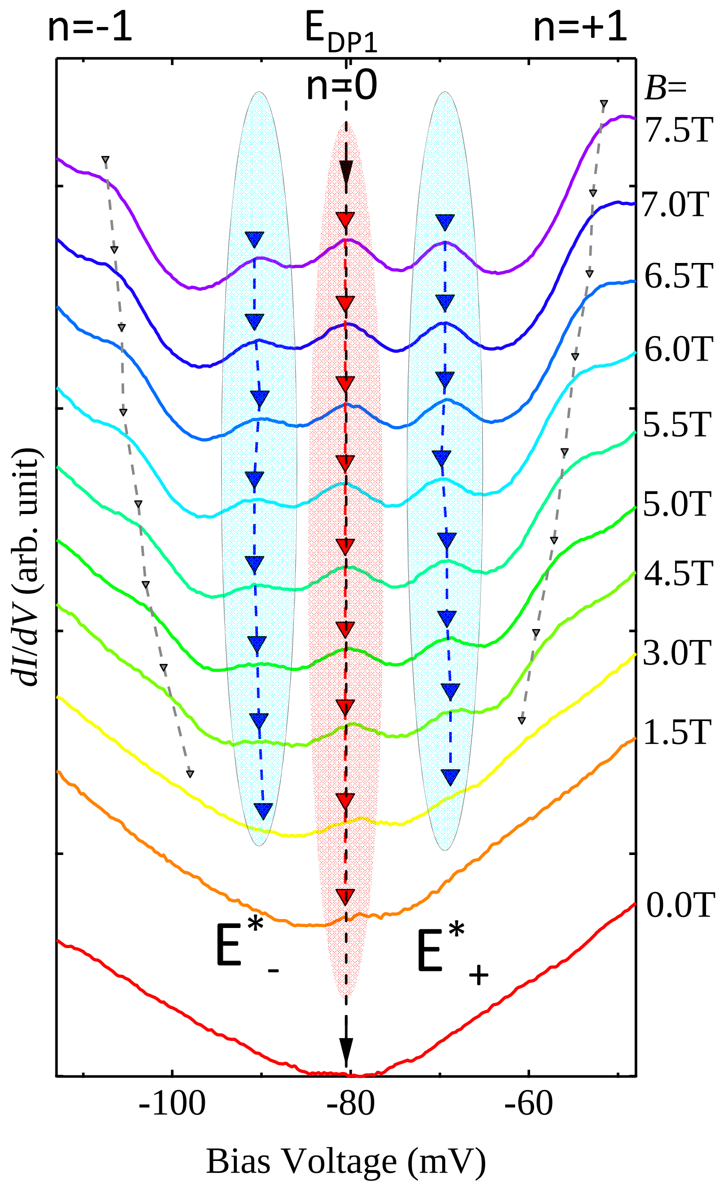


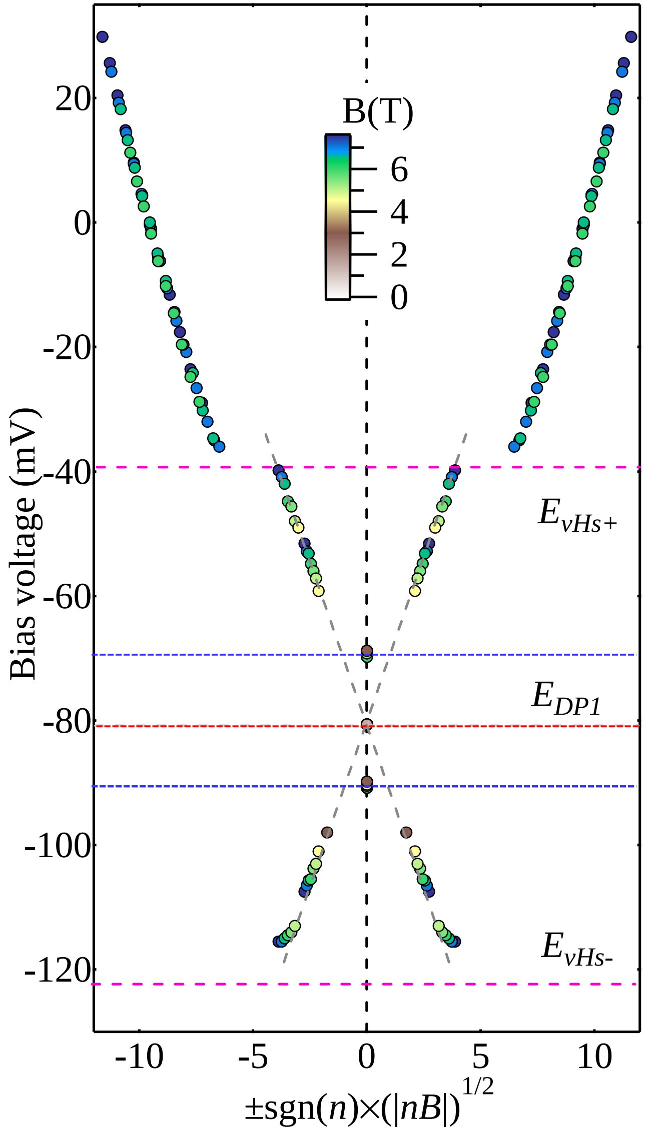


Okada *et al. Science* (2013)

