Wigner Functions

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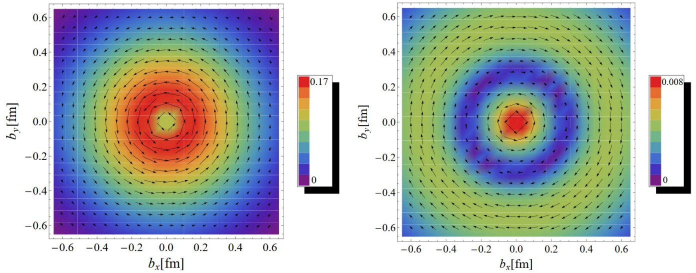
- femtography: generate images of the nucleon
- unified framework ($\rightsquigarrow$ global fit) for
  - Generalized Parton Distributions (GPDs)
  - Transverse Momentum Dependent PDFs (TMDs)
  - Orbital Angular Momentum (OAM)

Wigner function in LF constituent quark model (Lorcé & Pasquini)



Outline

- Jaffe-Manohar vs. Ji decomposition of spin

↪ Wigner function definition for OAM

↪ $\mathcal{L}^q_{JM} - L^q_{Ji}$ = change in OAM as quark

- twist-3 PDFs $g_2(x) \longrightarrow \perp$ force

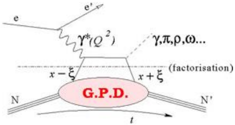
↪ twist-3 GPDs $\longrightarrow \perp$ force tomography

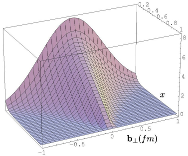
Motivation: why twist-3 GPDs

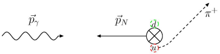
  - twist-3 GPD $G_2^q \longrightarrow L^q$
  - twist 3 PDF $g_2(x) \longrightarrow \perp$ force
  - twist 2 GPDs $\longrightarrow \perp$ imaging (of quark densities)

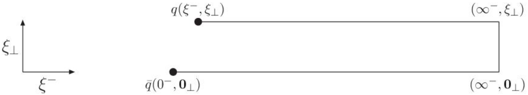
  ↪ twist 3 GPDs $\longrightarrow \perp$ imaging of $\perp$ forces

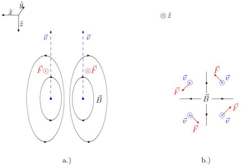
- Summary











Nucleon Spin Puzzle

spin sum rule

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + \mathcal{L}$$

Longitudinally polarized DIS:

- $\Delta\Sigma = \sum_q \Delta q \equiv \sum_q \int_0^1 dx\, [q_\uparrow(x) - q_\downarrow(x)] \approx 30\%$

$\hookrightarrow$ only small fraction of proton spin due to quark spins

Gluon spin ΔG

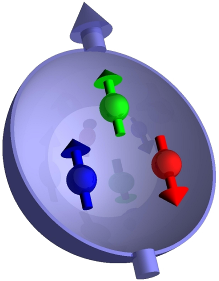
could possibly account for remainder of nucleon spin, but still large uncertainties $\rightarrow$ EIC

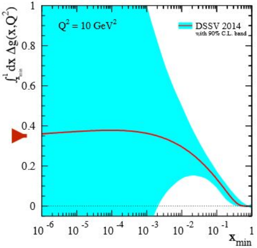
Quark Orbital Angular Momentum

- how can we measure $\mathcal{L}_{q,g}$

$\hookrightarrow$ need correlation between position & momentum

- how exactly is $\mathcal{L}_{q,g}$ defined





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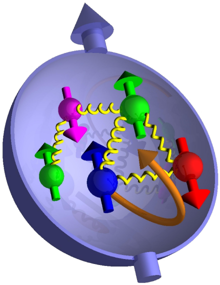
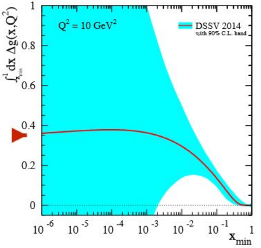
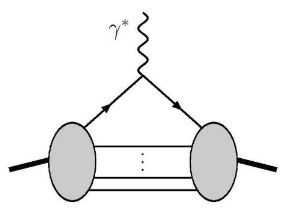




form factor



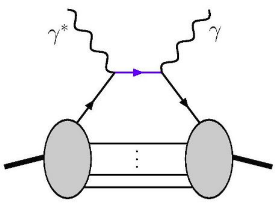
- electron hits nucleon & nucleon remains intact

$\hookrightarrow$ form factor $F(q^2)$

- position information from Fourier trafo
- no sensitivity to quark momentum
- $F(q^2) = \int dx GPD(x, q^2)$

$\hookrightarrow$ GPDs provide momentum disected form factors

Compton scattering



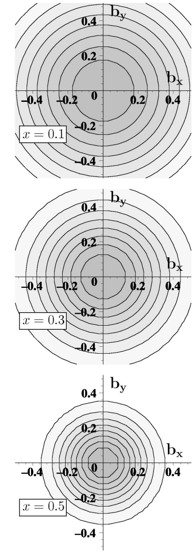
- electron hits nucleon, nucleon remains intact & photon gets emitted
- additional quark propagator

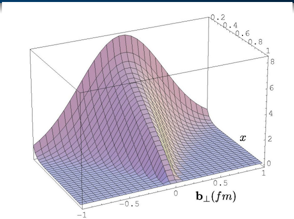
$\hookrightarrow$ additional information about momentum fraction x of active quark

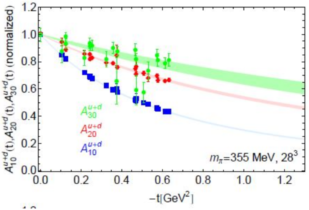
$\hookrightarrow$ generalized parton distributions $GPD(x, q^2)$

