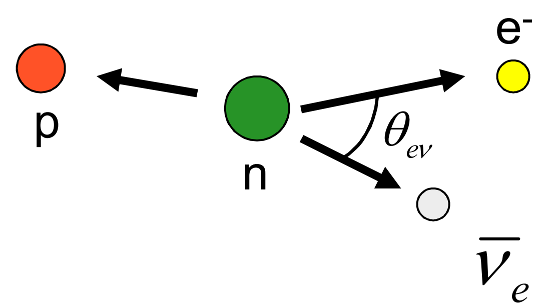

Precise measurement of the electron-antineutrino correlation in neutron beta decay

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Outline

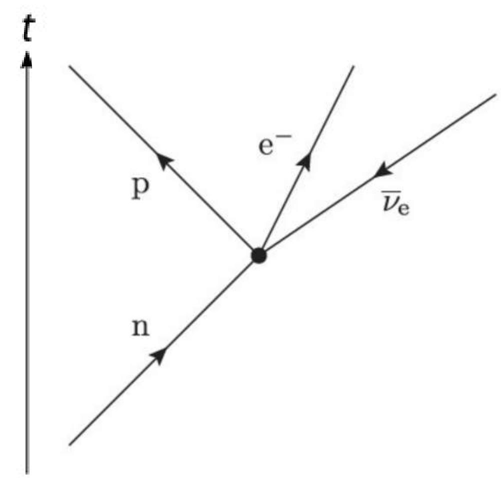
1. Motivation of the Nab experiment
2. Principle of the measurement of the electron-antineutrino correlation parameter “a”
3. Fitting method of “a” using Geant4 simulated data
4. Simulating uncertainty analysis in Geant4 simulation for “a”
5. Summary and Perspective

Motivation of the Nab experiment

Fermi's decay theory

In 1934, Enrico Fermi proposed a starting point for the study of neutron beta decays, the 4-fermion theory.

$$H_{\text{int}} = g(\overline{\psi}_p \gamma_\mu \psi_n)(\overline{\psi}_e \gamma_\mu \psi_{\nu_e}) + \text{H.C.}$$



E. Fermi. "Versuch einer Theorie der β-Strahlen. I." Zeitschrift für Physik 88 (1934), pp. 161–177. doi: 10.1007/BF01351864.

Parity-violating interaction terms

In 1956, Lee and Yang proposed expanding the Hamiltonian of prior Fermi theory in the weak interaction neutron decay with parity-violating terms:

$$
\begin{aligned}
H_{\text{int}} = {} & (\overline{\psi}_p \psi_n)(C_S \overline{\psi}_e \psi_{\nu_e} + C'_S \overline{\psi}_e \gamma_5 \psi_{\nu_e}) \\
& + (\overline{\psi}_p \gamma_\mu \psi_n)(C_V \overline{\psi}_e \gamma_\mu \psi_{\nu_e} + C'_V \overline{\psi}_e \gamma_\mu \gamma_5 \psi_{\nu_e}) \\
& + \frac{1}{2}(\overline{\psi}_p \sigma_{\lambda\mu} \psi_n)(C_T \overline{\psi}_e \sigma_{\lambda\mu} \psi_{\nu_e} + C'_T \overline{\psi}_e \sigma_{\lambda\mu} \gamma_5 \psi_{\nu_e}) \\
& - (\overline{\psi}_p \gamma_\mu \gamma_5 \psi_n)(C_A \overline{\psi}_e \gamma_\mu \gamma_5 \psi_{\nu_e} + C'_A \overline{\psi}_e \gamma_\mu \psi_{\nu_e}) \\
& - (\overline{\psi}_p \gamma_5 \psi_n)(C_P \overline{\psi}_e \gamma_5 \psi_{\nu_e} + C'_P \overline{\psi}_e \psi_{\nu_e}) + \text{H.C.},
\end{aligned}
$$

in which the constants prefix the Scalar, Vector, Tensor, Axial-vector, and Pseudo-scalar terms.

T. D. Lee and C. N. Yang. "Question of Parity Conservation in Weak Interactions." Physical Review 104.1 (Oct. 1956), pp. 254–258.

Decay rate

In 1957, Jackson et al. proposed that the β-decay rate would be (neglecting nucleon recoil and radiative corrections) :

$$\frac{\mathrm{d}^3\Gamma}{\mathrm{d}E_e \mathrm{d}\Omega_e \mathrm{d}\Omega_\nu} \simeq p_e E_e (E_0 - E_e)^2$$
$$\times \left[1 + a \frac{\vec{p}_e \cdot \vec{p}_\nu}{E_e E_\nu} + b \frac{m}{E_e} + \langle \vec{\sigma}_n \rangle \cdot \left(A \frac{\vec{p}_e}{E_e} + B \frac{\vec{p}_\nu}{E_\nu} \right) + \ldots \right]$$

Where $E_0 \approx 782\mathrm{keV}$ is the maximal electron energy

a, the electron-antineutrino correlation parameter, and b, the Fierz interference term, are measurable in decays of unpolarized neutrons. All except b depend on the ratio of axial-vector (g_A) to vector coupling constant (g_V) of the nucleon $\lambda = g_A/g_V$ in the following way (given here at the tree level):

$$a = \frac{1 - |\lambda|^2}{1 + 3|\lambda|^2}, \qquad A = -2\frac{|\lambda|^2 + Re(\lambda)}{1 + 3|\lambda|^2}, \qquad B = 2\frac{|\lambda|^2 - Re(\lambda)}{1 + 3|\lambda|^2}$$

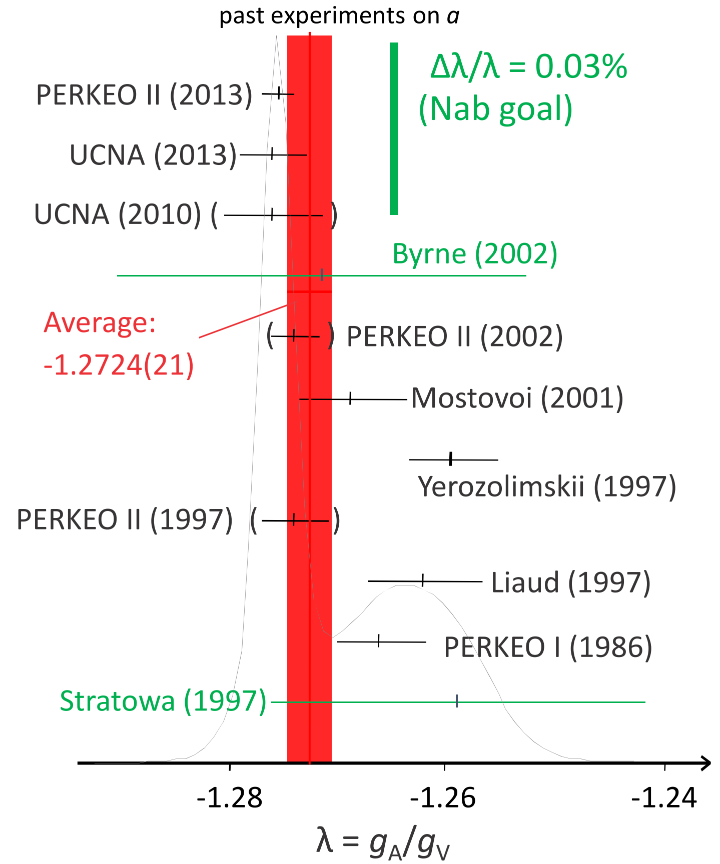
J. D. Jackson, S. B. Treiman, and H. W. Wyld Jr. “Possible Tests of Time Reversal Invariance in Beta Decay.” Physical Review 106 (3 May 1957), pp. 517–521. doi: 10.1103/PhysRev.106.517.

Nab Goals

- Measure "a" with a relative uncertainty of about 10^{-3}, and "b" with absolute uncertainty of 3×10^{-3}.

$$\frac{\sigma_a}{a} \approx 10^{-3} \qquad \sigma_b \approx 3 \times 10^{-3}$$

- Provide an independent measurement of the ratio λ to an uncertainty of 0.03%.



Data in black are indirect results from “A”, no direct measurement of “a” within this error range yet!