Surprise: Extra non-dispersing peaks

Non dispersive 3 LLs around DP



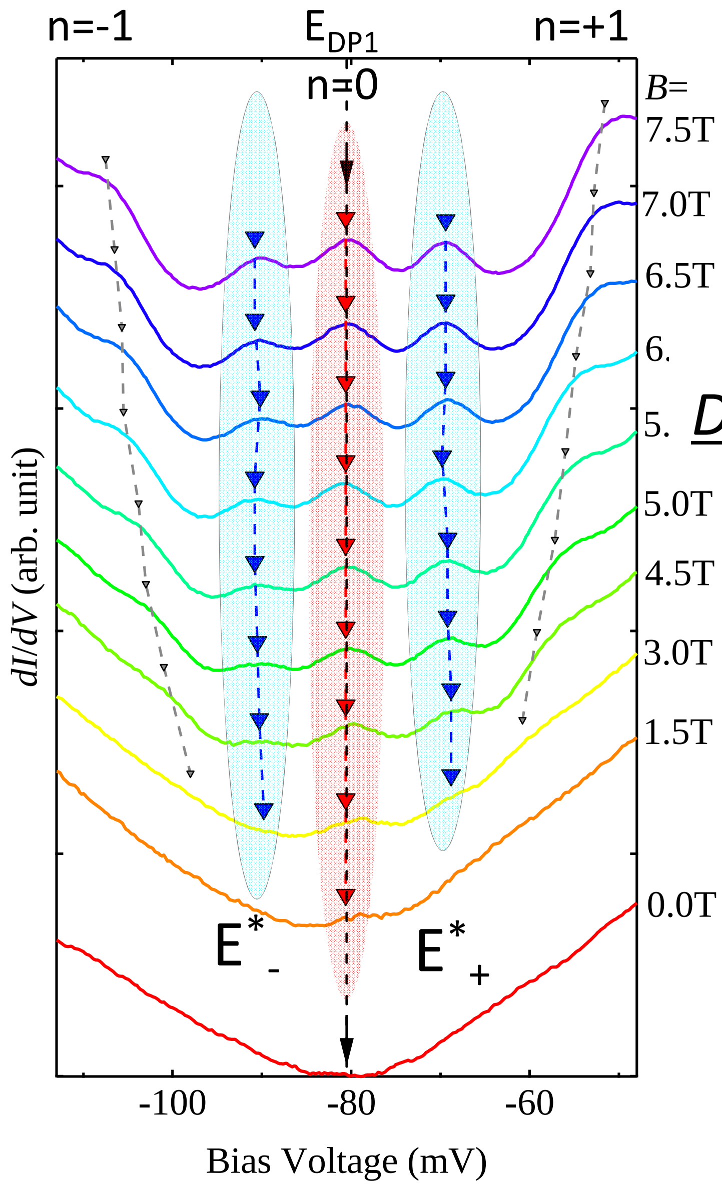


E^*_+

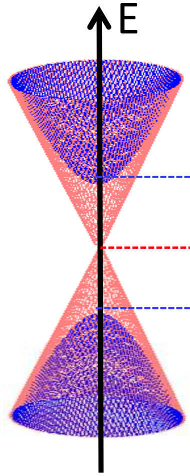
E^*_-

Extra non-dispersing peaks

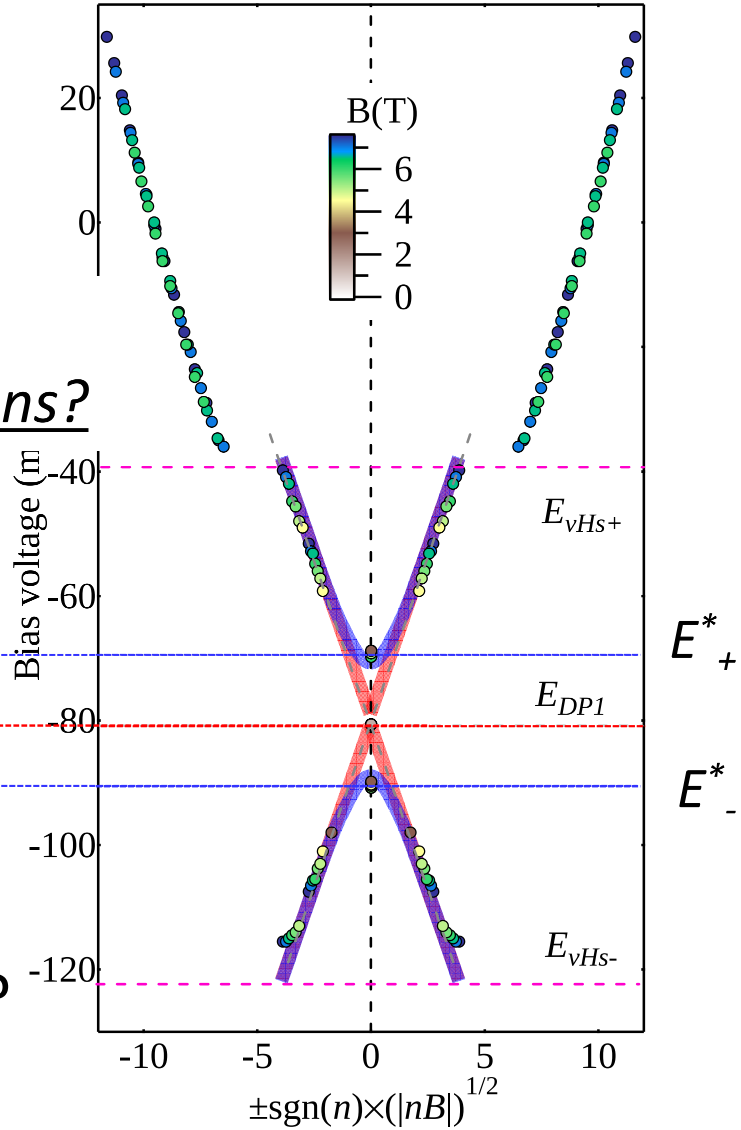
Non dispersive 3 LLs around DP



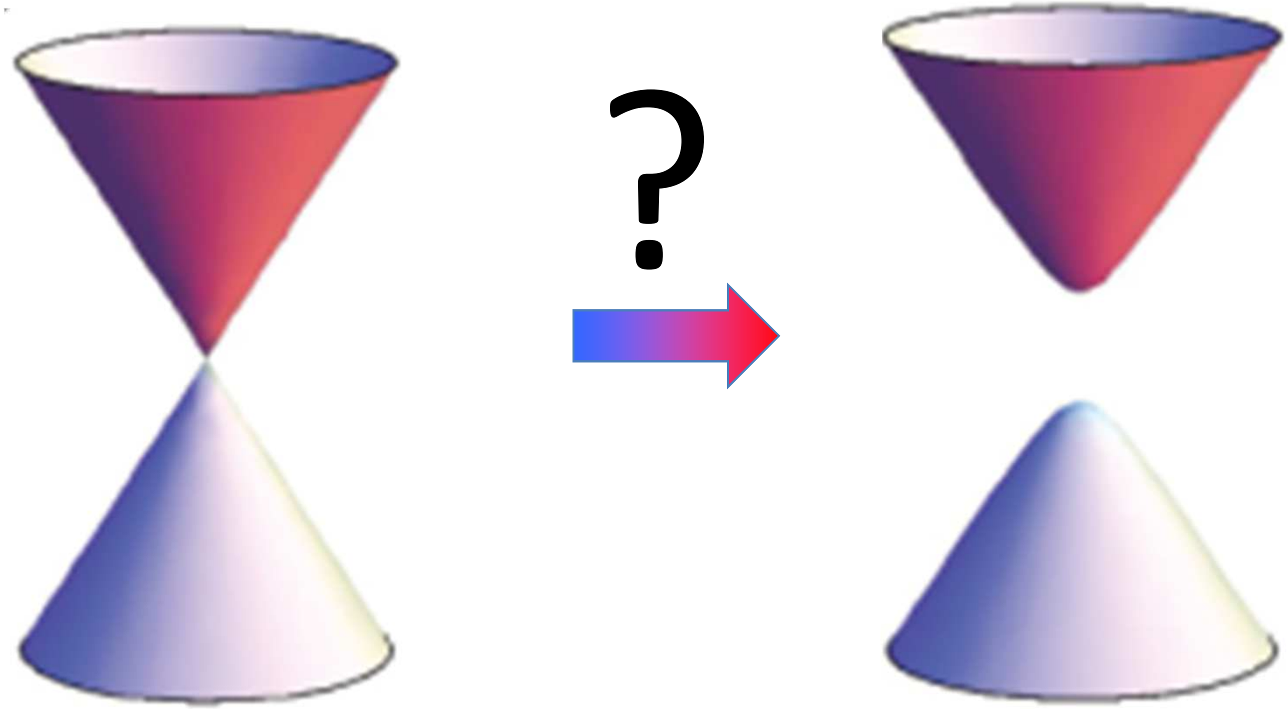
Massive Dirac Fermions?



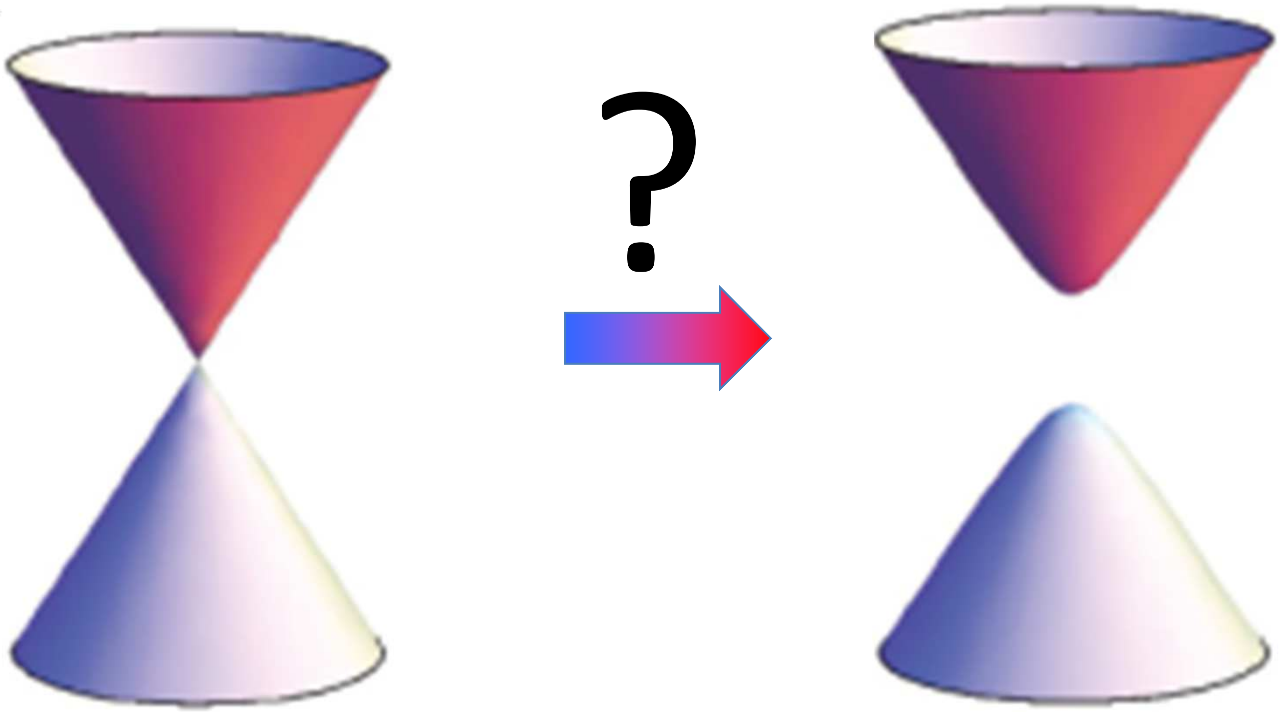
But how?