- info about both position and momentum of active quark

$q(x, \mathbf{b}_\perp)$ for unpol. p







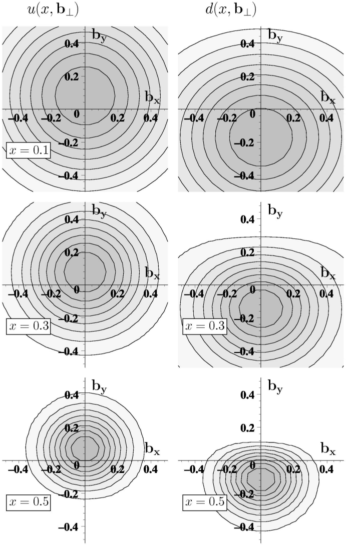
unpolarized proton

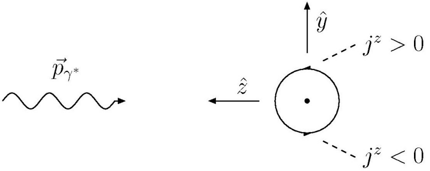
- $q(x, \mathbf{b}_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} H(x, 0, -\mathbf{\Delta}_\perp^2) e^{-i\mathbf{b}_\perp \cdot \mathbf{\Delta}_\perp}$

$\hookrightarrow$ probabilistic interpretation

- $F_1(-\mathbf{\Delta}_\perp^2) = \int dx H(x, 0, -\mathbf{\Delta}_\perp^2)$
- x = momentum fraction of the quark
- $\mathbf{b}_\perp$ relative to $\perp$ center of momentum
- small x: large 'meson cloud'
- larger x: compact 'valence core'
- $x \to 1$: active quark = center of momentum

$\hookrightarrow$ $\vec{b}_\perp \to 0$ (narrow distribution) for $x \to 1$





proton polarized in $+\hat{x}$ direction

$$q(x, \mathbf{b}_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} H_q(x,0,-\Delta_\perp^2) e^{-i\mathbf{b}_\perp \cdot \Delta_\perp}$$

$$-\frac{1}{2M}\frac{\partial}{\partial b_y} \int \frac{d^2\Delta_\perp}{(2\pi)^2} E_q(x,0,-\Delta_\perp^2) e^{-i\mathbf{b}_\perp \cdot \Delta_\perp}$$

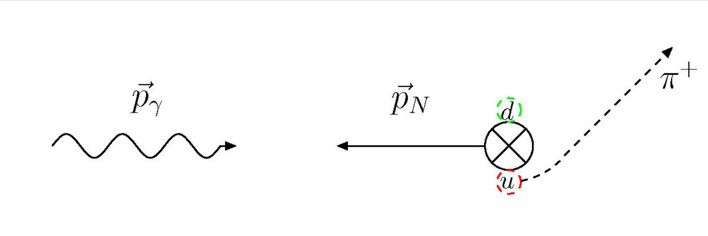
- relevant density in DIS is $j^+ \equiv j^0 + j^z$ and left-right asymmetry from j^z
- av. shift model-independently related to anomalous magnetic moments:

$$\langle b_y^q \rangle \equiv \int dx \int d^2 b_\perp q(x, \mathbf{b}_\perp) b_y$$

$$= \frac{1}{2M} \int dx E_q(x, 0, 0) = \frac{\kappa_q}{2M}$$

GPD $\longleftrightarrow$ Single Spin Asymmetries (SSA)

example: $\gamma p \to \pi X$



- u, d distributions in $\perp$ polarized proton have left-right asymmetry in $\perp$ position space (T-even!); sign "determined" by κ_u & κ_d
- attractive final state interaction (FSI) deflects active quark towards the center of momentum

$\hookrightarrow$ FSI translates position space distortion (before the quark is knocked out) in $+\hat{y}$-direction into momentum asymmetry that favors $-\hat{y}$ direction$\rightarrow$ chromodynamic lensing

$\Rightarrow$ $\kappa_p \, , \kappa_n \quad \longleftrightarrow \quad$ sign of SSA!!!!!!!! (MB,2004)

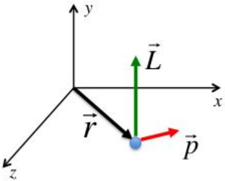
- qualitative connection GPDs $\rightsquigarrow$ TMDs (or $\mathbf{b}_\perp \rightsquigarrow k_\perp$)
- confirmed by HERMES & COMPASS data

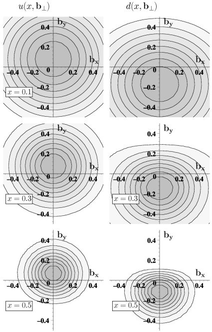
- Wigner functions!
  $L_z = \int dx \int d^2b_\perp \int d^2k_\perp (b_x k_y - b_y k_x)\, W(x,\mathbf{b}_\perp,\mathbf{k}_\perp)$
- Alternative: $L_x = yp_z - zp_y$
- if state invariant under rotations about $\hat{x}$ axis then $\langle yp_z \rangle = -\langle zp_y \rangle$

$\hookrightarrow$ $\langle L_x \rangle = 2\langle yp_z \rangle$

- GPDs provide simultaneous information about longitudinal momentum and transverse position

$\hookrightarrow$ use quark GPDs to determine angular momentum carried by quarks





Ji sum rule (1996)

$$J_q^x = \frac{1}{2}\int dx\, x\,[H(x,0,0) + E(x,0,0)]$$

- parton interpretation in terms of 3D distributions only for $\perp$ component (MB,2001,2005)