Nab Goals

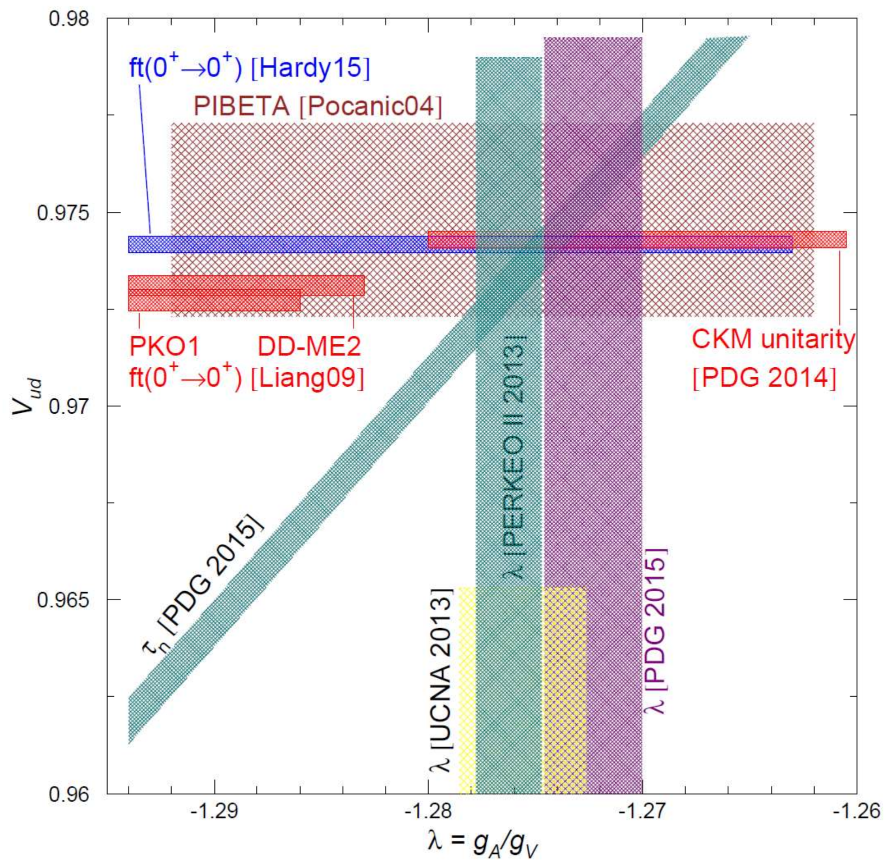
- Test the unitarity of the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix. Measurement of the neutron decay rate allows a determination of V_{ud}, the upper left matrix element, independent of nuclear models.

$$\Gamma = \frac{1}{\tau_n} \propto |V_{ud}|^2(1+3|\lambda|^2)$$

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$

For nab data to be competitive, we need:

$$\sigma_{\tau_n} \sim 0.3s \Rightarrow \frac{\sigma_\lambda}{\lambda} \sim 0.03\%$$



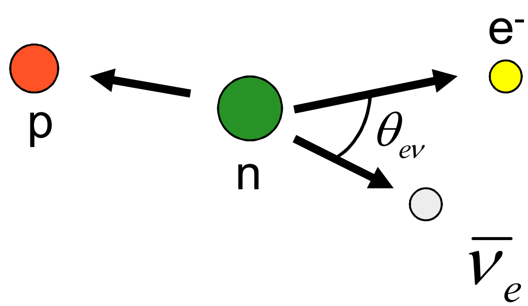
- Search for Beyond Standard Model Physics

Principle of the measurement of "a"

Principle of the measurement of "a"

$$\frac{d^3\Gamma}{dE_e d\Omega_e d\Omega_\nu} \simeq p_e E_e (E_0 - E_e)^2$$
$$\times \left[1 + a\frac{\vec{p}_e \cdot \vec{p}_\nu}{E_e E_\nu} + b\frac{m}{E_e} + \langle\vec{\sigma}_n\rangle \cdot \left(A\frac{\vec{p}_e}{E_e} + B\frac{\vec{p}_\nu}{E_\nu}\right) + \dots\right]$$

Where $E_0 \approx 782\text{keV}$ is the maximal electron energy



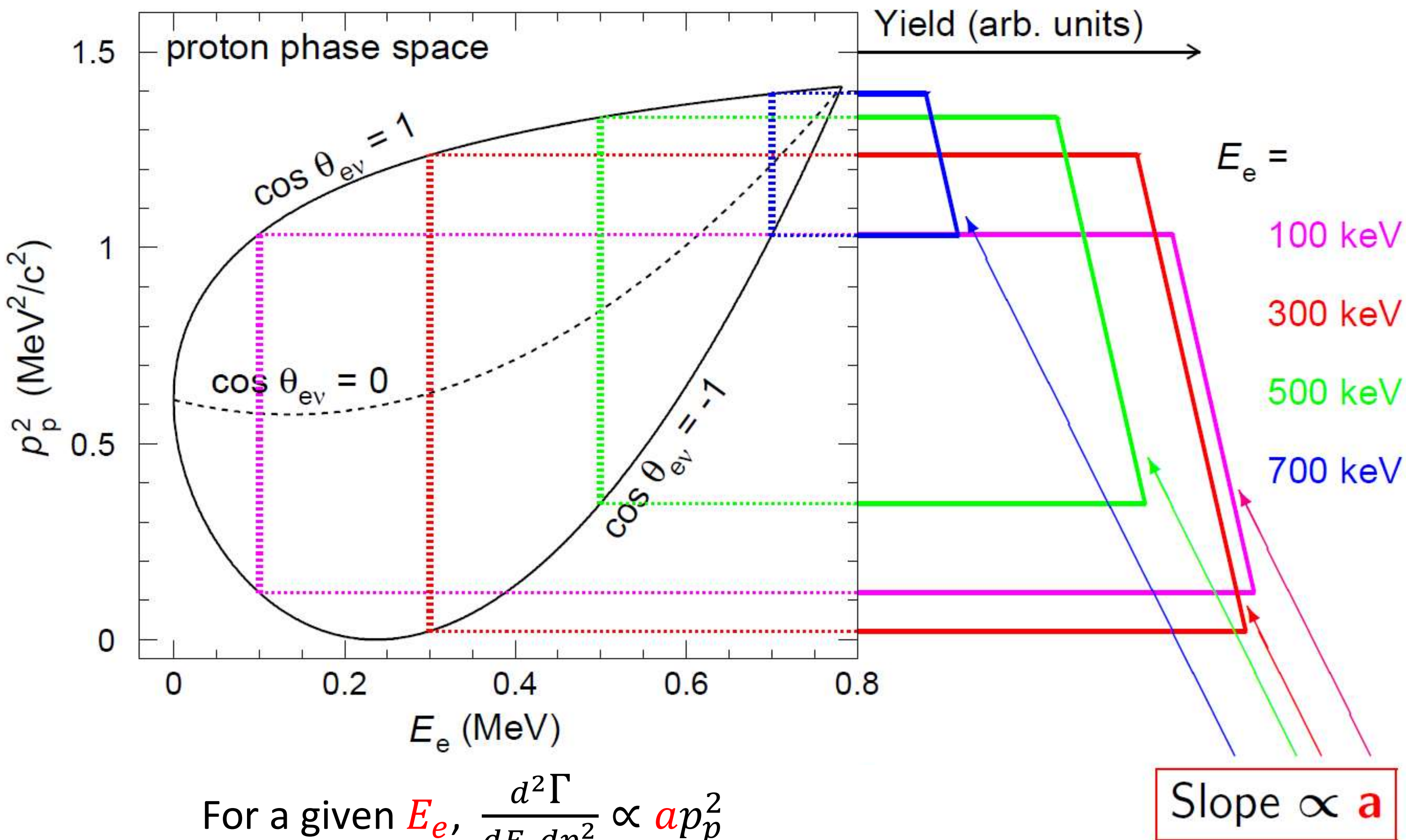
Kinematics (neglecting proton recoil):

- Energy Conservation $E_\nu = E_0 - E_e$
- Momentum Conservation $p_p^2 = p_e^2 + p_\nu^2 + 2p_e p_\nu \cos\theta_{e\nu}$

$$\frac{d^2\Gamma}{dE_e dp_p^2} = N\left[E_e(E_0 - E_e) + \frac{1}{2}a\left(p_p^2 - (2E_e^2 + E_0^2 - 2E_0E_e)\right)c^2\right]$$ Probability Density Function

Principle of the measurement of “a”

$$\frac{d^2\Gamma}{dE_e dp_p^2} = N\left[E_e(E_0 - E_e) + \frac{1}{2}a\left(p_p^2 - (2E_e^2 + E_0^2 - 2E_0E_e)\right)c^2\right]$$ Probability Density Function