How do you provide mass to massless Dirac Fermions?

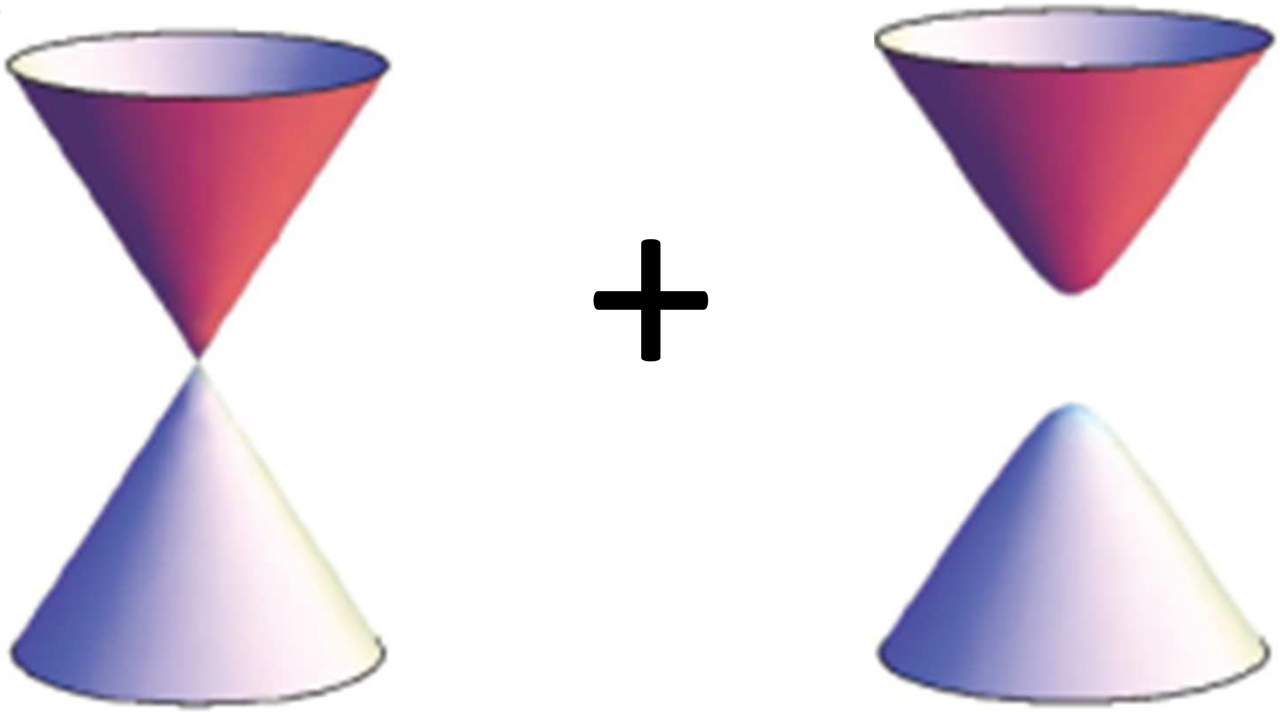


How do you provide mass to massless Dirac Fermions?



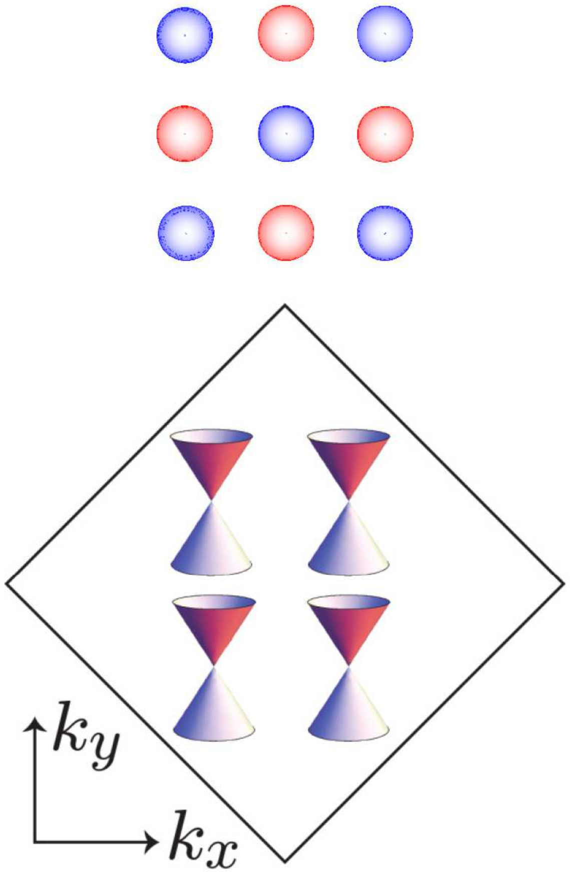
Break the Symmetry protecting the Dirac point

Data: Must have both Massless and Massive electrons

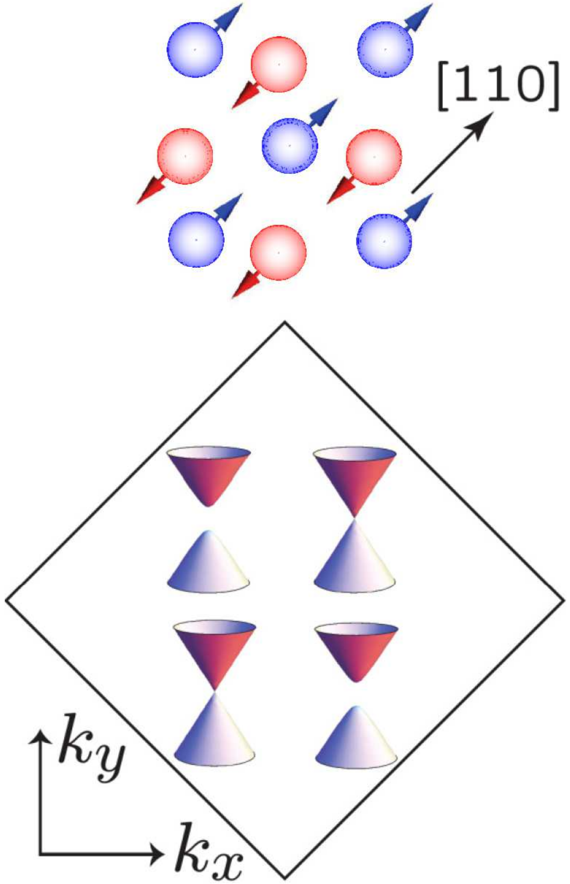


Our Idea: Generate mass in one pair of cones by breaking Mirror symmetry along one plane