QED with electrons

$$\begin{aligned}\vec{J}_\gamma &= \int d^3r\,\vec{r}\times\left(\vec{E}\times\vec{B}\right) = \int d^3r\,\vec{r}\times\left[\vec{E}\times\left(\vec{\nabla}\times\vec{A}\right)\right] \\ &= \int d^3r\left[E^j\left(\vec{r}\times\vec{\nabla}\right)A^j - \vec{r}\times(\vec{E}\cdot\vec{\nabla})\vec{A}\right] \\ &= \int d^3r\left[E^j\left(\vec{r}\times\vec{\nabla}\right)A^j + \left(\vec{r}\times\vec{A}\right)\vec{\nabla}\cdot\vec{E} + \vec{E}\times\vec{A}\right]\end{aligned}$$

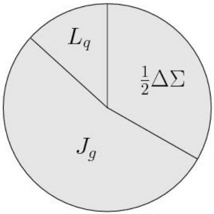
- replace 2^{nd} term (eq. of motion $\vec{\nabla}\cdot\vec{E} = ej^0 = e\psi^\dagger\psi$), yielding

$$\vec{J}_\gamma = \int d^3r\left[\psi^\dagger\vec{r}\times e\vec{A}\psi + E^j\left(\vec{x}\times\vec{\nabla}\right)A^j + \vec{E}\times\vec{A}\right]$$

- $\psi^\dagger\vec{r}\times e\vec{A}\psi$ cancels similar term in electron OAM $\psi^\dagger\vec{r}\times(\vec{p}-e\vec{A})\psi$

$\hookrightarrow$ decomposing $\vec{J}_\gamma$ into spin and orbital also shuffles angular momentum from photons to electrons!

Ji decomposition



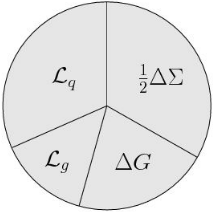
'pizza tre stagioni'

$$\frac{1}{2} = \sum_q \left(\frac{1}{2}\Delta q + L_q\right) + J_g$$

$$L_q = \int d^3x \langle P,S| q^\dagger(\vec{x}) \left(\vec{x} \times i\vec{D}\right)^3 q(\vec{x}) |P,S\rangle$$

- $i\vec{D} = i\vec{\partial} - g\vec{A}$
- DVCS $\longrightarrow$ GPDs $\longrightarrow L^q$

Jaffe-Manohar decomposition



'pizza quattro stagioni'

$$\frac{1}{2} = \sum_q \left(\frac{1}{2}\Delta q + \mathcal{L}_q\right) + \Delta G + \mathcal{L}_g$$

$$\mathcal{L}_q = \int d^3r \langle P,S| \bar{q}(\vec{r}) \gamma^+ \left(\vec{r} \times i\vec{\partial}\right)^z q(\vec{r}) |P,S\rangle$$

- light-cone gauge $A^+ = 0$
- $\vec{p}\overleftarrow{p} \longrightarrow \Delta G \longrightarrow \mathcal{L} \equiv \sum_{i\in q,g} \mathcal{L}^i$
- manifestly gauge inv. def. exists

How large is difference $\mathcal{L}_q - L_q$ in QCD and what does it represent?

- in Bjorken limit, many relevant observables (PDFs, GPDs, TMDs) equal x^+ correlators

$\hookrightarrow$ consider 6-D Wigner functions

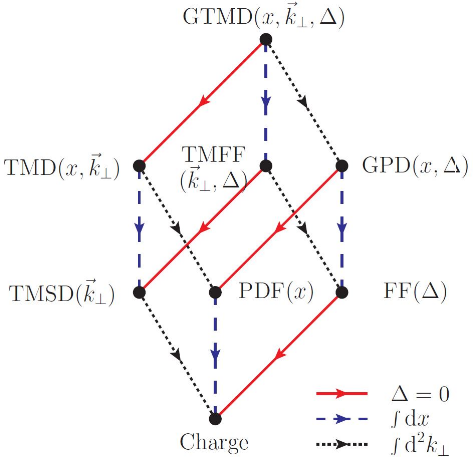
6-D Wigner Functions (Belitsky, Ji, Yuan)?

- indroduced Wigner functions that depend on $k^+, \vec{k}_\perp, \xi^-, \vec{\xi}_\perp$
- so far no insights from ξ^- dependence

$\hookrightarrow$ throw away that information ($\int d\xi^-$)

$\hookrightarrow$ 5-D Wigner functions

5-D Wigner Functions (Lorcé, Pasquini)



$$W(x, \vec{b}_\perp, \vec{k}_\perp) \equiv \int \frac{d^2\vec{\Delta}_\perp}{(2\pi)^2} e^{-i\vec{\Delta}_\perp \cdot \vec{b}_\perp} GTMD(x, \vec{k}_\perp, \vec{\Delta}_\perp)$$

Quark OAM from Wigner Functions

5-D Wigner Functions (Lorcé, Pasquini)

$$W(x,\vec{b}_\perp,\vec{k}_\perp) \equiv \int \frac{d^2\vec{\Delta}_\perp}{(2\pi)^2} \int \frac{d^2\xi_\perp d\xi^-}{(2\pi)^3} e^{ik\cdot\xi} e^{-i\vec{\Delta}_\perp\cdot\vec{b}_\perp} \langle P'S'|\bar{q}(0)\gamma^+ q(\xi)|PS\rangle.$$

- TMDs: $f(x,\mathbf{k}_\perp) = \int d^2\mathbf{b}_\perp W(x,\vec{b}_\perp,\vec{k}_\perp)$
- GPDs: $q(x,\mathbf{b}_\perp) = \int d^2\mathbf{k}_\perp W(x,\vec{b}_\perp,\vec{k}_\perp)$
- $L_z = \int dx \int d^2\mathbf{b}_\perp \int d^2\mathbf{k}_\perp W(x,\vec{b}_\perp,\vec{k}_\perp)(b_x k_y - b_y k_x)$
- need to include Wilson-line gauge link $\mathcal{U}_{0\xi} \sim \exp\left(i\frac{g}{\hbar}\int_0^\xi \vec{A}\cdot d\vec{r}\right)$ to connect 0 and ξ