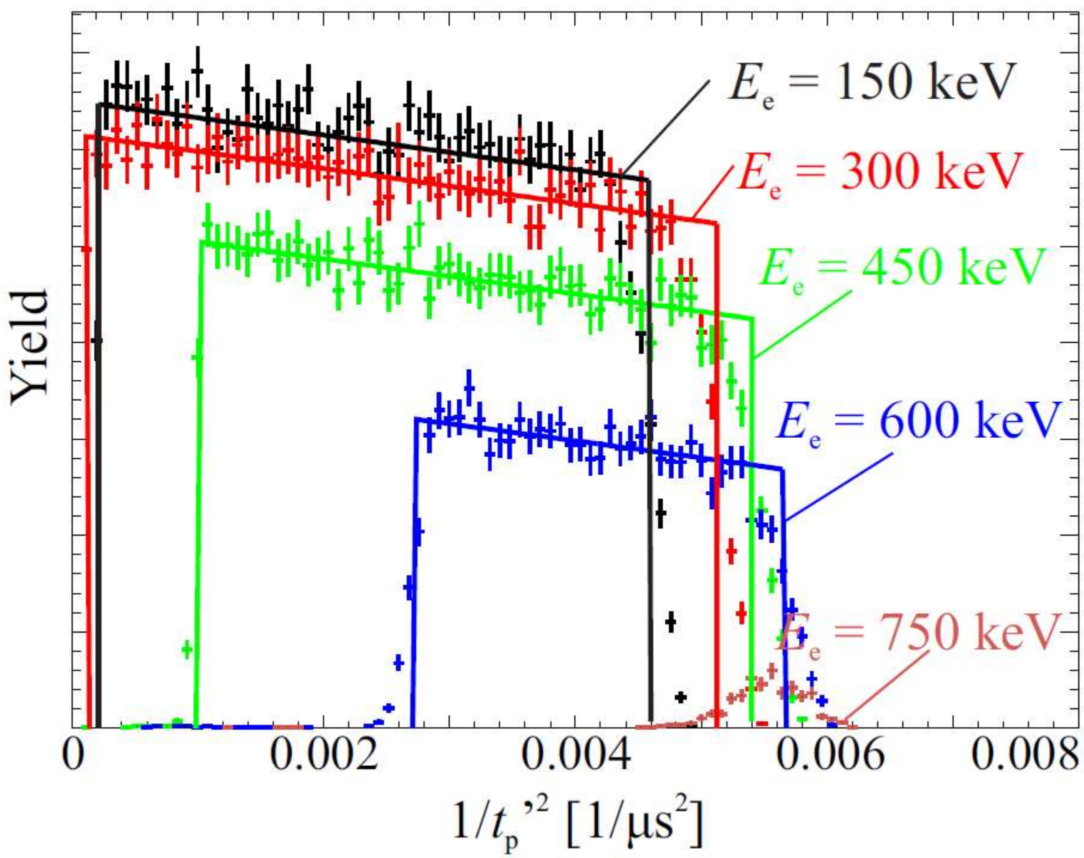
For a given E_e, $\frac{d^2\Gamma}{dE_e dp_p^2} \propto a p_p^2$

Principle of the measurement of "a"

$E_p < 750\text{eV}$
Hard to detect!

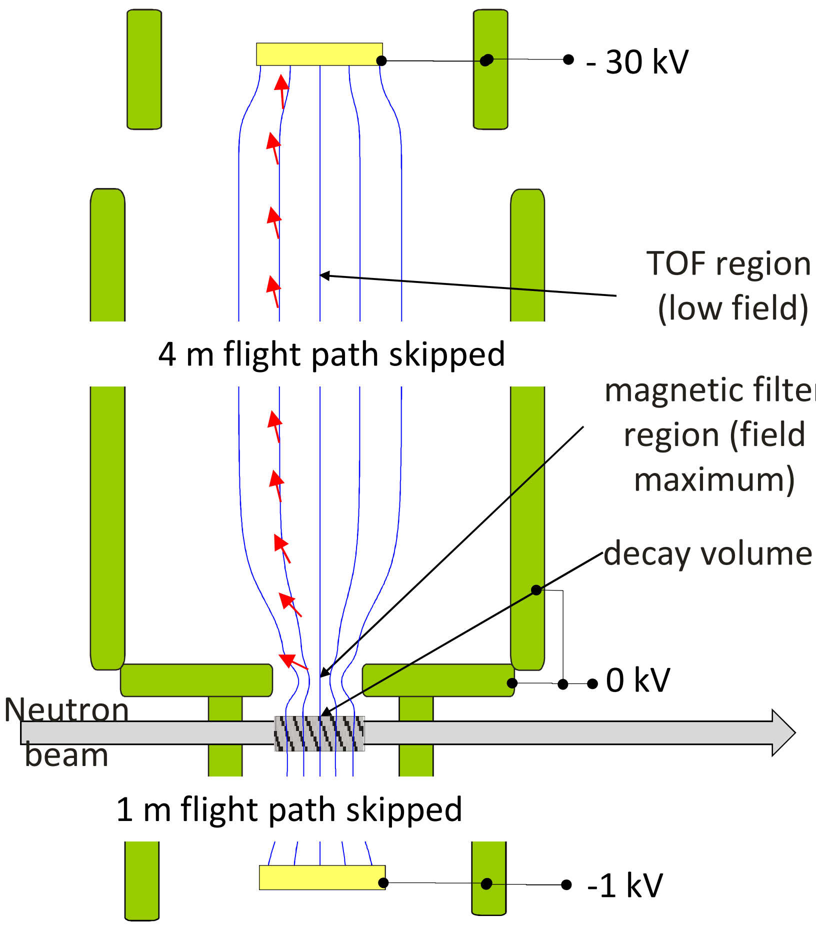
Another technique is to use $t_p = L\frac{m_p}{p_p}$

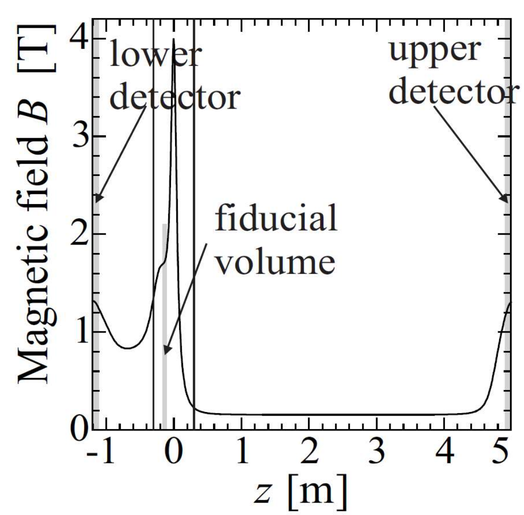
In the Nab experiment, "a" is determined by combined precise measurement of the electron energy E_e and the proton time of flight (TOF) t_p.

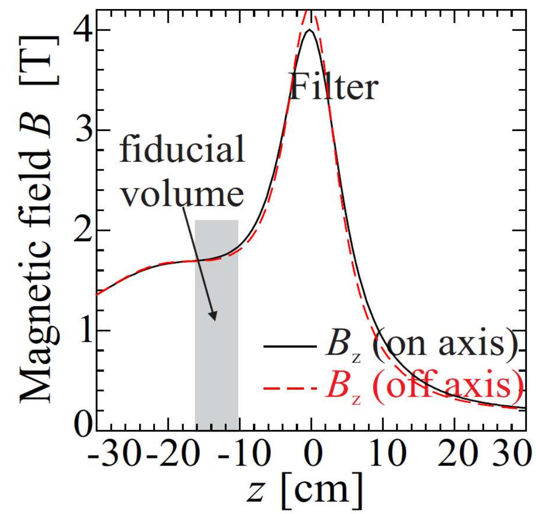


- Use edges to determine and verify shape of detection function $\Phi(p_p, 1/t_p)$;
- Use central part of $P_t(1/t_p^2)$ ($\sim 70\%$) to extract **a**.