NO MASS TERM

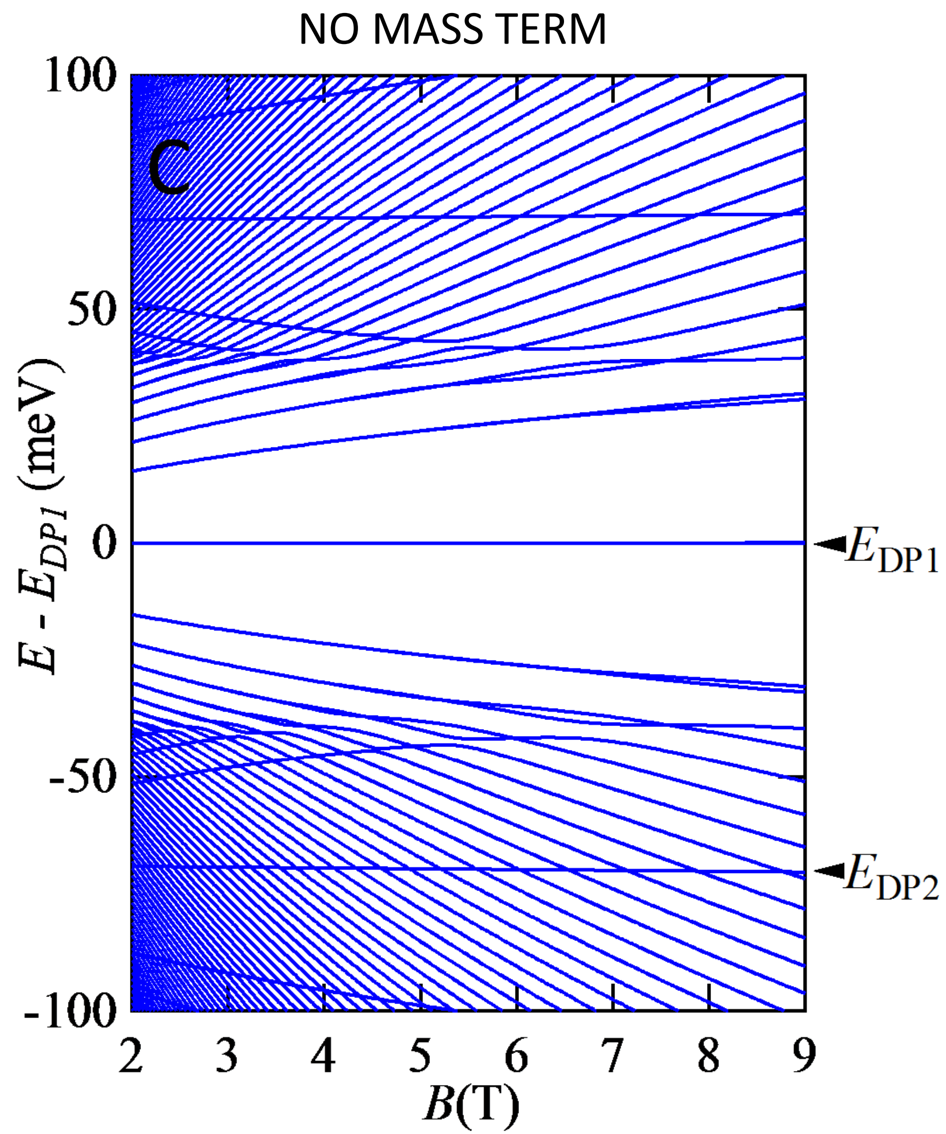


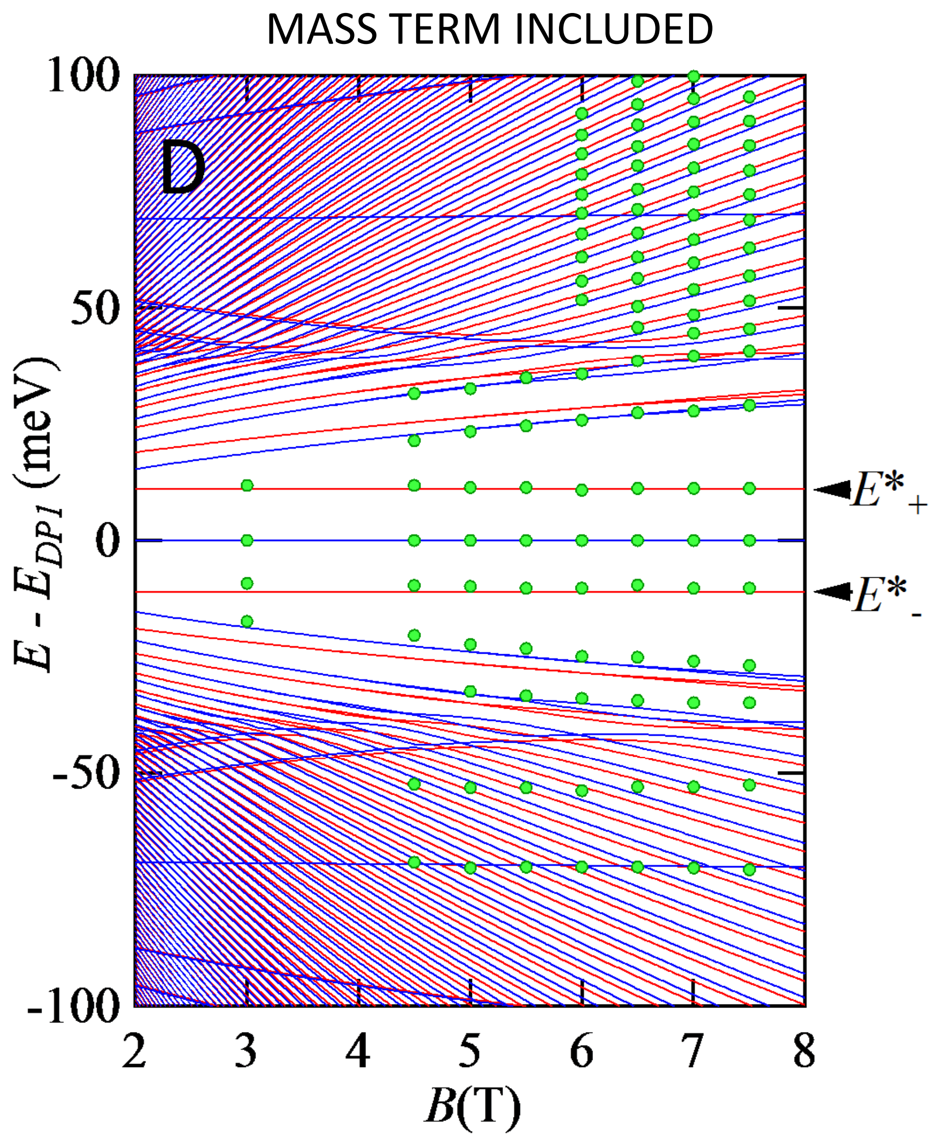
MASS TERM INCLUDED



Okada *et al. Science* (2013)