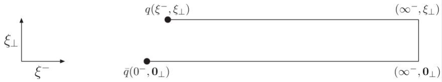
$\hookrightarrow$ 'light-cone staple' crucial for SSAs in SIDIS & DY

straight line for $\mathcal{U}_{0\xi}$

straigth Wilson line from 0 to ξ yields Ji-OAM:

$$L^q = \int d^3x \langle P,S|q^\dagger(\vec{x})\left(\vec{x}\times i\vec{D}\right)^z q(\vec{x})|P,S\rangle$$

Light-Cone Staple for $\mathcal{U}_{0\xi}$



'light-cone staple' yields $\mathcal{L}_{Jaffe-Manohar}$

$\mathcal{L}_{\sqsupset}/\mathcal{L}_{\sqsubset}$

$\mathcal{L}$ with light-cone staple at $x^- = \pm\infty$

PT (Hatta)

- PT$\longrightarrow \mathcal{L}_{\sqsupset} = \mathcal{L}_{\sqsubset}$

(different from SSAs due to factor $\vec{x}$ in OAM)

Bashinsky-Jaffe

- $A^+ = 0$ no complete gauge fixing

$\hookrightarrow$ residual gauge inv. $A^\mu \rightarrow A^\mu + \partial^\mu \phi(\vec{x}_\perp)$

- $\vec{x} \times i\vec{\partial} \quad \rightarrow \quad \mathcal{L}_{JB} \equiv \vec{x} \times \left[i\vec{\partial} - g\vec{\mathcal{A}}(\vec{x}_\perp) \right]$
- $\vec{\mathcal{A}}_\perp(\vec{x}_\perp) = \frac{\int dx^- \vec{A}_\perp(x^-, \vec{x}_\perp)}{\int dx^-}$

Bashinsky-Jaffe ↔ light-cone staple

- $A^+ = 0$

$\hookrightarrow$ $\mathcal{L}_{\sqsupset/\sqsubset} = \vec{x} \times \left[i\vec{\partial} - g\vec{A}_\perp(\pm\infty, \vec{x}_\perp) \right]$

- $\mathcal{L}_{JB} = \vec{x} \times \left[i\vec{\partial} - g\vec{\mathcal{A}}(\vec{x}_\perp) \right]$
- $\vec{\mathcal{A}}_\perp(\vec{x}_\perp) = \frac{\int dx^- \vec{A}_\perp(x^-, \vec{x}_\perp)}{\int dx^-} = \frac{1}{2} \left(\vec{A}_\perp(\infty, \vec{x}_\perp) + \vec{A}_\perp(-\infty, \vec{x}_\perp) \right)$

$\hookrightarrow$ $\mathcal{L}_{JB} = \frac{1}{2} \left(\mathcal{L}_{\sqsupset} + \mathcal{L}_{\sqsubset} \right) = \mathcal{L}_{\sqsupset} = \mathcal{L}_{\sqsubset}$

straight line (→Ji)

$$\frac{1}{2} = \sum_q \frac{1}{2}\Delta q + L_q + J_g$$

$$L_q = \int d^3x \langle P,S|\bar{q}(\vec{x})\gamma^+\left(\vec{x}\times i\vec{D}\right)^z q(\vec{x})|P,S\rangle$$

- $i\vec{D} = i\vec{\partial} - g\vec{A}$

light-cone staple (→ Jaffe-Manohar)

$$\frac{1}{2} = \sum_q \frac{1}{2}\Delta q + \mathcal{L}_q + \Delta G + \mathcal{L}_g$$

$$\mathcal{L}^q = \int d^3x \langle P,S|\bar{q}(\vec{x})\gamma^+\left(\vec{x}\times i\vec{\mathcal{D}}\right)^z q(\vec{x})|P,S\rangle$$

$$i\vec{\mathcal{D}} = i\vec{\partial} - g\vec{A}(x^- = \infty, \mathbf{x}_\perp)$$

difference $\mathcal{L}^q - L^q$

$$\mathcal{L}^q - L^q = -g\int d^3x \langle P,S|\bar{q}(\vec{x})\gamma^+\left[\vec{x}\times\int_{x^-}^{\infty} dr^- F^{+\perp}(r^-,\mathbf{x}_\perp)\right]^z q(\vec{x})|P,S\rangle$$

$$\sqrt{2}F^{+y} = F^{0y} + F^{zy} = -E^y + B^x$$

straight line ($\rightarrow$Ji)

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$$i\mathcal{D}^j = i\partial^j - gA^j(x^-,\mathbf{x}_\perp) - g\int_{x^-}^{\infty} dr^- F^{+j}$$

difference $\mathcal{L}^q - L^q$

$$\mathcal{L}^q - L^q = -g\int d^3x \langle P,S|\bar{q}(\vec{x})\gamma^+\left[\vec{x}\times\int_{x^-}^{\infty} dr^- F^{+\perp}(r^-,\mathbf{x}_\perp)\right]^z q(\vec{x})|P,S\rangle$$

color Lorentz Force on ejected quark (MB, PRD 88 (2013) 014014)

$$\sqrt{2}F^{+y} = F^{0y} + F^{zy} = -E^y + B^x = -\left(\vec{E} + \vec{v}\times\vec{B}\right)^y \text{ for } \vec{v} = (0,0,-1)$$

Quark OAM from Wigner Functions

straight line (→Ji)

$$\frac{1}{2} = \sum_q \frac{1}{2}\Delta q + L_q + J_g$$

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- $i\vec{D} = i\vec{\partial} - g\vec{A}$

light-cone staple (→ Jaffe-Manohar)

$$\frac{1}{2} = \sum_q \frac{1}{2}\Delta q + \mathcal{L}_q + \Delta G + \mathcal{L}_g$$