Novel “time of flight” spectrometer



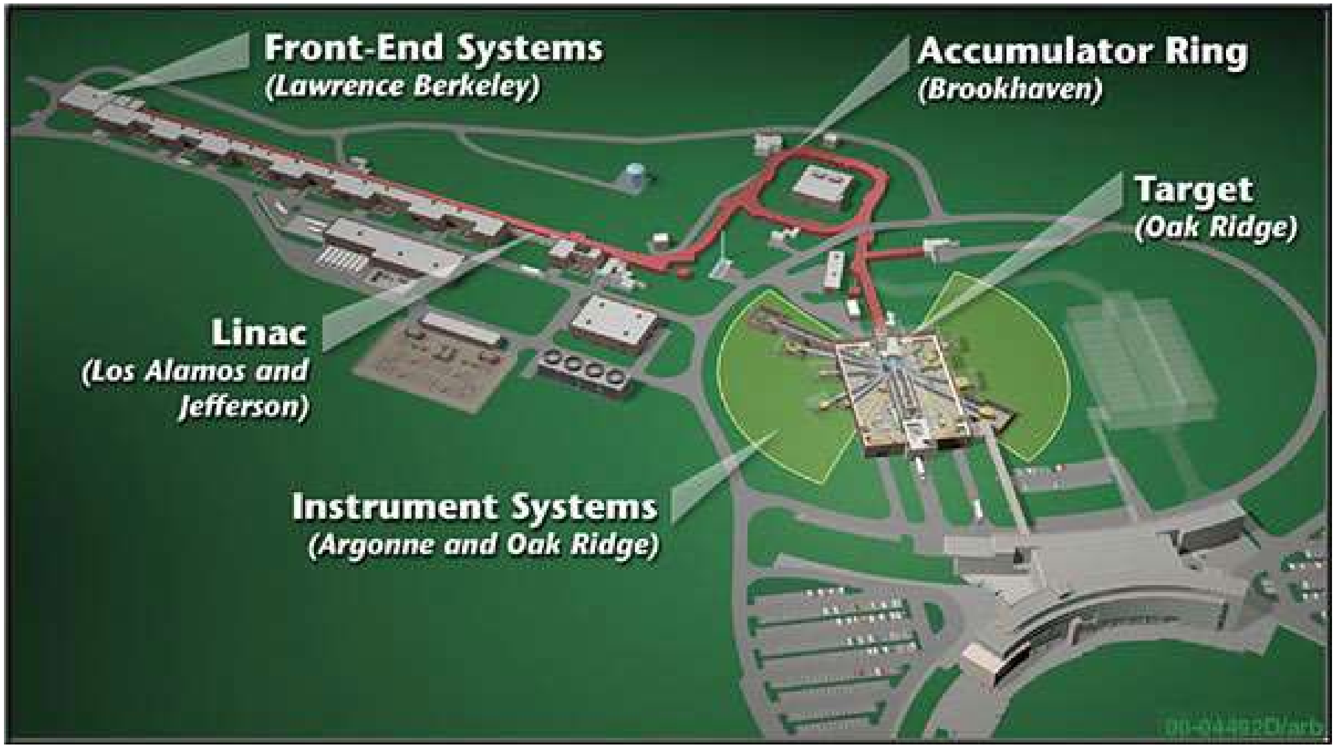


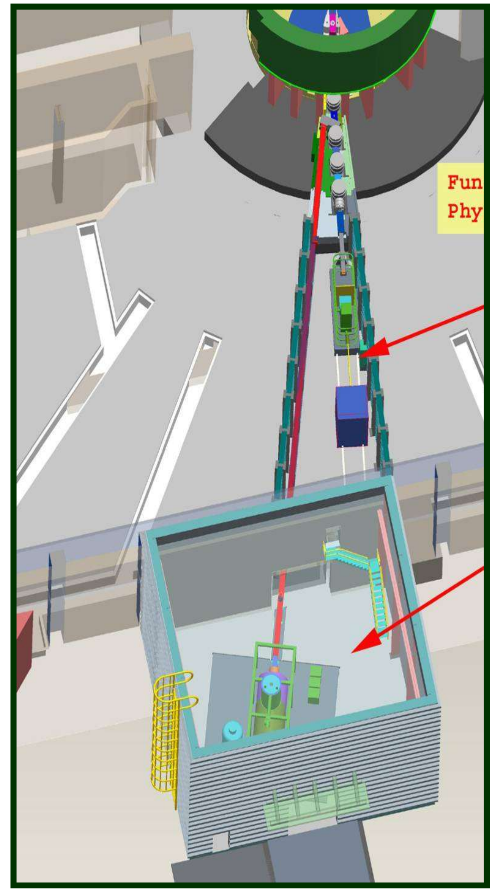


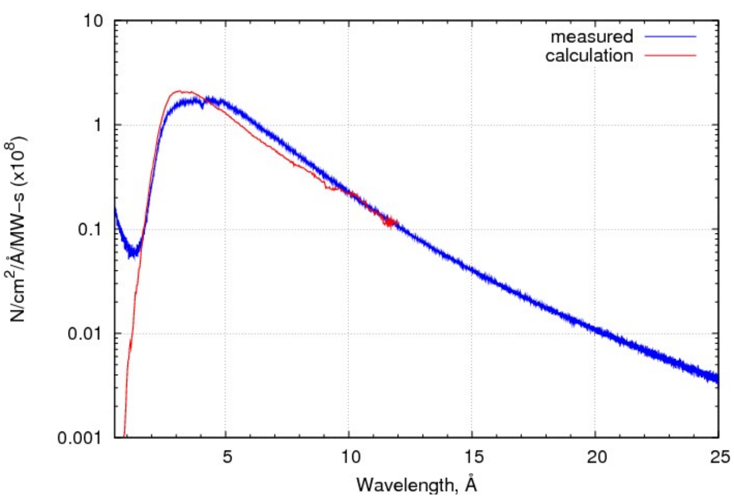
Main requirements:

1. Neutrons must decay in a region of large magnetic field. The decay protons and electrons spiral around a magnetic field line.
2. The momentum of the proton rapidly becomes parallel to the magnetic field in the TOF region, so that $t_p \cong L\frac{m_p}{p_p}$ dominates the total time of flight.
3. An electric field is required to accelerate the proton to a detectable energy.
4. Back to back silicon detector is used to take clean coincident signal and have good control over backscattering electrons.

Neutrons at the SNS in ORNL

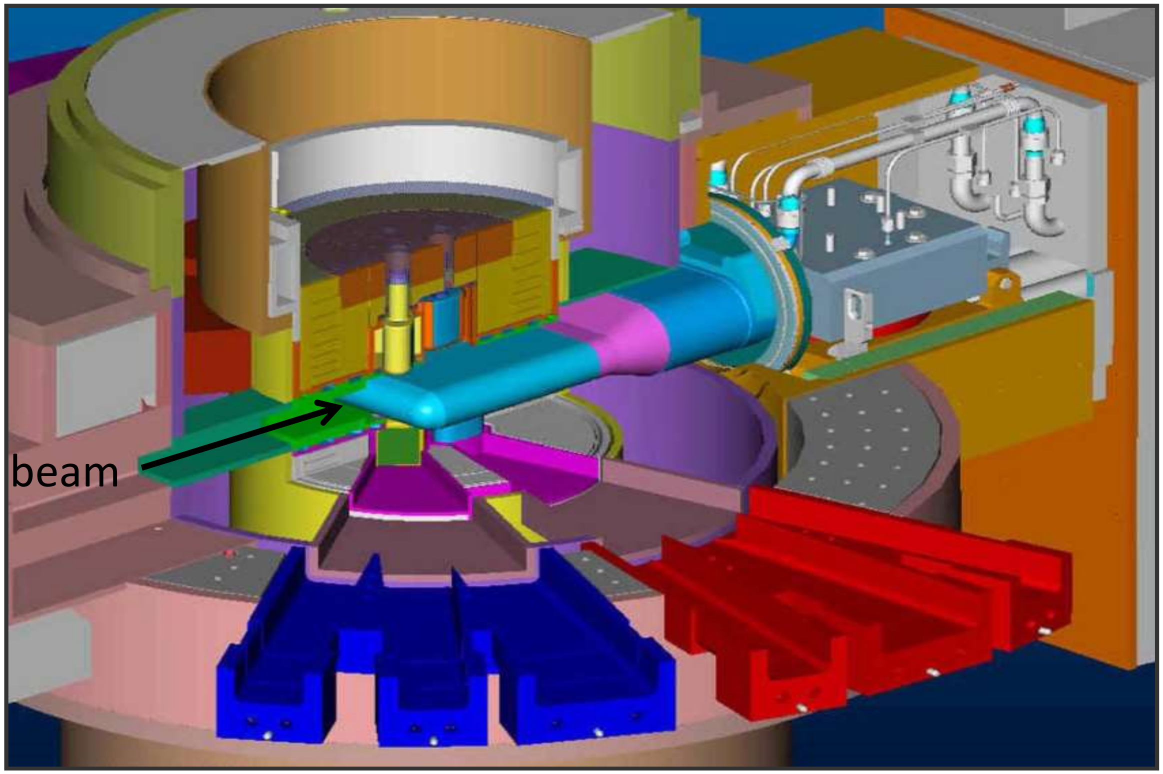


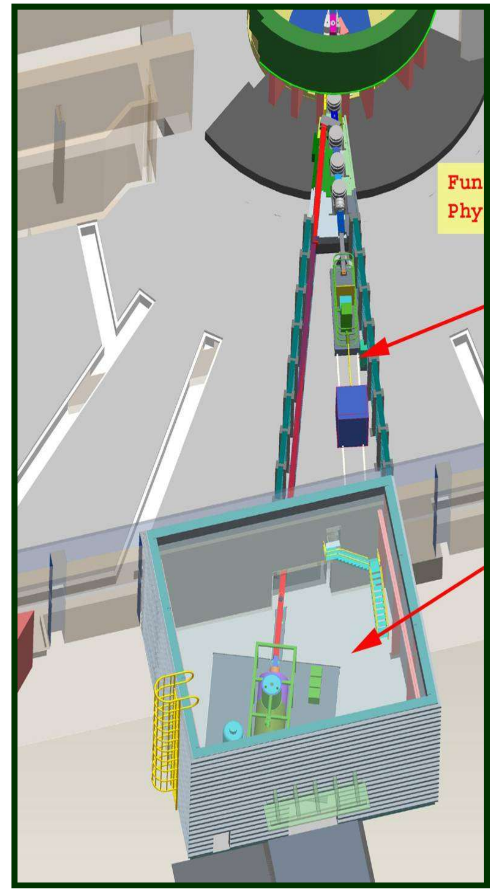


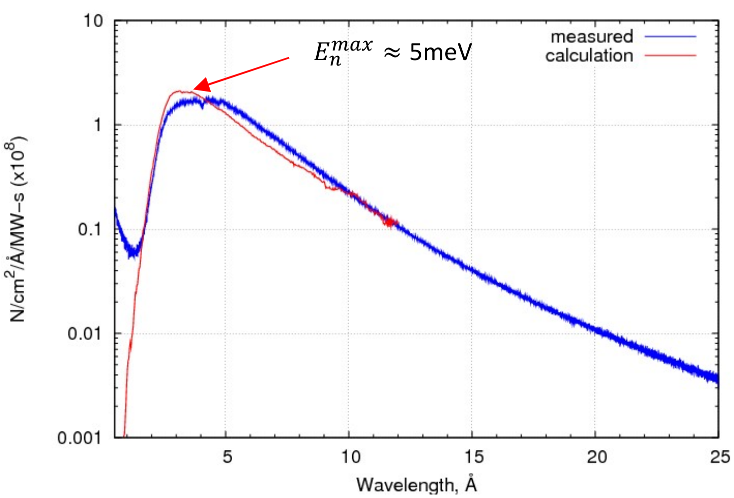


- Proton energy of 940MeV incident on a circulating target of mercury
- 60Hz rep rate with time-averaged proton power of 1.4MW
- Neutrons moderated by four H_2 moderators, H_2O moderators, and Be reflector