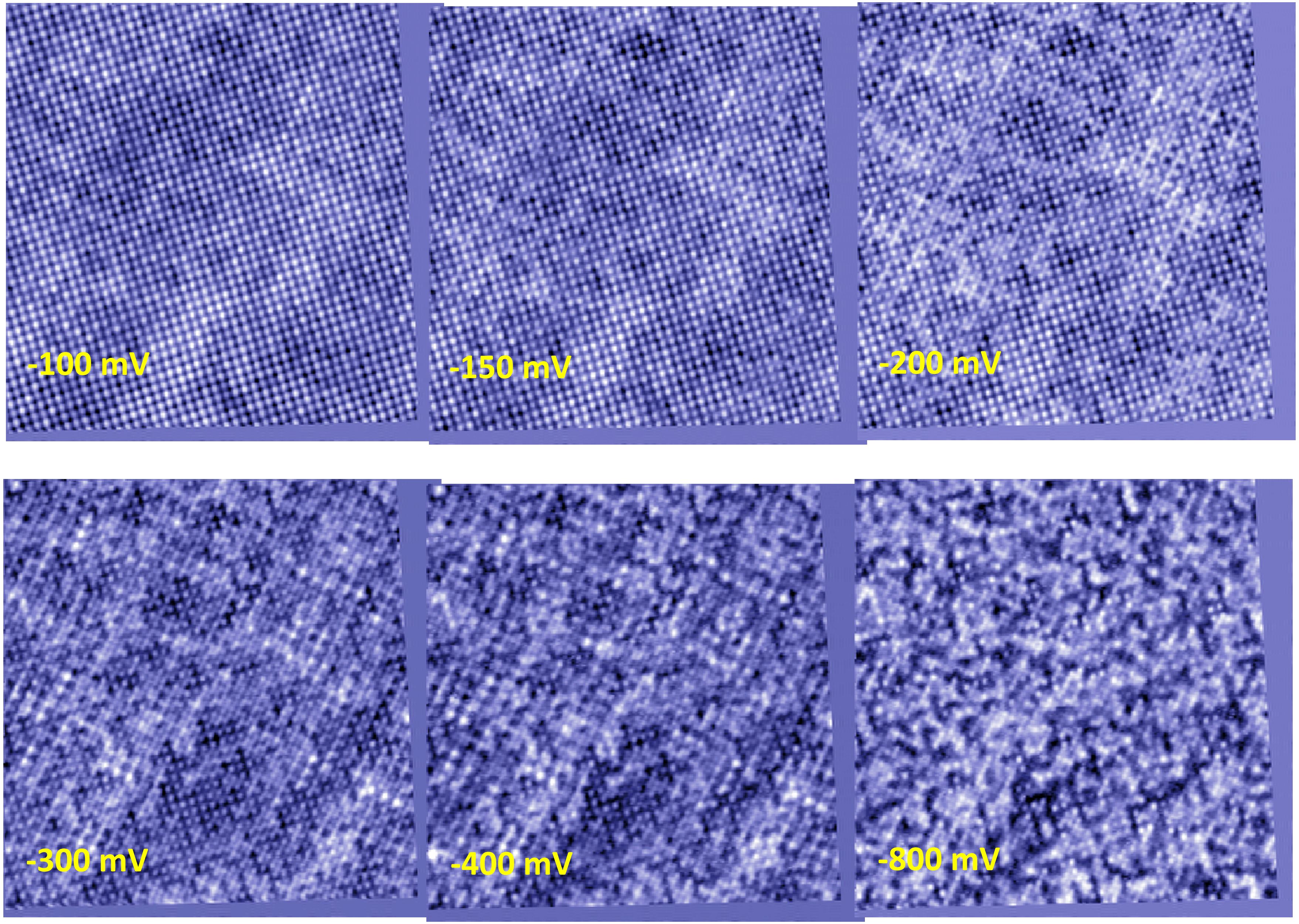
Massive Dirac Fermions





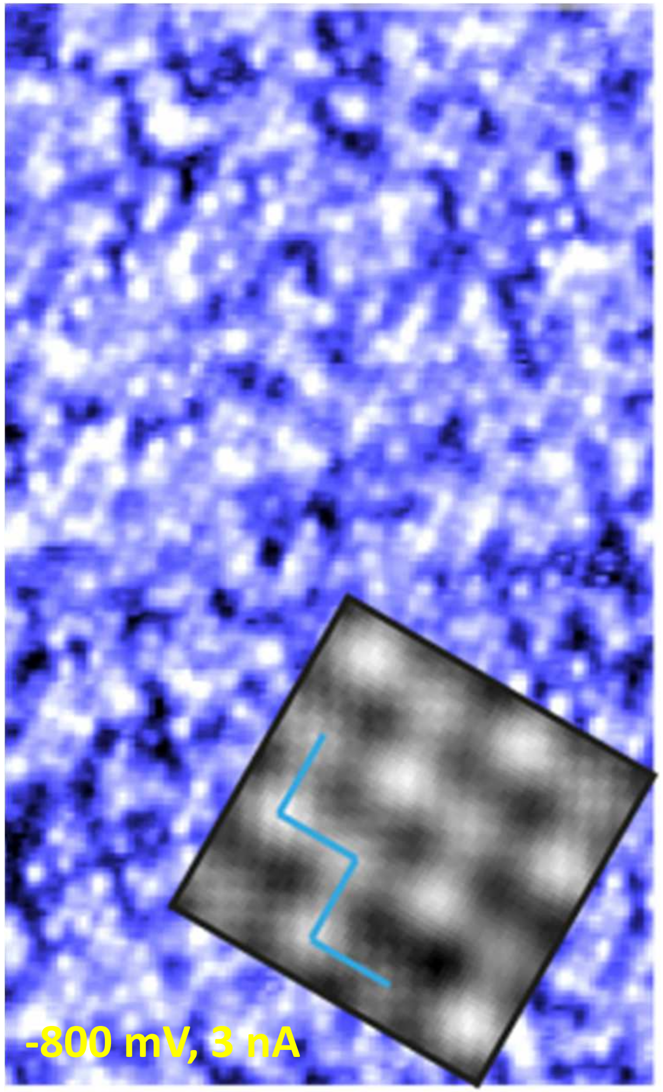
Okada *et al.* *Science* (2013)