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$$i\mathcal{D}^j = i\partial^j - gA^j(x^-,\mathbf{x}_\perp) - g\int_{x^-}^\infty dr^- F^{+j}$$

difference $\mathcal{L}^q - L^q$

$$\mathcal{L}^q - L^q = -g\int d^3x \langle P,S|\bar{q}(\vec{x})\gamma^+\left[\vec{x}\times\int_{x^-}^\infty dr^- F^{+\perp}(r^-,\mathbf{x}_\perp)\right]^z q(\vec{x})|P,S\rangle$$

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$$\sqrt{2}F^{+y} = F^{0y} + F^{zy} = -E^y + B^x = -\left(\vec{E} + \vec{v}\times\vec{B}\right)^y \quad \text{for } \vec{v} = (0,0,-1)$$

Torque along the trajectory of q

$$T^z = \left[\vec{x}\times\left(\vec{E} - \hat{\vec{z}}\times\vec{B}\right)\right]^z$$

Change in OAM

$$\Delta L^z = \int_{x^-}^\infty dr^- \left[\vec{x}\times\left(\vec{E} - \hat{\vec{z}}\times\vec{B}\right)\right]^z$$

‘Torque’ in Scalar Diquark Model M.B. & C.Lorcé

$\mathcal{L}_{JM} - L_{Ji} = \langle \bar{q}\gamma^+ \left(\vec{r} \times \vec{A}\right)^z q\rangle$ in scalar diquark model

- pert. evaluation of $\langle \bar{q}\gamma^+\left(\vec{r}\times\vec{A}\right)^z q\rangle$

$\hookrightarrow$ $\mathcal{L}_{JM} - L_{Ji} = \mathcal{O}(\alpha)$

- same order as Sivers

$\hookrightarrow$ $\mathcal{L}_{JM} - L_{Ji}$ as significant as SSAs

(Ji et al., 2016)

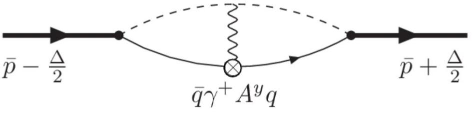
- for e^-: $\mathcal{L}_{JM} - L_{Ji} = 0$ to $\mathcal{O}(\alpha)$
  - earlier paper by M.B. & H.BC overlooked $h = 0$ component of Pauli-Villars γ
- $\mathcal{L}_{JM} - L_{Ji} \stackrel{?}{=} 0$ in general?
- how significant is $\mathcal{L}_{JM} - L_{Ji}$?

why scalar diquark model?

- Lorentz invariant
- 1^{st} to illustrate: FSI→SSAs (Brodsky,Hwang,Schmidt 2002)

$\hookrightarrow$ Sivers $\neq 0$

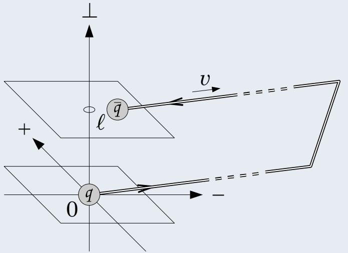
calculation M.B. & C.Lorcé



- nonforward matrix elem. of $\bar{q}\gamma^+ A^y q$
- $\left.\frac{d}{d\Delta^x}\right|_{\Delta=0}$

$\hookrightarrow$ $\langle k_\perp^q \rangle = \frac{3m_q+M}{12}\pi\langle \bar{q}\gamma^+ \left(\vec{r} \times \vec{A}\right)^z q\rangle$

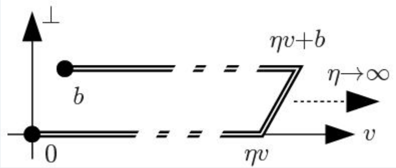
challenge



- TMDs/Wigner functions relevant for SIDIS require (near) light-like Wilson lines
- on Euclidean lattice, all distances are space-like

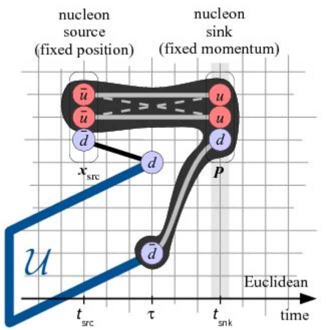
TMDs in lattice QCD

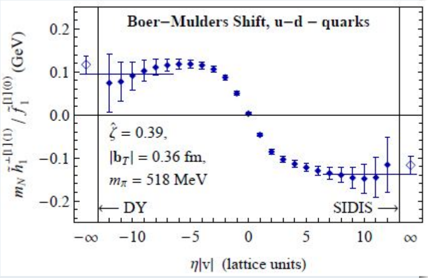
M.Engelhardt, P.Hägler, B.Musch, J.Negele, A.Schäfer



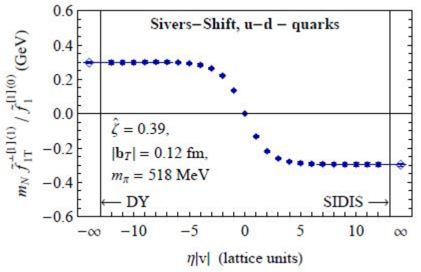
- calculate space-like staple-shaped Wilson line pointing in $\hat{z}$ direction; length $L \to \infty$
- momentum projected nucleon sources/sinks
- remove IR divergences by considering appropriate ratios