Neutrons at the SNS in ORNL

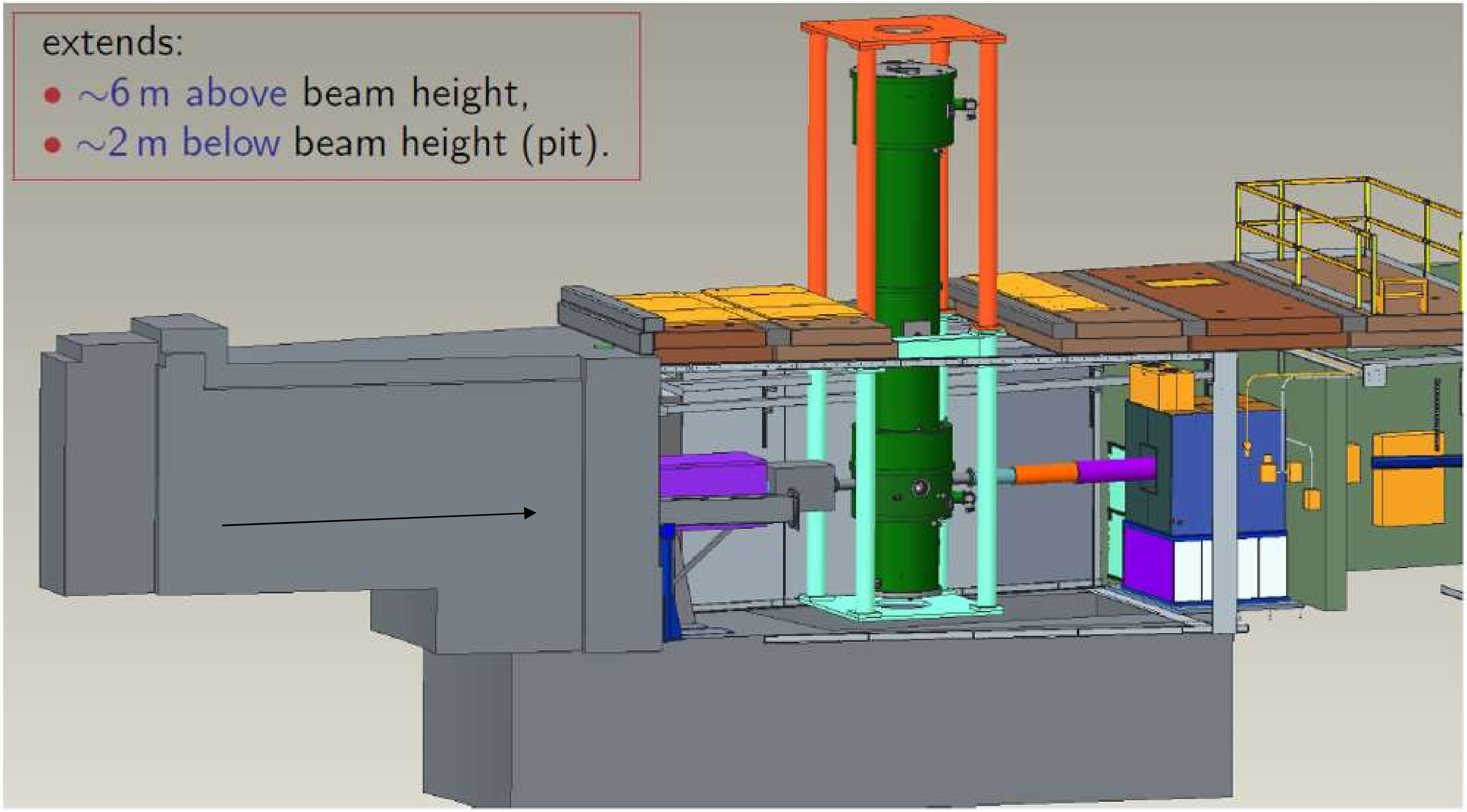






- Proton energy of 940MeV incident on a circulating target of mercury
- 60Hz rep rate with time-averaged proton power of 1.4MW
- Neutrons moderated by four H_2 moderators, H_2O moderators, and Be reflector

Nab apparatus in FNPB/SNS



Will be starting to install at the end of this summer!

Fitting method of “a” using Geant4 simulated data

Fitting method of “a”

$$\boxed{\frac{d^2\Gamma}{dE_e dp_p^2} = N\left[E_e(E_0 - E_e) + \frac{1}{2}a\left(p_p^2 - (2E_e^2 + E_0^2 - 2E_0E_e)\right)c^2\right] \quad \text{Probability Density Function}}$$

$$p_p = L\frac{m_p}{t_p}$$

With EM fields

$$p_p = \frac{m_p}{t_p}\int \frac{dz}{\sqrt{1 - \frac{B(z)}{B_0}\sin^2(\theta_0) + \frac{q(V(z) - V_0)}{E_{p0}}}}$$

Fitting method of "a"

$$p_p = \frac{m_p}{t_p}\int \frac{dz}{\sqrt{1-\frac{B(z)}{B_0}\sin^2(\theta_0)+\frac{q(V(z)-V_0)}{E_{p0}}}}$$

Expansion

$$p_p = \frac{m_p}{t_p}\left[\begin{array}{l} L-\eta\ln\frac{\cos(\theta_0)-\cos(\theta_0)_{min}}{1-\cos(\theta_0)_{min}}+\rho(1-\cos(\theta_0))+\beta(1-\cos(\theta_0))^2 \\ \qquad +\gamma(1-\cos(\theta_0))^3+\left(\delta' E_{p0}+\varepsilon' E_{p0}^2+\zeta' E_{p0}^3\right)\end{array}\right]$$

$$\boxed{p_p = \frac{m_p}{t_p}\left[\begin{array}{l} L-\eta\ln\frac{\cos(\theta_0)-\cos(\theta_0)_{min}}{1-\cos(\theta_0)_{min}}+\rho(1-\cos(\theta_0))+\beta(1-\cos(\theta_0))^2 \\ \qquad +\gamma(1-\cos(\theta_0))^3+\left(\delta\frac{1}{t_p^2}+\varepsilon\left(\frac{1}{t_p^2}\right)^2+\zeta\left(\frac{1}{t_p^2}\right)^3\right)\end{array}\right]}$$

Fitting method of "a"

$$\frac{d^2\Gamma}{dE_e dp_p^2} = N\left[E_e(E_0 - E_e) + \frac{1}{2}a\left(p_p^2 - (2E_e^2 + E_0^2 - 2E_0E_e)\right)c^2\right]$$

$$p_p = \frac{m_p}{t_p}\left[L - \eta \ln\frac{\cos(\theta_0) - \cos(\theta_0)_{min}}{1 - \cos(\theta_0)_{min}} + \rho(1 - \cos(\theta_0)) + \beta(1 - \cos(\theta_0))^2 + \gamma(1 - \cos(\theta_0))^3 + \delta\frac{1}{t_p^2} + \varepsilon\left(\frac{1}{t_p^2}\right)^2 + \zeta\left(\frac{1}{t_p^2}\right)^3\right]$$

- L = Length of spectrometer.
- $\eta = r\sqrt{2/3}$ Corrections for curvature of magnetic field from coils of radius r.
- $\rho, \beta, \gamma, \delta, \varepsilon, \zeta$ = Coefficients of expansion.
- N = Normalization factor.
- a = Electron-antineutrino correlation parameter.

This gives us a total of 10 free parameters for fitting! Impossible to fit without starting values!

Fitting method of “a”

$$\frac{d^2\Gamma}{dE_e dp_p^2} = N\left[E_e(E_0 - E_e) + \frac{1}{2}a\left(p_p^2 - (2E_e^2 + E_0^2 - 2E_0E_e)\right)c^2\right]$$

$$p_p = \frac{m_p}{t_p}\left[L - \eta \ln\frac{\cos(\theta_0) - \cos(\theta_0)_{min}}{1 - \cos(\theta_0)_{min}} + \rho(1-\cos(\theta_0)) + \beta(1-\cos(\theta_0))^2 + \gamma(1-\cos(\theta_0))^3 + \delta\frac{1}{t_p^2} + \varepsilon(\frac{1}{t_p^2})^2 + \zeta(\frac{1}{t_p^2})^3\right]$$

Steps to get starting values:

1. Neglecting E field to fit the starting values of the $\cos(\theta_0)$ dependent parameters.
2. Considering E field to fit the $\frac{1}{t_p^2}$ dependent parameters.
3. Putting these starting values back to the decay rate density function to fit a.