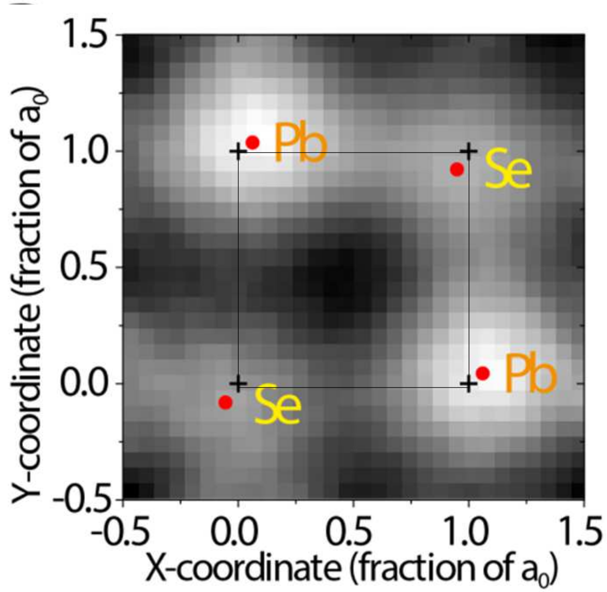
Imaging Broken mirror symmetry

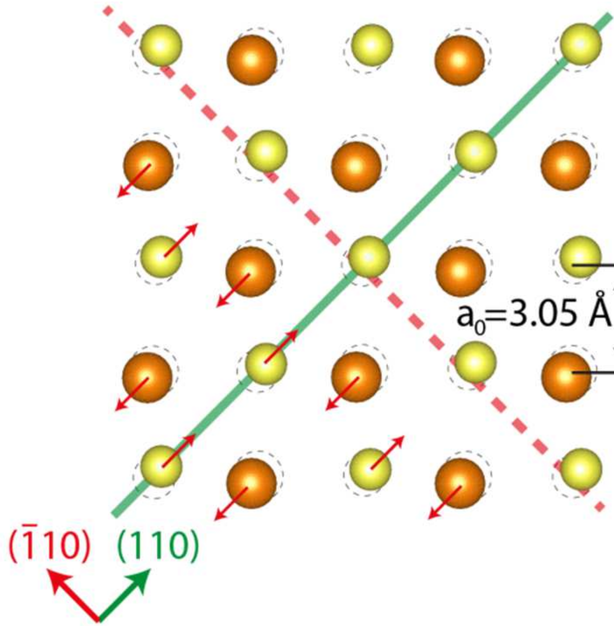


Average 4 x 4 and 2 x 2 unit cells showing Ortho Distortion

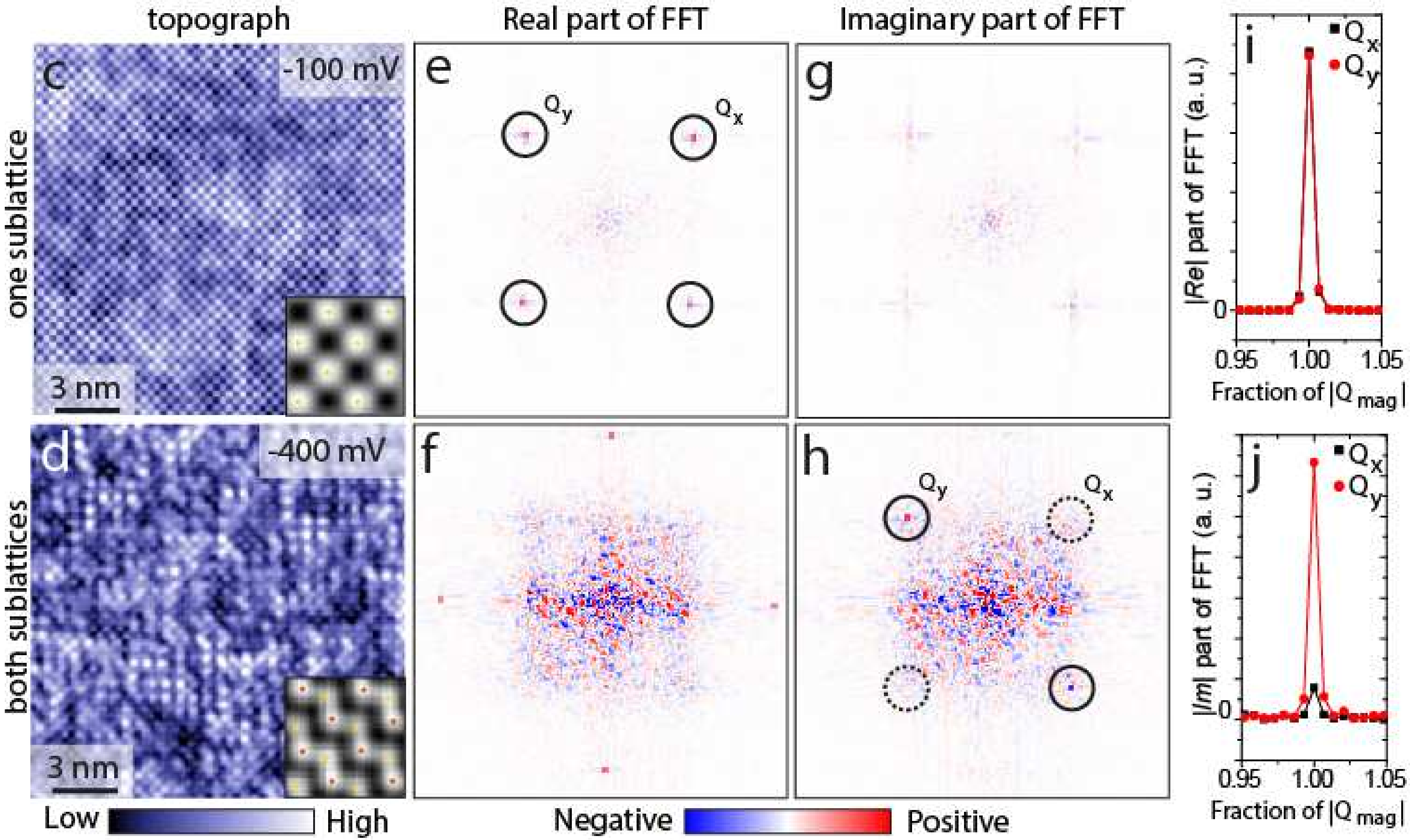
~10% unit cell distortion







Broken C_4 Symmetry from Fourier transform

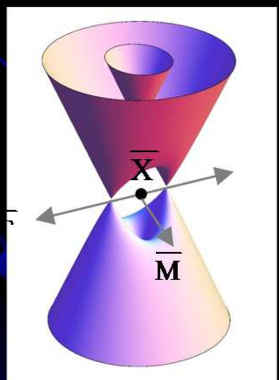


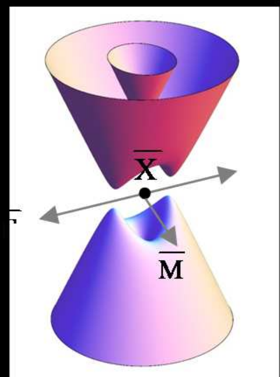
Summary

Fundamental experimental proof of the symmetry protecting the Dirac point

Mass generation is due to a spontaneously broken symmetry (crystalline symmetry in our case)

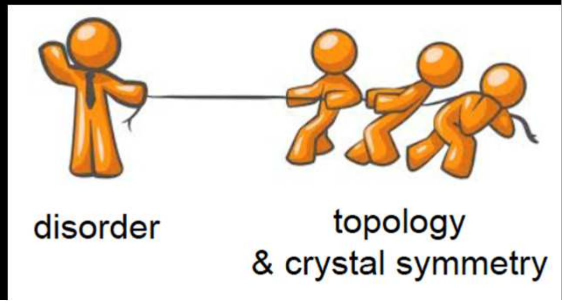
Analogous to the Higgs mechanism in particle physics where particles acquire mass by a spontaneous broken symmetry





Topological crystalline insulators

- Band Inversion known
- Is the dispersion linear near the `Dirac point'?
- Are the surface states topologically protected by mirror symmetry? Can you open up a gap by breaking mirror symmetry?
- Disorder necessarily violates crystal symmetry
- Symmetry is restored after disorder averaging?
- Are TCI surface states robust against strong disorder?



Outline

- The System: Topological Insulators
  - Unique properties of Conventional Z2 Topological Insulators
  - New Topological Material: Topological Crystalline Insulator
- The Technique: Scanning Tunneling Microscopy
  - Interference Patterns
  - Landau Level Spectroscopy
- The Experiment and Results: Breaking Mirror Symmetry to impart Mass to Massless Dirac Fermions
- Outstanding Questions and the Future

Implications of generating Massive Dirac Fermions

Fundamental Physics

Realization of exotic particles..magnetic mono- pole induced by a point charge

One dimensional domain wall states at boundaries between mass with different signs

Condensed matter realization domain wall modes discussed in cosmology and high energy physics

Device applications

Massless Dirac Fermions are notoriously difficult to confine...escape from high potential barriers. Surrounding with massive area could help confine them

Tunable gap implies tunable transport

Future

There may be a host of new TCIs with different lattice symmetries to be discovered

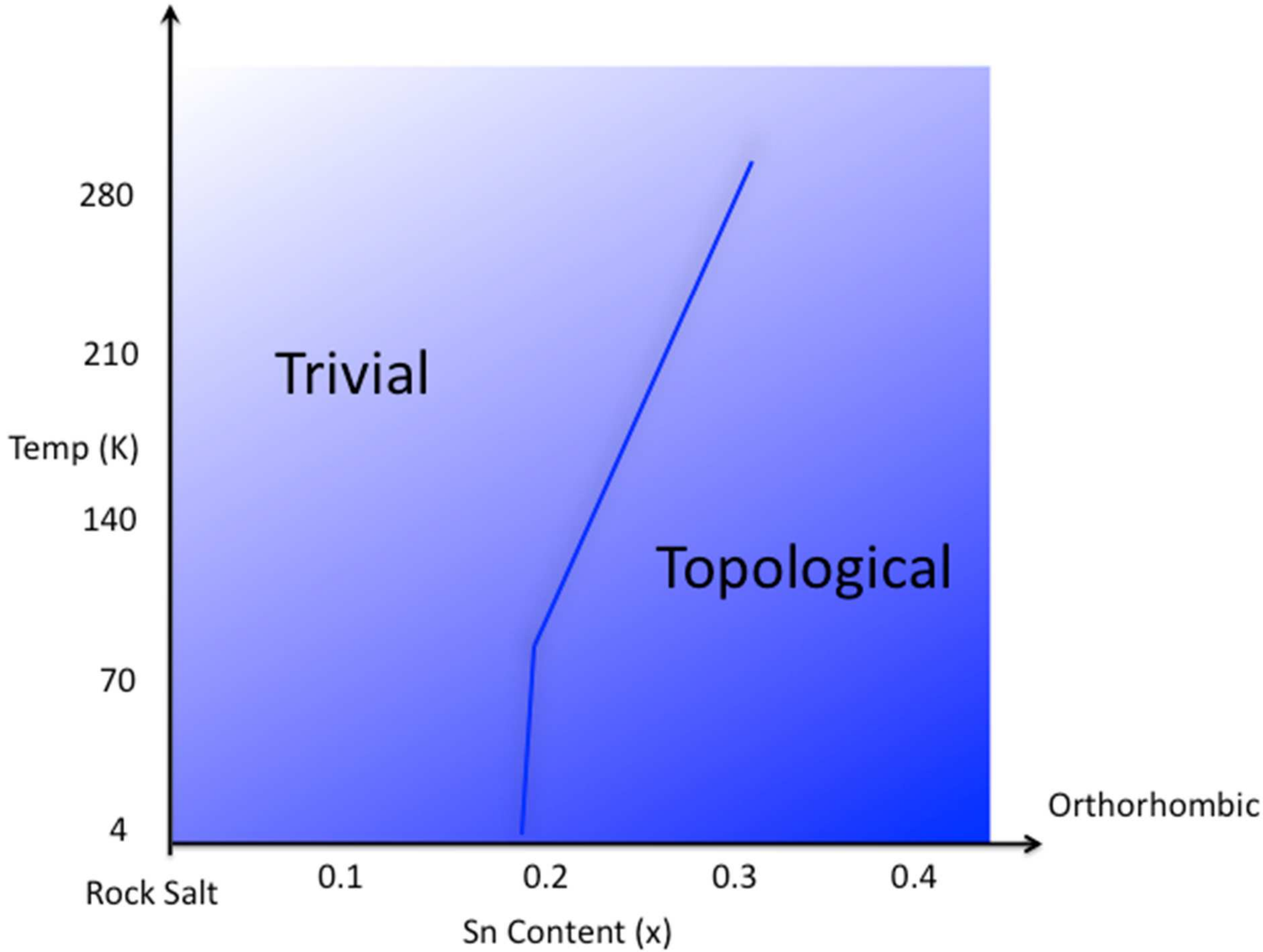
The introduction of electron interactions may drive the emergence of new phases arising from non-interacting TIs