$\hookrightarrow$ extrapolate/evolve to $P_z \to \infty$

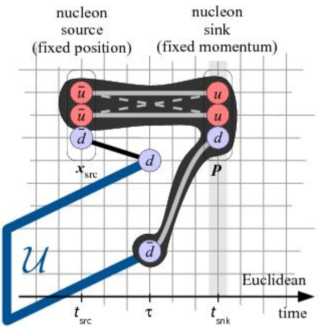


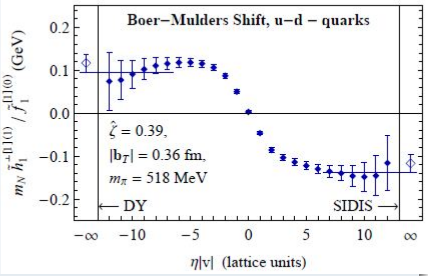


$f_{1T,SIDIS}^{\perp} = -f_{1T,DY}^{\perp}$ Collins



$f_{1T}^{\perp}(x, \mathbf{k}_\perp)$ is $\mathbf{k}_\perp$-odd term in quark-spin averaged momentum distribution in $\perp$ polarized target





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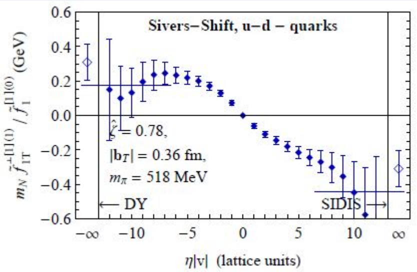

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difference $\mathcal{L}^q - L^q$

$\mathcal{L}^q_{JM} - L^q_{Ji} = \Delta L^q_{FSI}$ = change in OAM as quark leaves nucleon

$$\mathcal{L}^q_{JM} - L^q_{Ji} = -g \int d^3x \langle P,S|\bar{q}(\vec{x})\gamma^+ \left[\vec{x}\times \int_{x^-}^{\infty} dr^- F^{+\perp}(r^-,\mathbf{x}_\perp)\right]^z q(\vec{x})|P,S\rangle$$

e^+ moving through dipole field of e^-

- consider e^- polarized in $+\hat{z}$ direction

$\hookrightarrow$ $\vec{\mu}$ in $-\hat{z}$ direction (Figure)

- e^+ moves in $-\hat{z}$ direction

$\hookrightarrow$ net torque negative

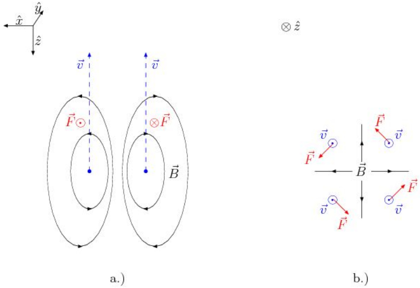
sign of $\mathcal{L}^q - L^q$ in QCD

- color electric force between two q in nucleon attractive

$\hookrightarrow$ same as in positronium

- spectator spins positively correlated with nucleon spin

$\hookrightarrow$ expect $\mathcal{L}^q - L^q < 0$ in nucleon



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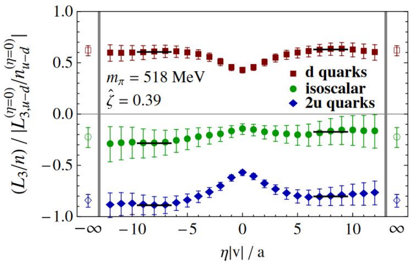
lattice QCD M.Engelhardt et al.

- L_{staple} vs. staple length

$\hookrightarrow$ L^q_{Ji} for length $= 0$

$\hookrightarrow$ $\mathcal{L}^q_{JM}$ for length $\to \infty$



Transverse Force Tomography

Matthias Burkardt

New Mexico State University

May 1, 2019

This is not stamp collecting

- twist 3 may have to be included to fit JLab data
- twist 3 necessary to understand what makes nucleon structure

$\hookrightarrow$ need to understand 'the force'

- alternative angular momentum sum rule (M.Polyakov; A.Rajan, M.Engelhardt, S.Liuti)
- transverse force tomography (this talk)
- there's really weird stuff going on at twist 3 *(F.Aslan + M.B. arXiv:1811.00938)*

yes, this will be hard, but...

- lattice QCD can provide (genuine) twist 3 info much sooner

$d_2 \leftrightarrow$ average $\perp$ force on quark in DIS from $\perp$ pol target

polarized DIS:

- $\sigma_{LL} \propto g_1 - \frac{2Mx}{\nu} g_2$
- $\sigma_{LT} \propto g_T \equiv g_1 + g_2$

$\hookrightarrow$ 'clean' separation between g_2 and $\frac{1}{Q^2}$ corrections to g_1

- $g_2 = g_2^{WW} + \bar{g}_2$ with $g_2^{WW}(x) \equiv -g_1(x) + \int_x^1 \frac{dy}{y} g_1(y)$

$$d_2 \equiv 3 \int dx\, x^2 \bar{g}_2(x) = \frac{1}{2MP^{+2}S^x} \left\langle P,S \left| \bar{q}(0)\gamma^+ gF^{+y}(0)q(0) \right| P,S \right\rangle$$

color Lorentz Force on ejected quark (MB, PRD 88 (2013) 114502)

$$\sqrt{2}F^{+y} = F^{0y} + F^{zy} = -E^y + B^x = -\left(\vec{E} + \vec{v} \times \vec{B}\right)^y \text{ for } \vec{v} = (0,0,-1)$$

matrix element defining $d_2 \leftrightarrow 1^{st}$ integration point in QS-integral
$d_2 \Rightarrow \perp$ force $\leftrightarrow$ QS-integral $\Rightarrow \perp$ impulse

sign of d_2

- $\perp$ deformation of $q(x, \mathbf{b}_\perp)$

$\hookrightarrow$ sign of d_2^q: opposite Sivers

magnitude of d_2

- $\langle F^y \rangle = -2M^2 d_2 = -10 \frac{GeV}{fm} d_2$
- $|\langle F^y \rangle| \ll \sigma \approx 1 \frac{GeV}{fm} \Rightarrow d_2 = \mathcal{O}(0.01)$