Initialize $\cos(\theta_0)$ parameters without E field

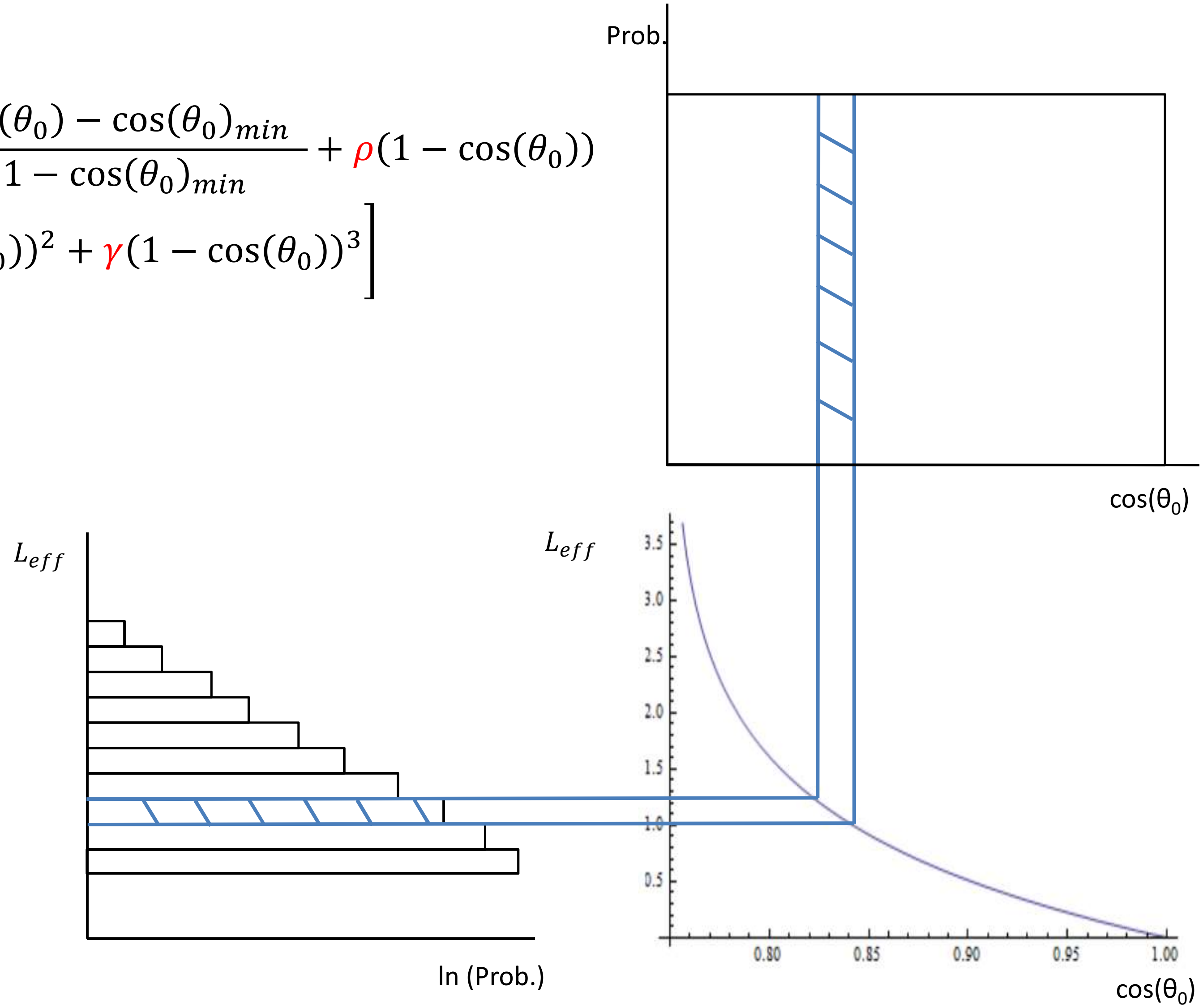
Let's first neglecting the E field, which only gives us the $\cos(\theta_0)$ dependent terms:

$$L_{eff} = \frac{p_p t_p}{m_p} = \left[L - \eta \ln \frac{\cos(\theta_0) - \cos(\theta_0)_{min}}{1 - \cos(\theta_0)_{min}} + \rho(1 - \cos(\theta_0)) + \beta(1 - \cos(\theta_0))^2 + \gamma(1 - \cos(\theta_0))^3 \right]$$

This is a one-to-one function from $\cos(\theta_0)$ to L_{eff} for given fitting parameters. Because $\cos(\theta_0)$ has a uniform distribution, we can use this relationship to get a probability density function for L_{eff}.

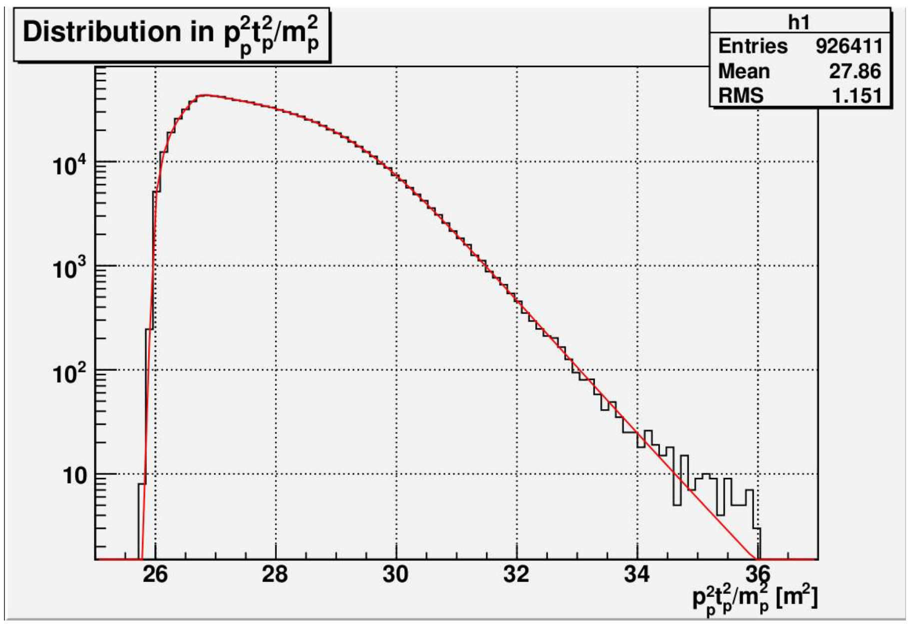
Initialize $\cos(\theta_0)$ parameters without E field

$$L_{eff} = \left[L - \eta \ln \frac{\cos(\theta_0) - \cos(\theta_0)_{min}}{1 - \cos(\theta_0)_{min}} + \rho(1 - \cos(\theta_0)) + \beta(1 - \cos(\theta_0))^2 + \gamma(1 - \cos(\theta_0))^3\right]$$



Initialize $\cos(\theta_0)$ parameters without E field

$$L_{eff} = \left[L - \eta \ln \frac{\cos(\theta_0) - \cos(\theta_0)_{min}}{1 - \cos(\theta_0)_{min}} + \rho(1 - \cos(\theta_0)) + \beta(1 - \cos(\theta_0))^2 + \gamma(1 - \cos(\theta_0))^3\right]$$