Domain wall states at boundaries between mass with different signs waiting to be measured

What determines the magnitude of the mass?

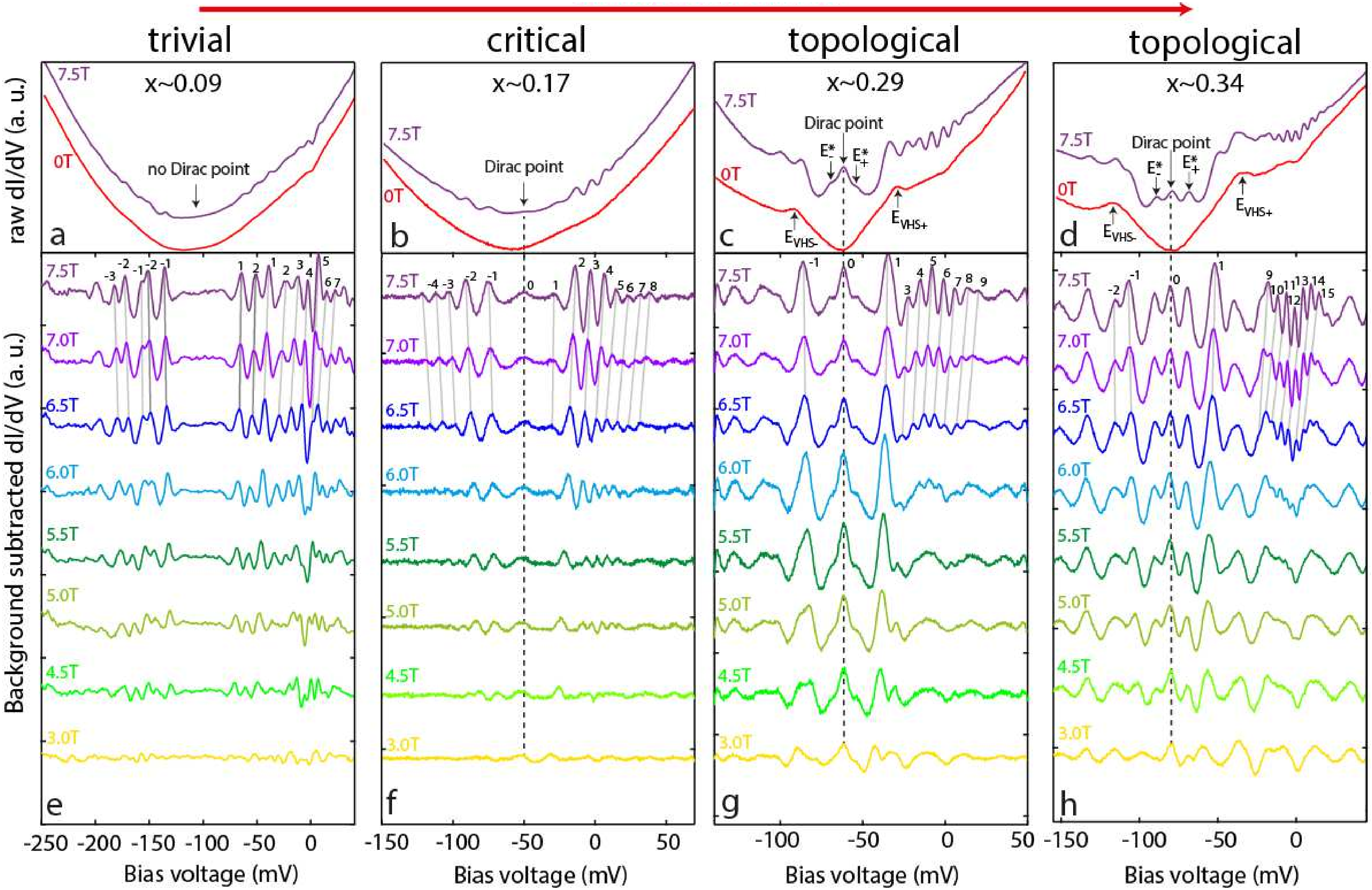
What is the role of electron-phonon coupling?

$Pb_{1-x}Sn_xSe$: Phase Diagram

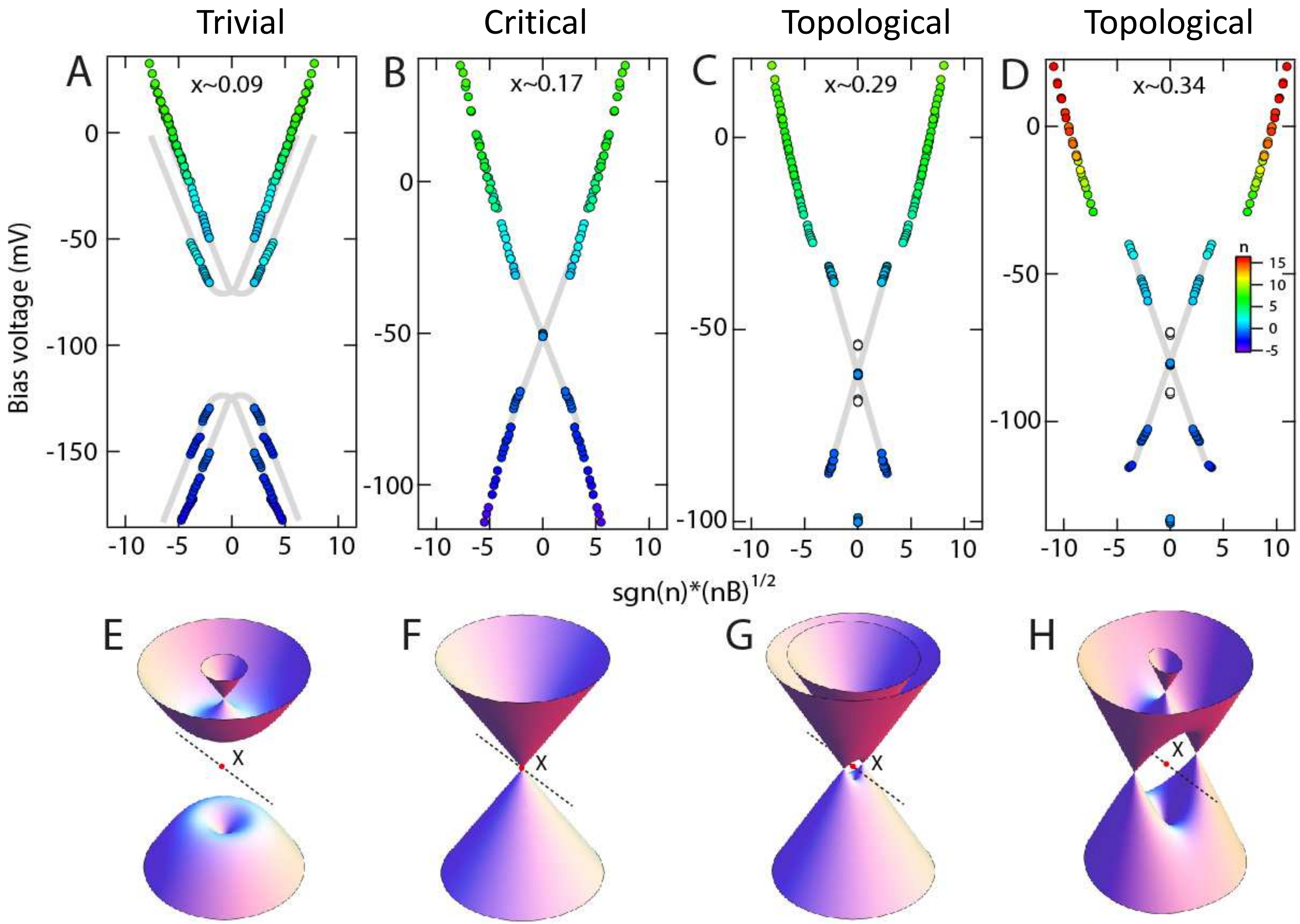


Nature Materials 11, 1023–1027 (2012)