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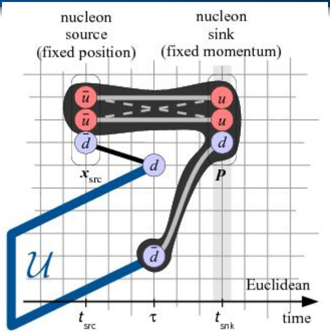
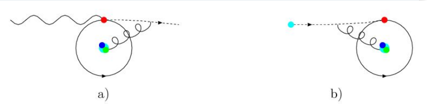
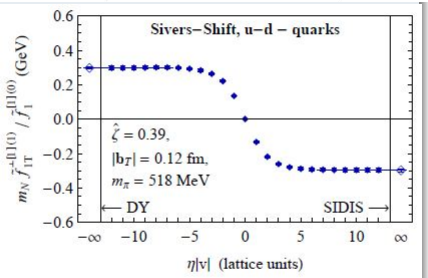
consitent with experiment (JLab,SLAC), model calculations (Weiss), and lattice QCD calculations (Göckeler et al., 2005)







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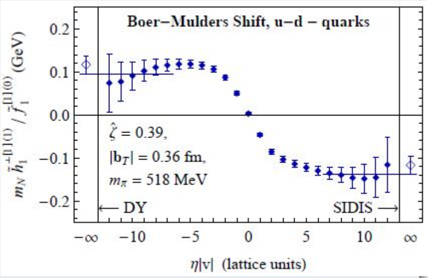

The Force

slope at length =0

Twist-3 PDFs $\longrightarrow \perp$ Force on Quarks in DIS

chirally even spin-dependent twist-3 PDF $g_2(x)$

MB, PRD 88 (2013) 114502

- $\int dx\, x^2 g_2(x) \Rightarrow \perp$ force on unpolarized quark in $\perp$ polarized target

$\hookrightarrow$ 'Sivers force'

scalar twist-3 PDF $e(x)$

MB, PRD 88 (2013) 114502

- $\int dx\, x^2 e(x) \Rightarrow \perp$ force on $\perp$ polarized quark in unpolarized target

$\hookrightarrow$ 'Boer-Mulders force'

chirally odd spin-dependent twist-3 PDF $h_2(x)$

M.Abdallah & MB, PRD94 (2016) 094040

- $\int dx\, x^2 h_2(x) = 0$

$\hookrightarrow$ $\perp$ force on $\perp$ pol. quark in long. pol. target vanishes due to parity

- $\int dx\, x^3 h_2(x) \Rightarrow$ long. gradient of $\perp$ force on $\perp$ polarized quark in long. polarized target

$\hookrightarrow$ chirally odd 'wormgear force'

$\perp$ localized state

$$|\mathbf{R}_\perp = 0, p^+, \Lambda\rangle \equiv \mathcal{N} \int d^2\mathbf{p}_\perp |\mathbf{p}_\perp, p^+, \Lambda\rangle$$

$\perp$ charge distribution (unpolarized quarks)

$$\begin{aligned} \rho_{\Lambda'\Lambda}(\mathbf{b}_\perp) &\equiv \langle \mathbf{R}_\perp = 0, p^+, \Lambda' | \bar{q}(\mathbf{b}_\perp)\gamma^+ q(\mathbf{b}_\perp) | \mathbf{R}_\perp = 0, p^+, \Lambda\rangle \\ &= |\mathcal{N}|^2 \int d^2\mathbf{p}_\perp \int d^2\mathbf{p}'_\perp \langle \mathbf{p}'_\perp, p^+, \Lambda' | \bar{q}(0)\gamma^+ q(0) | \mathbf{p}_\perp, p^+, \Lambda\rangle e^{i\mathbf{b}_\perp \cdot (\mathbf{p}_\perp - \mathbf{p}'_\perp)} \\ &= \int d^2\mathbf{\Delta}_\perp F_{\Lambda'\Lambda}(-\mathbf{\Delta}_\perp^2) e^{-i\mathbf{b}_\perp \cdot \mathbf{\Delta}_\perp} \end{aligned}$$

- crucial: $\langle \mathbf{p}'_\perp, p^+, \Lambda' | \bar{q}(0)\gamma^+ q(0) | \mathbf{p}_\perp, p^+, \Lambda\rangle$ depends only on $\mathbf{\Delta}_\perp$
- $F_{\Lambda'\Lambda}(-\mathbf{\Delta}_\perp^2)$ some linear combination of F_1 & F_2 - depending on Λ, Λ'
- similar for various polarized quark densities
- similar for x-dependent densities $\longrightarrow$ GPDs

$\perp$ force distribution (unpolarized quarks)

$$F^i_{\Lambda'\Lambda}(\mathbf{b}_\perp) \equiv \langle \mathbf{R}_\perp = 0, p^+, \Lambda' | \bar{q}(\mathbf{b}_\perp)\gamma^+ g F^{+i}(\mathbf{b}_\perp) q(\mathbf{b}_\perp) | \mathbf{R}_\perp = 0, p^+, \Lambda\rangle$$
$$= |\mathcal{N}|^2 \int d^2\mathbf{p}_\perp \int d^2\mathbf{p}'_\perp \langle \mathbf{p}'_\perp, p^+, \Lambda | \bar{q}(0)\gamma^+ g F^{+i}(0) q(0) | \mathbf{p}_\perp, p^+, \Lambda\rangle e^{i\mathbf{b}_\perp\cdot(\mathbf{p}_\perp - \mathbf{p}'_\perp)}$$