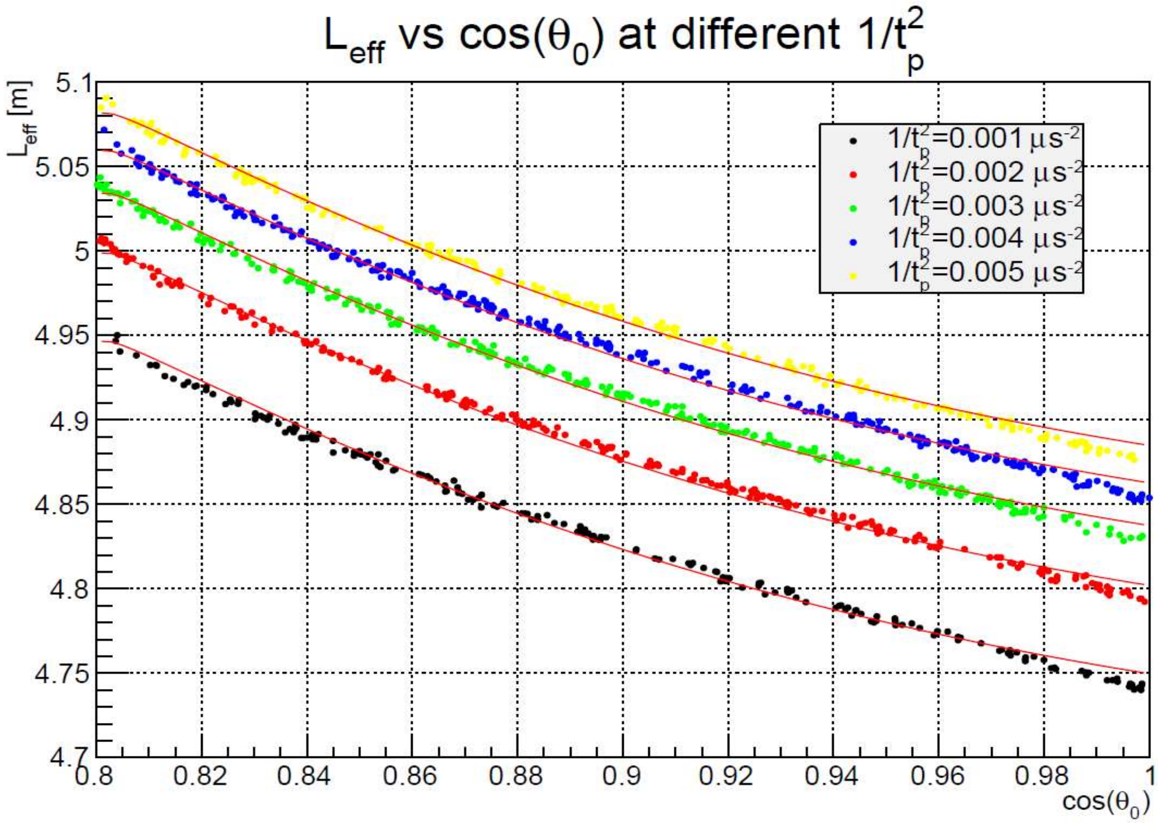


EXT NO.	PARAMETER NAME	VALUE	ERROR
1	N	3.65483e+00	8.38122e-03
2	a	0.00000e+00	fixed
3	Exp(L/m)	1.69008e+02	4.10135e-03
4	eta	5.82570e+04	2.07632e+01
5	rho	6.96179e+05	2.36730e+02
6	beta	-2.69737e+04	2.09954e+03
7	gamma	1.65895e+05	3.20104e+03

Initialize $\frac{1}{t_p^2}$ parameters with E field

If there is no E field, all curves are lined up together. With E field, they are shifted by the $\frac{1}{t_p^2}$ terms.

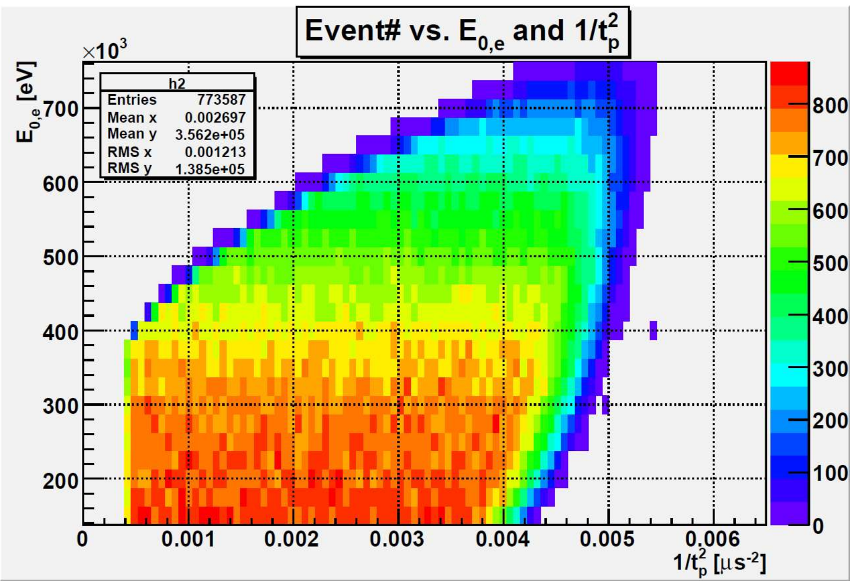
$$L_{eff} = \left[L - \eta \ln \frac{\cos(\theta_0) - \cos(\theta_0)_{min}}{1 - \cos(\theta_0)_{min}} + \rho(1 - \cos(\theta_0)) + \beta(1 - \cos(\theta_0))^2 + \gamma(1 - \cos(\theta_0))^3 + \delta \frac{1}{t_p^2} + \varepsilon(\frac{1}{t_p^2})^2 + \zeta(\frac{1}{t_p^2})^3\right]$$



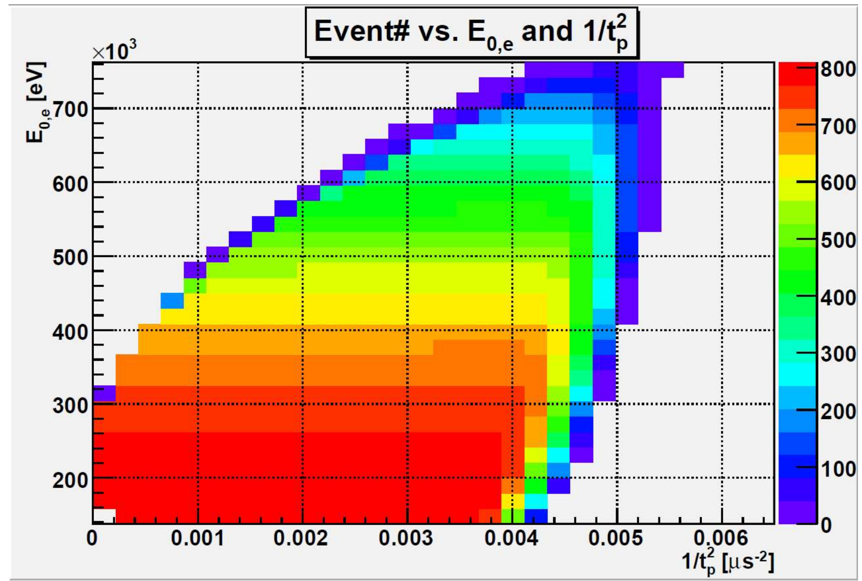
Fitting method of “a”

$$\frac{d^2\Gamma}{dE_e dp_p^2} = N\left[E_e(E_0 - E_e) + \frac{1}{2}a\left(p_p^2\left(\frac{1}{t_p^2}, \cos(\theta_0)\right) - (2E_e^2 + E_0^2 - 2E_0E_e)\right)c^2\right]$$

G4 Simulation



Fitting



EXT NO.	PARAMETER NAME	VALUE	ERROR
1	Log(N)	-6.39307e+01	4.62503e-02
2	a	8.35737e-03	9.92592e-02
3	Exp(L/m)	1.68858e+02	4.20006e-02
4	eta	6.14451e+04	1.41443e+02
5	rho	6.96166e+05	1.06304e+03
6	beta	-2.73937e+04	4.24721e+03
7	gamma	1.64554e+05	2.00312e+04

Statistical uncertainties for **a**

$E_{e,min}$	0	100 keV	100 keV	100 keV	300 keV
$t_{p,max}$	–	–	40 μs	30 μs	40 μs
σ_a	$2.4/\sqrt{N_u}$	$2.4/\sqrt{N_u}$	$2.6/\sqrt{N_u}$	$2.8/\sqrt{N_u}$	$3.1/\sqrt{N_u}$
$\sigma_a{}^\dagger$	$2.6/\sqrt{N_u}$	$2.6/\sqrt{N_u}$	$2.8/\sqrt{N_u}$	$3.1/\sqrt{N_u}$	$3.5/\sqrt{N_u}$
$\sigma_a{}^\S$	$3.3/\sqrt{N_u}$	$3.4/\sqrt{N_u}$	$3.6/\sqrt{N_u}$	$4.0/\sqrt{N_u}$	$4.6/\sqrt{N_u}$

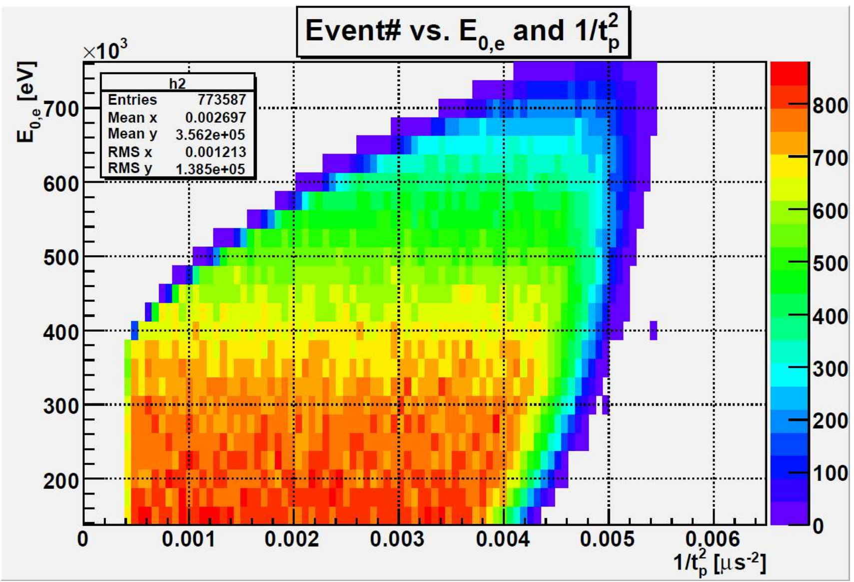
† with $\mathbf{E}_{calib}$ and $\mathbf{L}_{TOF}$ variable; § using inner 75% of $\mathbf{p}_p^2$ data.

[N_u … number of protons detected in upper detector.]