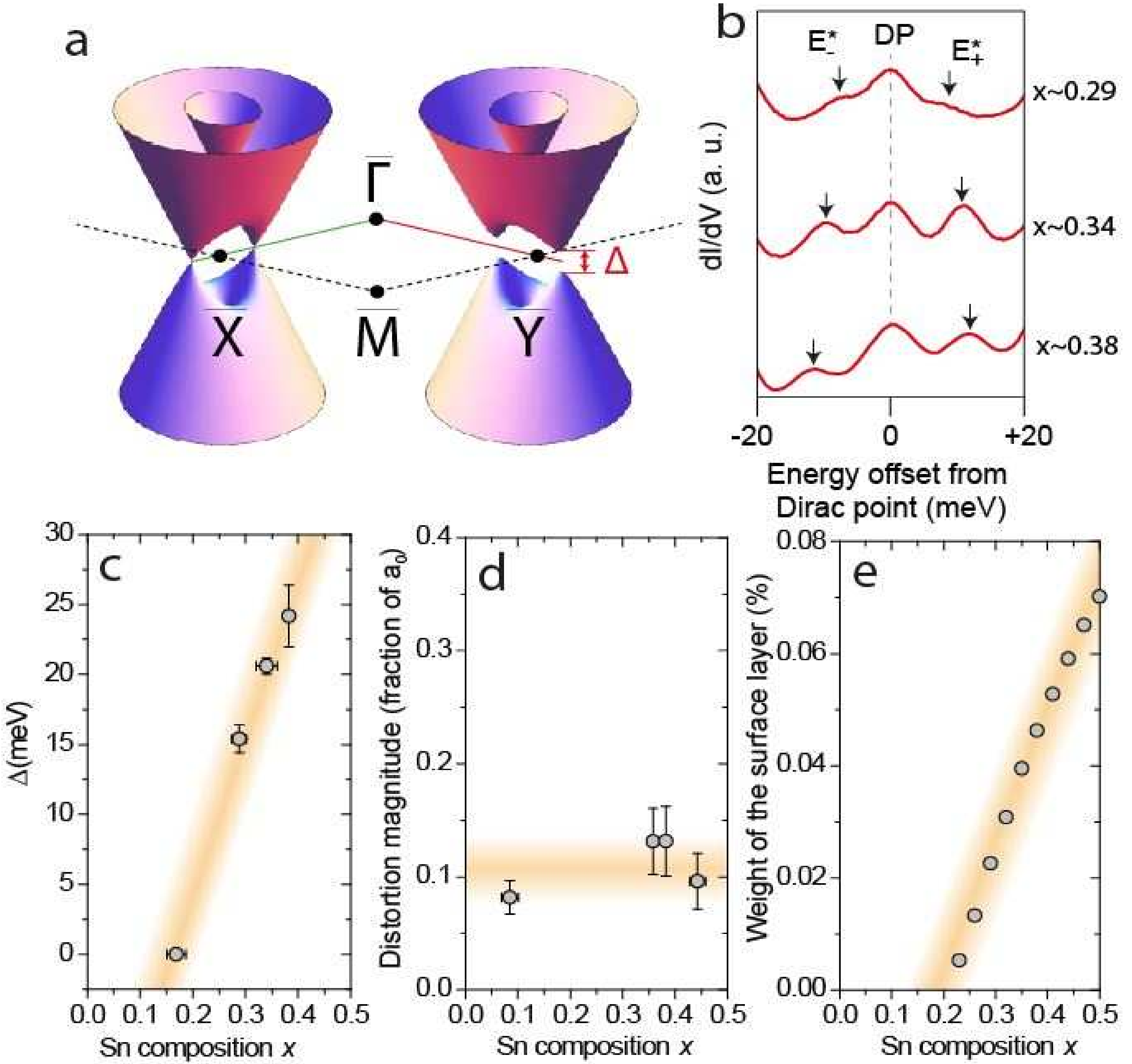
Topological Phase Transition



Topological Phase Transition



Mass decrease: SS Penetration



Summary

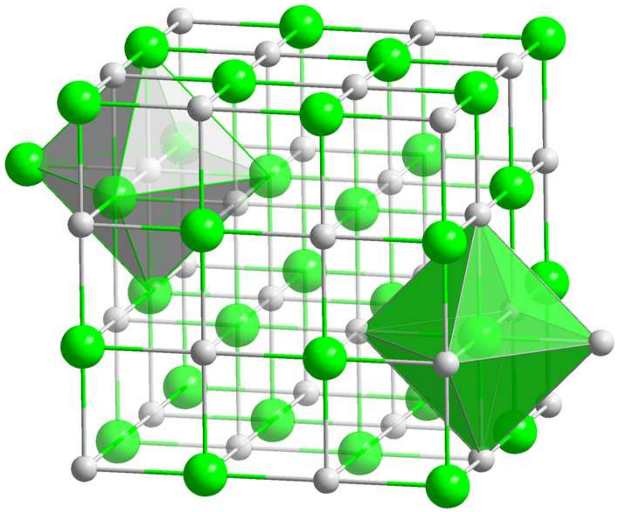
Coexistence of both massless and massive Dirac particles in the same system

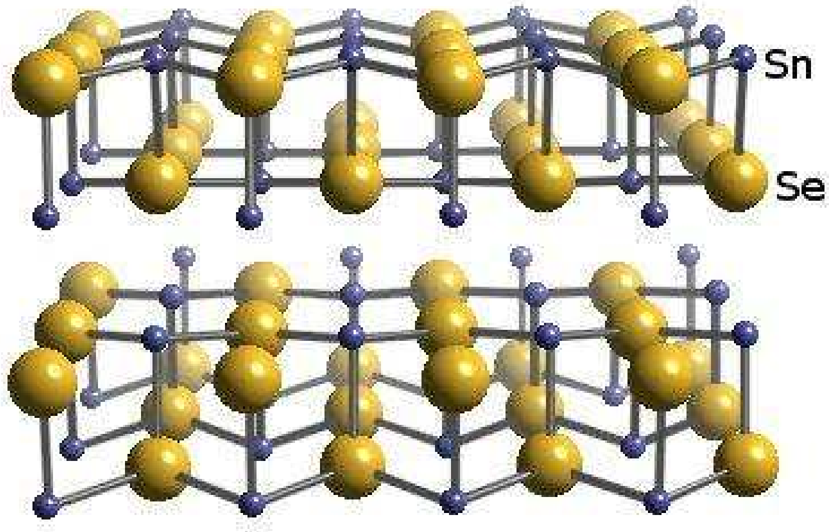
Mass generation is due to a spontaneously broken symmetry (crystalline symmetry in our case)

Analogous to the Higgs mechanism in particle physics where particles acquire mass by a spontaneous broken symmetry

Selenides

PbSe :	SnSe:
• Halite Structure • $a_0 = 6.128$ A • Trivial insulator	• Orthorhombic , Quasi tetragonal Structure • Trivial Insulator tin(II) selenide





Spatial Distribution of Dirac Point