Form factors of qgq correlator (F.Aslan, M.B., M.Schlegel arXiv:1904.03494)

$$\langle p', \lambda' | \bar{q}(0)\gamma^+ i g F^{+i}(0) q(0) | p, \lambda\rangle = \bar{u}(p', \lambda')\Big[\frac{P^+}{M}\gamma^+\frac{\Delta^i}{M}\Phi_1(t) + \frac{P^+}{M} i\sigma^{+i}\Phi_2(t)$$
$$+ \frac{P^+}{M}\frac{\Delta^i}{M}\frac{i\sigma^{+\Delta}}{M}\Phi_3(t) + \frac{P^+}{M}\frac{\Delta^+}{M}\frac{i\sigma^{i\Delta}}{M}\Phi_4(t) + \frac{P_\perp \Delta^+ i\sigma^{+\Delta}}{M^3}\Phi_5(t)\Big]u(p,\lambda).$$

crucial:

- for $p^{+\prime} = p^+$, $\langle p', \lambda' | \bar{q}(0)\gamma^+ i g F^{+i}(0) q(0) | p, \lambda\rangle$ only depends on $\mathbf{\Delta}_\perp$

$\hookrightarrow$ similar to $\perp$ charge density ...

$\perp$ force distribution (unpolarized quarks)

$$F^i_{\Lambda'\Lambda}(\mathbf{b}_\perp) \equiv \langle \mathbf{R}_\perp = 0, p^+, \Lambda' | \bar{q}(\mathbf{b}_\perp)\gamma^+ g F^{+i}(\mathbf{b}_\perp) q(\mathbf{b}_\perp) | \mathbf{R}_\perp = 0, p^+, \Lambda\rangle$$
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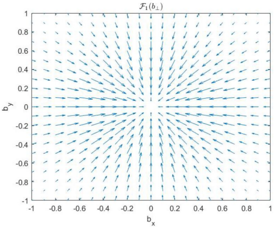
Φ_1

- unpolarized target
- axially symmetric 'radial' force



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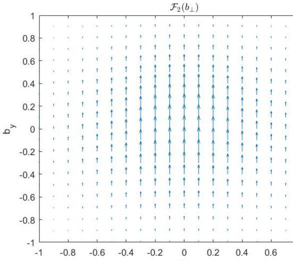
Φ_2

- $\perp$ polarized target; force $\perp$ to target spin

$\hookrightarrow$ spatially resolved Sivers force



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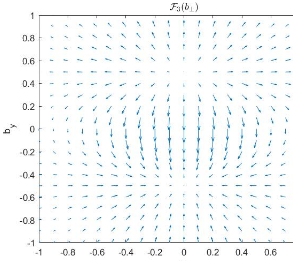
Φ_3

- tensor type force
- similar to charged particle flying through magnetic dipole field



Transverse Force Tomography

$\perp$ force distribution (unpolarized quarks)

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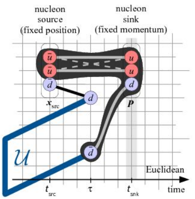
Φ_4 & Φ_5

- no conmtribution for $\Delta^+ = 0$

determining Φ_i

- match with x^2 moments of twist-3 GPDs (minus WW parts)
  F.Aslan, M.B. in progress
- experiments may take a few years, or immediately
- lattice QCD: fit to nonforward matrix elements of 'the force'-operator
  in progress (J.Bickerton, R.Young, J.Zanotti)

the force operator



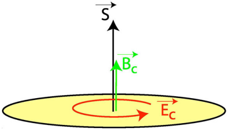
- form factor with quark denstity involving Wilson line staple
- take derivative w.r.t. staple length at length =0

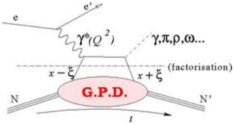
Summary

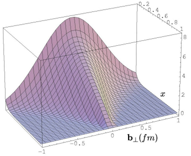
- GPDs $\xrightarrow{FT} q(x, \mathbf{b}_\perp)$ '3d imaging'
- 5-D Wigner functions
- application: Ji vs. Jaffe-Manohar OAM
- x^2 moment of twist-3 PDFs $\rightarrow$ force
- x^2 moment of twist-3 GPDs

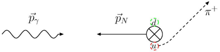
$\hookrightarrow$ $\bar{q}\gamma^+ F^{+\perp}\Gamma q$ distribution

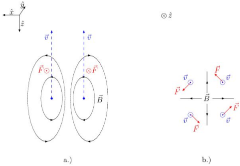
$\hookrightarrow$ $\perp$ force tomography









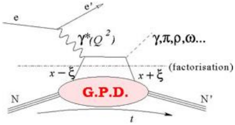


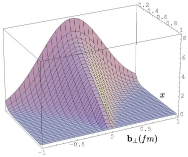
Summary

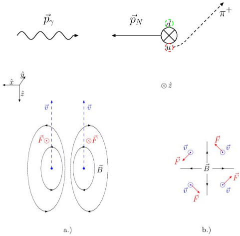
- GPDs $\xrightarrow{FT} q(x, \mathbf{b}_\perp)$ '3d imaging'
- 5-D Wigner functions
- application: Ji vs. Jaffe-Manohar OAM
- x^2 moment of twist-3 PDFs $\rightarrow$ force
- x^2 moment of twist-3 GPDs

$\hookrightarrow$ $\bar{q}\gamma^+ F^{+\perp}\Gamma q$ distribution

$\hookrightarrow$ $\perp$ force tomography







- evaluate GTMDs in lattice QCD (include disconnected diagrams)

↪ force tomography implicitly included in calcs. of GTMDs with quasi light-like staples!

- develop flexible parameterizations of GPDs & GTMDs that include rigorous theory constraints, such as polynomiality and Lorentz invariance relations
- combine with QCD evolution

↪ constrain parameterization using 'global fits'

↪ 5-D images of the nucleon (actually 6, if Q^2 is included)

- visualize by performing reduction to 2 or 3 variables, or 'travel' through 6-D by dialing some variables while viewing the others