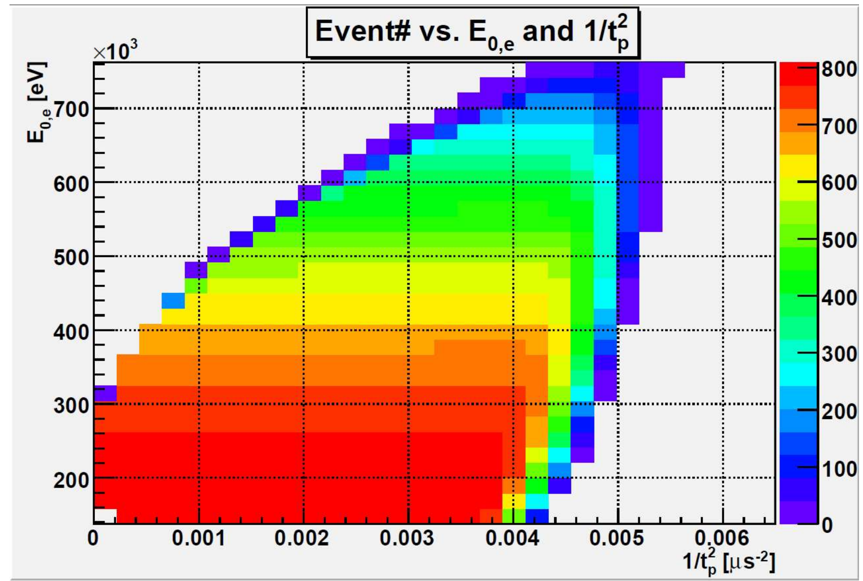
Fitting method of “a”

$$\frac{d^2\Gamma}{dE_e dp_p^2} = N\left[E_e(E_0 - E_e) + \frac{1}{2}a\left(p_p^2\left(\frac{1}{t_p^2}, \cos(\theta_0)\right) - (2E_e^2 + E_0^2 - 2E_0E_e)\right)c^2\right]$$

G4 Simulation



Fitting




```
EXT PARAMETER
NO.   NAME        VALUE            ERROR
 1   Log(N)     -6.39307e+01    4.62503e-02
 2   a           8.35737e-03    9.92592e-02
 3   Exp(L/m)    1.68858e+02    4.20006e-02
 4   eta         6.14451e+04    1.41443e+02
 5   rho         6.96166e+05    1.06304e+03
 6   beta       -2.73937e+04    4.24721e+03
 7   gamma       1.64554e+05    2.00312e+04
```

Statistical uncertainties for **a**

$E_{e,\min}$	0	100 keV	100 keV	100 keV	300 keV
$t_{p,\max}$	–	–	40 μs	30 μs	40 μs
σ_a	2.[illegible]	[illegible]	[illegible]	[illegible]	$3.1/\sqrt{N_u}$
σ_a †	2.[illegible]	[illegible]	[illegible]	[illegible]	$3.5/\sqrt{N_u}$
σ_a §	3.[illegible]	[illegible]	[illegible]	[illegible]	$4.6/\sqrt{N_u}$

$$\sigma_a \approx 3 \times 10^{-3}$$

† with $\mathbf{E}_{\text{calib}}$ and $\mathbf{L}_{\text{TOF}}$ variable; § using inner 75% of $\mathbf{p}_\mathbf{p}^2$ data.

[N_u . . . number of protons detected in upper detector.]

Statistical uncertainty of "a"

At 1.4 MW SNS beam power there will be ~1600 decays/s, or ~200 protons/s detected in the upper detector.
In a typical ~ 10-day run of beam time we would achieve

$$\frac{\sigma_a}{a} \simeq 2 \times 10^{-3} \quad \text{and} \quad \sigma_b \simeq 6 \times 10^{-4}$$

We plan to collect samples of ~ 2×10^9 events in several 4–6-week runs.

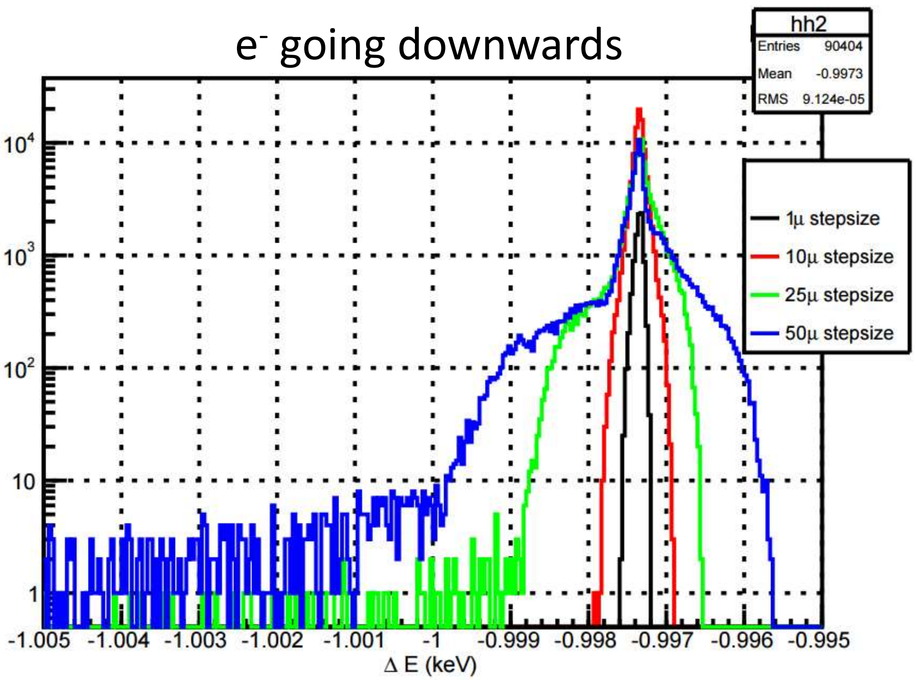
Overall accuracy will not be statistics-limited.

Simulating uncertainty analysis in Geant4 simulation for “a”

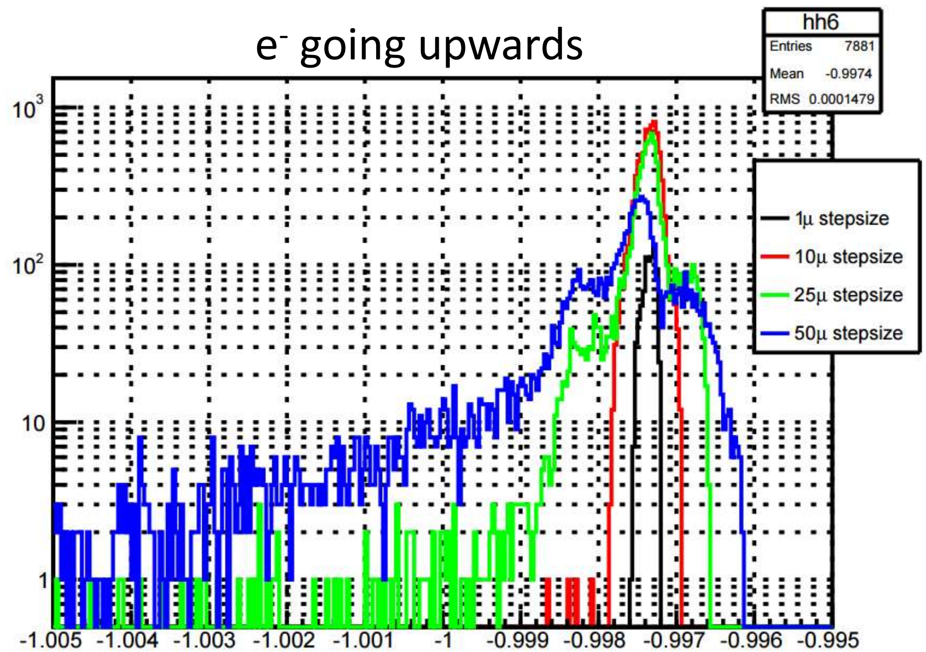
E_e simulating uncertainty analysis

Tolerance for E_e is $0.1keV$.

e⁻ going downwards



e⁻ going upwards



t_p simulating uncertainty analysis

Tolerance for t_p is $0.2\ ns$.

Initial condition	R (mm)	Z (mm)	$\cos\theta_z$	E_p (eV)
Sample event 1	0	-131.89	1	400
Sample event 2	8.15388344	-128.259824	0.844805068	649.228655
Sample 3 (Less than 0.1%)	18.50940873	-131.3936125	0.764213089	236.527382

Different method of generating EM fields	3rd order stepper	4th/5th order stepper	4th order stepper	8th order Stepper	2D fieldmap (Bilinear)	1D fieldmap (Bilinear)
t_p (μs)	17.2657	17.2657	17.2657	17.2657	17.2657	17.2657
t_p (μs)	14.0341	14.0341	14.0341	14.0341	14.0341	14.0341
t_p (μs)	25.6924	25.6924	25.6924	25.7012	25.7209	25.6889

Longer computational time, higher precision!

Shorter computational time, lower precision!

Summary and Perspective

- Developed a fitting method of “a”
- Optimized the accuracy of the electron energy spectrum in Geant4 simulation to $0.1\ keV$
- Studying field mapping techniques to get a precise TOF in Geant4 simulation while keeping the expected computational speed
- Will continue to optimize the fitting method of “a”
- Will participate in the setup
- Will take the data and analyze them in the fitting method

The Nab collaboration

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